SPCHART

This chapter discusses the capability and performance statistics that can be requested through SPCHART. For details on the computation of the charts, see Appendix 13.

Notation

$\bar{x}$	the total sample mean.
s	the total sample/process standard deviation.
$\hat{\sigma}$	the estimated sigma in the Process Capability Indices.
μ_o	the nominal or the target value, given by the user.
LSL	the lower specification limit, given by the user.
USL	the upper specification limit, given by the user.

Assumptions

- The process is in control. ($\bar{x}$ and s are finitely estimated.)
- The measured variable is normally distributed.

Prerequisites

- For the Process Capability Indices except *CpK* and the Process Performance Indices except *PpK*, both LSL and USL must be specified by the user, satisfying LSL < USL. For *CpK* and *PpK*, at least one of LSL and USL must be specified by the user.
- A target value μ_o such that $\text{LSL} \le \mu_o \le \text{USL}$ must be given by the user for *CpM* and *PpM* to be computed.

Process Capability Indices

The estimated capability sigma $\hat{\sigma}$ may be computed in one of four ways.

Let

k	the number of subgroups
n_i	the number of units in subgroup *i*
x_{ij}	the measurement of the *j*th unit in subgroup *i*
$\bar{x}_i = (\sum_{j=1}^{n_i} x_{ij}) / n_i$	the sample mean in subgroup *i*

R_i	the range in subgroup i
$s_i = (\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2) / (n_i - 1)$	the sample standard deviation in subgroup i.

(1). If it is to be based on the sample within-subgroup variance,

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2 \Big/ \sum_{i=1}^{k}(n_i - 1)}.$$

(2). If it is to be based on the mean range,

$$\hat{\sigma} = \frac{\sum_{i=1}^{k}\frac{R_i}{d_2(n_i)}}{k},$$

where $d_2(n_i) = \int_{-\infty}^{\infty} 1 - (1 - \Phi(x))^{n_i} - (\Phi(x))^{n_i}\,dx$ with $\Phi(x) = \int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-u^2/2}du$.

Note that n_i may or may not be equal for different subgroups. If they are all equal, we may write

$$\hat{\sigma} = \frac{\bar{R}}{d_2(n_i)},$$

where $\bar{R} = \sum_{i=1}^{k} R_i / k$, the mean range.

(3). If it is to be based on the mean standard deviation,

$$\hat{\sigma} = \frac{\sum_{i=1}^{k}\frac{s_i}{c_4(n_i)}}{k},$$

where $c_4(n_i) = \sqrt{\frac{2}{n_i - 1}}\frac{\Gamma(n_i / 2)}{\Gamma((n_i - 1)/2)}$ with the complete Gamma function $\Gamma(\cdot)$.

Note that n_i may or may not be equal for different subgroups. If they are all equal, we may write

$$\hat{\sigma} = \frac{\bar{s}}{c_4(n_i)},$$

where $\bar{s} = \sum_{i=1}^{k} s_i / k$, the mean standard deviation.

Next, let

n	the total sample size
m	the length of span (given by the user)
MR_i	the *i*th moving range for the data

$$MR_i = \begin{cases} \max(y_{i-m+1}, \ldots, y_i) - \min(y_{i-m+1}, \ldots, y_i), & \text{if } i = m, \ldots, n \\ \text{sysmis}, & \text{if } i = 1, \ldots, m-1 \end{cases}$$

(4). If it is to be based on the mean moving range,

$$\hat{\sigma} = \frac{\sum_{i=m}^{n} \frac{MR_i}{d_2(m)}}{n-m+1} = \frac{\overline{MR}}{d_2(m)}.$$

All of the capability indices, except *K*, require $\hat{\sigma}$, and in order to define them, we must have $\hat{\sigma} > 0$.

CP: Capability of the process.

$$CP = \frac{\text{USL - LSL}}{6\hat{\sigma}}.$$

CpL: The distance between the process mean and the lower specification limit scaled by capability sigma.

$$CpL = \frac{\bar{x} - \text{LSL}}{3\hat{\sigma}}.$$

CpU: The distance between the process mean and the upper specification limit scaled by capability sigma.

$$CpU = \frac{\text{USL} - \bar{x}}{3\hat{\sigma}}.$$

K: The deviation of the process mean from the midpoint of the specification limits.

$$K = \frac{2\left|(\text{USL} + \text{LSL})/2 - \bar{x}\right|}{\text{USL} - \text{LSL}}.$$

Note this is computed independently of the estimated capability sigma, .$\hat{\sigma}$ So, $\hat{\sigma}$ does not need to be greater than 0 or even specified.

CpK: Capability of process related to both dispersion and centeredness.

$$CpK = \min\left(CpU, CpL\right).$$

If only one specification limit is provided, we compute and report a unilateral *CpK* instead of taking the minimum.

CR: The reciprocal of CP.

$$CR = \frac{1}{CP}.$$

CpM: An index relating capability sigma and the difference between the process mean and the target value.

$$CpM = \frac{\text{USL} - \text{LSL}}{6\sqrt{\hat{\sigma}^2 + (\bar{x} - \mu_o)^2}}.$$

Note in particular that μ_o must be given by the user.

Z-lower (Cap): The number of capability sigmas between the process mean and the lower specification limit.

$$CZ_L = \frac{\bar{x} - \text{LSL}}{\hat{\sigma}}.$$

Z-upper (Cap): The number of capability sigmas between the process mean and the upper specification limit.

$$CZ_U = \frac{\text{USL} - \bar{x}}{\hat{\sigma}}.$$

Z-min (Cap): The minimum number of capability sigmas between the process mean and the specification limits.

$$CZmin = \min\left(CZ_U, CZ_L\right).$$

Note that unlike *CpK*, this index is undefined unless both specification limits are given and valid.

Z-max (Cap): The maximum number of capability sigmas between the process mean and the specification limits.

$$CZmax = \max\left(CZ_U, CZ_L\right).$$

Note that unlike *CpK*, this index is undefined unless both specification limits are given and valid.

The estimated percentage outside the specification limits (Cap)

$$\left(1 - \Phi\left(CZ_U\right) + \Phi\left(-CZ_L\right)\right) \times 100\% ,$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

The Process Performance Indices

The estimated performance sigma is always the process standard deviation s. None of the indices in this chapter is defined unless $s > 0$.

PP: Performance of the process.

$$PP = \frac{\text{USL - LSL}}{6s}.$$

PpL: The distance between the process mean and the lower specification limit scaled by process standard deviation.

$$PpL = \frac{\bar{x} - \text{LSL}}{3s}.$$

PpU: The distance between the process mean and the upper specification limit scaled by process standard deviation.

$$PpU = \frac{\text{USL} - \bar{x}}{3s}.$$

PpK: Performance of process related to both dispersion and centeredness.

$$PpK = \min(PpU, PpL).$$

If only one specification limit is provided, we compute and report a unilateral *PpK* instead of taking the minimum.

PR: The reciprocal of PP.

$$PR = \frac{1}{PP}.$$

PpM: An index relating process variance and the difference between the process mean and the target value.

$$PpM = \frac{\text{USL} - \text{LSL}}{6\sqrt{s^2 + (\bar{x} - \mu_o)^2}}.$$

Note in particular that μ_o must be given by the user.

Z-lower (Perf): The number of standard deviations between the process mean and the lower specification limit.

$$PZ_L = \frac{\bar{x} - \text{LSL}}{s}.$$

Z-upper (Perf): The number of standard deviations between the process mean and the upper specification limit.

$$PZ_U = \frac{\text{USL} - \bar{x}}{s}.$$

Z-min (Perf): The minimum number of standard deviations between the process mean and the specification limits.

$$PZmin = \min\left(PZ_U, PZ_L\right).$$

Note that unlike *PpK*, this index is undefined unless both specification limits are given and valid.

Z-max (Perf): The maximum number of standard deviations between the process mean and the specification limits.

$$PZmax = \max\left(PZ_U, PZ_L\right).$$

Note that unlike *PpK*, this index is undefined unless both specification limits are given and valid.

The estimated percentage outside the specification limits (Perf)

$$\left(1 - \Phi\left(PZ_U\right) + \Phi\left(-PZ_L\right)\right) \times 100\% \text{ ,}$$

where $\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

Measure(s) for Assessing Normality

The observed percentage outside the specification limits

This is the percentage of individual observations in the process which lie outside the specification limits. A point is defined as outside the specification limits when its value is greater than the USL or is less than the LSL.