

# Portfolio Optimization Service Methodology Documentation

Bluemix Experimental Services

2017/08/11



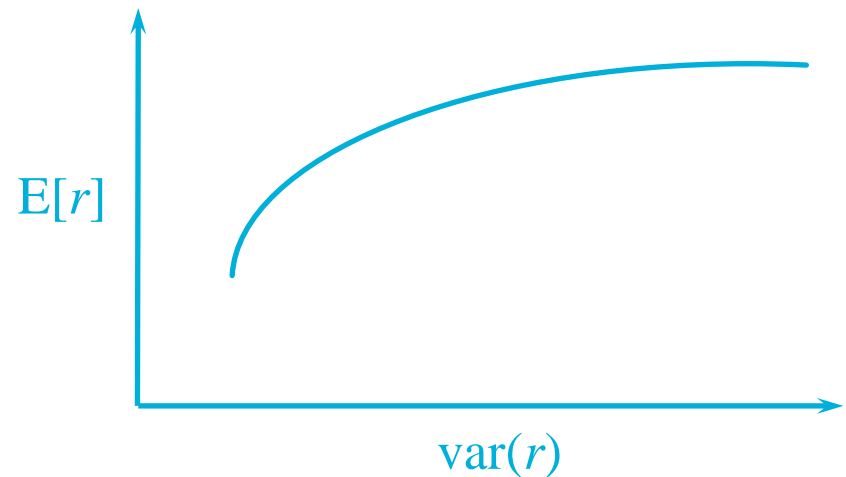
# What is Optimization?

- Optimization attempts to construct a portfolio that best meets a given set of requirements
  - Objective function – quantity to be minimized or maximized
  - Constraints – restrictions on the portfolio's composition
- The objective function and constraints are expressed in terms of
  - Input data (coefficients)
  - Decision variables
- An optimization algorithm adjusts the decision variables and tries to find an optimal solution
  - Minimizes (maximizes) the objective function and satisfies the constraints
  - If an optimal solution exists, optimal trades are returned



# Example – Markowitz (1952)

- Mean-Variance Problem
  - Minimize the variance of the portfolio's return
  - Earn an expected return of at least  $R$
  - No short positions
  - Input data for a set of instruments  $P$  (portfolio):
    - $\sigma_{jk}$  – covariance of returns of assets  $j$  and  $k$
    - $\tilde{r}_j$  – expected return of asset  $j$
  - Decision variables:
    - $w_j$  – weight of asset  $j$



# Markowitz (Mean-Variance) Problem

Objective

$$\min_{\mathbf{w}} \sum_{j \in P} \sum_{k \in P} \sigma_{jk} w_j w_k$$

Minimize variance of the return

Constraints

subject to  $\sum_{j \in P} \tilde{r}_j w_j \geq R$

Vary to construct efficient frontier

Expected return is at least  $R$

$$\sum_{j \in P} w_j = 1$$

Portfolio is fully allocated

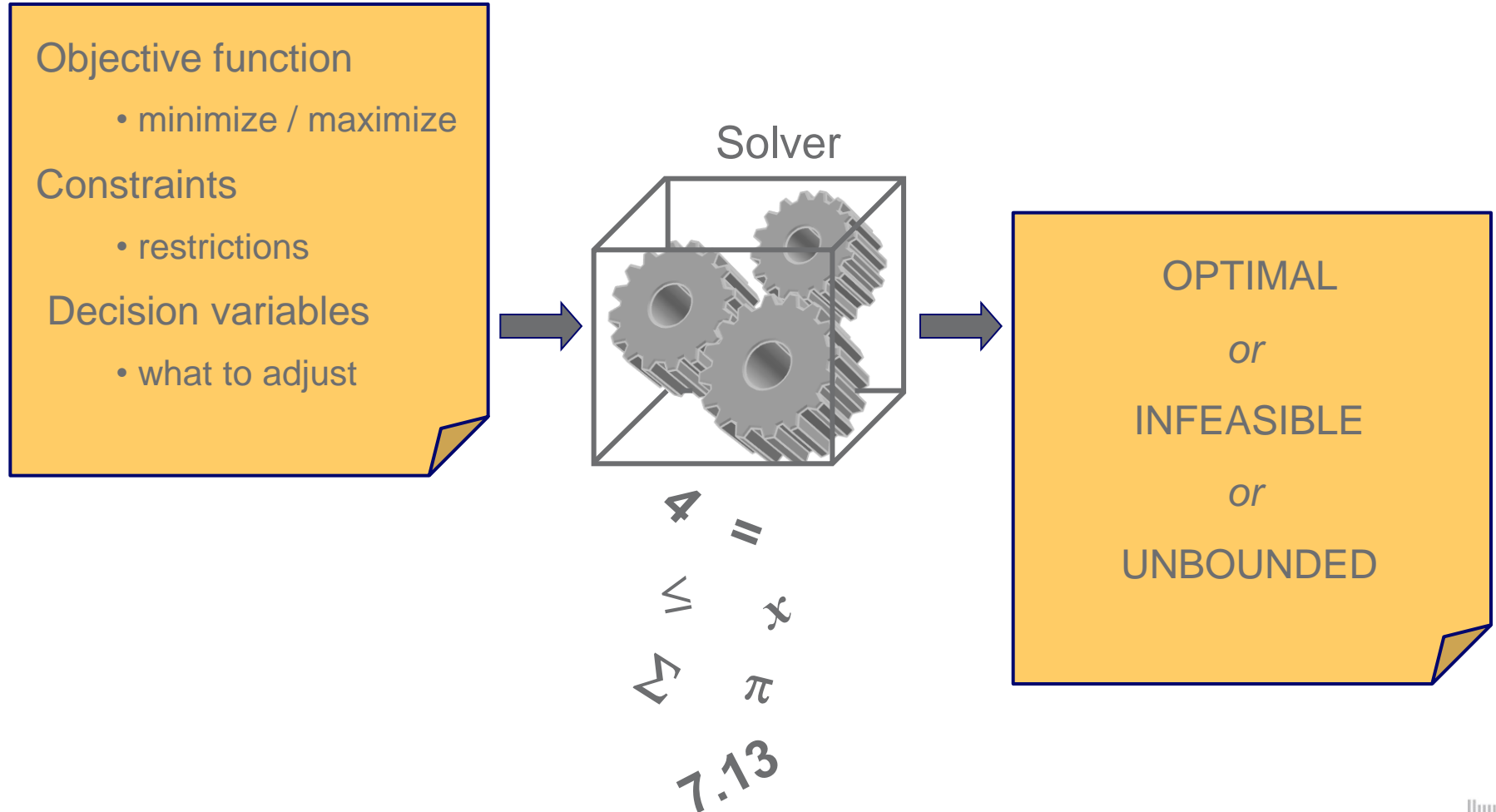
$$w_j \geq 0 \text{ for all } j \in P$$

No short positions

Weight of instrument  $j$  (decision variable)



# Optimization Basics



# Portfolio Optimization Service on Bluemix

The Portfolio Optimization service helps investment managers understand the optimal trade-offs between risk and reward based on changes in the portfolio. The framework is built upon a flexible mathematical model which allows for solving a wide range of investment problems based on the objectives and constraints.

Some applications of the optimizer for investment management include

- Index Tracking
- Hedging and Pricing
- Asset Allocation

The optimizer supports state-of-the-art algorithms for

- Linear optimization
- Quadratic and quadratically-constrained optimization
- Mixed-integer optimization

The service translates the expressions into an algebraic form that can be understood by the solver IBM ILOG CPLEX<sup>®</sup> and applies adjustments to improve solution time and numerical stability.

**To provision the service go to the Finance Tab of the Bluemix experimental services catalogue and select Portfolio Optimizer.**

**[https://console.bluemix.net/catalog/labs/?env\\_id=ibm:yp:us-south&category=finance](https://console.bluemix.net/catalog/labs/?env_id=ibm:yp:us-south&category=finance)**



# Formulating an Optimization Problem

Formulating a problems consists of 3 main sections

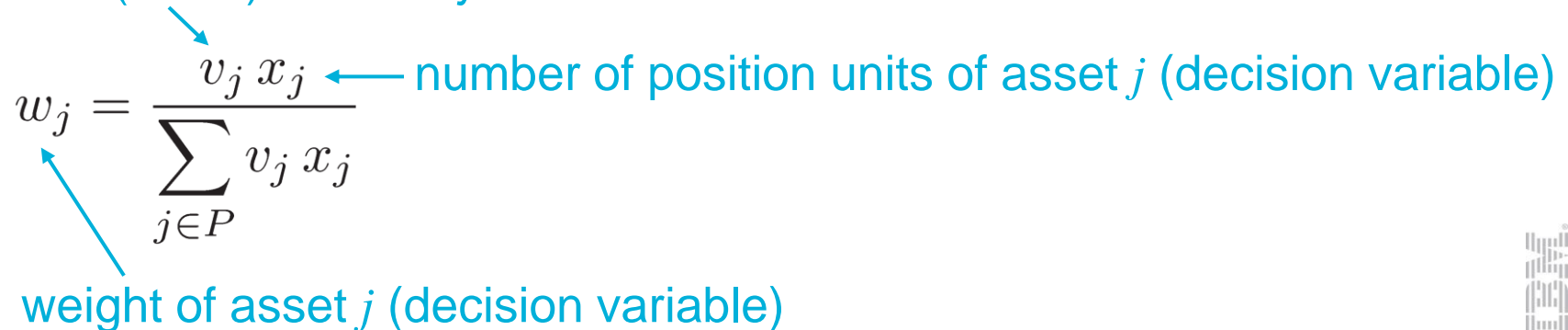
- **Primary inputs:**
  - Position units in root portfolio and benchmark portfolios
  - Expected (mean) returns for a single time horizon (30 days)
  - Return variance-covariance (VCV) matrix for a single time horizon (30 days)
- **Objectives:**
  - Maximize expected return
  - Minimize variance of return
  - Minimize tracking error with respect to a benchmark
- **Constraints:**
  - Weights of individual positions
  - Weights of groups for various aggregations (asset type, issuer, currency, etc.)
  - Bounds on expected return, variance of return, tracking error

price (value) of asset  $j$

$$w_j = \frac{v_j x_j}{\sum_{j \in P} v_j x_j}$$

weight of asset  $j$  (decision variable)

number of position units of asset  $j$  (decision variable)

The diagram shows the formula for the weight of asset j, w\_j. The numerator is v\_j x\_j, where v\_j is the price of asset j and x\_j is the number of position units of asset j. The denominator is the sum of v\_j x\_j over all assets j in the portfolio P. Blue arrows point from the text labels to the corresponding variables in the formula.



# Primary Inputs



# Portfolios and benchmarks

The Bluemix experimental service comes package with a small sample dataset that can be used to build custom portfolios and benchmarks.

ASSET ID	ISSUER NAME	CURRENT MARKET VALUE	SECTOR	ASSET CLASS	ASSET TYPE	SAMPLE GROUPINGS	
						ASSET SUBTYPE	ISSUER COUNTRY
CX_IE00BY7QL619_NYQ	JOHNSON CONTROLS INTERNATIONAL PLC	44.00	Industrials	Common Stock	Equities	Ordinary shares	US
CX_US02079KAA51_USD	ALPHABET INC	105.39	Technology	Fixed Income	Corporate	Financials	US
CX_US031162BG42_USD	AMGEN INC	106.49	Consumer Non-Cyclical	Fixed Income	Corporate	Services	US
CX_US031162BK53_USD	AMGEN INC	116.05	Consumer Non-Cyclical	Fixed Income	Corporate	Services	US
CX_US03232PAD06_USD	ENVISION HEALTHCARE CORP (DELAWARE	104.00	Consumer Non-Cyclical	Fixed Income	Corporate	Services	US
CX_US0533321024_NYQ	AUTOZONE INC	495.69	Consumer Discretionary	Common Stock	Equities	Ordinary shares	US
CX_US0584981064_NYQ	BALL CORP	42.42	Materials	Common Stock	Equities	Ordinary shares	US
CX_US1696561059_NYQ	CHIPOTLE MEXICAN GRILL INC	374.98	Consumer Discretionary	Common Stock	Equities	Ordinary shares	US
CX_US1912161007_NYQ	COCA-COLA	44.67	Consumer Staples	Common Stock	Equities	Ordinary shares	US
CX_US29379VAY92_USD	ENTERPRISE PRODUCTS OPERATING LLC	99.53	Energy	Fixed Income	Corporate	Industrials	US
CX_US30231GAN25_USD	EXXON MOBIL CORP	96.35	Energy	Fixed Income	Corporate	Industrials	US
CX_US46120E6023_NSQ	INTUITIVE SURGICAL INC	946.65	Health Care	Common Stock	Equities	Ordinary shares	US
CX_US4642851053_NYQ	iShares Gold Trust	11.95	Commodity	ETF	Commodities	Precious Metals	US
CX_US4642871762_NYQ	iShares TIPS Bond	113.60	Treasuries	ETF	Govt/Treasury/Central Bank	Government Notes/Bonds	US
CX_US4642872000_NYQ	iShares Core S&P 500	247.36	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642872422_NYQ	iShares iBoxx \$ Investment Grade Corporate Bond	121.20	Diversified	ETF	Corporate	Credit	US
CX_US4642874329_NYQ	iShares 20+ Year Treasury Bond	124.75	Treasuries	ETF	Govt/Treasury/Central Bank	Government Notes/Bonds	US
CX_US4642874402_NYQ	iShares Core S&P Mid-Cap ETF	106.92	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642874576_NYQ	iShares 1-3 Year Treasury Bond	84.54	Treasuries	ETF	Govt/Treasury/Central Bank	Government Notes/Bonds	US
CX_US4642874659_NYQ	iShares MSCI EAFE	66.25	Diversified	ETF	Equities	Ordinary shares	Non-US
CX_US4642875078_NYQ	iShares Core S&P Mid-Cap	175.68	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642875987_NYQ	iShares Russell 1000 Value	116.25	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642876142_NYQ	iShares Russell 1000 Growth ETF	121.70	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642878049_NYQ	iShares Core S&P Small-Cap ETF	70.69	Diversified	ETF	Equities	Ordinary shares	US
CX_US4642882819_NYQ	Shares J.P. Morgan USD Emerging Markets Bond	114.85	Diversified	ETF	Corporate	Credit	Non-US
CX_US4642884146_NYQ	iShares National Muni Bond ETF	110.57	Municipal	ETF	Municipal	Municipal	US
CX_US4642885135_NYQ	iShares iBoxx \$ High Yield Corporate Bond ETF	88.60	Diversified	ETF	Corporate	Credit	US
CX_US4642885887_NYQ	iShares MBS ETF	106.99	Structured Products	ETF	Mortgage Backed	Credit	US
CX_US4642886463_NYQ	iShares 1-3 Year Credit Bond	105.40	Diversified	ETF	Corporate	Credit	US
CX_US4642886612_NYQ	iShares 3-7 Year Treasury Bond	123.90	Treasuries	ETF	Govt/Treasury/Central Bank	Government Notes/Bonds	US
CX_US4642886877_NYQ	iShares U.S. Preferred Stock ETF	39.13	Diversified	ETF	Preferreds	Preferred Shares	US
CX_US46434G1031_NYQ	iShares Core MSCI Emerging Markets	52.23	Diversified	ETF	Equities	Ordinary shares	Non-US



# Portfolio hierarchy

universe of assets

## portfolios

		Root Portfolio "Root"	Benchmark 1	Benchmark 2
CX_US02079KAA51_USD	ALPHABET INC	-	-	-
CX_US031162BK53_USD	AMGEN INC	1231	-	1000
CX_US03232PAD06_USD	ENVISION HEALTHCARE CORP (DELAWARE	1374	-	-
CX_US1696561059_NYQ	CHIPOTLE MEXICAN GRILL INC	-	-	-
CX_US1912161007_NYQ	COCA-COLA	3198	-	-
CX_US30231GAN25_USD	EXXON MOBIL CORP	-	-	-
CX_US4642872000_NYQ	iShares Core S&P 500	-	2021	-
CX_US4642872422_NYQ	iShares iBoxx \$ Investment Grade Corporate Bond	0	4125	-
CX_US4642874329_NYQ	iShares 20+ Year Treasury Bond	-	-	-
CX_US4642874576_NYQ	iShares 1-3 Year Treasury Bond	-	-	-
CX_US4642874659_NYQ	iShares MSCI EAFE	-	-	-
CX_US4642875987_NYQ	iShares Russell 1000 Value	0	-	-
CX_US4642882819_NYQ	Shares J.P. Morgan USD Emerging Markets Bond	-	-	-
CX_US4878361082_NYQ	KELLOGG	2156	-	-
CX_US70450Y1038_NSQ	PAYPAL HOLDINGS INC	-	-	-
CX_US912810EJ35_USD	TREASURY BOND	1154	-	-

root portfolio (tradable assets)      benchmarks (fixed portfolios)

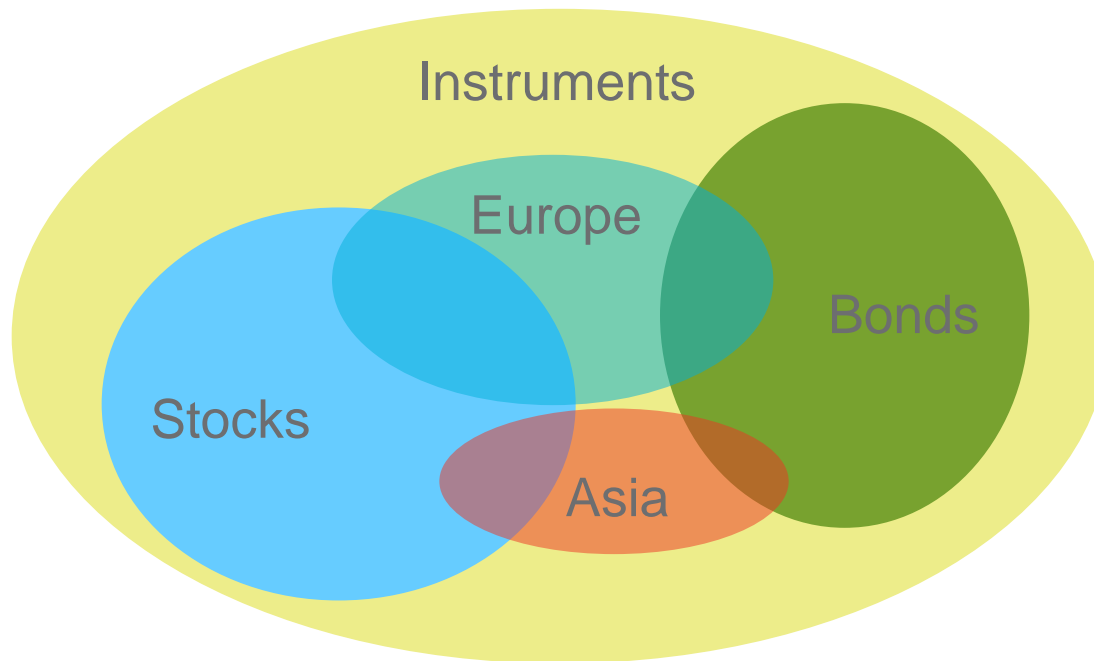
## aggregations (tags)

Sector	Asset Class	Asset Type	User Defined
Technology	Fixed Income	Corporate	My Strategy 2
Consumer Non-Cyclical	Fixed Income	Corporate	My Strategy 1
Consumer Non-Cyclical	Fixed Income	Corporate	My Strategy 1
Consumer Discretionary	Common Stock	Equities	My Strategy 2
Consumer Staples	Common Stock	Equities	My Strategy 2
Energy	Fixed Income	Corporate	My Strategy 2
Diversified	ETF	Equities	My Strategy 2
Diversified	ETF	Corporate	My Strategy 3
Treasuries	ETF	Govt/Treasury/Central Bank	My Strategy 3
Treasuries	ETF	Govt/Treasury/Central Bank	My Strategy 3
Diversified	ETF	Equities	My Strategy 3
Diversified	ETF	Equities	My Strategy 3
Diversified	ETF	Corporate	My Strategy 3
Consumer Staples	Common Stock	Equities	My Strategy 1
Information Technology	Common Stock	Equities	My Strategy 2
Treasuries	Fixed Income	Govt/Treasury/Central Bank	My Strategy 2



# Why create sub-portfolios

- Instruments/assets can be grouped based on various characteristics
- These groupings are represented as sub-portfolios and are used in constraints in the optimization problem.
- Depending on the type of constraint being setup a sub-portfolio can be setup for benchmark or portfolio instruments



# Portfolio hierarchy (Corporate sub-portfolios)

universe of assets

		Root Portfolio "Root"	Benchmark 1	Benchmark 2
CX_US02079KAA51_USD	ALPHABET INC	-	-	-
CX_US031162BK53_USD	AMGEN INC	1231	-	1000
CX_US03232PAD06_USD	ENVISION HEALTHCARE CORP (DELAWARE)	1374	-	-
CX_US1696561059_NYQ	CHIPOTLE MEXICAN GRILL INC	-	-	-
CX_US1912161007_NYQ	COCA-COLA	3198	-	-
CX_US30231GAN25_USD	EXXON MOBIL CORP	-	-	-
CX_US4642872000_NYQ	iShares Core S&P 500	-	2021	-
CX_US4642872422_NYQ	iShares iBoxx \$ Investment Grade Corporate Bond	0	4125	-
CX_US4642874329_NYQ	iShares 20+ Year Treasury Bond	-	-	-
CX_US4642874576_NYQ	iShares 1-3 Year Treasury Bond	-	-	-
CX_US4642874659_NYQ	iShares MSCI EAFE	-	-	-
CX_US4642875987_NYQ	iShares Russell 1000 Value	0	-	-
CX_US4642882819_NYQ	Shares J.P. Morgan USD Emerging Markets Bond	-	-	-
CX_US4878361082_NYQ	KELLOGG	2156	-	-
CX_US70450Y1038_NSQ	PAYPAL HOLDINGS INC	-	-	-
CX_US912810EJ35_USD	TREASURY BOND	1154	-	-

Sector	Asset Class	Asset Type	User Defined
Technology	Fixed Income	Corporate	My Strategy 2
Consumer Non-Cyclical	Fixed Income	Corporate	My Strategy 1
Consumer Non-Cyclical	Fixed Income	Corporate	My Strategy 1
Consumer Discretionary	Common Stock	Equities	My Strategy 2
Consumer Staples	Common Stock	Equities	My Strategy 2
Energy	Fixed Income	Corporate	My Strategy 2
Diversified	ETF	Equities	My Strategy 2
Diversified	ETF	Corporate	My Strategy 3
Treasuries	ETF	Govt/Treasury/Central Bank	My Strategy 3
Treasuries	ETF	Govt/Treasury/Central Bank	My Strategy 3
Diversified	ETF	Equities	My Strategy 3
Diversified	ETF	Equities	My Strategy 3
Diversified	ETF	Corporate	My Strategy 3
Consumer Staples	Common Stock	Equities	My Strategy 1
Information Technology	Common Stock	Equities	My Strategy 2
Treasuries	Fixed Income	Govt/Treasury/Central Bank	My Strategy 2

```
{
  "name": "Corporate_in_Root",
  "type": "subportfolio",
  "ParentPortfolio": "Root",
  "holdings": [
    {
      "asset": "CX_US031162BK53_USD",
      "quantity": 1231
    },
    {
      "asset": "CX_US03232PAD06_USD",
      "quantity": 1374
    },
    {
      "asset": "CX_US4642872422_NYQ",
      "quantity": 0
    }
  ]
}
```

```
{
  "name": "Corporate_in_Benchmark1",
  "type": "subportfolio",
  "ParentPortfolio": "Benchmark1",
  "holdings": [
    {
      "asset": "CX_US4642872422_NYQ",
      "quantity": 4125
    }
  ]
}
```

```
{
  "name": "Corporate_in_Benchmark2",
  "type": "subportfolio",
  "ParentPortfolio": "Benchmark2",
  "holdings": [
    {
      "asset": "CX_US031162BK53_USD",
      "quantity": 1000
    }
  ]
}
```



# Expected Returns

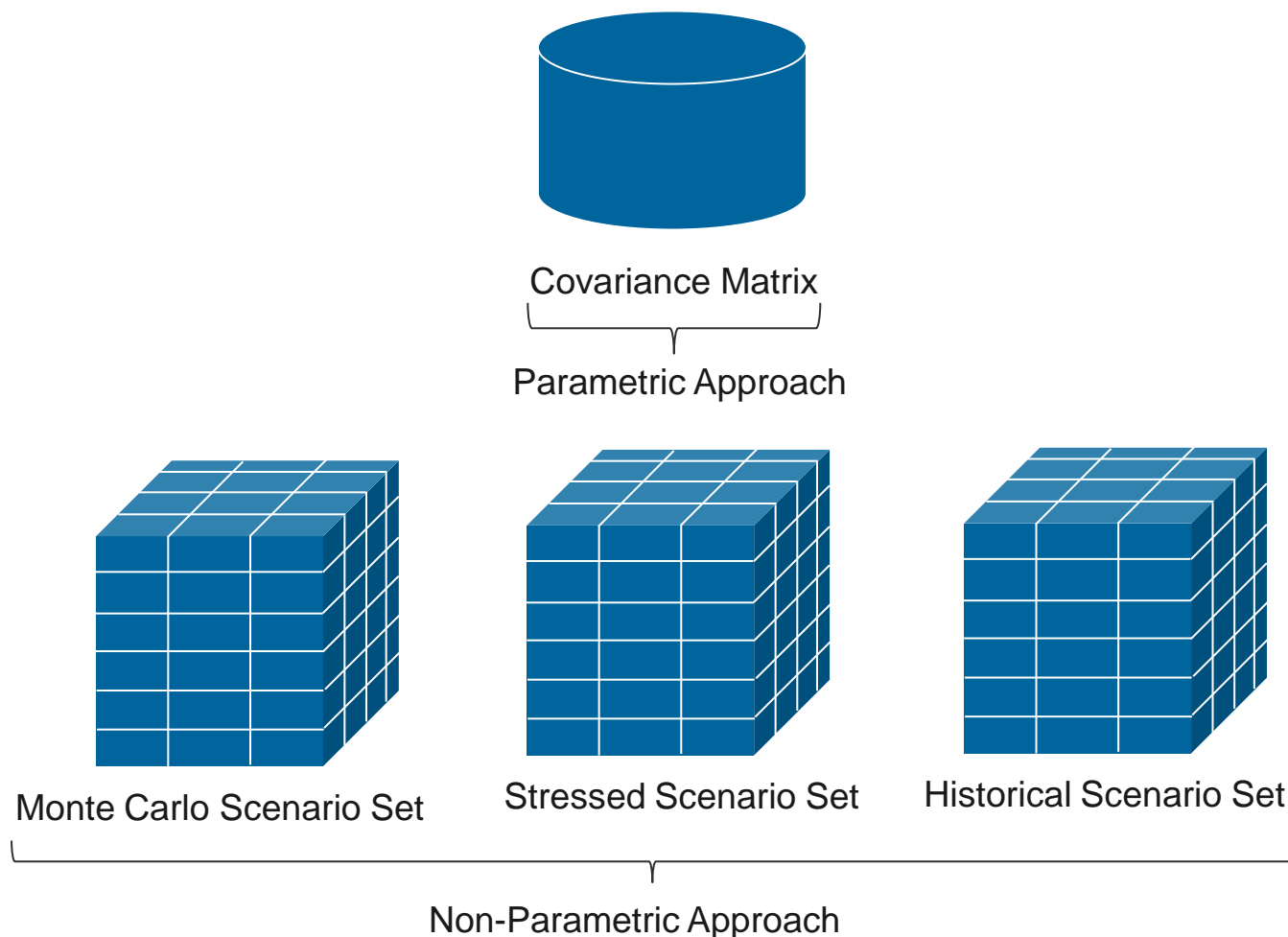
- In order to estimate the future returns for individual assets and the portfolio the optimization problem must have an expected return data.
- Clients can input their own returns expectation.

**For the Sample Data Set the expected returns in 30 days, have been estimated based on 2 years monthly historical values for each instrument.**



# Returns Variance and Covariances

The optimizer support both parametric (covariance matrix) and non-parametric (scenario-based) datasets. **The sample dataset provided with the service uses a Covariance matrix computed based on 2 year, monthly returns.**



# Objectives

# Objective Functions

The objective function denotes the function the optimizer will minimize or maximize. The objective may be setup on an absolute basis (i.e. only on the portfolio) or on a relative basis (i.e. the portfolio in relation to a benchmark). Users can set the following objectives:

- Minimize variance of the return of the root portfolio

$$\min_{\mathbf{w}} \sum_{j \in P} \sum_{k \in P} \sigma_{jk} w_j w_k$$

- Minimize tracking error squared, which is the variance of the differences between the root portfolio and benchmark portfolio returns

$$\min_{\mathbf{w}} \sum_{j \in P} \sum_{k \in P} \sigma_{jk} (w_j - w_j^B) (w_k - w_k^B)$$

- Maximize expected return of the root portfolio

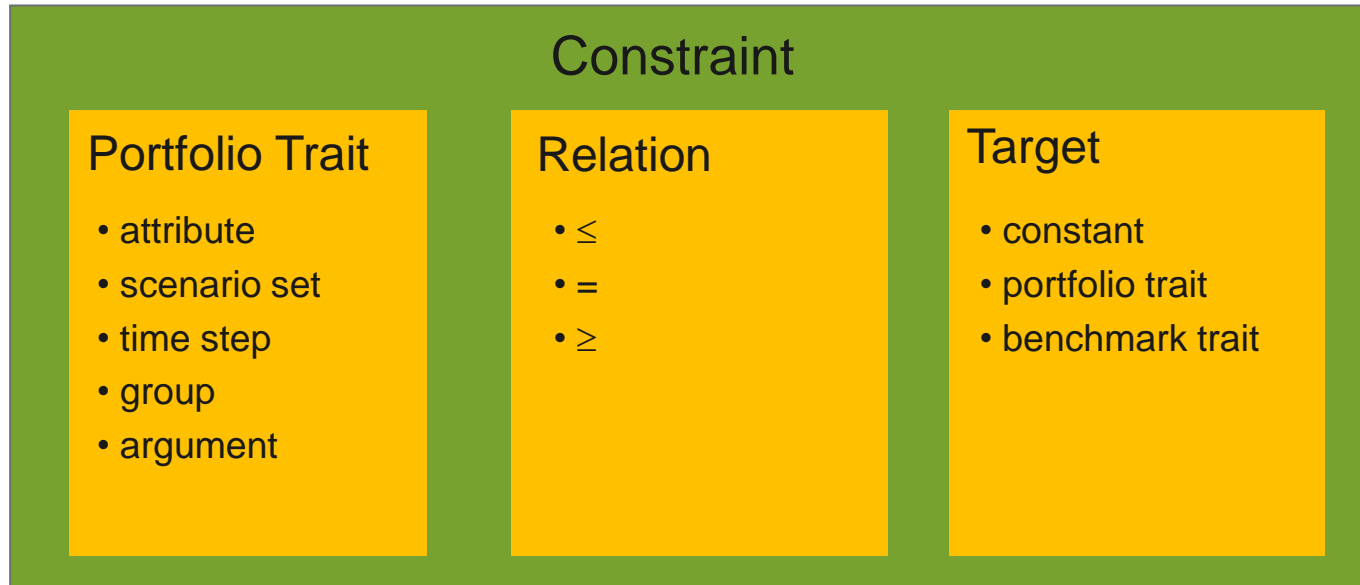
$$\max_{\mathbf{w}} \sum_{j \in P} \tilde{r}_j w_j$$





# Constraints

# A constraint is a relation between a trait and a target



## Examples of the types of constraints

- The weight of corporate bonds is greater than or equal to 40%
- The weight of corporate bonds is at least as large as the weight of equities
- The duration of the portfolio equals the duration of the benchmark
- The total value traded is at most EUR 1 million

# Portfolio Optimization Experimental Service

The types of constraints currently supported in the portfolio optimization experimental service are the following:

- The **expected return** of a portfolio at a given time period
- The **weight allocation of a particular group** of assets. For example the allocation to fixed income must be greater than or equal to 20% of the optimal portfolio
- The **weight allocation of individual assets**. For example, weight of each asset in the fixed income sub-portfolio must be greater than or equal to 5% of the optimal portfolio.
- The number of assets which have an optimal quantity of non-zero. This is known as a **cardinality constraint**.
- **No short-selling**, which means that in the optimal portfolio no assets will have a negative quantity.
- **Cash Inflows**. This represents an amount to be invested into the portfolio (in addition to the portfolio value itself).



# Constraint – Expected Returns

For the sample data set the expected returns constraint is defined at the 30 day time step

$$\sum_{j \in P} \tilde{r}_j w_j \geq R$$

expected return of the portfolio is at least R

vary to construct efficient frontier



# Constraint – Expected Returns (JSON)

```
{  
  "measure": "expectation",  
  "attribute": "return",  
  "timestep": 30,  
  "relation": "greater-or-equal",  
  "constant": 0.0075,  
  "description": "expected return of  
the portfolio is at  
least 0.75%"  
}
```

## Expectation

### Trait

Monthly returns of  
the root portfolio

### Time

- Now
- 1 day
- **1 month**
- 1 quarter

### Attribute

- Value
- Weight
- **Return**
- Count

### Relation

- $\leq$
- $=$
- $\geq$

### Target

- **constant**
- portfolio trait
- benchmark trait

### Portfolio

- **Root Portfolio**
- Benchmark
- Bonds
- EUR

"portfolio": "Root"

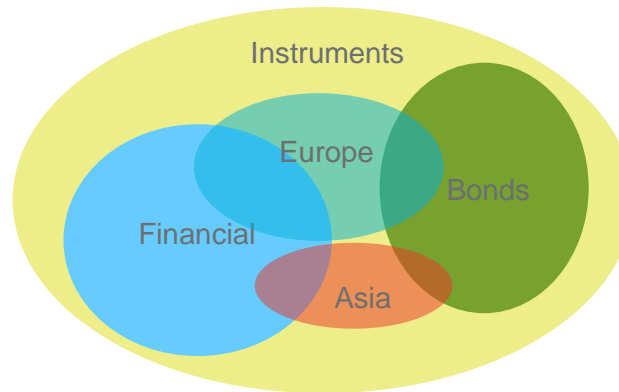
by default

# Constraint – Weight allocation on a group

The Portfolio Optimization experimental service supports portfolio groups (subportfolios) vs. a constant target. For example stocks less-than-or-equal to 30%. Constraints relative to portfolio or benchmark target are currently not available.

Example restrictions on sector weights, where the Financial sector is less-than or equal to 50%

$$\sum_{j \in \text{Fin}} w_j \leq 0.5$$



# Constraint – Weight allocation to individual assets

The Portfolio Optimization experimental service supports

Restrictions on weights for each asset in a portfolio or a sub-portfolio, e.g., hold at least 10% and at most 30% of each asset  $j$  of portfolio  $P$

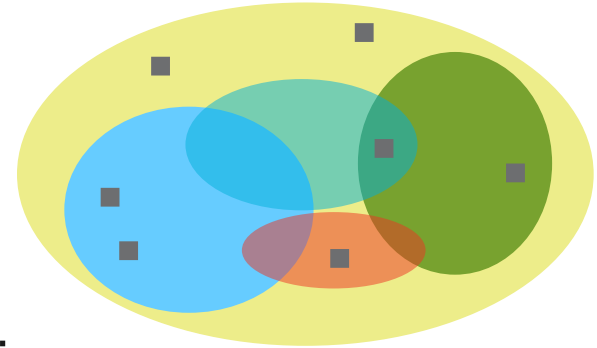
$$0.1 \leq w_j \leq 0.3 \quad \text{for all } j \in P$$



# Constraint – Cardinality

The Portfolio Optimization experimental service supports limits on the number of Positions.

- In general limit the **number** of
  - Trades
  - Positions



e.g., Limit the portfolio to at most 4 positions

Suppose  $0 \leq x_j \leq u_j$  and consider the constraints

$$\left. \begin{array}{l} x_j - u_j z_j \leq 0 \quad \text{for all } j \in P \\ \sum_{j \in P} z_j \leq 4 \\ z_j \in \{0, 1\} \quad \text{for all } j \in P \end{array} \right\} \begin{array}{l} z_j = 0 \quad \rightarrow \quad x_j = 0 \\ z_j = 1 \quad \rightarrow \quad 0 \leq x_j \leq u_j \end{array}$$

Binary decision variable



# Constraint – No short-selling

No short-selling, which means that in the optimal portfolio no assets will have a negative quantity.

$$w_j \geq 0 \quad \text{for all } j \in P$$

no short positions



weight of instrument  $j$  (decision variable)



IBM Institute for Business Value