Parallel Engineering and Scientific
Subroutine Library for AIX

Guide and Reference

SA22-7273-02

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Notes!

- Special notices are included in “Special Notices” on page xiii.
- For a summary of changes for Parallel ESSL Version 2 Release 1.2, see page xxv.

Third Edition (October 1999)

This edition applies to Version 2 Release 1.2 of the IBM* Parallel Engineering and Scientific Subroutine Library (Parallel ESSL) for Advanced Interactive Executive (AIX*) licensed program, program number 5765-C41 and all subsequent releases and modifications until otherwise indicated by new editions. Significant changes or additions to the text and illustrations are marked by a vertical line (|) to the left of the change.

Changes are periodically made to the information herein.

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- Page number or topic related to your comment

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Programming Interfaces

This *Parallel Engineering and Scientific Subroutine Library (Parallel ESSL) Guide and Reference* manual is intended to help the customer to do application programming. This manual documents General-use Programming Interface and Associated Guidance Information provided by Parallel ESSL.

General-use programming interfaces allow the customer to write programs that obtain the services of Parallel ESSL.

About This Book

The IBM Parallel Engineering and Scientific Subroutine Library (Parallel ESSL) is a set of high-performance mathematical subroutines. This book applies to Parallel ESSL for AIX.

This book is a guide and reference manual for use in doing application programming in Fortran, C, C++, and High Performance Fortran (HPF). It includes:

- An overview of Parallel ESSL and guidance information for coding and running your program, as well as using error handling
- Reference information for coding each subroutine calling sequence

This book is meant to be used in conjunction with the *ESSL Version 3 Guide and Reference*. Where information is identical between Parallel ESSL and ESSL for AIX, such as matrix storage modes, this book references the appropriate section of the *ESSL Version 3 Guide and Reference.*

This book is written for a wide class of users: scientists, mathematicians, engineers, statisticians, computer scientists, and system programmers. It assumes a basic knowledge of mathematics, Single Program Multiple Data (SPMD) parallel processing concepts and familiarity with Fortran, C, C++, or HPF.

How to Use This Book

Front Matter consists of the Table of Contents, Special Notices, and Preface. Use these to find or interpret information in the book.

Part 1. “Guide Information” provides guidance information for using Parallel ESSL.

- **Chapter 1, “Overview, Requirements, and List of Subroutines”** gives an overview of Parallel ESSL and lists required hardware and software products. Read this chapter first to determine the aspects of Parallel ESSL you want to use.
- **Chapter 2, “Distributing Your Data”** describes how to distribute your data across processes for various types of data structures: vectors, matrices, and sequences. Use this information when designing and coding your program.
- **Chapter 3, “Coding and Running Your Program”** explains coding requirements for calling Parallel ESSL from Fortran, C, C++, and HPF programs, performance coding tips, and how to run your program in the Parallel Environment. Use this information when coding or running your program.
- **Chapter 4, “Migrating Your Program”** describes how to migrate your program to Parallel ESSL. Use this information when updating your program for a new release of Parallel ESSL or when moving from ScaLAPACK to Parallel ESSL.
- **Chapter 5, “Using Error Handling”** describes how to use error handling in Parallel ESSL to retrieve information about errors that occur in your program and diagnose problems. Use this information when designing and coding your program as well as diagnosing your problems.

Part 2. “Reference Information (Message Passing)” provides reference information you need to code calling sequences for the Parallel ESSL message passing subroutines. Each chapter contains an introduction and subroutine descriptions. To understand the information in the subroutine descriptions, see “Interpreting the Subroutine Descriptions” on page xxii. Use the appropriate chapter when coding your program:

- **Chapter 6, “Level 2 PBLAS (Message Passing)”**
- **Chapter 7, “Level 3 PBLAS (Message Passing)”**
- **Chapter 8, “Linear Algebraic Equations (Message Passing)”**
- **Chapter 9, “Eigensystem Analysis and Singular Value Analysis (Message Passing)”**
- **Chapter 10, “Fourier Transforms (Message Passing)”**
- **Chapter 11, “Random Number Generation (Message Passing)”**
- **Chapter 12, “Utilities (Message Passing)”**

Part 3. “Reference Information (HPF)” provides reference information you need to code Parallel ESSL calling sequences in an HPF program. Each chapter contains an introduction and subroutine descriptions. To understand the information in the subroutine descriptions, see “Interpreting the Subroutine Descriptions” on page xxii. Use the appropriate chapter when coding your program:

- **Chapter 13, “PBLAS (HPF)”**
- **Chapter 14, “Linear Algebraic Equations (HPF)”**
- **Chapter 15, “Eigensystem Analysis and Singular Value Analysis (HPF)”**
- **Chapter 16, “Fourier Transforms (HPF)”**
- **Chapter 17, “Random Number Generation (HPF)”**

Appendix A. Basic Linear Algebra Communication Subprograms (BLACS) Quick Reference Guide provides a list of calling sequences for the BLACS subroutines.

Appendix B. Sample Programs contains a sample message passing Fortran 90 and a sample HPF application program, both using Parallel ESSL. It also contains sample message passing application programs using the Fortran 90 and Fortran 77 sparse linear algebraic equation subroutines.

Glossary contains definitions of terms used in this book.

Bibliography provides information about publications related to Parallel ESSL. Use it to identify and order publications with supporting information.

How to Find a Subroutine Description

If you want to locate a subroutine description and you know the subroutine name, you can use the “Looking for a Subroutine?” on page xi, following the Table of Contents. You can also find them listed individually or under the entry “subroutines” in the Index.

Where to Find Related Publications

If you have a question about the SP, PSSP, or a related product, the following online information resources make it easy to find the information you are looking for:

- If you have installed the RS/6000 SP Resource Center available with Parallel System Support Programs (PSSP) Version 3 Release 1 or later, you can access the SP Resource Center by issuing the command:

  /usr/lpp/ssp/bin/resource_center

  If you have the SP Resource Center on CD ROM, see the readme.txt file for information on how to run it.

- Access the RS/6000 Web site at:

  http://www.rs6000.ibm.com

All Parallel ESSL publications, as well as related programming and hardware publications, are listed in the bibliography. Also included is a list of math background publications you may find helpful, along with the necessary information for ordering them from independent sources. See "Bibliography" on page BIB-1.

How to Look Up a Bibliography Reference

Special references are made throughout this book to mathematical background publications and software libraries, available through IBM, publishers, or other companies. All of these are described in detail in the bibliography. A reference to one of these is made by using a number enclosed in square brackets. The number refers to the item listed under that number in the bibliography. For example, reference [1] cites the first item listed in the bibliography.

Special Terms

Standard data processing and mathematical terms are used in this book. Terminology is generally consistent with that used for Fortran. See the Glossary for more definitions of terms used in this book.

Distribution: Used to describe the method in which global data structures are divided among processes. Reference reports may use the term **decomposition** to mean the same thing.

Global: Used to identify arguments that must have the same value on all processes.

Local: Used to identify arguments that may have different values on different processes.

LOCp(): For block-cyclic data distribution, LOCp(M_) represents the number of rows that a process would receive if M_ was distributed block-cyclically over *p* rows of its process column.

The *ScaLAPACK Users' Guide* uses LOCr, which is equivalent to LOCp.

LOCq(): LOCq() can be used in three ways:

- For block-cyclic data distribution, LOCq(N_) represents the number of columns that a process would receive if N_ was distributed block-cyclically over *q* columns of its process row.
- For block-column data distribution, LOCq(*n*) represents the number of columns that a process would receive if *n* was distributed block over *q* processes.
- For block-plane data distribution, LOCq(*n*) represents the number of planes that a process would receive if *n* was distributed block over *q* processes.

The *ScaLAPACK Users' Guide* uses LOCc, which is equivalent to LOCq.

Optional: Indicates an argument does not have to be coded and is assigned a default value if the argument is not present.

Process: Indicates the logical CPUs identified in the process grid. Referenced reports may also use the terms **processor** or **node** to mean the same thing.

Process Grid: Indicates a way to view a parallel machine as a logical one- or two-dimensional rectangular grid.

For one-dimensional process grids, the variables *p* and *np* are used interchangeably to indicate the number of processes in a row or column of the process grid.

For two-dimensional process grids, the variables *p* and *nprow* are used interchangeably to indicate the number of rows in the process grid. The variables *q* and *npcol* are used interchangeably to indicate the number of columns in the process grid.

Referenced reports or manuals may also use the terms **processor mesh**, **processor template**, **processor shape**, or **processor grid**. These all mean the same thing.

Required: Indicates an argument must be coded in the calling sequence.

Scope: Scope can be used in two ways:

1. Refers to the portion of the parallel computer program within which the definition of an argument remains unchanged. When the scope of an argument is defined as global, the argument must have the same value on all processes. When the scope of an argument is defined as local, the argument may have different values on different processes.
2. In Appendix A on page 995, scope indicates the processes that participate in the broadcast and global operations. It can equal `'all'`, `'row'`, or `'column'`.

Short and Long Precision: Because Parallel ESSL can be used with more than one programming language, the terms **short precision** and **long precision** are used in place of the Fortran terms **single precision** and **double precision**.

Subroutines and Subprograms: A **subroutine** is a named sequence of instructions within the Parallel ESSL library, whose execution is invoked by a call. A subroutine can be called in one or more user programs and at one or more times within each program. The Parallel ESSL subroutines are referred to as **subprograms** in the areas of Level 2 and 3 Parallel Basic Linear Algebra

Subprograms (PBLAS). The term subprograms is used because it is consistent with the Basica Linear Algebra Subprograms (BLAS).

How to Interpret Product Names Used in This Document

Parallel ESSL refers to the Parallel Engineering and Scientific Subroutine Library for AIX product.

ESSL refers to the Engineering and Scientific Subroutine Library for AIX product.

MPI refers to the Message Passing Interface provided by Parallel Environment (PE).

Abbreviated Names

The abbreviated names used in this book are defined below.

Short Name	Full Name
AIX	Advanced Interactive Executive
BLACS	Basic Linear Algebra Communication Subprograms
BLAS	Basic Linear Algebra Subprograms
ESSL	Engineering and Scientific Subroutine Library
FDDI	Fiber Distributed Data Interface
HPF	High Performance Fortran
HTML	Hypertext Markup Language
IP	Internet Protocol
LAPACK	Linear Algebra Package
LAPI	Low-level Application Programming Interface
MPI	Message Passing Interface
MPL	Message Passing Library
NLS	National Language Support
PE	Parallel Environment
PBLAS	Parallel Basic Linear Algebra Subprograms
POWER, POWER2, POWER3 and PowerPC processors	RS/6000 processors
PSSP	Parallel System Support Programs
ScaLAPACK	Scalable Linear Algebra Package
SMP	Symmetric Multi-Processing
SPMD	Single Program Multiple Data
US	User Space

Fonts

This book uses a variety of special fonts to distinguish between many mathematical and programming items. These are defined below.

Special Font	Example	Description
Italic with no subscripts	*m*, *incx*, *uplo*	A calling sequence argument or mathematical variable
Italic with subscripts	x_1, a_{ij}, $y_{k1,\ k2}$	An element of a vector, matrix, or sequence
Bold italic lowercase	$\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$	A vector or sequence
Bold italic lowercase with subscripts	$\boldsymbol{x}_{ix:ix+n-1}$	A vector, with defined bounds
Bold italic uppercase	$\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$	A matrix
Bold italic uppercase with subscripts	$\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$ $\boldsymbol{X}_{ix:ix+n-1,\ ja:ja}$	A submatrix, with defined bounds A vector (a special form of submatrix), with defined limits

Gothic uppercase	`A`, `B`, `C`, `AGB`
	`NPROW=2`
		An array
		A Fortran statement

Scalar Data Notations

Following are the special notations used in this book for scalar data items. These notations do not imply usage of any precision, short or long.

Data Item	Example	Description
Character item	`'T'`	Character(s) in single quotation marks

Logical item	`.TRUE.`
	`.FALSE.`
		True or false logical value, as indicated

Integer data	`1`	Number with no decimal point
Real data	`1.6`	Number with a decimal point
Complex data	`(1.0,−2.9)`	Real part followed by the imaginary part

Special Characters, Symbols, Expressions, and Abbreviations

The mathematical and programming notations used in this book are consistent with traditional mathematical and programming usage. These conventions are explained below, along with special abbreviations that are associated with specific values.

Item	Description
Greek letters: α, σ, ω, Ω	Symbolic scalar values
$\lvert a \rvert$	The absolute value of *a*
$\boldsymbol{a} \bullet \boldsymbol{b}$	The dot product of $\boldsymbol{a}$ and $\boldsymbol{b}$
x_i	The *i*-th element of vector $\boldsymbol{x}$
c_{ij}	The element in matrix $\boldsymbol{C}$ at row *i* and column *j*

Item	Description
$x_1 \ldots x_n$	Elements from x_1 to x_n
$i = 1, n$	i is assigned the values 1 to n
$\boldsymbol{y} \leftarrow \boldsymbol{x}$	Vector $\boldsymbol{y}$ is replaced by vector $\boldsymbol{x}$
$\boldsymbol{xy}$	Vector $\boldsymbol{x}$ times vector $\boldsymbol{y}$
a^k	a raised to the k power
e^x	Exponential function of x
$\boldsymbol{A}^T$; $\boldsymbol{x}^T$	The transpose of matrix $\boldsymbol{A}$; the transpose of vector $\boldsymbol{x}$
$\bar{\boldsymbol{x}}; \bar{A}$	The complex conjugate of vector $\boldsymbol{x}$; the complex conjugate of matrix $\boldsymbol{A}$
$\bar{x}_i; \bar{c}_{jk}$	The complex conjugate of the complex vector element x_i, where: $\text{if } x_i = (a_i, b_i),$ $\text{then } \bar{x}_i = (a_i, -b_i)$ The complex conjugate of the complex matrix element c_{jk}
$\boldsymbol{x}^H$; $\boldsymbol{A}^H$	The complex conjugate transpose of vector $\boldsymbol{x}$; the complex conjugate transpose of matrix $\boldsymbol{A}$
I	Identity matrix
$\sum_{i=1}^{n} x_i$	The sum of elements x_1 to x_n
$\sqrt{a+b}$	The square root of $a+b$
$\boldsymbol{A}^{-1}$	The inverse of matrix $\boldsymbol{A}$
$\boldsymbol{A}^{-T}$	The transpose of $\boldsymbol{A}$ inverse
$\lvert\boldsymbol{A}\rvert$	The determinant of matrix $\boldsymbol{A}$
m by n matrix $\boldsymbol{A}$	Matrix $\boldsymbol{A}$ has m rows and n columns
sin a	The sine of a
cos b	The cosine of b
SIGN (a)	The sign of a; the result is either + or –
address {a}	The storage address of a
size(a, *dim*)	The result equals the number of elements in a along a specified dimension *dim* or if *dim* is not present the total number of array elements in a.
max($\boldsymbol{x}$)	The maximum element in vector $\boldsymbol{x}$
min($\boldsymbol{x}$)	The minimum element in vector $\boldsymbol{x}$
ceiling(x)	The smallest integer that is greater than or equal to x
floor(x)	The largest integer that is not greater than x
iceil(m,n)	The smallest integer that is greater than or equal to m/n; that is, iceil(m,n) = ceiling(m/n)
int(x), $x > 0$	The largest integer that is less than or equal to x
$m \mapsto (p, i)$	m is mapped into (p, i)
mod(x, m)	x modulo m; the remainder when x is divided by m

Item	Description
∞	Infinity
π	Pi, 3.14159265

Interpreting the Subroutine Descriptions

This section explains how to interpret the information in the subroutine descriptions in Part 2 and 3 of this book. Each subroutine description explains the function(s) performed by the subroutine(s). It provides a data types table, showing how the data differs for each subroutine. It also contains sections that are described below.

Syntax

This section shows the syntax for the Fortran, C, C++, and HPF calling statements.

Fortran, C, and C++ Syntax

This section shows the syntax for the Fortran, C, and C++ calling statements.

Fortran	CALL NAME-1 | NAME-2 | ... | NAME-n (*arg-1*, *arg-2*, ... , *arg-m*)
C and C++	name-1 | name-2 | ... | name-n (*arg-1*, ... , *arg-m*);

The syntax indicates:

- The programming language (Fortran, C, or C++)
- Each possible subroutine name that you can code in the calling sequence. Each name is separated by the **|** (or) symbol. You specify only one of these names in your calling sequence. (You do not code the **|** in the calling sequence.)
- The arguments, listed in the order in which you code them in the calling sequence. You must code them all in your calling sequence.

  You can distinguish between input arguments and output arguments by looking at the “On Entry” and “On Return” sections, respectively. An argument used for both input and output is described in both the “On Entry” and “On Return” sections. In this case, the input value for the argument is overlaid with the output value.

Fortran 90 or HPF Syntax

This shows the syntax for the Fortran 90 and HPF calling statements.

Fortran 90 or HPF	**Equations** or **Cases**	CALL NAME (*req-1*, ... , *req-m*) CALL NAME (*req-1*, ... , *req-m*, *opt-1*, ... , *opt-l*)

The syntax indicates:

- The programming language (Fortran 90 or HPF)
- The mathematical equations or subroutines associated with a calling sequence (HPF only)

- The Parallel ESSL subroutine name, which is a generic name for one or more functions.
- The arguments in the calling sequence.

  The first calling sequence shows the arguments required when coding your program. The second calling sequence shows all the arguments, required and optional. The subroutine assigns a default value for any optional argument that is not present.

  You can distinguish between input arguments and output arguments by looking at the “On Entry” and “On Return” sections, respectively. An argument used for both input and output is described in both the “On Entry” and “On Return” sections. In this case, the input value for the argument is overlaid with the output value.

On Entry

This lists the input arguments, which are the arguments you pass to the subroutine. Each argument description first gives the meaning of the argument, and then gives the form of data required for the argument. (To help you avoid errors, output arguments are included, with a reference to the On Return section.)

On Return

This lists the output arguments, which are the arguments passed back to your program from the subroutine. Each argument description first gives the meaning of the argument, and then gives the form of data passed back to your program for the argument.

Notes and Coding Rules

The notes describe any programming considerations and restrictions that apply to the arguments or the data for the arguments. There may be references to other parts of the book for further information.

Error Conditions

These are all the Parallel ESSL run-time errors that can occur in the subroutine. They are organized under the headings, “Computational Errors,” “Input Argument Errors,” “Resource Errors,” “Communications Errors,” and “Miscellaneous Errors.”

Example

The two reference sections in this book contain different types of examples.

Fortran Examples

The examples in Part 2 of this book show how you would call the subroutine in a Fortran program. Each example includes:

- A description of the salient features of the example
- The calling sequence, coded in Fortran
- The input and output data distributed across a process grid

HPF Examples

The examples in Part 3 of this book show how you would call the subroutine in an HPF program. Each example includes:

- A description of the salient features of the example
- The data directives
- One or more ways to code the calling sequence
- The input and output data

What's New For Parallel ESSL

This section summarizes the changes made to each release of Parallel ESSL.

What's New for Parallel ESSL Version 2 Release 1.2

- The Parallel ESSL POWER Libraries and the Parallel ESSL SMP Libraries are tuned for the RS/6000 POWER3 SMP thin, wide, and high nodes.
- New Level 2 and Level 3 PBLAS long-precision complex message-passing subroutines are provided.
- New Dense Linear Algebraic Equation long-precision real and complex combined factorization and solve message-passing subroutines are provided.
- Updated Banded Linear Algebraic Equation Subroutines, PDDTTRS and DTTRS, support Diagonally-Dominant General Tridiagonal Matrix Transpose Solve
- Banded Linear Algebraic Equation Subroutines, PDPBSV, PDGTSV, PDDTSV and PDPTSV have been modified for the case where N is greater than zero and NRHS is zero so that the matrix is factored. Previously, this was a quick return condition and the matrix was not factored.

Changes for Parallel ESSL Version 2 Release 1.1

Parallel ESSL for AIX provides distinct libraries for AIX 4.2.1 and AIX 4.3.2:

- The AIX 4.2.1 **Parallel ESSL Thread-Tolerant POWER2 Library** and the **Parallel ESSL SMP Library** were built using the pthreads draft 7 library supplied on AIX 4.2.1. This is the same as Parallel ESSL 2.1.
- The AIX 4.3.2 **Parallel ESSL Thread-Tolerant POWER2 Library** and the **Parallel ESSL SMP Library** were built using the pthreads library that conforms to the IEEE POSIX 1003.1-1996 specification supplied on AIX 4.3.

Changes for Parallel ESSL Version 2.1

- Parallel ESSL provides two new run-time libraries:
  - The **Parallel ESSL SMP Library** is provided for use with the Parallel ESSL message passing subroutines and the PE MPI threaded library. You may run single or multithreaded applications on all types of nodes. However, you cannot simultaneously call Parallel ESSL from multiple threads. Use this library if you are using both Parallel Environment (PE) Message Passing Interface (MPI) and the Communications Low-level Application Programming Interface (LAPI). The SMP library is for use on RS/6000* POWER* and PowerPC* (for example, 604 or 604e High Nodes) processors.
  - The **Parallel ESSL Thread-Tolerant POWER2 Library** is provided for use with the Parallel ESSL message passing subroutines and the PE MPI threaded library. You may run single or multithreaded applications on POWER2 nodes. However, you cannot simultaneously call Parallel ESSL from multiple threads. Use this library if you are using both PE MPI and

LAPI. The Thread-Tolerant POWER2 library is tuned for the RS/6000 POWER2* processors.

- The Linear Algebraic Equations Subroutines now include iterative solutions to linear systems of equations for real general sparse matrices.
- The Fourier Transforms subroutines have had the transform length restriction removed. The transform lengths in each direction no longer need to be divisible by the number of processes.
- Parallel ESSL supports an environment variable, PESSL_ERROR_SYNC, which allows you to disable error synchronization. This may improve performance of production level codes.
- Format of array descriptors

  Migration Note

  If you are using the Message Passing Level 2 or 3 PBLAS, Linear Algebraic Equations, or the Eigensystem Analysis and Singular Value Analysis subroutines, you must now use the array descriptors defined in Chapter 4 on page 113 for your programs to continue to run with Parallel ESSL Version 2.

  The format of the array descriptors was changed in Parallel ESSL Version 1.2.1 to maintain compatibility with ScaLAPACK. Therefore, all application programs previously migrated to accommodate the new array descriptor, can run unchanged with Parallel ESSL Version 2.1 However, if you were dependent upon the PESSL_DESC_TYPE environment variable, you must change the array descriptors as defined in Chapter 4 on page 113. If you do not change the array descriptors, Parallel ESSL Version 2.1 issues an error message and terminates your program.
- The files for the Hypertext Markup Language (HTML) version of the *Parallel ESSL Version 2 Guide and Reference* are packaged with the Parallel ESSL product.

Changes for Parallel ESSL Release 2.1 for AIX

- The format of the array descriptors has been changed in Parallel ESSL Release 2.1 to maintain compatibility with ScaLAPACK 1.2. Therefore, you must either change the way you code array descriptors in your existing application programs, or set and export the PESSL_DESC_TYPE environment variable. If you do not change the array descriptors or use the environment variable, Parallel ESSL Release 2.1 issues an error message and terminates your program.

  For more information on the new format for array descriptors, see Chapter 4 on page 113.
- The Banded Linear Algebraic Equations subroutines now include solutions to linear systems of equations for real positive definite symmetric tridiagonal matrices.

Changes for Parallel ESSL Release 2.0 for AIX

- Parallel ESSL Release 2.0 provides a new set of subroutines that are callable from application programs written in High Performance Fortran (HPF). The HPF subroutines covers the same range of mathematical function that the message passing subroutines cover.
- The Dense Linear Algebraic Equations subroutines now include solutions to linear systems of equations for complex general and complex Hermitian matrices.
- The Banded Linear Algebraic Equations subroutines now include solutions to linear systems of equations for real positive definite symmetric band matrices, real general tridiagonal matrices, and diagonally-dominant real general tridiagonal matrices.
- New short-precision versions for the Fourier transforms subroutines are provided in the product package.
- Parallel ESSL Release 2.0 supports clusters of IBM RS/6000* workstations.

Changes for Parallel ESSL Release 1 for AIX

- Parallel ESSL Release 1 for AIX uses the Parallel Environment (PE) Message Passing Interface (MPI) for communication.
- An InfoExplorer version of the *Parallel ESSL Version 2 Guide and Reference* manual is provided with the product package.
- Sample programs are provided with the product package.

In Brief—What's Provided in Parallel ESSL

IBM Parallel Engineering and Scientific Subroutine Library (Parallel ESSL) for AIX has the following characteristics:

- Parallel ESSL provides these run-time libraries:
  - The **Parallel ESSL SMP Library** is provided for use with the Parallel ESSL message passing subroutines and the PE MPI threaded library. You may run single or multithreaded applications on all types of nodes. However, you cannot simultaneously call Parallel ESSL from multiple threads. Use this Parallel ESSL library if you are using both PE MPI and LAPI. The SMP library is for use on the RS/6000 POWER and PowerPC (for example, POWER3 SMP Thin, Wide, or High Nodes) processors.
  - The **Parallel ESSL Thread-Tolerant POWER2 Library** is provided for use with the Parallel ESSL message passing subroutines and the PE MPI threaded library. You may run single or multithreaded applications on POWER2 nodes. However, you cannot simultaneously call Parallel ESSL from multiple threads. Use this library if you are using both PE MPI and LAPI. The Thread-Tolerant POWER2 library is tuned for the RS/6000 POWER2 processors.
  - The **Parallel ESSL POWER Libraries** are provided for use with the Parallel ESSL message-passing and HPF subroutines, and the MPI signal handling library. The POWER libraries are tuned for the RS/6000 POWER and PowerPC processors.
  - The **Parallel ESSL POWER2 Libraries** are provided for use with the Parallel ESSL message-passing and HPF subroutines, and the MPI signal handling library. The POWER2 libraries are tuned for the RS/6000 POWER2 processors.
- Parallel processing subroutines (distributed memory versions) provided in key math areas:
  - Subset of Level 2 and Level 3 Parallel BLAS (PBLAS)
  - Linear Algebraic Equations
    - Subset of ScaLAPACK (dense and banded)
    - Sparse subroutines and their utilities
  - Subset of ScaLAPACK Eigensystem Analysis and Singular Value Analysis
  - Fourier transforms
  - Uniform random number generation

  For a list of subroutines, refer to “Looking for a Subroutine?” on page xi.
- Supports the IBM RS/6000 SP and clusters of RS/6000 workstations
- Includes the Basic Linear Algebra Communication Subprograms (BLACS) which provides ease of use for message passing.
- Supports the SPMD programming model:
  - Uses the ESSL subroutines for computations on each processor node
  - Uses the MPI signal handling or threaded library for communication:
    - US—High Performance Switch, SP Switch

  - IP—Ethernet, Token Ring, Fiber Distributed Data Interface (FDDI), High Performance Switch, SP Switch
- Callable from application programs written in Fortran, C, C++, and HPF.

Part 1. Guide Information

This part of the book is organized into five chapters, providing guidance information on how to use Parallel ESSL. It is organized as follows:

- Overview, Requirements, and List of Subroutines
- Distributing Your Data
- Coding and Running Your Program
- Migrating Your Program
- Using Error Handling

Chapter 1. Overview, Requirements, and List of Subroutines

This chapter introduces you to IBM* Parallel Engineering and Scientific Subroutine Library (Parallel ESSL) for Advanced Interactive Executive (AIX*) products.

Overview of Parallel ESSL

Parallel ESSL is a scalable mathematical subroutine library that supports parallel processing applications on IBM RS/6000* SP* Systems and clusters of IBM RS/6000 workstations. Parallel ESSL supports the Single Program Multiple Data (SPMD) programming model using either the Message Passing Interface (MPI) signal handling library or the MPI threaded library. Parallel ESSL provides subroutines in six major areas of mathematical computations.

Parallel ESSL provides subroutines in the following computational areas:

- Level 2 Parallel Basic Linear Algebra Subprograms (PBLAS)
- Level 3 PBLAS
- Linear Algebraic Equations
- Eigensystem Analysis and Singular Value Analysis
- Fourier Transforms
- Random Number Generation

The subroutines run under the AIX operating system and can be called from application programs written in Fortran, C, C++, and High Performance Fortran (HPF). On the SP, Parallel System Support Programs (PSSP) is also required.

For communication, Parallel ESSL includes the Basic Linear Algebra Communications Subprograms (BLACS), which use the Parallel Environment (PE) Message Passing Interface (MPI). Communications using the User Space (US) require either the High Performance Switch or SP Switch. Communications using the Internet Protocol (IP) may use Ethernet, Token Ring, Fiber Distributed Data Interface (FDDI), High Performance Switch or SP Switch. For computations, Parallel ESSL uses the ESSL for AIX subroutines.

To order the IBM Parallel ESSL for AIX, specify program number 5765-C41.

How Parallel ESSL Works under the Parallel Environment (PE)

Parallel ESSL uses PE for communication during parallel processing, supporting the SPMD programming model, running on the SP or workstation clusters. In other words, your application program must be using PE if you want to call Parallel ESSL subroutines.

The RS/6000 processors are called **processor nodes**. A parallel program, such as yours with calls to the Parallel ESSL subroutines, executes as a number of individual, but related, **parallel tasks** on a number of your system's processor nodes. The group of parallel tasks is called a **partition**. The parallel tasks of your partition can communicate to exchange data or synchronize execution.

Your SP may have an optional high-performance switch for communication. The switch increases the speed of communication between nodes. It supports a high volume of message passing with increased bandwidth and low latency. This helps your application program, as well as the Parallel ESSL subroutines, achieve maximum performance.

Parallel ESSL assumes that the application program is using the SPMD programming model, where the programs running the parallel tasks of your partition are identical. The tasks, however, work on different sets of data.

Parallel ESSL Message Passing Subroutines

The following sections describe how to use the Message Passing subroutines supplied in Parallel ESSL.

Coding Your Program: The application developer begins by creating a parallel program's source code, including calls to the Parallel ESSL subroutines. The application developer might create this program from scratch and then places calls to BLACS or MPI or MPL routines so that it can run as a number of parallel tasks. These calls enable the parallel processes of your partition to communicate data and coordinate their execution. As part of each parallel process, the Parallel ESSL subroutines also perform these types of functions.

Details on what other specific coding additions are required when using Parallel ESSL are given in Chapter 3 on page 83.

Distributing Your Data: Your global data structures (vectors, matrices, or sequences) must be distributed across your processes prior to calling the Parallel ESSL subroutines.

Because data is distributed for both input and output, no implicit bottleneck is created by an initial scatter or ending gather operation. Parallel ESSL works in true SPMD mode, where each process operates only on a portion of the data. Also, the input and output data may be too large to collectively reside on a single node; therefore, problems associated with the storage limitations of a single processor node are eased by performing the computation in actual SPMD fashion.

See Chapter 2 on page 17 for details on distributing your data.

Running and Testing: After writing the parallel application program containing calls to the Parallel ESSL subroutines, the developer then begins a cycle of modification and testing. The application program is run using the **Parallel Operating Environment** (POE). The POE includes a number of **compiler scripts**, **environment variables**, and **command-line flags**, which may be used to set up your PE execution environment. (For example, before you execute a program, you need to set the size of your partition—the number of parallel tasks—by setting the appropriate environment variables or their command-line flags.) You can use all of these capabilities of POE with Parallel ESSL.

Tuning for Performance: Once the parallel program is debugged, you now want to tune the program for optimal performance. This is an important step of the process, because performance is the key reason for using the Parallel ESSL subroutines. To tune and analyze programs with calls to the Parallel ESSL subroutines, you may wish to use the tools provided by PE. For details, see the PE manuals listed in “Parallel Environment Version 2” on page BIB-4.

Parallel ESSL HPF Subroutines

XL HPF provides an easy way to develop parallel software with the SPMD programming model on your SP or cluster configuration. The XL HPF compiler, guided by XL HPF directives in your source code, handles the distribution of data and communication on multiple processes. There are three steps involved in getting a Fortran program ready to run in a parallel environment:

- Specify how many physical processors are required to run the program or let the compiler perform this function for you. It is to this arrangement, either physical or abstract, that the data is mapped.
- Align the data. Determine which data elements interact at runtime and designate that those elements be grouped to ensure they are assigned to the same process.
- Distribute the data. The aligned data is allocated to the processes specified in the process grid.

These three steps are accomplished using XL HPF directives. To parallelize your Fortran program, you must do the following:

- Analyze your data and algorithm
- Direct the data to different processes, using XL HPF directives

Parallel Environment is required to compile and run Parallel ESSL HPF programs.

For more details about the XL HPF language, see the XL HPF manuals. For more information on distributing your data, see Chapter 2 on page 17. For more information on coding your program, see Chapter 3 on page 83.

Where to Find Information on PE

For further details on PE and its various capabilities, see the PE manuals listed in “Parallel Environment Version 2” on page BIB-4. For more information about MPI, see references [38] and [46].

Accuracy of the Computations

Parallel ESSL provides accuracy comparable to libraries using equivalent algorithms with identical precision formats. The data types operated on are RS/6000 architecture precisions: ANSI/IEEE 64-bit binary floating-point format, and 32-bit integer. See the *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*, for more detail.

The Fortran Language Interface to the Message Passing Subroutines

The Parallel ESSL subroutines follow standard Fortran calling conventions. When Parallel ESSL subroutines are called from a program in a language other than Fortran, such as C or C++, the Fortran conventions must be used. This applies to all aspects of the interface, such as the linkage conventions and the data conventions. For example, array ordering must be consistent with Fortran array ordering techniques. Data and linkage conventions for each language are given in the *ESSL Version 3 Guide and Reference*.

Hardware and Software Products That Can Be Used with Parallel ESSL

This section describes the hardware and software products you can use with Parallel ESSL.

Parallel ESSL—Hardware

Parallel ESSL runs on the IBM RS/6000 SP and clusters of RS/6000 workstations supported by the operating systems listed under “Parallel ESSL—System Software.”

Parallel ESSL—System Software

Parallel ESSL for AIX is supported in the following operating system environments:

- AIX Version 4.2.1 or later modification levels of AIX Version 4.2 (program number 5765-655 or 5765-C34)

  On the SP, you also need the following along with AIX:

  – Parallel System Support Programs (PSSP) for AIX, Version 2.3 or later modification levels (program number 5765-529)

  – Any additional AIX 4.2.1 PTFs required for running on the SP

- AIX Version 4.3.2 or later modification levels of AIX Version 4.3 (program number 5765-C34)

  On the SP, you also need the following along with AIX:

  – PSSP for AIX, Version 3.1 or later modification levels (program number 5765-D51)

  – Any additional AIX 4.3.2 PTFs required for running on the SP

Parallel ESSL—Software Products

Parallel ESSL for AIX requires the software products shown in Table 1 on page 7 for compiling and running.

ESSL for AIX must be ordered separately.

To assist C and C++ users, a header file is provided with the Parallel ESSL product. Use of this file is described in “Running Your Message Passing Program” on page 100.

To assist Fortran 90 sparse linear algebraic equation users, a module file is provided with the Parallel ESSL product. Use of this file is described in “Using Extrinsic Procedures—The Fortran 90 Sparse Linear Algebraic Equation Subroutines” on page 95.

To assist HPF users, a module file is provided with the Parallel ESSL product. Use of this file is described in “Using Extrinsic Procedures—The Parallel ESSL Subroutines” on page 105.

Table 1. Software Products Required for Use with Parallel ESSL

For Compiling	**For Linking, Loading, or Running**
XL Fortran for AIX, Version 5.1.1 or later (program number 5808-AAR part number 04L2110) *–or–* IBM C, C++ compilers Version 3.6.4[1] *–or–* C for AIX, Version 4.4 or later (program number 5765-C64 with part number 31L0497 with feature 0002)	XL Fortran Run-time Environment for AIX, Version 5.1.1 or later (program number 5808-AAR part number 04L2123) *–or–* XL High Performance Fortran Run-time Environment for AIX, Version 1.3.1 or later (program number 5765-612) **–and–** Parallel Environment for AIX, Version 2.4 or later (program number 5765-543) on AIX 4.3.2 *–or–* Parallel Environment for AIX, Version 2.3 (program number 5765-543 and APAR IX72055) on AIX 4.2.1 **–and–** ESSL for AIX, Version 3.1.2 (program number 5765-C42) **–and–** C libraries[2]
XL High Performance Fortran for AIX, Version 1.3.1 or later (program number 5765-613)[3,4]	XL High Performance Fortran Run-Time Environment for AIX, Version 1.3.1 or later (program number 5765-612)[1] **–and–** Parallel Environment for AIX, Version 2.4 or later (program number 5765-543) on AIX 4.3.2 *–or–* Parallel Environment for AIX, Version 2.3 (program number 5765-543 and APAR IX72055) on AIX 4.2.1 **–and–** ESSL for AIX, Version 3.1.2 (program number 5765-C42) **–and–** C libraries[2]

[1] Available as a component of the VisualAge C++ Professional for AIX, Version 4, product. If using Parallel Environment 2.3 commands to compile with C++ Version 3.6, PE requires APAR IX76163.

[2] AIX includes the C libraries and math libraries in the Application Development Toolkit.

[3] XL HPF for AIX is only needed when you call Parallel ESSL HPF subroutines.

[4] If using Parallel Environment 2.3 commands to compile non-HPF programs with XL HPF, PE requires APAR IX80634.

Thread Safety

Parallel ESSL is not thread safe; however, Parallel ESSL is thread-tolerant and can therefore be called from a single thread of a multithreaded application. Multiple simultaneous calls to Parallel ESSL from different threads of a single process causes unpredictable results.

For more information on Thread Programming Concepts, see *IBM AIX Version 4 General Programming Concepts: Writing and Debugging Programs*.

Installation and Customization

Parallel ESSL is distributed on an 4-millimeter cartridge or a 8-millimeter cartridge. The *Parallel ESSL Installation Memo* provides the detailed information you need to install Parallel ESSL on AIX.

The Parallel ESSL product is packaged in accordance with the AIX guidelines. The product can be installed using the **smit** command, as described in the *IBM Parallel System Support Programs for AIX: Administration Guide*. The product can be installed on multiple nodes using the **dsh** command, as described in the *IBM Parallel System Support Programs for AIX: Administration Guide* and the **installp** command, as described in the *IBM AIX Version 4 Commands Reference*.

Software Products for Displaying Parallel ESSL Online Information

The *Parellel ESSL Version 2 Guide and Reference* is available in PostScript and HTML on the product media.

To view the online publications shipped on the product media, you need the following:

- Access to a common HTML document browser (such as Netscape Navigator).
- The location of the HTML index file provided with the file sets. Contact your system administrator or installer for this location.

Parallel ESSL—PostScript File

A PostScript file for the *Parallel ESSL Version 2 Guide and Reference* is provided with Parallel ESSL on the product medium. You can print it on your PostScript printer without any special setup, using whatever printing procedures you normally use for PostScript files. Duplex printing is suggested, due to the size of the book.

ESSL Internet Resources

This section describes how you can use the ESSL resources available over the Internet.

Obtaining Documentation

The *Parallel ESSL Guide and Reference* is available in PDF and HTML format at the IBM RS/6000 Web site at:

http://www.rs6000.ibm.com/resource/aix_resource/sp_books

To view the Parallel ESSL PDF publication, you need to access the Adobe Acrobat Reader 3.0.1. The Acrobat Reader is shipped with the AIX Version 4.3 Bonus Pack and is also freely available for downloading from th Adobe Web site at:

http://www.adobe.com

Accessing ESSL's Home Pages

The following home pages contain information on Parallel ESSL and ESSL:

- For Parallel ESSL, use:

  http://www.rs6000.ibm.com/software/sp_products/esslpara.html

- For ESSL for AIX, use:

  http://www.rs6000.ibm.com/software/Apps/essl.html

Getting on the ESSL Mailing List

Information concerning ESSL's home pages and other home pages available for the RS/6000 family of products, plus late breaking information about ESSL, can be obtained by being placed on the ESSL mailing list. In addition, users on the mailing list will receive information about new ESSL function and may receive customer satisfaction surveys and requirements surveys, to provide feedback to ESSL Development on the product and user requirements.

You can be placed on the mailing list by sending a request to either of the following, asking to be placed on the ESSL mailing list:

International Business Machines Corporation
ESSL Development
Department LQJA / MS P963
522 South Rd.
Poughkeepsie, N.Y. 12601-5400

e-mail: essl@us.ibm.com

Note: You should send us e-mail if you would like to be withdrawn from the ESSL mailing list.

When requesting to be placed on the mailing list or asking any questions, please provide the following information:

- Your name
- The name of your company
- Your mailing address
- Your Internet address
- Your phone number

BLACS—Usage in Parallel ESSL for Communication

The Basic Linear Algebra Communication Subprograms (BLACS) provide the same ease-of-use and portability for message passing in parallel linear algebra programs as the Basic Linear Algebra Subprograms (BLAS) provide for computation in such programs. The BLACS efficiently support not only point-to-point operations between processes on a logical two-dimensional process grid, but also collective communications on such grids, or within just a grid row or column (a one-dimensional process grid).

Most communication packages, such as PE, require an address and a length to be sent; therefore, they are classified as having operations based on vectors. In programming linear algebra problems, however, it is preferable to express all operations in terms of matrices. Vectors and scalars are simply subclasses of matrices. The BLACS operate on matrices, as defined by an address, column size, row size, leading dimension, and so forth.

Parallel ESSL includes the BLACS. Any public domain interface that calls the BLACS can be used compatibly with Parallel ESSL.

A BLACS quick reference guide can be found in Appendix A on page 995.

An example of the usage of BLACS in a message passing Fortran 90 program is shown in Appendix B on page 999.

The BLACS interface is documented in references [6], [32], and [33].

List of Parallel ESSL Subroutines (Message Passing and HPF)

This section provides an overview of the subroutines in each of the areas of Parallel ESSL.

Level 2 PBLAS

The Level 2 PBLAS include a subset of the standard set of distributed memory parallel versions of the Level 2 BLAS.

Note: The message passing subroutines were designed in accordance with the proposed Level 2 PBLAS standard. (See references [14], [15], and [17].) If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so.

The HPF subroutines were designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so.

If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 2 (Page 1 of 2). List of Level 2 PBLAS

Descriptive Name	Long-Precision Subprogram	Page
Matrix-Vector Product for a General Matrix or Its Transpose	PDGEMV PZGEMV GEMM*	163 811
Matrix-Vector Product for a Real Symmetric or a Complex Hermitian Matrix	PDSYMV PZHEMV SYMM*	189 822
Rank-One Update of a General Matrix	PDGER PZGERC PZGERU GEMM*	204 811
Rank-One Update of a Real Symmetric or a Complex Hermitian Matrix	PDSYR PZHER SYRK*	224 840

Table 2 (Page 2 of 2). List of Level 2 PBLAS

Descriptive Name	Long-Precision Subprogram	Page
Rank-Two Update of a Real Symmetric or a Complex Hermitian Matrix	PDSYR2 PZHER2 SYR2K*	236 846
Matrix-Vector Product for a Triangular Matrix or Its Transpose	PDTRMV PZTRMV TRMM*	252 828
Solution of Triangular System of Equations with a Single Right-Hand Side	PDTRSV PZTRSV TRSM*	335 834
* Only long-precision real data types are supported.		

Level 3 PBLAS

The Level 3 PBLAS include a subset of the standard set of distributed memory parallel versions of the Level 3 BLAS.

Note: The message passing subroutines were designed in accordance with the proposed Level 3 PBLAS standard. (See references [14], [15], and [17].) If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so.

The HPF subroutines were designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so.

If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 3 (Page 1 of 2). List of Level 3 PBLAS

Descriptive Name	Long-Precision Subprogram	Page
Matrix-Matrix Product for a General Matrix, Its Transpose, or Its Conjugate Transpose	PDGEMM PZGEMM GEMM	281 811
Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian	PDSYMM PZSYMM PZHEMM SYMM*	299 822
Triangular Matrix-Matrix Product	PDTRMM PZTRMM TRMM*	321 828
Solution of Triangular System of Equations with Multiple Right-Hand Sides	PDTRSM PZTRSM TRSM*	335 834

Table 3 (Page 2 of 2). List of Level 3 PBLAS

Descriptive Name	Long-Precision Subprogram	Page
Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix	PDSYRK PZSYRK PZHERK SYRK*	349 840
Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix	PDSYR2K PZSYR2K PZHER2K SYR2K*	365 846
Matrix Transpose for a General Matrix	PDTRAN PZTRANC PZTRANU TRAN*	386 852
* Only long-precision real data types are supported.		

Linear Algebraic Equations

These subroutines consist of dense, banded, and sparse subroutines, and include a subset of the ScaLAPACK subroutines.

Note: The message passing dense and banded linear algebraic equations subroutines were designed in accordance with the proposed ScaLAPACK standard. See references [10], [16], [18], [27], and [28]. If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so.

The HPF dense and banded linear algebraic equations subroutines were designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so.

If IBM updates these subroutines, the update could require modifications of the calling application program.

Dense Linear Algebraic Equations

The dense linear algebraic equation subroutines provide solutions to linear systems of equations for real and complex general matrices and their transposes, and for positive definite real symmetric and complex Hermitian matrices.

Table 4 (Page 1 of 2). List of Dense Linear Algebraic Equation Subroutines

Descriptive Name	Long-Precision Subroutine	Page
General Matrix Factorization and Solve	PDGESV PZGESV	405
General Matrix Factorization	PDGETRF PZGETRF GETRF	422 858

Table 4 (Page 2 of 2). List of Dense Linear Algebraic Equation Subroutines

Descriptive Name	Long-Precision Subroutine	Page
General Matrix Solve	PDGETRS PZGETRS GETRS	434 863
Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization and Solve	PDPOSV PZPOSV	446
Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization	PDPOTRF PZPOTRF POTRF	461 868
Positive Definite Real Symmetric or Complex Hermitian Matrix Solve	PDPOTRS PZPOTRS POTRS	471 873

Banded Linear Algebraic Equations

The banded linear algebraic equation subroutines provide solutions to linear systems of equations for real positive definite symmetric band matrices, real general tridiagonal matrices, diagonally-dominant real general tridiagonal matrices, and real positive definite symmetric tridiagonal matrices.

Table 5. List of Banded Linear Algebraic Equation Subroutines

Descriptive Name	Long- Precision Subroutine	Page
Positive Definite Symmetric Band Matrix Factorization and Solve	PDPBSV PBSV	484 879
Positive Definite Symmetric Band Matrix Factorization	PDPBTRF PBTRF	497 884
Positive Definite Symmetric Band Matrix Solve	PDPBTRS PBTRS	507 889
General Tridiagonal Matrix Factorization and Solve	PDGTSV GTSV	519 894
General Tridiagonal Matrix Factorization	PDGTTRF GTTRF	535 899
General Tridiagonal Matrix Solve	PDGTTRS GTTRS	553 908
Diagonally-Dominant General Tridiagonal Matrix Factorization and Solve	PDDTSV DTSV	519 894
Diagonally-Dominant General Tridiagonal Matrix Factorization	PDDTTRF DTTRF	535 899
Diagonally-Dominant General Tridiagonal Matrix Solve	PDDTTRS DTTRS	553 908
Positive Definite Symmetric Tridiagonal Matrix Factorization and Solve	PDPTSV PTSV	573 916
Positive Definite Symmetric Tridiagonal Matrix Factorization	PDPTTRF PTTRF	588 921
Positive Definite Symmetric Tridiagonal Matrix Solve	PDPTTRS PTTRS	602 927

Fortran 90 Sparse Linear Algebraic Equation Subroutines

The Fortran 90 sparse linear algebraic equation subroutines provide solutions to linear systems of equations for a real general sparse matrix. The sparse utility subroutines provided in Parallel ESSL must be used in conjunction with the sparse linear algebraic equation subroutines.

Table 6. List of Fortran 90 Sparse Linear Algebraic Equation Subroutines

Descriptive Name	**Long-Precision Subroutine**	**Page**
Allocates Space for an Array Descriptor for a General Sparse Matrix	PADALL	619
Allocates Space for a General Sparse Matrix	PSPALL	621
Allocates Space for a Dense Vector	PGEALL	623
Inserts Local Data into a General Sparse Matrix	PSPINS	625
Inserts Local Data into a Dense Vector	PGEINS	630
Assembles a General Sparse Matrix	PSPASB	632
Assembles a Dense Vector	PGEASB	635
Preconditioner for a General Sparse Matrix	PSPGPR	637
Iterative Linear System Solver for a General Sparse Matrix	PSPGIS	640
Deallocates Space for a Dense Vector	PGEFREE	645
Deallocates Space for a General Sparse Matrix	PSPFREE	646
Deallocates Space for an Array Descriptor for a General Sparse Matrix	PADFREE	648

Fortran 77 Sparse Linear Algebraic Equation Subroutines

The Fortran 77 sparse linear algebraic equation subroutines provide solutions to linear systems of equations for a real general sparse matrix. The sparse utility subroutines provided in Parallel ESSL must be used in conjunction with the sparse linear algebraic equation subroutines.

Table 7. List of The Fortran 77 Sparse Linear Algebraic Equation Subroutines

Descriptive Name	**Long-Precision Subroutine**	**Page**
Initializes an Array Descriptor for a General Sparse Matrix	PADINIT	658
Initializes a General Sparse Matrix	PDSPINIT	660
Inserts Local Data into a General Sparse Matrix	PDSPINS	662
Inserts Local Data into a Dense Vector	PDGEINS	667
Assembles a General Sparse Matrix	PDSPASB	670
Assembles a Dense Vector	PDGEASB	674
Preconditioner for a General Sparse Matrix	PDSPGPR	676
Iterative Linear System Solver for a General Sparse Matrix	PDSPGIS	679

Eigensystem Analysis and Singular Value Analysis

The eigensystem analysis and singular value analysis subroutines include a subset of the ScaLAPACK subroutines. See references [19] and [20].

Note: The message passing subroutines were designed in accordance with the proposed ScaLAPACK standard. If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so.

The HPF subroutines were designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so.

If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 8. List of Eigensystem Analysis and Singular Value Analysis Subroutines

Descriptive Name	Long- Precision Subroutine	Page
Selected Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix	PDSYEVX SYEVX	693 935
Reduce a Real Symmetric Matrix to Tridiagonal Form	PDSYTRD SYTRD	711 946
Reduce a General Matrix to Upper Hessenberg Form	PDGEHRD GEHRD	722 952
Reduce a General Matrix to Bidiagonal Form	PDGEBRD GEBRD	732 957

Fourier Transforms

The Fourier transform subroutines perform mixed-radix transforms in two and three dimensions. See references [1] and [3].

Table 9. List of Fourier Transform Subroutines

Descriptive Name	Short-Precision Subroutine	Long-Precision Subroutine	Page
Complex Fourier Transforms in Two Dimensions	PSCFT2 FFT	PDCFT2 FFT	748 968
Real-to-Complex Fourier Transforms in Two Dimensions	PSRCFT2 FFT	PDRCFT2 FFT	755 968
Complex-to-Real Fourier Transforms in Two Dimensions	PSCRFT2 FFT	PDCRFT2 FFT	761 968
Complex Fourier Transforms in Three Dimensions	PSCFT3 FFT	PDCFT3 FFT	767 976
Real-to-Complex Fourier Transforms in Three Dimensions	PSRCFT3 FFT	PDRCFT3 FFT	776 976
Complex-to-Real Fourier Transforms in Three Dimensions	PSCRFT3 FFT	PDCRFT3 FFT	783 976

Random Number Generation

The random number generation subroutine generates uniformly distributed random numbers.

Table 10. List of Random Number Generation Subroutines

Descriptive Name	Long- Precision Subroutine	Page
Uniform Random Number Generator	PDURNG URNG	793 989

Utilities

The message passing utility subroutines perform general service functions that support Parallel ESSL, rather than mathematical computations.

Table 11. List of Utility Subroutines

Descriptive Name	Integer Subroutine	Page
Determine the Level of Parallel ESSL Installed on Your System	IPESSL	801
Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process	NUMROC	803

Chapter 2. Distributing Your Data

This chapter provides information on how to distribute your data for message passing programs and for HPF programs. The sections include:

- “Concepts”
- “Specifying and Distributing Data in a Message Passing Program” on page 23
- “Distributing Data in an HPF Program” on page 79

Concepts

This section describes the general concepts used in distributing data.

About Global Data Structures

Because the Parallel ESSL subroutines support the SPMD programming model, your global data structures (vectors, matrices, or sequences) must be distributed across your processes prior to calling the Parallel ESSL subroutines.

Conceptually, global data structures have a defined storage mode consistent with those used by the serial ESSL library, **except for symmetric tridiagonal matrices**. For Parallel ESSL, you must store symmetric tridiagonal matrices as described in this chapter in “Block-Cyclically Distributing a Symmetric Tridiagonal Matrix” on page 49. For how to store all other data structures when using Parallel ESSL, you should see the appropriate section in the *ESSL Version 3 Guide and Reference*. The FFT-packed storage mode is a new storage mode for Parallel ESSL and is described in “Specifying Sequences for the Fourier Transforms” on page 66.

Global data structures must be mapped to local (distributed memory) data structures, according to the data distribution technique supported by the Parallel ESSL subroutines that you are using. These local data structures are called local arrays.

These data distribution techniques are described throughout this chapter and apply equally to real and complex data structures.

About Process Grids

A parallel machine with *k* processes is often thought of as a one-dimensional linear array of processes labeled 0, 1, ..., $k-1$. For performance reasons, it is sometimes useful to map this one-dimensional array into a logical two-dimensional rectangular grid, which is also referred to as process grid, of processes. The process grid can have *p* process rows and *q* process columns, where $p \times q = k$. A process can now be indexed by row and column, (i,j), where $0 \le i < p$ and $0 \le j < q$.

Table 12 on page 18 shows six processes mapped into a process grid using row-major order. For message passing subroutines, the BLACS_GRIDINIT default to map processes is row-major order. In this example, process t_3 is mapped to P_{10}.

Table 12. Six Processes Mapped to a 2 × 3 Process Grid Using Row-Major Order

p,q	0	1	2
0	t_0	t_1	t_2
1	t_3	t_4	t_5

Table 13 shows six processes mapped into a process grid using column-major order. For HPF subroutines, the XL HPF compiler default to map processes is column-major order. In this example, process t_3 is mapped to P_{11}.

Table 13. Six Processes Mapped to a 2 × 3 Process Grid Using Column-Major Order

p,q	0	1	2
0	t_0	t_2	t_4
1	t_1	t_3	t_5

All the subroutines, except the Banded Linear Algebraic Equations and Fourier transform subroutines, can view the processes as a logical one- or two-dimensional process grid. The Banded Linear Algebraic Equations support one-dimensional process grids. The Fourier transform subroutines support one-dimensional, row-oriented process grids.

Each process has local memory, and all the processes are connected by a communication network (for example, a switch or Ethernet). In most cases *k* is less than or equal to the number of processor nodes that your job is running on. In special cases, however, the number of processes can be greater than the number of processor nodes.

What to Do in Your Program

Prior to calling any of the subroutines, you must define your process grid and distribute your data according to the distribution technique required by the Parallel ESSL subroutine you are using.

The size and shape of the process grid and the way global data structures are distributed over the processes has a major impact on performance and scalability. For details, see "Coding Tips for Optimizing Parallel Performance" on page 83. Block-cyclic data distribution generally provides good load balancing for many linear algebra computations. All subroutines support block-cyclic data distributions, except the Fourier Transforms, the HPF versions of the Banded Linear Algebraic Equations, and the HPF version of the Random Number Generation subroutine (URNG). These subroutines support only block distribution, which is a special case of block-cyclic data distribution.

Some of the message passing and HPF data distribution techniques described in this chapter are illustrated in Appendix B on page 999.

For more information using High Performance Fortran, see reference [44], and the *XL High Performance Fortran* manuals.

Block, Cyclic, and Block-Cyclic Data Distributions

In this section, three types of data distribution are described in algorithmic terms: block, cyclic, and block-cyclic. How these data distribution methods are used by Parallel ESSL is explained later in this chapter.

The example notation means the following:

- **B** represents the global block row numbers.
- **D** represents the global block column numbers.
- **p** represents the process row index.
- **q** represents the process column index.

Distribution Techniques

An important aspect of the data distributions described here is that independent distributions are applied over each dimension of the data structure. The algorithms presented here for the vector in one dimension can, therefore, be used for the rows and columns of a matrix, or even for data structures with more dimensions.

Consider the distribution of a vector $\boldsymbol{x}$ of M data objects (elements) over P processes. This can be described by a mapping of the global index m ($0 \leq m < M$) of a data object to an index pair (p,i), where p ($0 \leq p < P$) specifies the process to which the data object is mapped, and i specifies its location in the local array.

Two common distributions are the **block** and **cyclic**. The block distribution is often used when the computational load is distributed homogeneously over a regular data structure, such as a Cartesian grid. It assigns blocks of size r of the global vector to the processes. For block distribution, the mapping $m \mapsto (p, i)$ is defined as:

$m \mapsto (\text{floor}(m/L), m \text{ mod } L)$

where $L = \text{ceiling}(M/P)$. The cyclic distribution (also known as the wrapped or scattered decomposition) is commonly used to improve load balance when the computational load is distributed inhomogeneously over a regular data structure. The cyclic distribution assigns consecutive entries of the global vector to successive processes. For cyclic distribution, the mapping $m \mapsto (p, i)$ is defined as:

$m \mapsto (m \text{ mod } P, \text{floor}(m/P))$

Examples of block and cyclic distribution are shown in Figure 1 and Figure 2 on page 20, where $M = 23$ data objects are distributed over $P = 3$ processes, using $r = 8$ block size. As shown in the examples, there can be uneven distribution, where the last block is smaller than the others. A global block number B is shown for block distribution. For cyclic distribution, there is no concept of block numbers.

m	0 1 2 3 4 5 6 7	8 9 10 11 12 13 14 15	16 17 18 19 20 21 22
p	0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1	2 2 2 2 2 2 2
i	0 1 2 3 4 5 6 7	0 1 2 3 4 5 6 7	0 1 2 3 4 5 6
B	0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1	2 2 2 2 2 2 2

Figure 1. Block Distribution

Following are HPF statements you could use to perform this block distribution of your data:

```
!HPF$ PROCESSORS P(3)
!HPF$ DISTRIBUTE X (BLOCK) ONTO P
```

m	0 1 2	3 4 5	6 7 8	9 10 11	12 13 14	15 16 17	18 19 20	21 22
p	0 1 2	0 1 2	0 1 2	0 1 2	0 1 2	0 1 2	0 1 2	0 1
i	0 0 0	1 1 1	2 2 2	3 3 3	4 4 4	5 5 5	6 6 6	7 7

Figure 2. Cyclic Distribution

Following are HPF statements you could use to perform this cyclic distribution of your data:

```
!HPF$ PROCESSORS P(3)
!HPF$ DISTRIBUTE X (CYCLIC) ONTO P
```

The block-cyclic distribution is a generalization of the block and cyclic distributions, in which blocks of *r* consecutive data objects are distributed cyclically over the *p* processes. This can be described by a mapping of the global index *m* ($0 \le m < M$) of a data object to an index triplet (*p,b,i*), where *p* ($0 \le p < P$) specifies the process to which the data object is mapped, *b* is the block number in process *p*, and *i* is the location in the block. For block-cyclic distribution, the mapping $m \mapsto (p, b, i)$ is defined as:

$$m \mapsto (\text{floor}((m \bmod T)/r), \text{floor}(m/T), m \bmod r)$$

where $T = rP$. (It should be noted that this reverts to the cyclic distribution when $r = 1$ and a block distribution when $r = L$.) The inverse mapping to a global index $(p, b, i) \mapsto m$ is defined by:

$$(p, b, i) \mapsto Br+i = pr+bT+i$$

where $B = p+bP$ is the global block number. An example of block-cyclic distribution is shown in Figure 3, where $M = 23$ data objects are distributed over $P = 3$ processes, using $r = 2$ block size. As shown in the example, there can be uneven distribution, where the last block is smaller than the others. The inverse mapping is shown in the second part of the example. (This shows what is stored in the local array on each of the three processes.)

m	0 1 2 3 4 5	6 7 8 9 10 11	12 13 14 15 16 17	18 19 20 21 22
p	0 0 1 1 2 2	0 0 1 1 2 2	0 0 1 1 2 2	0 0 1 1 2
b	0 0 0 0 0 0	1 1 1 1 1 1	2 2 2 2 2 2	3 3 3 3 3
i	0 1 0 1 0 1	0 1 0 1 0 1	0 1 0 1 0 1	0 1 0 1 0
B	0 0 1 1 2 2	3 3 4 4 5 5	6 6 7 7 8 8	9 9 10 10 11

Figure 3. Block-Cyclic Distribution

m	0 1 6 7 12 13 18 19	2 3 8 9 14 15 20 21	4 5 10 11 16 17 22
p	0 0 0 0 0 0 0 0	1 1 1 1 1 1 1 1	2 2 2 2 2 2 2
b	0 0 1 1 2 2 3 3	0 0 1 1 2 2 3 3	0 0 1 1 2 2 3
i	0 1 0 1 0 1 0 1	0 1 0 1 0 1 0 1	0 1 0 1 0 1 0
B	0 0 3 3 6 6 9 9	1 1 4 4 7 7 10 10	2 2 5 5 8 8 11

Figure 4. Inverse Mapping of Block-Cyclic Distribution

Following are HPF statements you could use to perform this block-cyclic distribution of your data:

```
!HPF$ PROCESSORS P(3)
!HPF$ DISTRIBUTE X (CYCLIC(2)) ONTO P
```

In decomposing an $m \times n$ matrix, ***A***, independent block-cyclic distributions are applied in the row and column directions. Thus, suppose the matrix rows are distributed with block size r over P processes by the $\lambda_{r,P}$ block-cyclic mapping, and the matrix columns are distributed with block size s over Q processes by the $\psi_{s,Q}$ block-cyclic mapping. Then the matrix element indexed globally by (m, n) is mapped as follows:

$$m \overset{\lambda}{\mapsto} (p,b,i)$$

$$n \overset{\psi}{\mapsto} (q,d,j)$$

The distribution of the matrix can be regarded as the tensor product of the row and column distributions, which can be expressed as:

$$(m, n) \mapsto ((p, q),(b, d),(i, j))$$

The block-cyclic matrix distribution expressed above distributes blocks of size $r \times s$ to a grid of $P \times Q$ processes.

An example of block-cyclic distribution of an $m \times n = 16 \times 30$ matrix with block size $r \times s = 3 \times 4$ and a $P \times Q = 2 \times 3$ process grid is shown in Figure 5 on page 22 and Figure 6 on page 22. The numbers in the leftmost column and on the top of the matrix represent the global row and column numbers **B** and **D**, respectively. Figure 5 on page 22 shows the assignment of global blocks (**B,D**) to processes (P,Q). Figure 6 on page 22 shows which global blocks each process contains.

In this example, the global matrix dimensions are not divisible by the respective block sizes. All the row blocks are of size 3, except the last row block, which only contains 1 row. All column blocks are of size 4, except the last column block, which contains 2 columns. For example, global block (5,0) is 1×4, global block (1,7) is 3×2, and global block (0,0) is 3×4. The global block (5,7) is 1×2. The asterisk (*) in Figure 5 on page 22 denotes which global blocks contain left over data; that is, the blocks that are not 3×4.

B,D	0	1	2	3	4	5	6	7
0	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}^{*}
1	P_{10}	P_{11}	P_{12}	P_{10}	P_{11}	P_{12}	P_{10}	P_{11}^{*}
2	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}^{*}
3	P_{10}	P_{11}	P_{12}	P_{10}	P_{11}	P_{12}	P_{10}	P_{11}^{*}
4	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}	P_{02}	P_{00}	P_{01}^{*}
5	P_{10}^{*}	P_{11}^{*}	P_{12}^{*}	P_{10}^{*}	P_{11}^{*}	P_{12}^{*}	P_{10}^{*}	P_{11}^{*}

Figure 5. Block Distribution Over a 2 by 3 Process Grid

B,D	0	3	6	1	4	7	2	5
0						*		
2		P_{00}			P_{01}	*		P_{02}
4						*		
1						*		
3		P_{10}			P_{11}	*		P_{12}
5	*	*	*	*	*	*	*	*

Figure 6. Data Distribution from a Process Point-of-View

B,D	0	3	6	1	4	7	2	5
0	$a_{0:2,0:3}$	$a_{0:2,12:15}$	$a_{0:2,24:27}$	$a_{0:2,4:7}$	$a_{0:2,16:19}$	$a_{0:2,28:29}^{*}$	$a_{0:2,8:11}$	$a_{0:2,20:23}$
2	$a_{6:8,0:3}$	$a_{6:8,12:15}$	$a_{6:8,24:27}$	$a_{6:8,4:7}$	$a_{6:8,16:19}$	$a_{6:8,28:29}^{*}$	$a_{6:8,8:11}$	$a_{6:8,20:23}$
4	$a_{12:14,0:3}$	$a_{12:14,12:15}$	$a_{12:14,24:27}$	$a_{12:14,4:7}$	$a_{12:14,16:19}$	$a_{12:14,28:29}^{*}$	$a_{12:14,8:11}$	$a_{12:14,20:23}$
1	$a_{3:5,0:3}$	$a_{3:5,12:15}$	$a_{3:5,24:27}$	$a_{3:5,4:7}$	$a_{3:5,16:19}$	$a_{3:5,28:29}^{*}$	$a_{3:5,8:11}$	$a_{3:5,20:23}$
3	$a_{9:11,0:3}$	$a_{9:11,12:15}$	$a_{9:11,24:27}$	$a_{9:11,4:7}$	$a_{9:11,16:19}$	$a_{9:11,28:29}^{*}$	$a_{9:11,8:11}$	$a_{9:11,20:23}$
5	$a_{15,0:3}^{*}$	$a_{15,12:15}^{*}$	$a_{15,24:27}^{*}$	$a_{15,4:7}^{*}$	$a_{15,16:19}^{*}$	$a_{15,28:29}^{*}$	$a_{15,8:11}^{*}$	$a_{15,20:23}^{*}$

Figure 7. Distributed Matrix Elements from a Process Point-of-View

Following are HPF statements you could use to perform this block-cyclic distribution of your data:

```
!HPF$ PROCESSORS P(2,3)
!HPF$ DISTRIBUTE A (CYCLIC(3),CYCLIC(4)) ONTO P
```

Special Usage

The block-cyclic distribution can reproduce most of the data distributions commonly used in linear algebra computations on parallel computers. Some examples are:

- Block distribution in the row direction is obtained by $Q = 1$ and $r = \text{ceiling}(M/P)$.
- Block distribution in the column direction is obtained by $P = 1$ and $s = \text{ceiling}(N/Q)$.

- Block-cyclic distribution in the row direction is obtained by $Q = 1$ and $r <$ ceiling(M/P). (You might use this for distributing a single block column to pass to Parallel ESSL.)
- Block-cyclic distribution in the column direction is obtained by $P = 1$ and $s <$ ceiling(N/Q). (You might use this for distributing a single block row to pass to Parallel ESSL.)
- To achieve **fine** granularity of distribution in the following directions, specify:
  - For the row direction, $r = 1$
  - For the column direction, $s = 1$
  - In both directions, $r = 1$ and $s = 1$
- To achieve **coarse** granularity of distribution in the following directions, specify:
  - For the row direction, $r =$ ceiling(M/P).
  - For the column direction, $s =$ ceiling(N/Q).
  - In both directions, $r =$ ceiling(M/P) and $s =$ ceiling(N/Q).

This section provided a detailed description of the distribution of vectors—one-dimensional data structures. Those same techniques were then applied to matrices—two-dimensional data structures—in the row and column directions. If you have data structures with three or more dimensions, you can use these same techniques by applying them in the direction of each dimension. For example, the block distribution of a three-dimensional sequence is described in “Three-Dimensional Sequences” on page 71.

Specifying and Distributing Data in a Message Passing Program

This section describe the calling sequence arguments for vectors and matrices, and shows how to distribute vectors, matrices and sequences in a message passing program for the following areas:

- For the Level 2 and 3 PBLAS, Dense Linear Algebraic Equations, and Eigensystem Analysis and Singular Value Analysis subroutines, see “Specifying Block-Cyclically-Distributed Vectors and Matrices” on page 24.
- For the Banded Linear Algebraic Equations, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.
- For the Sparse Linear Algebraic Equations, see “Specifying Sparse Matrices for the Fortran 90 and Fortran 77 Sparse Linear Algebraic Equations” on page 60.
- For the Fourier Transforms, see “Specifying Sequences for the Fourier Transforms” on page 66.

An example of block-cyclic distribution of a global matrix in a Fortran 90 program in a message passing environment is shown in Appendix B. See the following:

- The subroutine get_diffusion_matrix in “Module Fourier (Message Passing)” on page 1017, which shows how a local array can be assigned values.
- The subroutine rlocal_to_rglobal in “Module Scale (Message Passing)” on page 1025, which shows gathering the local portions of the block-cyclically-distributed real array to generate the corresponding global matrix.

Specifying Block-Cyclically-Distributed Vectors and Matrices

For the Level 2 and 3 PBLAS, Dense Linear Algebraic Equations, and Eigensystem Analysis and Singular Value Analysis subroutines, certain calling sequence arguments are used to specify block-cyclically-distributed vectors or matrices.

Calling Sequence Arguments for Block-Cyclically-Distributed Vectors and Matrices

Table 14 describes the arguments associated with a vector ***X***. Table 15 describes the arguments associated with a matrix ***A***.

Table 14. Calling Sequence Arguments for a Block-Cyclically-Distributed Vector

Argument	**Meaning**
x	is the local part of the global matrix ***X***. To determine the size of the local array for ***X***, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25.
ix	is the row index of global matrix ***X***.
jx	is the column index of global matrix ***X***.
desc_x	is the array descriptor for global matrix ***X***. (See Table 16 on page 25.)
incx	Stride for global vector ***X***.

Note: A global vector of length *n* is distributed across process rows the same way as an $n \times 1$ matrix is (in this case M_X is *n* and N_X is 1). A global vector of length *n* is distributed across process columns the same way as a $1 \times n$ matrix is (in this case M_X is 1 and N_X is *n*).

Table 15. Calling Sequence Arguments for a Block-Cyclically-Distributed Matrix

Argument	**Meaning**
a	is the local part of the global matrix ***A***. To determine the size of the local array for ***A***, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25.
ia	is the row index of the global matrix ***A***.
ja	is the column index of the global matrix ***A***.
desc_a	is the array descriptor for global matrix ***A***. (See Table 16 on page 25.)

Array Descriptors for Block-Cyclically-Distributed Matrices

An array descriptor, which is an integer array, is needed for each block-cyclically-distributed vector or matrix. The process grid definition and array descriptor are used to establish the mapping between the global vector or matrix and its corresponding process and distributed memory location.

Throughout this book, the _ (underscore) symbol in the array descriptor is followed by an ***X*** to indicate a vector or an ***A*** to indicate a matrix.

An example of setting up descriptor arrays in a Fortran 90 program is shown in Appendix B. See the subroutines initialize_rarray and initialize_carray in "Module Scale (Message Passing)" on page 1025.

Table 16 on page 25 shows the type-1 array descriptor, as it is used in the Level 2 and 3 PBLAS, Dense Linear Algebraic Equations, and Eigensystem Analysis and Singular Value Analysis subroutines.

Table 16. Type-1 Array Descriptor for Block-Cyclically Distributed Vector or Matrix

DESC_( )	Symbolic name	Meaning
1	DTYPE_	Descriptor type, where DTYPE_=1
2	CTXT_	BLACS context in which the global matrix is defined. (See “Initializing the BLACS” on page 87.)
3	M_	Number of rows in the global matrix
4	N_	Number of columns in the global matrix
5	MB_	Row block size
6	NB_	Column block size
7	RSRC_	The process row of the $p \times q$ process grid over which the first row of the global matrix is distributed
8	CSRC_	The process column of the $p \times q$ process grid over which the first column of the global matrix is distributed
9	LLD_	Leading dimension of the local array. (See “Determining the Number of Rows and Columns in Your Local Arrays.”) This value may be different on each process.

Specifying Submatrices

After a global vector or matrix is block-cyclically distributed over a process grid, you may decide to use only a portion of the global data structure. This is called a submatrix. For examples of how to specify the calling sequence arguments, listed in Table 14 and Table 15, for a submatrix, see:

- “Example 1” on page 174
- “Example 2” on page 177
- “Example 1” on page 212
- “Example 1” on page 271
- “Example 1” on page 440
- “Example 1” on page 477

Suppose you decide to distribute your global vector or matrix over the process grid, starting at a process other than 0,0. For examples of how to set the array descriptor values, listed in Table 16, see:

- “Example 1” on page 427
- “Example 1” on page 440
- “Example 1” on page 477

Determining the Number of Rows and Columns in Your Local Arrays

In a Parallel ESSL calling sequence, you specify an array that contains the local part of the global vector or matrix. To determine LOCp(M_) or LOCq(N_), which are used in the subroutines descriptions in Part 2 of this book, you must make a call to NUMROC:

- For LOCp(M_), which represents the number of rows that a process would receive if M_ was distributed block-cyclically over the *p* rows of its process column, you specify:

  ```
  LOCp(M_) = NUMROC (M_, MB_, myrow, RSRC_, p)
  ```

  where:

  M_ is the number of rows in the global matrix.
  MB_ is the row block size.
  myrow is the process row index. See "Initializing the BLACS" on page 87.
  RSRC_ is the process row over which the first row of the global matrix is distributed.
  p is the number of rows in the $p \times q$ process grid.

- For LOCq(N_), which represents the number of columns that a process would receive if N_ was distributed block-cyclically over the *q* columns of its process row, you specify:

  ```
  LOCq(N_) = NUMROC (N_, NB_, mycol, CSRC_, q)
  ```

  where:

  N_ is the number of columns in the global matrix.
  NB_ is the column block size.
  mycol is the process column index. See "Initializing the BLACS" on page 87.
  CSRC_ is the process column over which the first column of the global matrix is distributed.
  q is the number of columns in the $p \times q$ process grid.

Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations

For the Banded Linear Algebraic Equations, certain calling sequence arguments are used to specify block-cyclically distributed matrices on one-dimensional process grids.

Although the global array is block-cyclically distributed, the actual submatrix used in computation is either block-row or block-column distributed. See the appropriate subroutine for restrictions.

Symmetric Band Matrix

A symmetric band matrix must be distributed over a one-dimensional process grid:

- On a $1 \times p$ process grid, the symmetric band matrix is block-cyclically distributed. In this case, either type-501 or type-1 array descriptor may be specified.
- On a $p \times 1$ process grid, the symmetric band matrix is block-cyclically distributed as if the process grid is $1 \times p$. In this case, the type-501 array descriptor must be specified.

Table 17 on page 27 describes the calling sequence arguments associated with a symmetric band matrix.

Table 17. Calling Sequence Arguments for a Distributed Symmetric Band Matrix

Argument	Meaning
n	is the order of the global symmetric band submatrix ***A***.
a	is the local part of the global symmetric band matrix ***A***.
ja	is the column index of the global symmetric band matrix ***A***.
desc_a	is the array descriptor for the global symmetric band matrix ***A***. For more details, see Table 21 on page 29 and Table 16 on page 25.

General Tridiagonal Matrix

A general tridiagonal matrix, represented as three vectors, must be distributed over a one-dimensional process grid using a block-cyclic data distribution. Because vectors are one-dimensional data structures, you can use type-501, type-502, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. Table 18 describes the calling sequence arguments associated with a general tridiagonal matrix.

Table 18. Calling Sequence Arguments for General Tridiagonal Matrix

Argument	Meaning
n	is the order of the global general tridiagonal submatrix ***A***.
dl, d, du	is the local part of the global vectors. (The general tridiagonal matrix ***A*** is stored in tridiagonal storage mode in *dl, d*, and *du*.)
ia	is the row index of the global general tridiagonal matrix ***A***.
desc_a	is the array descriptor for the global general tridiagonal matrix ***A***. For more details, see Table 21 on page 29, Table 16 on page 25, or Table 22 on page 29.

Symmetric Tridiagonal Matrix

A symmetric tridiagonal matrix, represented as two vectors, must be distributed over a one-dimensional process grid using block-cyclic data distribution.

Note: For both serial ESSL and Parallel ESSL, the $n-1$ elements of the equal off-diagonals of a symmetric tridiagonal matrix are stored in a one-dimensional vector of length *n*. To be compatible with ScaLAPACK, in Parallel ESSL, the off-diagonal is chosen to be the superdiagonal and is stored in elements *ia* through $ia+n-2$. In the serial ESSL library, the off-diagonal is chosen to be the subdiagonal and is stored in elements 2 through *n*.

Because vectors are one-dimensional data structures, you can use a type-501, type-502, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. Table 19 describes the calling sequence arguments associated with a symmetric tridiagonal matrix.

Table 19 (Page 1 of 2). Calling Sequence Arguments for a Symmetric Tridiagonal Matrix

Argument	Meaning
n	is the order of the global symmetric tridiagonal submatrix ***A***.

Table 19 (Page 2 of 2). Calling Sequence Arguments for a Symmetric Tridiagonal Matrix

Argument	Meaning
d, e	is the local part of the global vectors. (The symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode in *d* and *e*.)
ia	is the row index of the global symmetric tridiagonal matrix ***A***.
desc_a	is the array descriptor for the global symmetric tridiagonal matrix ***A***. For more details, see Table 21 on page 29, Table 16 on page 25, or Table 22 on page 29.

General Matrix Consisting of Multiple Right-Hand Sides

For the Banded Linear Algebraic Equations subroutines, a general matrix consisting of multiple right-hand sides must be distributed over a one-dimensional process grid:

- On a $p \times 1$ process grid, the multiple right-hand sides is block-cyclically distributed. In this case either type-502 or type-1 array descriptor may be specified.
- On a $1 \times p$ process grid, the multiple right-hand sides is block-cyclically distributed as if the process grid is $p \times 1$. In this case type-502 array descriptor must be specified.

Table 20 describes the calling sequence arguments associated with the general matrix.

Table 20. Calling Sequence Arguments for a Matrix Containing the Multiple Right-Hand Sides

Argument	Meaning
n	is the number of rows in the global general submatrix ***B***.
b	is the local part of the global general matrix ***B***.
ib	is the row index of the global general matrix ***B***.
desc_b	is the array descriptor for the global general matrix ***B***. For more details, see Table 22 on page 29 and Table 16 on page 25.

Array Descriptors for Banded Matrices

An array descriptor, which is an integer array, is needed for each block-distributed matrix. The process grid definition and the array descriptor are used to establish the mapping between the global matrix and its corresponding process and distributed memory location.

In the Banded Linear Algebraic Equations sections throughout this book, the _ (underscore) symbol in the array descriptor is followed by an ***A*** or a ***B***. ***A*** indicates a banded, tridiagonal, or symmetric tridiagonal matrix. ***B*** indicates a matrix containing the multiple right-hand sides matrix.

When you place a call to the banded or tridiagonal subroutines, you must be careful to choose consistent combinations of array descriptor types for matrix ***A*** and matrix ***B***, and process grids. For consistent combinations, see the “Notes and Coding Rules” in the subroutine descriptions in Part 2 of this book.

Therefore, depending on which subroutine you are using in the Banded Linear Algebraic Equations, you may choose different array descriptors in the same subroutine calling sequence. Keep in mind **you must only create one process grid**; that is, CTXT_A = CTXT_B.

For example, when calling PDPBSV suppose you choose DTYPE_A = 501 for the band matrix ***A*** and DTYPE_B = 502 for matrix ***B***. If you specify CTXT_A as 1 × *p*, you must also specify CTXT_B as 1 × *p*. Or if you specify CTXT_A as *p* × 1, you must also specify CTXT_B as *p* × 1. For an example of how to set the array descriptor values, see "Example" on page 492.

Table 21. Type-501 Array Descriptor

DESC_( )	Symbolic name	Value
1	DTYPE_	DTYPE_ = 501 for 1 × *p* or *p* × 1, where *p* is the number of processes in a process grid.
2	CTXT_	BLACS context in which the global matrix is defined. The BLACS process grid can be defined as 1 × *p* or *p* × 1. (See "Initializing the BLACS" on page 87.)
3	N_	Number of columns in the global matrix
4	NB_	Column block size.
5	CSRC_	The process column over which the first column of the global matrix is distributed
6	LLD_	Leading dimension of the local array. (See "Determining the Number of Rows or Columns in Your Local Arrays" on page 30.) This value may be different on each process. For the tridiagonal subroutines, this argument is ignored.
7	—	Reserved.

Table 22 (Page 1 of 2). Type-502 Array Descriptor

DESC_( )	Symbolic name	Value
1	DTYPE_	DTYPE_ = 502 for *p* × 1 or 1 × *p*, where *p* is the number of processes in a process grid.
2	CTXT_	BLACS context in which the global matrix is defined. The BLACS process grid can be defined as 1 × *p* or *p* × 1. (See "Initializing the BLACS" on page 87.)
3	M_	Number of rows in the global matrix
4	MB_	Row block size.
5	RSRC_	The process row over which the first row of the global matrix is distributed
6	LLD_	Leading dimension of the local array. (See "Determining the Number of Rows or Columns in Your Local Arrays" on page 30.) This value may be different on each process. For the tridiagonal subroutines, this argument is ignored for matrix ***A***.

Table 22 (Page 2 of 2). Type-502 Array Descriptor

DESC_( )	Symbolic name	Value
7	—	Reserved.

Determining the Number of Rows or Columns in Your Local Arrays

For local arrays described by type-501 array descriptor, the number of rows in the local matrix is always equal to the number of rows in the global matrix. The number of columns in the local array is determined as follows:

- For a 1 × *q* process grid:

  ```
  LOCq(N_) = NUMROC(N_,NB_,mycol,CSRC_,q)
  ```

- For *q* × 1 process grid:

  ```
  LOCq(N_) = NUMROC(N_,NB_,myrow,CSRC_,q)
  ```

where:

N_ is the number of columns in the global matrix.
NB_ is the column block size.
mycol, for a 1 × *q* process grid, is the process column index. See “Initializing the BLACS” on page 87.
myrow, for a *q* × 1 process grid, is the process row index. See “Initializing the BLACS” on page 87.
CSRC_ is element 5 of type-501 array descriptor.
q is the number of columns in the process grid.

For local arrays described by type-502 array descriptor, the number of columns in the local matrix is always equal to the number of columns in the global matrix. The number of rows in the local array is determined as follows:

- For a *p* × 1 process grid:

  ```
  LOCp(M_) = NUMROC(M_,MB_,myrow,RSRC_,p)
  ```

- For a 1 × *p* process grid:

  ```
  LOCp(M_) = NUMROC(M_,MB_,mycol,RSRC_,p)
  ```

where:

M_ is the number of rows in the global matrix.
MB_ is the row block size.
myrow, for a *p* × 1 process grid, is the process row index. See “Initializing the BLACS” on page 87.
mycol, for a 1 × *p* process grid, is the process column index. See “Initializing the BLACS” on page 87.
RSRC_ is element 5 of type-502 array descriptor.
p is the number of rows in the process grid.

Distributing Data Structures

You must distribute your data before calling Parallel ESSL from your message passing program. This section shows how you how to distribute your data.

All the Parallel ESSL message passing subroutines, except the Banded Linear Algebraic Equations and Fourier transform subroutines, support block-cyclic distribution. The Banded Linear Algebraic Equations and the Fourier transform subroutines only support block distribution.

The following sections provide examples for distributing data over one- or two-dimensional process grids:

- "Vectors"
- "Matrices" on page 40
- "Specifying Sequences for the Fourier Transforms" on page 66

Vectors

Parallel ESSL supports block-cyclic distribution for vectors over one- or two-dimensional process grids. A vector is distributed over a single row or column of the process grid, except for PDURNG. For PDURNG, vectors are distributed block-cyclically over the entire one- or two-dimensional process grid using row-major order, where the length *n* of the vector *x* must be evenly divisible by the available processes *np* multiplied by the block size *nb*. In other words, *n*/(*np*)(*nb*) must be an integer.

Block-Cyclic Distribution over One-Dimensional Process Grids

This example shows how a global vector of length 24 with blocks of size 3 is distributed block-cyclically over one-dimensional process grids. Assume the following:

X = (8, 2, 3, 6, 5, 1, 9, 5, 3, 6, 2, 4, 10, 7, 4, 2, 8, 2, 8, 9, 2, 3, 11, 10)

Global vector ***x***:

$$\begin{array}{cc}
B,D & 0 \\
\begin{array}{c} \\ 0 \\ \\ \\ \\ 1 \\ \\ \\ \\ 2 \\ \\ \\ \\ 3 \\ \\ \\ \\ 4 \\ \\ \\ \\ 5 \\ \\ \\ \\ 6 \\ \\ \\ \\ 7 \\ \\ \end{array} & \left[\begin{array}{c} 8 \\ 2 \\ 3 \\ \text{--} \\ 6 \\ 5 \\ 1 \\ \text{--} \\ 9 \\ 5 \\ 3 \\ \text{--} \\ 6 \\ 2 \\ 4 \\ \text{--} \\ 10 \\ 7 \\ 4 \\ \text{--} \\ 2 \\ 8 \\ 2 \\ \text{--} \\ 8 \\ 9 \\ 2 \\ \text{--} \\ 3 \\ 11 \\ 10 \end{array}\right]
\end{array}$$

Column-oriented, 4 × 1 process grid:

B,D	**0**
0 **4**	P_{00}
1 **5**	P_{10}
2 **6**	P_{20}
3 **7**	P_{30}

Local arrays:

```
p,q  | 0
-----|----
     | 8
     | 2
     | 3
 0   | 10
     | 7
     | 4
-----|----
     | 6
     | 5
     | 1
 1   | 2
     | 8
     | 2
-----|----
     | 9
     | 5
     | 3
 2   | 8
     | 9
     | 2
-----|----
     | 6
     | 2
     | 4
 3   | 3
     | 11
     | 10
```

For the column-oriented example, the array descriptor DESC_X contains the following:

DESC_X( )	**Symbolic name**	**Value**
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	24
4	N_X	1
5	MB_X	3
6	NB_X	1
7	RSRC_X	0
8	CSRC_X	0
9	LLD_X	6

Row-oriented, 1 × 4 process grid:

B,D	**0 4**	**1 5**	**2 6**	**3 7**
0	P_{00}	P_{01}	P_{02}	P_{03}

Local array:

```
p,q  |       0        |        1       |       2        |        3
-----|----------------|----------------|----------------|----------------
 0   | 8 2 3 10 7 4   |  6 5 1 2 8 2   |  9 5 3 8 9 2   |  6 2 4 3 11 10
```

For the row-oriented example, the array descriptor DESC_X contains the following:

DESC_X( )	Symbolic name	Value
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	1
4	N_X	24
5	MB_X	1
6	NB_X	3
7	RSRC_X	0
8	CSRC_X	0
9	LLD_X	1

Note: The same global vector was distributed over a 4 × 1 grid and then over a 1 × 4 grid. Notice the values contained in the corresponding local arrays are identical.

Block-Cyclic Distribution over Two-Dimensional Process Grids

This example shows how a global vector of length 18 with block size of 3 is distributed over two-dimensional grids. When a two-dimensional process grid is used, the global vector can be distributed over any single row or any single column of the grid. Assume the following:

X = (4, 11, 17, 21, 3, 7, 12, 5, 3, 15, 3, 4, 9, 17, 1, 10, 9, 25)

Global vector ***x***:

```
B,D    0
      ┌    ┐
      |  4 |
 0    | 11 |
      | 17 |
      | -- |
      | 21 |
 1    |  3 |
      |  7 |
      | -- |
      | 12 |
 2    |  5 |
      |  3 |
      | -- |
      | 15 |
 3    |  3 |
      |  4 |
      | -- |
      |  9 |
 4    | 17 |
      |  1 |
      | -- |
      | 10 |
 5    |  9 |
      | 25 |
      └    ┘
```

Two-dimensional, 2 × 3 process grid:

B,D	**—**	**—**	**0**
0 **2** **4**	P_{00}	P_{01}	P_{02}
1 **3** **5**	P_{10}	P_{11}	P_{12}

If the global vector is distributed over the third column of a 2 × 3 process grid, then P_{02} and P_{12} contain the following local arrays:

```
p,q  | 2
-----|----
     |  4
     | 11
     | 17
     | 12
 0   |  5
     |  3
     |  9
     | 17
     |  1
-----|----
     | 21
     |  3
     |  7
     | 15
 1   |  3
     |  4
     | 10
     |  9
     | 25
```

For the single column example, the array descriptor `DESC_X` contains the following:

DESC_X( )	**Symbolic name**	**Value**
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	18
4	N_X	1
5	MB_X	3
6	NB_X	1
7	RSRC_X	0
8	CSRC_X	2
9	LLD_X	9

If the global vector is distributed over the second row of a 2 × 3 process grid, then P_{10}, P_{11}, and P_{12} contain the following local arrays:

```
p,q  |        0          |        1         |        2
-----|-------------------|------------------|-----------------
 1   |  4 11 17 15 3 4   |  21 3 7 9 17 1   |  12 5 3 10 9 25
```

For the single row example, the array descriptor `DESC_X` contains the following:

DESC_X( )	Symbolic name	Value
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	1
4	N_X	18
5	MB_X	1
6	NB_X	3
7	RSRC_X	1
8	CSRC_X	0
9	LLD_X	1

For PDURNG, the global vector is distributed block-cyclically over the **entire** 2 × 3 process grid using row-major order, as follows:

```
p,q  |    0      |     1      |     2
-----|-----------|------------|-----------
 0   |  4 11 17  |  21  3  7  |  12  5  3
-----|-----------|------------|-----------
 1   | 15  3  4  |   9 17  1  |  10  9 25
```

Notes:

1. For PDURNG, the length *n* of the vector *x* must be evenly divisible by the number of available processes *np* multiplied by the block size *nb*. For this example, 18 = (6)(3).
2. For PDURNG, the array descriptor is not used.

Following is an example of uneven block-cyclic distribution for a global vector of length 20 with block size of 3, where the two local arrays are different sizes. In this case, a fragment of a block with two elements occurs at the end of the vector. Assume the following:

```
X = (0, 5, 6, 3, 21, 5, 6, 1, 8, 9, 13, 11, 12, 15, 14, 15, 11, 17, 18, 19)
```

Following is a global vector ***x*** with block size 3:

```
B,D    0
     ┌    ┐
     │  0 │
 0   │  5 │
     │  6 │
     │ -- │
     │  3 │
 1   │ 21 │
     │  5 │
     │ -- │
     │  6 │
 2   │  1 │
     │  8 │
     │ -- │
     │  9 │
 3   │ 13 │
     │ 11 │
     │ -- │
     │ 12 │
 4   │ 15 │
     │ 14 │
     │ -- │
     │ 15 │
 5   │ 11 │
     │ 17 │
     │ -- │
 6   │ 18 │
     │ 19 │
     └    ┘
```

Two-dimensional, 2 × 3 process grid:

B,D	**0**	**—**	**—**
0 **2** **4** **6**	P_{00}	P_{01}	P_{02}
1 **3** **5**	P_{10}	P_{11}	P_{12}

If the vector is distributed over the first column of a 2 × 3 process grid, then P_{00} and P_{10} contain the following local arrays:

```
p,q  | 0
-----|----
     |  0
     |  5
     |  6
     |  6
     |  1
 0   |  8
     | 12
     | 15
     | 14
     | 18
     | 19
-----|----
     |  3
     | 21
     |  5
     |  9
 1   | 13
     | 11
     | 15
     | 11
     | 17
```

Array descriptor `DESC_X` contains the following:

DESC_X( )	Symbolic name	Value
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	20
4	N_X	1
5	MB_X	3
6	NB_X	1
7	RSRC_X	0
8	CSRC_X	0
9	LLD_X	11 (For P_{00}) 9 (For P_{10})

If the vector is distributed over the first row of the 2 × 3 process grid, then P_{00}, P_{01}, and P_{02} contain the following local arrays:

```
p,q  |            0              |         1          |         2
-----|---------------------------|--------------------|-------------------
 0   | 0  5  6  9 13 11 18 19  |  3 21  5 12 15 14  |  6  1  8 15 11 17
```

Array descriptor `DESC_X` contains the following:

DESC_X( )	Symbolic name	Value
1	DTYPE_X	1
2	CTXT_X	BLACS context
3	M_X	1
4	N_X	20
5	MB_X	1
6	NB_X	3
7	RSRC_X	0
8	CSRC_X	0
9	LLD_X	1

Matrices

The Parallel ESSL subroutines, except the Banded Linear Algebraic Equations, support block-cyclic data distribution for matrices using one- or two-dimensional process grids. The Banded Linear Algebraic Equations support only block data distribution using one-dimensional process grids.

The following terminology is used when it is necessary to distinguish special types of matrices:

- Full block matrix — a matrix of blocks distributed over the whole process grid.
- Block row matrix — a matrix of blocks distributed over a single row of the process grid.
- Block column matrix — a matrix of blocks distributed over a single column of the process grid.
- Single block matrix — a matrix consisting of a single block lying in a single process of the process grid.

Distributed over One-Dimensional Process Grids

This section describes how to distribute a matrix block-cyclically over a one-dimensional process grid. It also shows how matrices for the Banded Linear Algebraic Equations are distributed over a one-dimensional process grid using block distribution.

Block-Cyclically Distributing a Matrix: The examples that follow show how a 6 × 8 global matrix ***A*** with blocks of size 2 × 2 is distributed block-cyclically over one-dimensional process grids. Assume the following global matrix ***A***:

```
B,D      0           1           2           3
     ┌                                              ┐
 0   │   0   1  │    2   3  │    4   5  │    6   7 │
     │  10  11  │   12  13  │   14  15  │   16  17 │
     │ ---------|-----------|-----------|--------- │
 1   │  20  21  │   22  23  │   24  25  │   26  27 │
     │  30  31  │   32  33  │   34  35  │   36  37 │
     │ ---------|-----------|-----------|--------- │
 2   │  40  41  │   42  43  │   44  45  │   46  47 │
     │  50  51  │   52  53  │   54  55  │   56  57 │
     └                                              ┘
```

Column-oriented, 3 × 1 process grid:

B,D	**0 1 2 3**
0	P_{00}
1	P_{10}
2	P_{20}

Local arrays:

```
p,q  |               0
-----|--------------------------------
 0   |   0   1   2   3   4   5   6   7
     |  10  11  12  13  14  15  16  17
-----|--------------------------------
 1   |  20  21  22  23  24  25  26  27
     |  30  31  32  33  34  35  36  37
-----|--------------------------------
 2   |  40  41  42  43  44  45  46  47
     |  50  51  52  53  54  55  56  57
```

For the column-oriented example, the array descriptor `DESC_A` contains:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	1
2	CTXT_A	BLACS context
3	M_A	6
4	N_A	8
5	MB_A	2
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	2

Row-oriented, 1 × 2 process grid:

B,D	**0 2**	**1 3**
0	P_{00}	P_{01}
1		
2		

Local arrays:

```
p,q  |        0         |        1
-----|------------------|------------------
     |   0   1   4   5  |    2   3   6   7
     |  10  11  14  15  |   12  13  16  17
     |  20  21  24  25  |   22  23  26  27
 0   |  30  31  34  35  |   32  33  36  37
     |  40  41  44  45  |   42  43  46  47
     |  50  51  54  55  |   52  53  56  57
```

For the row-oriented example, the array descriptor DESC_A:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	1
2	CTXT_A	BLACS context
3	M_A	6
4	N_A	8
5	MB_A	2
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	6

For an example of distributing a matrix over a one-dimensional process grid in a Fortran 90 program, see matrix ***F*** in Appendix B, which is:

- Created in subroutine initialize_carray in "Module Scale (Message Passing)" on page 1025.
- Assigned values in subroutine get_diffusion_matrix in "Module Fourier (Message Passing)" on page 1017.
- Used in "Program Main (Message Passing)" on page 1006.

Block-Cyclically Distributing a Symmetric Band Matrix: This section shows how to distribute a symmetric band matrix ***A*** over a one-dimensional process grid using block-cyclic distribution.

Assume the following symmetric band matrix ***A*** of size 9 × 9 with a half bandwidth of 2:

$$A = \begin{bmatrix} 11 & 21 & 31 & 0 & 0 & 0 & 0 & 0 & 0 \\ 21 & 22 & 32 & 42 & 0 & 0 & 0 & 0 & 0 \\ 31 & 32 & 33 & 34 & 53 & 0 & 0 & 0 & 0 \\ 0 & 42 & 34 & 44 & 54 & 64 & 0 & 0 & 0 \\ 0 & 0 & 53 & 54 & 55 & 65 & 75 & 0 & 0 \\ 0 & 0 & 0 & 64 & 65 & 66 & 76 & 86 & 0 \\ 0 & 0 & 0 & 0 & 75 & 76 & 77 & 87 & 97 \\ 0 & 0 & 0 & 0 & 0 & 86 & 87 & 88 & 98 \\ 0 & 0 & 0 & 0 & 0 & 0 & 97 & 98 & 99 \end{bmatrix}$$

Matrix ***A*** must be stored in upper- or lower-band-packed storage mode. The sections that follow contain examples describing these two storage modes. In these examples, matrix ***A*** is stored in an array with dimensions 3 × 9.

Upper-Band-Packed Storage Mode: The global matrix ***A*** with block size of 2 is stored in upper-band-packed storage mode, as follows:

$$\begin{array}{c|cc|cc|cc|cc|c|} \text{B,D} & \multicolumn{2}{c}{0} & \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} & \multicolumn{1}{c}{4} \\ & * & * & 31 & 42 & 53 & 64 & 75 & 86 & 97 \\ 0 & * & 21 & 32 & 34 & 54 & 65 & 76 & 87 & 98 \\ & 11 & 22 & 33 & 44 & 55 & 66 & 77 & 88 & 99 \end{array}$$

Following is a row-oriented, 1 × 3 process grid:

B,D	0 3	1 4	2
0	P_{00}	P_{01}	P_{02}

The following local arrays A are distributed block-cyclically over the 1 × 3 process grid:

```
p,q  |      0       |     1     |    2
-----|--------------|-----------|--------
     |  *  *  75 86 |  31 42 97 |  53 64
 0   |  * 21  76 87 |  32 34 98 |  54 65
     | 11 22  77 88 |  33 44 99 |  55 66
```

where * means you do not have to store a value in that position in the local array. However, these storage positions are required and overwritten during the computation.

The type-501 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 501 for 1 × *p*
2	CTXT_A	BLACS context
3	N_A	9
4	NB_A	2
5	CSRC_A	0
6	LLD_A	3
7	—	Reserved

Alternately, the type-1 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for 1 × *p*
2	CTXT_A	BLACS context
3	M_A	3
4	N_A	9
5	MB_A	1
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	3

Lower-Band-Packed Storage Mode: The global matrix ***A*** with block size of 2 is stored in lower-band-packed storage mode, as follows:

```
B,D      0       1       2       3      4
     ┌                                     ┐
     │ 11 22 │ 33 44 │ 55 66 │ 77 88 │ 99 │
 0   │ 21 32 │ 34 54 │ 65 76 │ 87 98 │  * │
     │ 31 42 │ 53 64 │ 75 86 │ 97  * │  * │
     └                                     ┘
```

Following is a row-oriented, 1 × 3 process grid:

B,D	0 3	1 4	2
0	P_{00}	P_{01}	P_{02}

The following local arrays A are distributed block-cyclically over the 1 × 3 process grid:

```
p,q  │      0      │     1    │    2
-----│-------------│----------│--------
     │ 11 22 77 88 │ 33 44 99 │ 55 66
 0   │ 21 32 87 98 │ 34 54  * │ 65 76
     │ 31 42 97  * │ 53 64  * │ 75 86
```

where * means you do not have to store a value in that position in the local array. However, these storage positions are required and overwritten during the computation.

The type-501 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 501 for 1 × *p*
2	CTXT_A	BLACS context
3	N_A	9
4	NB_A	2
5	CSRC_A	0
6	LLD_A	3
7	—	Reserved

Alternately, the type-1 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for 1 × *p*
2	CTXT_A	BLACS context
3	M_A	3
4	N_A	9
5	MB_A	1
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	3

For more information on how to store symmetric band matrices, see the *ESSL Version 3 Guide and Reference* manual.

Block-Cyclically Distributing a General Tridiagonal Matrix: A general tridiagonal matrix, represented as three vectors, must be distributed over a one-dimensional process grid using a block-cyclic data distribution. Because vectors are one-dimensional data structures, you can use a type-501, type-502, or type-1 array descriptor regardless of whether the process grid is $1 \times p$ or $p \times 1$.

The first part of this section shows how to distribute a general tridiagonal matrix ***A*** over a $p \times 1$ process grid. The second part shows how to distribute the same matrix over a $1 \times p$ process grid. **In both cases, the values contained in the corresponding local arrays are identical**.

Assume the following general tridiagonal matrix ***A*** of size 7×7:

$$\begin{bmatrix} 11 & 12 & 0 & 0 & 0 & 0 & 0 \\ 21 & 22 & 23 & 0 & 0 & 0 & 0 \\ 0 & 32 & 33 & 34 & 0 & 0 & 0 \\ 0 & 0 & 43 & 44 & 45 & 0 & 0 \\ 0 & 0 & 0 & 54 & 55 & 56 & 0 \\ 0 & 0 & 0 & 0 & 65 & 66 & 67 \\ 0 & 0 & 0 & 0 & 0 & 76 & 77 \end{bmatrix}$$

Matrix ***A*** is stored in tridiagonal storage mode in the following three vectors:

dl = (*, 21, 32, 43, 54, 65, 76)

d = (11, 22, 33, 44, 55, 66, 77)

du = (12, 23, 34, 45, 56, 67, *)

Block-Cyclic Distribution on a p × 1 Process Grid: The general tridiagonal matrix ***A*** is stored in tridiagonal storage mode in vectors ***dl***, ***d***, and ***du***.

Following is global vector ***dl***:

```
B,D    0
      ┌    ┐
 0    │  * │
      │ 21 │
      │ -- │
 1    │ 32 │
      │ 43 │
      │ -- │
 2    │ 54 │
      │ 65 │
      │ -- │
 3    │ 76 │
      └    ┘
```

Following is global vector ***d***:

```
B,D    0
     ┌    ┐
 0   | 11 |
     | 22 |
     | -- |
 1   | 33 |
     | 44 |
     | -- |
 2   | 55 |
     | 66 |
     | -- |
 3   | 77 |
     └    ┘
```

Following is global vector ***du***:

```
B,D    0
     ┌    ┐
 0   | 12 |
     | 23 |
     | -- |
 1   | 34 |
     | 45 |
     | -- |
 2   | 56 |
     | 67 |
     | -- |
 3   |  * |
     └    ┘
```

Following is a column-oriented, 3 × 1 process grid:

B,D	**0**
0 **3**	P_{00}
1	P_{10}
2	P_{20}

The arrays are block-cyclically distributed over the 3 × 1 process grid.

Following are the local arrays for `DL`:

```
p,q  | 0
-----|----
 0   |  *
     | 21
     | 76
-----|----
 1   | 32
     | 43
-----|----
 2   | 54
     | 65
```

Following are the local arrays for `D`:

```
p,q  | 0
-----|----
 0   | 11
     | 22
     | 77
-----|----
 1   | 33
     | 44
-----|----
 2   | 55
     | 66
```

Following are the local arrays for DU:

```
p,q  | 0
-----|----
 0   | 12
     | 23
     |  *
-----|----
 1   | 34
     | 45
-----|----
 2   | 56
     | 67
```

where “*” means you do not have to store a value in that position in the local array. However, these storage positions are required.

The type-502 array descriptor DESC_A contains the following:

DESC_A( )	**Symbolic name**	**Value**
1	DTYPE_A	DTYPE_A = 502 for $p \times 1$
2	CTXT_A	BLACS context
3	M_A	7
4	MB_A	2
5	RSRC_A	0
6	LLD_A	Not used
7	–	Reserved

Alternately, the type-1 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for $p \times 1$
2	CTXT_A	BLACS context
3	M_A	7
4	N_A	1
5	MB_A	2
6	NB_A	1
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	Not used

Block-Cyclic Distribution on a 1 × p Process Grid: The general tridiagonal matrix ***A*** is stored in tridiagonal storage mode in vectors ***dl***, ***d***, and ***du***. Because vectors are one-dimensional data structures, the block-cyclically distributed arrays on a $1 \times p$ process grid are identical to the block-cyclically distributed arrays on a $p \times 1$ process grid.

Following is global vector ***dl***:

```
B,D     0       1       2      3
       ┌                        ┐
 0     │ * 21 │ 32 43 │ 54 65 │ 76 │
       └                        ┘
```

Following is global vector ***d***:

```
B,D      0       1       2      3
       ┌                          ┐
 0     │ 11 22 │ 33 44 │ 55 66 │ 77 │
       └                          ┘
```

Following is global vectors ***du***:

```
B,D      0       1       2      3
       ┌                           ┐
 0     │ 12 23 │ 34 45 │ 55 67 │  *  │
       └                           ┘
```

Following is a row-oriented, 1×3 process grid:

B,D	**0 3**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

The arrays are block-cyclically distributed over the 1×3 process grid.

Following are the local arrays for `DL`:

```
p,q  │     0    │   1   │   2
-----│---------│-------│------
 0   │ * 21 76 │ 32 43 │ 54 65
```

Following are the local arrays for `D`:

```
p,q  |     0    |   1   |   2
-----|----------|-------|------
 0   | 11 22 77 | 33 44 | 55 66
```

Following are the local arrays for DU:

```
p,q  |   0      |    1   |   2
-----|----------|--------|-------
 0   | 12 23  * | 34 45  | 55 67
```

where "*" means you do not have to store a value in that position in the local array. However, these storage positions are required.

The type-501 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 501 for 1 × *p*
2	CTXT_A	BLACS context
3	N_A	7
4	NB_A	2
5	CSRC_A	0
6	LLD_A	Not used
7	–	Reserved

Alternately, the type-1 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for 1 × *p*
2	CTXT_A	BLACS context
3	M_A	1
4	N_A	7
5	MB_A	1
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	Not used

For more information on how to store general tridiagonal matrices, see the *ESSL Version 3 Guide and Reference* manual.

Block-Cyclically Distributing a Symmetric Tridiagonal Matrix: A symmetric tridiagonal matrix, represented as two vectors, must be distributed over a one-dimensional process grid using a block-cyclic data distribution. Because vectors are one-dimensional data structures, you can use a type-501, type-502, or type-1 array descriptor regardless of whether the process grid is *p* × 1 or 1 × *p*.

Note: For both serial ESSL and Parallel ESSL, the *n*– 1 elements of the equal off-diagonals of a symmetric tridiagonal matrix are stored in a one-dimensional vector of length *n*. To be compatible with ScaLAPACK, in

Parallel ESSL, the off-diagonal is chosen to be the superdiagonal and is stored in elements 1 through *n*– 1. In the serial ESSL library, the off-diagonal is chosen to be the subdiagonal and is stored in elements 2 through *n*.

The first part of this section shows a how to distribute a symmetric tridiagonal matrix ***A*** over a *p* × 1 process grid. The second part shows how to distribute the same matrix over a 1 × *p* process grid. **In both cases, the values contained in the corresponding local arrays are identical**.

Assume the following symmetric tridiagonal matrix ***A*** of size 7 × 7:

$$\begin{bmatrix} 10 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 20 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 30 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 40 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 50 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 & 60 & 6 \\ 0 & 0 & 0 & 0 & 0 & 6 & 70 \end{bmatrix}$$

Matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode in the following two vectors:

d = (10, 20, 30, 40, 50, 60, 70)

e = (1, 2, 3, 4, 5, 6, *)

Block-Cyclic Distribution on a p × 1 Process Grid: The symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode in vectors ***d*** and ***e***.

Following is global vector ***d***:

```
B,D    0
     ┌    ┐
     | 10 |
 0   | 20 |
     | 30 |
     | -- |
 1   | 40 |
     | 50 |
     | 60 |
     | -- |
 2   | 70 |
     └    ┘
```

Following is global vector ***e***:

```
B,D    0
     ┌   ┐
     | 1 |
 0   | 2 |
     | 3 |
     | - |
 1   | 4 |
     | 5 |
     | 6 |
     | - |
 2   | * |
     └   ┘
```

Following is a column-oriented, 2 × 1 process grid:

B,D	0
0 2	P_{00}
1	P_{10}

The arrays are block-cyclically distributed over the 2 × 1 process grid.

Following are the local arrays for D:

```
p,q  | 0
-----|----
     | 10
 0   | 20
     | 30
     | 70
-----|----
 1   | 40
     | 50
     | 60
```

Following are the local arrays for E:

```
p,q  | 0
-----|---
     | 1
 0   | 2
     | 3
     | *
-----|---
 1   | 4
     | 5
     | 6
```

where * means you do not have to store a value in that position in the local array. However, these storage positions are required.

The type-502 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 502 for $p \times 1$
2	CTXT_A	BLACS context
3	M_A	7
4	MB_A	3
5	RSRC_A	0
6	LLD_A	Not used
7	–	Reserved

Alternately, the type-1 array descriptor `DESC_A` contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for $p \times 1$
2	CTXT_A	BLACS context
3	M_A	7
4	N_A	1
5	MB_A	3
6	NB_A	1
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	Not used

Block-Cyclic Distribution on a 1 × p Process Grid: The symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode in vectors ***d*** and ***e***. Because vectors are one-dimensional data structures, the block-cyclically distributed arrays on a $1 \times p$ process grid are identical to the block-cyclically distributed arrays on a $p \times 1$ process grid.

Following is global vector ***d***:

```
B,D       0          1       2
        ┌                         ┐
 0      │ 10 20 30 | 40 50 60 | 70 │
        └                         ┘
```

Following is global vector ***e***:

```
B,D     0       1     2
        ┌                 ┐
 0      │ 1 2 3 | 4 5 6 | * │
        └                 ┘
```

Following is a row-oriented, 1×2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}

The arrays are block-cyclically distributed over the 1×2 process grid.

Following are the local arrays for D:

```
p,q  |      0       |    1
-----|--------------|---------
 0   | 10 20 30 70  | 40 50 60
```

Following are the local arrays for E:

```
p,q  |    0    |   1
-----|---------|------
 0   | 1 2 3 * | 4 5 6
```

where "*" means you do not have to store a value in that position in the local array. However, these storage positions are required.

The type-501 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 501 for 1 × p
2	CTXT_A	BLACS context
3	N_A	7
4	NB_A	3
5	CSRC_A	0
6	LLD_A	Not used
7	–	Reserved

Alternately, the type-1 array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	DTYPE_A = 1 for 1 × p
2	CTXT_A	BLACS context
3	M_A	1
4	N_A	7
5	MB_A	1
6	NB_A	3
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	Not used

Block-Cyclically Distributing a General Matrix Containing the Right-Hand Sides: This section shows how to block-cyclically distribute a general matrix ***B*** containing the multiple right-hand sides for the Banded Linear Algebraic Equations subroutines.

Following is the global matrix ***B***:

```
B,D       0
     ┌           ┐
 0   │ 11 12 13 │
     │ 21 22 23 │
     │ -------- │
 1   │ 31 32 33 │
     │ 41 42 43 │
     │ -------- │
 2   │ 51 52 53 │
     │ 61 62 63 │
     │----------│
 3   │ 71 72 73 │
     └           ┘
```

Following is a 3 × 1 process grid:

B,D	0
0 **3**	P_{00}
1	P_{10}
2	P_{20}

Following are the local arrays:

```
p,q  │    0
-----│----------
 0   │ 11 12 13
     │ 21 22 23
     │ 71 72 73
-----│----------
 1   │ 31 32 33
     │ 41 42 43
-----│----------
 2   │ 51 52 53
     │ 61 62 63
```

The type-502 array descriptor `DESC_B` contains the following:

DESC_B( )	Symbolic name	Value
1	DTYPE_B	DTYPE_B = 502 for *p* × 1
2	CTXT_B	BLACS context
3	M_B	7
4	MB_B	2
5	RSRC_B	0
6	LLD_B	3 (For P_{00}) 2 (For P_{10} and P_{20})
7	—	Reserved

Alternately, the type-1 array descriptor `DESC_B` contains the following:

DESC_B( )	Symbolic name	Value
1	DTYPE_B	DTYPE_B = 1 for $p \times 1$
2	CTXT_B	BLACS context
3	M_B	7
4	N_B	3
5	MB_B	2
6	NB_B	1
7	RSRC_B	0
8	CSRC_B	0
9	LLD_B	3 (For P_{00}) 2 (For P_{10} and P_{20})

Block-Cyclically Distributing over Two-Dimensional Process Grids

This section shows how to distribute general, symmetric, and upper triangular matrices over a two-dimensional process grid using block-cyclic distribution.

Distributing a General Matrix: This example shows how the data for a global matrix ***A*** with block size of 2 × 3 is distributed block-cyclically over the entire 2 × 3 process grid. Assume the following 9 × 26 global matrix ***A*** with 45 blocks:

```
B,D      0          1          2          3          4           5          6          7          8
     ┌                                                                                              ┐
 0   │ 112 5 7 │  8 9 3  │  7 5 1  │  3 2 1  │  8 98   4  │  8 9 4  │  1 3 10  │  3 3 10  │  5 3 │
     │ 116 9 6 │  7 2 3  │  6 5 6  │  4 3 2  │  7  2 111  │  7 2 1  │  7 6 15  │  7 6 15  │  7 6 │
     │ ---------|---------|---------|---------|------------|---------|----------|----------|----- │
 1   │   1 5 7 │  1 9 3  │  1 5 1  │  1 2 1  │  1  9   4  │  1 9 4  │  5 8 10  │  3 3 11  │  5 3 │
     │   6 9 6 │  7 2 3  │  6 5 6  │  4 3 2  │  7  2   1  │  7 2 1  │  7 6 19  │  7 1 15  │  7 2 │
     │ ---------|---------|---------|---------|------------|---------|----------|----------|----- │
 2   │   2 5 7 │  2 9 3  │  2 5 1  │  2 2 1  │  2  9   4  │  2 9 4  │  1 8 10  │  2 3 11  │  3 3 │
     │   6 9 6 │  7 2 3  │  6 5 6  │  4 3 2  │  7  2   1  │  7 2 1  │  7 3 19  │  7 4 15  │  7 8 │
     │ ---------|---------|---------|---------|------------|---------|----------|----------|----- │
 3   │   3 5 7 │  3 9 3  │  3 5 1  │  3 2 1  │  3  9   4  │  3 9 4  │  9 8 10  │  2 3 11  │  3 3 │
     │   6 9 6 │  7 2 3  │  6 5 6  │  4 3 2  │  7  2   1  │  7 2 1  │  1 3 49  │  7 4 55  │  7 3 │
     │ ---------|---------|---------|---------|------------|---------|----------|----------|----- │
 4   │  20 1 9 │  4 5 6  │  9 8 7  │  1 4 3  │  1 15  21  │  4 7 6  │  9 8 12  │  3 9 18  │  2 4 │
     └                                                                                              ┘
```

Note: In this example, the global matrix dimensions are not divisible by the respective block size. As a result, all of the block sizes are 2 × 3, except for blocks in the last row and the last column of the blocked matrix.

Two-dimensional, 2 × 3 process grid:

B,D	**0 3 6**	**1 4 7**	**2 5 8**
0 **2** **4**	P_{00}	P_{01}	P_{02}
1 **3**	P_{10}	P_{11}	P_{12}

Local arrays:

```
p,q  |           0           |            1             |          2
-----|-----------------------|--------------------------|------------------
     | 112 5 7 3 2 1 1 3 10  |  8 9 3 8 98   4 3 3 10  |  7 5 1 8 9 4 5 3
     | 116 9 6 4 3 2 7 6 15  |  7 2 3 7  2 111 7 6 15  |  6 5 6 7 2 1 7 6
 0   |   2 5 7 2 2 1 1 8 10  |  2 9 3 2  9   4 2 3 11  |  2 5 1 2 9 4 3 3
     |   6 9 6 4 3 2 7 3 19  |  7 2 3 7  2   1 7 4 15  |  6 5 6 7 2 1 7 8
     |  20 1 9 1 4 3 9 8 12  |  4 5 6 1 15  21 3 9 18  |  9 8 7 4 7 6 2 4
-----|-----------------------|--------------------------|------------------
     |   1 5 7 1 2 1 5 8 10  |  1 9 3 1  9   4 3 3 11  |  1 5 1 1 9 4 5 3
     |   6 9 6 4 3 2 7 6 19  |  7 2 3 7  2   1 7 1 15  |  6 5 6 7 2 1 7 2
 1   |   3 5 7 3 2 1 9 8 10  |  3 9 3 3  9   4 2 3 11  |  3 5 1 3 9 4 3 3
     |   6 9 6 4 3 2 1 3 49  |  7 2 3 7  2   1 7 4 55  |  6 5 6 7 2 1 7 3
```

Array descriptor `DESC_A` contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	1
2	CTXT_A	BLACS context
3	M_A	9
4	N_A	26
5	MB_A	2
6	NB_A	3
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	5 (For P_{00}, P_{01}, and P_{02}) 4 (For P_{10}, P_{11}, and P_{12})

Distributing a Symmetric Matrix: This example shows how the data for a global symmetric matrix ***A*** with block size of 3 × 3 is distributed block-cyclically over a 2 × 3 process grid. Assume the following 18 × 18 global symmetric matrix ***A*** with 36 blocks:

```
B,D      0          1          2          3          4          5
       ┌                                                              ┐
       │  1  2  3 │  4  5  6 │  7  8  9 │ 10 11 12 │ 13 14 15 │ 16 17 18 │
 0     │  2 10 11 │ 12 13 14 │ 15 16 17 │ 18 19 20 │ 21 22 23 │ 24 25 26 │
       │  3 11 20 │ 21 22 23 │ 24 25 26 │ 27 28 29 │ 30 31 32 │ 33 34 35 │
       │ ---------|----------|----------|----------|----------|--------- │
       │  4 12 21 │  2  3  5 │  7 11 13 │ 17 19 23 │ 29 31 37 │ 41 43 47 │
 1     │  5 13 22 │  3  1  4 │  9 16 25 │ 36 49 64 │ 81 10 12 │ 14 16 19 │
       │  6 14 23 │  5  4  5 │  6 10 11 │ 15 16 20 │ 21 25 26 │ 30 31 35 │
       │ ---------|----------|----------|----------|----------|--------- │
       │  7 15 24 │  7  9  6 │  1  2  3 │  4  5  6 │  7  8  9 │ 10 11 12 │
 2     │  8 16 25 │ 11 16 10 │  2 11 13 │ 15 17 19 │ 21 23 25 │ 27 29 31 │
       │  9 17 26 │ 13 25 11 │  3 13  2 │  4  6  8 │ 10 12 14 │ 16 18 20 │
       │ ---------|----------|----------|----------|----------|--------- │
       │ 10 18 27 │ 17 36 15 │  4 15 4  │  3  6  9 │  2  4  6 │  3  6  9 │
 3     │ 11 19 28 │ 19 49 16 │  5 17 6  │  6  1  2 │  3  4  5 │  6  7  8 │
       │ 12 20 29 │ 23 64 20 │  6 19 8  │  9  2  1 │  3  5  7 │  9 11 13 │
       │ ---------|----------|----------|----------|----------|--------- │
       │ 13 21 30 │ 29 81 21 │  7 21 10 │  2  3  3 │ 20 22 21 │ 24 23 25 │
 4     │ 14 22 31 │ 31 10 25 │  8 23 12 │  4  4  5 │ 22  4  5 │  6  9 10 │
       │ 15 23 32 │ 37 12 26 │  9 25 14 │  6  5  7 │ 21  5  3 │  2  7  8 │
       │ ---------|----------|----------|----------|----------|--------- │
       │ 16 24 33 │ 41 14 30 │ 10 27 16 │  3  6  9 │ 24  6  2 │  4 11 15 │
 5     │ 17 25 34 │ 43 16 31 │ 11 29 18 │  6  7 11 │ 23  9  7 │ 11 17 13 │
       │ 18 26 35 │ 47 19 35 │ 12 31 20 │  9  8 13 │ 25 10  8 │ 15 13 21 │
       └                                                              ┘
```

Two-dimensional, 3 × 2 process grid:

B,D	**0 2 4**	**1 3 5**
0 **3**	P_{00}	P_{01}
1 **4**	P_{10}	P_{11}
2 **5**	P_{20}	P_{21}

The symmetric matrix is distributed block-cyclically in lower storage mode over a 3 × 2 process grid:

```
p,q  |             0              |              1
-----|----------------------------|----------------------------
     |  1  *  *  *  *  *  *  *  * |   *  *  *  *  *  *  *  *  *
     |  2 10  *  *  *  *  *  *  * |   *  *  *  *  *  *  *  *  *
     |  3 11 20  *  *  *  *  *  * |   *  *  *  *  *  *  *  *  *
 0   | 10 18 27  4 15  4  *  *  * |  17 36 15  3  *  *  *  *  *
     | 11 19 28  5 17  6  *  *  * |  19 49 16  6  1  *  *  *  *
     | 12 20 29  6 19  8  *  *  * |  23 64 20  9  2  1  *  *  *
-----|----------------------------|----------------------------
     |  4 12 21  *  *  *  *  *  * |   2  *  *  *  *  *  *  *  *
     |  5 13 22  *  *  *  *  *  * |   3  1  *  *  *  *  *  *  *
     |  6 14 23  *  *  *  *  *  * |   5  4  5  *  *  *  *  *  *
 1   | 13 21 30  7 21 10 20  *  * |  29 81 21  2  3  3  *  *  *
     | 14 22 31  8 23 12 22  4  * |  31 10 25  4  4  5  *  *  *
     | 15 23 32  9 25 14 21  5  3 |  37 12 26  6  5  7  *  *  *
-----|----------------------------|----------------------------
     |  7 15 24  1  *  *  *  *  * |   7  9  6  *  *  *  *  *  *
     |  8 16 25  2 11  *  *  *  * |  11 16 10  *  *  *  *  *  *
     |  9 17 26  3 13  2  *  *  * |  13 25 11  *  *  *  *  *  *
 2   | 16 24 33 10 27 16 24  6  2 |  41 14 30  3  6  9  4  *  *
     | 17 25 34 11 29 18 23  9  7 |  43 16 31  6  7 11 11 17  *
     | 18 26 35 12 31 20 25 10  8 |  47 19 35  9  8 13 15 13 21
```

where * means you do not have to store a value in that position in the local array. However, these storage positions are required.

Notice that the local arrays are not symmetric.

Array descriptor DESC_A contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	1
2	CTXT_A	BLACS context
3	M_A	18
4	N_A	18
5	MB_A	3
6	NB_A	3
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	6

For more information on how to store symmetric matrices, see the *ESSL Version 3 Guide and Reference* manual.

Distributing an Upper Triangular Matrix: This example shows how the data for a global upper triangular matrix ***A*** with block size of 2 × 2 is distributed block-cyclically over a 2 × 3 process grid. Assume the following 12 × 12 global upper triangular matrix ***A*** with 36 blocks:

```
B,D      0         1         2         3         4         5
      ┌                                                           ┐
 0    │  2  1  │   2 13  │  13 10  │  15 21  │  26 31  │   7  5  │
      │  0  3  │   4  4  │  11 23  │  41 45  │  59 67  │   1  8  │
      │ -------│---------│---------│---------│---------│-------  │
 1    │  0  0  │   5  9  │   6  9  │  33 65  │  21 14  │   9  4  │
      │  0  0  │   0  7  │  16  8  │   7 33  │   3  7  │   5  3  │
      │ -------│---------│---------│---------│---------│-------  │
 2    │  0  0  │   0  0  │  11 25  │  10  5  │  23  7  │  10  6  │
      │  0  0  │   0  0  │   0 13  │  36 12  │   3 13  │   5  6  │
      │ -------│---------│---------│---------│---------│-------  │
 3    │  0  0  │   0  0  │   0  0  │  17 49  │  14  1  │   7  2  │
      │  0  0  │   0  0  │   0  0  │   0 19  │  64 16  │   1  7  │
      │ -------│---------│---------│---------│---------│-------  │
 4    │  0  0  │   0  0  │   0  0  │   0  0  │  23 81  │   6 15  │
      │  0  0  │   0  0  │   0  0  │   0  0  │   0 29  │   9  4  │
      │ -------│---------│---------│---------│---------│-------  │
 5    │  0  0  │   0  0  │   0  0  │   0  0  │   0  0  │   5  3  │
      │  0  0  │   0  0  │   0  0  │   0  0  │   0  0  │   0  4  │
      └                                                           ┘
```

Two-dimensional, 2 × 3 process grid:

B,D	**0 3**	**1 4**	**2 5**
0	P_{00}	P_{01}	P_{02}
2			
4			
1	P_{10}	P_{11}	P_{12}
3			
5			

The following local arrays are distributed block-cyclically in upper-triangular storage mode over a 2 × 3 process grid:

```
p,q  │      0       │      1       │      2
-----│--------------│--------------│--------------
     │  2  1 15 21  │  2 13 26 31  │ 13 10  7  5
     │  *  3 41 45  │  4  4 59 67  │ 11 23  1  8
     │  *  * 10  5  │  *  * 23  7  │ 11 25 10  6
 0   │  *  * 36 12  │  *  *  3 13  │  * 13  5  6
     │  *  *  *  *  │  *  * 23 81  │  *  *  6 15
     │  *  *  *  *  │  *  *  * 29  │  *  *  9  4
-----│--------------│--------------│--------------
     │  *  * 33 65  │  5  9 21 14  │  6  9  9  4
     │  *  *  7 33  │  *  7  3  7  │ 16  8  5  3
     │  *  * 17 49  │  *  * 14  1  │  *  *  7  2
 1   │  *  *  * 19  │  *  * 64 16  │  *  *  1  7
     │  *  *  *  *  │  *  *  *  *  │  *  *  5  3
     │  *  *  *  *  │  *  *  *  *  │  *  *  *  4
```

where “*” means you do not have to store a value in that position in the local array. However, these storage positions are required.

Notice the local arrays are not upper triangular.

Array descriptor `DESC_A` contains the following:

DESC_A( )	Symbolic name	Value
1	DTYPE_A	1
2	CTXT_A	BLACS context
3	M_A	12
4	N_A	12
5	MB_A	2
6	NB_A	2
7	RSRC_A	0
8	CSRC_A	0
9	LLD_A	6

For more information on how to store triangular matrices, see the *ESSL Version 3 Guide and Reference* manual.

Specifying Sparse Matrices for the Fortran 90 and Fortran 77 Sparse Linear Algebraic Equations

For the Fortran 90 and Fortran 77 sparse linear algebraic equation subroutines, you must use the sparse utility subroutines provided with Parallel ESSL to build the sparse matrices on each process in the process grid. This sections shows the calling sequence arguments associated with the sparse matrix ***A***.

Fortran 90 Sparse Linear Algebraic Equation Subroutines

This section contains the following sections:

- “Calling Sequence Arguments for the Sparse Matrix”
- “Derived Data Types” on page 61

Calling Sequence Arguments for the Sparse Matrix: This section describes the calling sequence arguments associated with a sparse matrix ***A***.

Table 23. Calling Sequence Arguments for the Sparse Matrix

Arguments	Meaning
a	is the local part of the sparse matrix ***A*** and specified as derived data type D_SPMAT. For more details about D_SPMAT, see “Derived Data Type D_SPMAT” on page 61.
ia	is the row index of the sparse matrix ***A***.
ja	is the column index of the sparse matrix ***A***.
desc_a	is the array descriptor for the sparse matrix ***A*** and specified as derived data type DESC_TYPE. For more details about DESC_TYPE, see “Derived Data Type DESC_TYPE” on page 61.
parts	is a user-supplied subroutine that specifies a mapping between a global index for an element in the global sparse matrix and its corresponding storage location on one or more processes. For details about how you must define the PARTS subroutine, see “Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)” on page 65.

Derived Data Types: Some of the arguments of the Fortran 90 sparse linear algebraic equations and their utility subroutines are derived data types.

For more information on derived data types, see the XL Fortran manuals.

Derived Data Type D_SPMAT: Table 24 describes the components of D_SPMAT that you must provide as input to the PSPINS subroutine. In addition to the components you provide, Parallel ESSL creates other components as necessary that are only for internal use.

Table 24. Components of D_SPMAT

Components of D_SPMAT	**Description**	**Scope**
M	Number of local rows	**Local**
N	Number of local columns	**Local**
FIDA	Storage mode for the submatrix	Global
AS	Pointer to the submatrix, which contains the coefficients.	**Local**
IA1	Pointer to the column numbers of each non-zero element in the submatrix.	**Local**
IA2	Pointer to the starting positions of each row of the submatrix and one position past the end of the submatrix.	**Local**
The AS, IA1, and IA2 components, which are described in this table depend on how you specify the FIDA component. This description assumes you are using storage by rows. For details about how these components must be specified and their special restrictions, see the appropriate argument descriptions in “PSPINS—Inserts Local Data into a General Sparse Matrix” on page 625.		

Derived Data Type DESC_TYPE: Parallel ESSL builds the array descriptor, *desc_a*, which is specified as derived data type DESC_TYPE, and its components, as follows:

- PADALL allocates space for the array descriptor and initializes its components.
- PSPINS updates some components of the array descriptor.
- PSPASB makes final updates to some components of the array descriptor.

MATRIX_DATA is one component of the array descriptor. Table 25 on page 62 describes the elements of DESC_A%MATRIX_DATA that you may want to reference. However, your application programs should not modify the components of the array descriptor directly. These components should only be updated with calls to PSPINS and PSPASB.

Table 25. Elements of DESC_A%MATRIX_DATA(_)

MATRIX_DATA(_)	Name	Description	Data Type	Limits	Scope
1	DEC_TYPE	Type of data distribution	Fullword integer	Internal format	Global
2	CTXT	BLACS context	Fullword integer	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M	Number of rows in the global general sparse matrix ***A***	Fullword integer	M ≥ 0 and M = N	Global
4	N	Number of columns in the global general sparse matrix ***A***	Fullword integer	N ≥ 0 and M = N	Global
5	N_ROW	Number of local rows	Fullword integer	N_ROW ≥ 1	**Local**
6	N_COL	Number of local columns†	Fullword integer	N_COL ≥ 1	**Local**

†DESC_A%MATRIX_DATA(6) is stable after you have placed a call to PSPASB.

Fortran 77 Sparse Linear Algebraic Equation Subroutines

This section contains the following sections:

- "Calling Sequence Arguments for the Sparse Matrix"
- "Array Descriptor" on page 63

Calling Sequence Arguments for the Sparse Matrix: This section describes the calling sequence arguments associated with a general sparse matrix ***A.***

Table 26 (Page 1 of 2). Calling Sequence Arguments for the Sparse Matrix

Arguments	Meaning
as	is the local part of a matrix
ia	is the row index of the sparse matrix.
ja	is the column index of the sparse matrix.
ia1	is the local part of an array containing the sparse matrix indices.
ia2	is the local part of an array containing the sparse matrix indices.
infoa	is an integer array for a matrix. For details about *infoa* see Table 27 on page 63.
desc_a	is an array descriptor for the sparse matrix. For details about *desc_a* see "Array Descriptor" on page 63.

Table 26 (Page 2 of 2). Calling Sequence Arguments for the Sparse Matrix

Arguments	Meaning
parts	is a user-supplied subroutine that specifies a mapping between a global index for an element in the global sparse matrix and its corresponding storage location on one or more processes. For details about how you must define the PARTS subroutine, see "Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)" on page 65.

Table 27. Elements of INFOA()

INFOA()	Description	Scope
1	Length of an array for a matrix	**Local**
2	Length of an array containing sparse matrix indices.	**Local**
3	Length of an array containing sparse matrix indices.	**Local**
4	Storage format of the matrix.	Global
5	Type of matrix.	Global
6	Number of local rows.	**Local**
7	Number of local columns.	**Local**
8 through 30	Reserved for internal use.	—
If *infoa* is in a subroutine calling sequence, you must always specify a value for INFOA(1), INFOA(2), and INFOA(3).		

Array Descriptor: An integer array descriptor, *desc_a*, is needed to establish a mapping between the global general sparse matrix ***A*** and its corresponding distributed memory location. You must specify an array descriptor length, DLEN, in DESC_A(11) on input to PADINIT:

- For the maximum length you should need, use the following formulas to calculate the length of the array descriptor, DLEN:

  If there is no overlap,
  DLEN = 33+3(*np*)+*n*+(N_COL)+(*np*–1)(N_ROW)+(N_COL–N_ROW)

  If there is no overlap, 33+3(*np*)+4*n* is an upper bound for DLEN.

  If overlap occurs, add at most to DLEN: 3(*np*)+1+2(*np*)(N_ROW)

  where:

  – N_ROW ≤ *n*

  – N_COL ≤ *n*

  – N_ROW is approximately *n*/*np*

  – *n* is the order of the global general sparse matrix ***A***.

  – *np* is the number of processes in the process grid.

- Use the following formula(s) to calculate a more typical value of the length of the array descriptor, DLEN:

$33+3(np)+\alpha n \le \text{DLEN} \le 33+6(np)+3n$

where:

- $1 < \alpha \le 2$
- n is the order of the global general sparse matrix ***A***.
- np is the number of processes in the process grid.

Note: The actual length of the array descriptor depends on the sparse matrix structure and therefore is known after a call to PDSPASB.

Parallel ESSL builds the remaining elements in the array descriptor, as follows:

- PADINIT initializes the array descriptor.
- PDSPINS updates parts of the array descriptor.
- PDSPASB makes final updates to some parts of the array descriptor.

You may want to use some of the values in *desc_a* to build vector ***b*** containing the right-hand side and vector ***x*** containing initial guess to the solution. (Parallel ESSL creates other elements in the array descriptor that are for internal use only.) Table 28 describes the elements of of the array descriptor that you may want to reference. Your application programs should not modify the elements of the array descriptor directly. The elements should only be updated with calls to PDSPINS and PDSPASB.

Table 28 (Page 1 of 2). Elements of DESC_A()

DESC_A()	**Name**	**Description**	**Data Type**	**Limits**	**Scope**
1	DEC_TYPE	Type of data distribution	Fullword integer	Internal format	Global
2	CTXT	BLACS context	Fullword integer	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M	Number of rows in the global general sparse matrix ***A***	Fullword integer	M ≥ 0 and M = N	Global
4	N	Number of columns in the global general sparse matrix ***A***	Fullword integer	N ≥ 0 and M = N	Global
5	N_ROW	Number of local rows†	Fullword integer	1 ≤ N_ROW ≤ *n*	**Local**
6	N_COL	Number of local columns‡	Fullword integer	1 ≤ N_COL ≤ *n*	**Local**
11	DLEN	Length of the array descriptor	Fullword integer	See the formulas shown in the beginning of this section.	Global

Table 28 (Page 2 of 2). Elements of DESC_A()

DESC_A()	Name	Description	Data Type	Limits	Scope
†DESC_A(5) is stable after you have placed a call to PADINIT. You can use this value to calculate *lprcs* in PDSPGPR.					
‡DESC_A(6) is stable after you have placed a call to PDSPASB. You can use this value to calculate *lprcs* in PDSPGPR.					
DESC_A(7) through DESC_A(10) are only for internal use.					
DESC_A(12) through DESC_A(DLEN) are only for internal use.					

Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)

This section describes how to design and code the *parts* subroutine for use by the Parallel ESSL Fortran 90 and Fortran 77 sparse linear algebraic equation subroutines and their utility subroutines.

You must supply a separate subroutine that is callable by Parallel ESSL. You must specify the name of the subroutine in the *parts* argument. This subroutine name is selected by you. You must declare *parts* as an external subroutine in your application program.

Coding and Designing the Parts Subroutine for the Sparse Subroutines: The *parts* subroutine specifies the mapping between a global index for an element in the global general sparse matrix ***A*** and its corresponding storage location on a process or processes (if overlap occurs).

You should design the *parts* subroutine so it receives, as input, *global_index*, *n*, and *np*. It also must return to Parallel ESSL, as output, the information in the *pv* and *nv* arguments indicating the storage location of *global_index* on one or more processes.

Syntax

Fortran	CALL PARTS (*global_index*, *n*, *np*, *pv*, *nv*)
C	parts (*&global_index*, *&n*, *&np*, *pv*, *&nv*);

C++	extern "Fortran" void parts(const int &, const int &, const int &, int *, const int &);
	parts (*global_index*, *n*, *np*, *pv*, *nv*);

On Entry

global_index
: is an input scalar argument containing an integer that indicates the global index for an element in the global general sparse matrix ***A***, where: $1 \le global\_index \le n$.

n
: is an input scalar argument containing an integer that indicates the order of the global general sparse matrix ***A***, where: $n \ge 0$.

np
: is an input scalar argument containing an integer that indicates the number of processes in the process grid, where: $np > 0$.

On Output

pv
: is an output array containing integers that identify which processes are receiving the global index, *global_index*, where: $0 \le pv(i) < np$ and $1 \le i \le nv$.

nv
: is an output scalar argument containing an integer that indicates the number of unique processes specified in the *pv* argument, where: $1 \le nv \le np$.

Notes

1. The *parts* subroutine can be coded in Fortran, C, or C++. However, for C and C++ programs, all the arguments must be passed by reference.

Examples for the PARTS Subroutine: Examples of how you could code *parts* for different types of data distribution are shown in:

- “PART_BLOCK (Block Data Distribution)” on page 1084
- “Block Data Distribution for a C Program”
- “PARTBCYC (Block-Cyclic Data Distribution)” on page 1085
- “PARTRAND (Random Data Distribution)” on page 1086

Block Data Distribution for a C Program

```
void part_block(global_indx,n,nnodes,pv,nv)
int  *global_indx,*n,*nnodes,*pv,*nv;
{
  int dim_block;
     dim_block = (*n + *nnodes -1)/(*nnodes);
     *nv = 1;
     pv[*nv-1] = (*global_indx - 1)/dim_block;
}
```

Specifying Sequences for the Fourier Transforms

This section shows how to use block-column distribution to distribute a two- or three-dimensional sequence over a one-dimensional process grid. It also describes how some of the two- and three-dimensional complex sequences are stored in FFT-packed storage mode.

Two-Dimensional Sequence

Following is a two-dimensional sequence using zero-based indexing where the first dimension is *n1*, and the second dimension is *n2*:

$$\begin{array}{cccc}
a_{0,0} & a_{0,1} & \cdots & a_{0,n2-1} \\
a_{1,0} & a_{1,1} & \cdots & a_{1,n2-1} \\
\vdots & \vdots & & \vdots \\
a_{n1-1,0} & a_{n1-1,1} & \cdots & a_{n1-1,n2-1}
\end{array}$$

Distributing Data: For the Fourier transform subroutines, a two-dimensional sequence is distributed over a one-dimensional process grid, using block-column distribution. The process grid must be arranged as a row ($1 \times q$, where q is the number of processes).

Note: Two-dimensional sequences can be thought of as two-dimensional matrices. The term sequence is used because it is traditional for Fourier transforms.

You must distribute the input sequence sequentially to the processes in the process grid, using block-column distribution. Parallel ESSL also returns the output sequence using block-column distribution. The output sequence may be returned in normal or transposed form.

A sequence can be distributed unevenly; that is, one process in the process grid can receive an array that is smaller than other processes. It can also happen that some processes receive no data. “Example 2” on page 753 shows an example of uneven data distribution.

LOCq(n) represents the number of columns that a process would receive if n is distributed block over q processes. You need to calculate LOCq(n) for each process, as follows:

- The number of columns, LOCq(n), that processes P_{00} through $P_{0,k\text{-}1}$ receive is calculated as follows:

  LOCq(n) = NB2 = $(n+q-1)/q$

- The number of columns, LOCq(n), that process $P_{0,k}$ receives is calculated as follows:

  LOCq(n) = $n-(q-1)$(NB2)

- Processes $P_{0,k+1}$ through $P_{0,q\text{-}1}$ would not receive any data. This may happen if there is not enough data to distribute to all the specified processes.

where:

n represents the following:

n is the second dimension, *n2*, of the sequence (for normal form)

n is the first dimension, *n1*, of the sequence (for transposed form and the sequence is not stored in FFT-packed storage mode)

n is *n1*/2 (for transposed form and the sequence is stored in FFT-packed storage mode)

q is the number processes in the process grid

$P_{0,k}$ is the process that receives the last block of data. For uneven data distribution, $P_{0,k}$ would receive an array that is smaller than the other processes receive.

Following is an example of block-column distribution for a two-dimensional sequence over a one-dimensional, row-oriented process grid.

Global sequence of size 8×12:

```
B,D      0           1            2            3
     ┌                                                ┐
     │ 0 10 20 │ 30 40 50 │ 60 70 80 │ 90 100 110 │
     │ 1 11 21 │ 31 41 51 │ 61 71 81 │ 91 101 111 │
     │ 2 12 22 │ 32 42 52 │ 62 72 82 │ 92 102 112 │
     │ 3 13 23 │ 33 43 53 │ 63 73 83 │ 93 103 113 │
 0   │ 4 14 24 │ 34 44 54 │ 64 74 84 │ 94 104 114 │
     │ 5 15 25 │ 35 45 55 │ 65 75 85 │ 95 105 115 │
     │ 6 16 26 │ 36 46 56 │ 66 76 86 │ 96 106 116 │
     │ 7 17 27 │ 37 47 57 │ 67 77 87 │ 97 107 117 │
     └                                                ┘
```

Row-oriented, 1 × 4 process grid:

B,D	0	1	2	3
0	P_{00}	P_{01}	P_{02}	P_{03}

Local arrays:

```
p,q  │      0       │       1       │       2       │       3
-----│-------------│--------------│--------------│-------------
     │ 0  10  20 │ 30  40  50 │ 60  70  80 │ 90 100 110
     │ 1  11  21 │ 31  41  51 │ 61  71  81 │ 91 101 111
     │ 2  12  22 │ 32  42  52 │ 62  72  82 │ 92 102 112
     │ 3  13  23 │ 33  43  53 │ 63  73  83 │ 93 103 113
 0   │ 4  14  24 │ 34  44  54 │ 64  74  84 │ 94 104 114
     │ 5  15  25 │ 35  45  55 │ 65  75  85 │ 95 105 115
     │ 6  16  26 │ 36  46  56 │ 66  76  86 │ 96 106 116
     │ 7  17  27 │ 37  47  57 │ 67  77  87 │ 97 107 117
```

An example of the distribution of a two-dimensional sequence in a Fortran 90 program is shown in Appendix B. See the following:

- The subroutine initialize-scale in “Module Scale (Message Passing)” on page 1025, which determines the parameters to be used for block distribution, ultimately setting up the correct parameters for distributing an FFT sequence.
- The subroutine get-diffusion_matrix in “Module Fourier (Message Passing)” on page 1017, which shows how a local array can be assigned values.
- The subroutine rlocal_to_rglobal in “Module Scale (Message Passing)” on page 1025, which shows gathering the local portions of the block-distributed real array to generate the corresponding global sequence/matrix.

FFT-Packed Storage Mode: The output sequence for PSRCFT2 and PDRCFT2, and the input sequence for PSCRFT2 and PDCRFT2 are stored in FFT-packed storage mode because they consist of complex-conjugate, even symmetric data.

For FFT-packed storage mode, only certain elements of the complex-conjugate, even symmetric data are stored. This section describes how the complex elements of sequence *y*, which is the output sequence for PSRCFT2 and PDRCFT2, and the input sequence for PSCRFT2 and PDCRFT2, are stored in global matrices ***Y*** and ***X***, respectively.

For example, suppose *y* is the two-dimensional sequence to be stored in FFT-packed storage mode for PDRCFT2. The following list describes how the elements in *y* correspond to the elements in the global matrix ***Y***:

- The real part of $y_{0,0}$ is stored in the real part of $\boldsymbol{Y}_{0,0}$
- The real part of $y_{0,n2/2}$ is stored in the imaginary part of $\boldsymbol{Y}_{0,0}$
- The real part of $y_{n1/2,0}$ is stored in the real part of $\boldsymbol{Y}_{n2/2,0}$
- The real part of $y_{n1/2,n2/2}$ is stored in the imaginary part of $\boldsymbol{Y}_{n2/2,0}$
- The elements $y_{0,1:n2/2-1}$ are stored in elements $\boldsymbol{Y}_{1:n2/2-1,0}$
- The elements $y_{n1/2,1:n2/2-1}$ are stored in elements $\boldsymbol{Y}_{n2/2+1:n2-1,0}$
- The rows $y_{1:n1/2-1,i}$ are stored in columns $\boldsymbol{Y}_{i,1:n1/2-1}$

  where:

  n1 is the first dimension of array *y*
  n2 is the second dimension of array *y*
  $i = 0, ..., n2-1$

The remaining elements of *y* are not stored because they are the complex conjugates of elements already stored. These relationships are shown in the following equations:

- $y_{0,n2-j} = \bar{y}_{0,j}$ where $j = 1, \dots, n2/2-1$
- $y_{n1/2,n2-j} = \bar{y}_{n1/2,j}$ where $j = 1, \dots, n2/2-1$
- $y_{n1-i,0} = \bar{y}_{i,0}$ where $i = 1, \dots, n1/2-1$
- $y_{n1-i,n2/2} = \bar{y}_{i,n2/2}$ where $i = 1, \dots, n1/2-1$
- $y_{n1-i,n2-j} = \bar{y}_{i,j}$ where $i = 1, \dots, n1/2-1$
  and $j = 1, \dots, n2/2-1, n2/2+1, \dots, n2-1$

where:

n1 is the first dimension of array *y*
n2 is the second dimension of array *y*

The following example, which uses zero-based indexing, has complex conjugate, even symmetry. The dimensions of array *y* are 8 × 8 (that is *n1* = *n2* = 8), where array *y* is:

$$\begin{bmatrix}
(111,0) & (-3,23) & (-8,10) & (-9,4) & (-9,0) & (-9,-4) & (-8,-10) & (-3,-23) \\
(10,-10) & (4,4) & (9,3) & (-6,2) & (-1,2) & (-2,1) & (-3,1) & (-5,-3) \\
(6,-4) & (1,3) & (0,2) & (-7,1) & (-1,9) & (-1,4) & (-2,-4) & (-2,-2) \\
(6,-2) & (6,2) & (-5,1) & (-8,8) & (-1,4) & (-1,-1) & (-1,-8) & (-1,-2) \\
(6,0) & (-3,2) & (-9,1) & (-1,5) & (-1,0) & (-1,-5) & (-9,-1) & (-3,-2) \\
(6,2) & (-1,2) & (-1,8) & (-1,1) & (-1,-4) & (-8,-8) & (-5,-1) & (6,-2) \\
(6,4) & (-2,2) & (-2,4) & (-1,-4) & (-1,-9) & (-7,-1) & (0,-2) & (1,-3) \\
(10,10) & (-5,3) & (-3,-1) & (-2,-1) & (-1,-2) & (-6,-2) & (9,-3) & (4,-4)
\end{bmatrix}$$

Because zero-based indexing is used, $y_{0,0}$ = (111,0), $y_{3,2}$ = (–5,1), and $y_{5,7}$ = (6,–2).

In this example, the real part of $y_{0,0}$ is 111, the real part of $y_{0,4}$ is–9, the real part of $y_{4,0}$ is 6, the real part of $y_{4,4}$ is–1, and their imaginary parts are all zero. For the FFT-packed storage mode, the imaginary parts at these particular positions are not stored. Therefore, the number stored at position $\boldsymbol{Y}_{0,0}$ is (111,–9), which represents the contents of both $y_{0,0}$ and $y_{0,4}$. The number stored at position $\boldsymbol{Y}_{4,0}$ is (6,–1), which represents the contents of both $y_{4,0}$ and $y_{4,4}$.

The elements $y_{0,1:3}$ are stored in $\boldsymbol{Y}_{1:3,0}$. The elements $y_{4,1:3}$ are stored in $\boldsymbol{Y}_{5:7,0}$. The rows $y_{1:3,0:7}$ are stored in columns $\boldsymbol{Y}_{0:7,1:3}$. For FFT-packed storage mode, the elements in positions $y_{0,5:7}$, $y_{4,5:7}$, and rows $y_{5:7,0:7}$ are not stored.

Following is the global matrix ***Y*** in FFT-packed storage mode:

```
B,D            0                   1
     ┌                                      ┐
     | (111,-9) (10,-10)  |   (6,-4)   (6,-2) |
     |  (-3,23)  (4,  4)  |   (1, 3)   (6, 2) |
     |  (-8,10)  (9,  3)  |   (0, 2)  (-5, 1) |
     |  (-9, 4) (-6,  2)  |  (-7, 1)  (-8, 8) |
 0   |   (6,-1) (-1,  2)  |  (-1, 9)  (-1, 4) |
     |  (-3, 2) (-2,  1)  |  (-1, 4)  (-1,-1) |
     |  (-9, 1) (-3,  1)  |  (-2,-4)  (-1,-8) |
     |  (-1, 5) (-5, -3)  |  (-2,-2)  (-1,-2) |
     └                                      ┘
```

Following is a 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

After the data has been distributed over the process grid, the following local arrays for ***Y*** are stored in FFT-packed storage mode:

```
p,q  |         0           |         1
-----|---------------------|-------------------
     | (111,-9)  (10,-10)  |   (6,-4)   (6,-2)
     |  (-3,23)   (4,  4)  |   (1, 3)   (6, 2)
     |  (-8,10)   (9,  3)  |   (0, 2)  (-5, 1)
     |  (-9, 4)  (-6,  2)  |  (-7, 1)  (-8, 8)
 0   |   (6,-1)  (-1,  2)  |  (-1, 9)  (-1,4)
     |  (-3, 2)  (-2,  1)  |  (-1, 4)  (-1,-1)
     |  (-9, 1)  (-3,  1)  |  (-2,-4)  (-1,-8)
     |  (-1, 5)  (-5, -3)  |  (-2,-2)  (-1,-2)
```

Example: The following example shows how to pack data from a two-dimensional array `X` into a global array `XG`, whose columns could then be block-column distributed among *q* processes. Array `X` must contain complex-conjugate even symmetric data.

Each of the *q* processes would get LOCq(*n*) consecutive columns of array `XG`. Array `X` is stored as *n1* rows by *n2* columns. Array `XG` is stored as *n2* rows by *n1*/2 columns. This is the transposed form required by PSCRFT2 and PDCRFT2 for the input array.

```
PROGRAM PACK2D
IMPLICIT NONE
INTEGER*4 N1,N2,INDEX,JINDEX
PARAMETER(N1 = 64, N2 = 32)
COMPLEX*16 XG(0:N2-1,0:N1/2-1)
COMPLEX*16 X(0:N1-1,0:N2-1)
XG(0,0)    = ( REAL(X(0,0))    , REAL(X(0,N2/2))    )
XG(N2/2,0) = ( REAL(X(N1/2,0)) , REAL(X(N1/2,N2/2)) )
DO INDEX = 1 , N2/2-1
   XG(INDEX,0) = X(0,INDEX)
   XG(N2/2+INDEX,0) = X(N1/2,INDEX)
ENDDO
DO JINDEX = 0,N2-1
 DO INDEX = 1,N1/2-1
    XG(JINDEX,INDEX) = X(INDEX,JINDEX)
 ENDDO
ENDDO
STOP
END
```

Three-Dimensional Sequences

Following is a three-dimensional sequence using zero-based indexing where the first dimension is *n1*, the second dimension is *n2*, and the third dimension is *n3*:

Plane 0:

$$\begin{array}{ccc} a_{0,0,0} & \cdots & a_{0,n2-1,0} \\ a_{1,0,0} & \cdots & a_{1,n2-1,0} \\ \vdots & & \vdots \\ a_{n1-1,0,0} & \cdots & a_{n1-1,n2-1,0} \end{array}$$

Plane 1:

$$\begin{array}{ccc} a_{0,0,1} & \cdots & a_{0,n2-1,1} \\ a_{1,0,1} & \cdots & a_{1,n2-1,1} \\ \vdots & & \vdots \\ a_{n1-1,0,1} & \cdots & a_{n1-1,n2-1,1} \end{array}$$

$$\vdots$$

Plane $(n3-1)$:

$$\begin{array}{ccc} a_{0,0,n3-1} & \cdots & a_{0,n2-1,n3-1} \\ a_{1,0,n3-1} & \cdots & a_{1,n2-1,n3-1} \\ \vdots & & \vdots \\ a_{n1-1,0,n3-1} & \cdots & a_{n1-1,n2-1,n3-1} \end{array}$$

Distributing Data: For the Fourier transform subroutines, a three-dimensional sequence is distributed over a one-dimensional process grid, using block-plane distribution. The process grid must be arranged as a row ($1 \times q$, where q is the number of processes).

Note: Three-dimensional sequences can be thought of as three-dimensional matrices. The term sequence is used because it is traditional for Fourier transforms.

You must distribute the three-dimensional input sequence sequentially to the processes in the process grid, using block-plane distribution. Parallel ESSL also returns the output sequence using block-plane distribution. The output sequence may be returned in normal or transposed form.

A sequence can be distributed unevenly; that is, one process in the process grid can receive an array that is smaller than other processes. It can also happen that some processes receive no data. “Example 2” on page 773 shows an example of when a process does not receive any data.

LOCq(*n*) represents the number of planes that a process would receive if *n* is distributed block over *q* processes. You need to calculate LOCq(*n*) for each process, as follows:

- The number of planes, LOCq(*n*), that processes P_{00} through $P_{0,k\text{-}1}$ receive is calculated as follows:

  LOCq(*n*) = NB3 = (*n*+*q*–1)/*q*

- The number of planes, LOCq(*n*), that process $P_{0,k}$ receives is calculated as follows:

  LOCq(*n*) = *n*–(*q*–1)(NB3)

- Processes $P_{0,k+1}$ through $P_{0,q\text{-}1}$ would not receive any data. This may happen if there is not enough data to distribute to all the specified processes.

where:

n represents the following:

n is the third dimension, *n3*, of the sequence (for normal form)

n is the first dimension, *n1*, of the sequence (for transposed form and the sequence is not stored in FFT-packed storage mode)

n is *n1*/2 (for transposed form and the sequence is stored in FFT-packed storage mode)

q is the number processes in the process grid

$P_{0,k}$ is the process that receives the last block of data. For uneven data distribution, $P_{0,k}$ would receive an array that is smaller than the other processes receive.

Following is an example of block plane distribution for a three-dimensional sequence over a one-dimensional process grid.

Three-dimensional, global sequence with four planes that are of size 2 × 2:

```
        Plane 0:      Plane 1:
----------------------------------------------
B,D                0
----------------------------------------------
      ┌                          ┐
 0    │    0  1    │     10 101  │
      │   10 11    │     11 111  │
      └                          ┘

        Plane 2:      Plane 3:
----------------------------------------------
B,D                1
----------------------------------------------
      ┌                          ┐
 0    │   20 21    │     30 31   │
      │   23 24    │     33 34   │
      └                          ┘
```

Row-oriented, 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays:

```
p,q  |       0        |       1
-----|----------------|---------------
 0   |  0  1  10 101  |  20 21  30 31
     | 10 11  11 111  |  23 24  33 34
```

FFT-Packed Storage Mode: The output sequence for PSRCFT3 and PDRCFT3, and the input sequence for PSCRFT3 and PDCRFT3 are stored in FFT-packed storage mode because they consist of complex-conjugate, even symmetric data.

For FFT-packed storage mode, only certain elements of the complex-conjugate, even symmetric data are stored. This section describes how the complex elements of sequence *y*, which is the output sequence for PSRCFT3 and PDRCFT3, and the input sequence for PSCRFT3 and PDCRFT3, are stored in global matrices ***Y*** and ***X***, respectively.

For example, suppose *y* is the three-dimensional sequence to be stored in FFT-packed storage mode for PDRCFT3. The following list describes how the elements in *y* correspond to the complex elements in the global matrix ***Y***:

- The real part of $y_{0,0,0}$ is stored the real part of $\boldsymbol{Y}_{0,0,0}$
- The real part of $y_{0,0,n3/2}$ is stored in the imaginary part of $\boldsymbol{Y}_{0,0,0}$
- The elements $y_{0,0,1:n3/2-1}$ are stored in elements $\boldsymbol{Y}_{1:n3/2-1,0,0}$
- The real part of $y_{0,n2/2,0}$ is stored in the real part of $\boldsymbol{Y}_{n3/2,0,0}$
- The real part of $y_{0,n2/2,n3/2}$ is stored in the imaginary part of $\boldsymbol{Y}_{n3/2,0,0}$
- The elements $y_{0,n2/2,1:n3/2-1}$ are stored in elements $\boldsymbol{Y}_{n3/2+1:n3-1,0,0}$
- The real part of $y_{n1/2,0,0}$ is stored in the real part of $\boldsymbol{Y}_{0,n2/2,0}$
- The real part of $y_{n1/2,0,n3/2}$ is stored in the imaginary part of $\boldsymbol{Y}_{0,n2/2,0}$
- The elements $y_{n1/2,0,1:n3/2-1}$ are stored in elements $\boldsymbol{Y}_{1:n3/2-1,n2/2,0}$
- The real part of $y_{n1/2,n2/2,0}$ is stored in the real part of $\boldsymbol{Y}_{n3/2,n2/2,0}$
- The real part of $y_{n1/2,n2/2,n3/2}$ is the imaginary part of $\boldsymbol{Y}_{n3/2,n2/2,0}$
- The elements $y_{n1/2,n2/2,1:n3/2-1}$ are stored in elements $\boldsymbol{Y}_{n3/2+1:n3-1,n2/2,0}$
- The rows $y_{0,1:n2/2-1,i}$ are stored in columns $\boldsymbol{Y}_{i,1:n2/2-1,0}$
- The rows $y_{n1/2,1:n2/2-1,j}$ are stored in columns $\boldsymbol{Y}_{j,n2/2+1:n2-1,0}$
- The planes $y_{1:n1/2-1,i,j}$ are stored in planes $\boldsymbol{Y}_{j,i,1:n1/2-1}$

where:
i = 0, ..., $n2$–1
j = 0, ..., $n3$–1
n1 is the first dimension of array *y*
n2 is the second dimension of array *y*
n3 is the third dimension of array *y*

The remaining elements of *y* are not stored because they are the complex conjugates of elements already stored. These relationships are shown in the following equations:

- $y_{0,0,n3-k} = \bar{y}_{0,0,k}$ where $k = 1, \ldots, n3/2-1$
- $y_{0,n2/2,n3-k} = \bar{y}_{0,n2/2,k}$ where $k = 1, \ldots, n3/2-1$
- $y_{n1/2,0,n3-k} = \bar{y}_{n1/2,0,k}$ where $k = 1, \ldots, n3/2-1$
- $y_{n1/2,n2/2,n3-k} = \bar{y}_{n1/2,n2/2,k}$ where $k = 1, \ldots, n3/2-1$
- $y_{0,n2-j,0} = \bar{y}_{0,j,0}$ where $j = 1, \ldots, n2/2-1$
- $y_{0,n2-j,n3/2} = \bar{y}_{0,j,n3/2}$ where $j = 1, \ldots, n2/2-1$
- $y_{n1/2,n2-j,0} = \bar{y}_{n1/2,j,0}$ where $j = 1, \ldots, n2/2-1$
- $y_{n1/2,n2-j,n3/2} = \bar{y}_{n1/2,j,n3/2}$ where $j = 1, \ldots, n2/2-1$
- $y_{n1-i,0,0} = \bar{y}_{i,0,0}$ where $i = 1, \ldots, n1/2-1$
- $y_{n1-i,0,n3/2} = \bar{y}_{i,0,n3/2}$ where $i = 1, \ldots, n1/2-1$

- $y_{n1-i,n2/2,0} = \bar{y}_{i,n2/2,0}$ where $i = 1, \ldots, n1/2-1$
- $y_{n1-i,n2/2,n3/2} = \bar{y}_{i,n2/2,n3/2}$ where $i = 1, \ldots, n1/2-1$
- $y_{0,n2-j,n3-k} = \bar{y}_{0,j,k}$ where $j = 1, \ldots, n2/2-1$ and $k = 1, \ldots, n3/2-1, n3/2+1, \ldots, n3-1$
- $y_{n1/2,n2-j,n3-k} = \bar{y}_{n1/2,j,k}$ where $j = 1, \ldots, n2/2-1$ and $k = 1, \ldots, n3/2-1, n3/2+1, \ldots, n3-1$
- $y_{n1-i,0,n3-k} = \bar{y}_{i,0,k}$ where $i = 1, \ldots, n1/2-1$ and $k = 1, \ldots, n3/2-1, n3/2+1, \ldots, n3-1$
- $y_{n1-i,n2/2,n3-k} = \bar{y}_{i,n2/2,k}$ where $i = 1, \ldots, n1/2-1$ and $k = 1, \ldots, n3/2-1, n3/2+1, \ldots, n3-1$
- $y_{n1-i,n2-j,0} = \bar{y}_{i,j,0}$ where $i = 1, \ldots, n1/2-1$ and $j = 1, \ldots, n2/2-1, n2/2+1, \ldots, n2-1$
- $y_{n1-i,n2-j,n3/2} = \bar{y}_{i,j,n3/2}$ where $i = 1, \ldots, n1/2-1$ and $j = 1, \ldots, n2/2-1, n2/2+1, \ldots, n2-1$
- $y_{n1-i,n2-1-j,n3-1-k} = \bar{y}_{i,j,k}$ where $i = 1, \ldots, n1/2-1$ and $j = 1, \ldots, n2/2-1, n2/2+1, \ldots, n2-1$ and $k = 1, \ldots, n3/2-1, n3/2+1, \ldots, n3-1$

where:

n1 is the first dimension of array *y*
n2 is the second dimension of array *y*
n3 is the third dimension of array *y*

The following example, which uses zero-based indexing, has complex-conjugate, even symmetry. The dimensions of array *y* are $4 \times 4 \times 4$ (that is $n1 = n2 = n3 = 4$).

Plane 0:

$$y_{0:3,0:3,0} = \begin{bmatrix} (30,0) & (2,-3) & (-0.3,0) & (2,3) \\ (-1,0.7) & (-1,-4) & (-2,-0.7) & (0.5,-2) \\ (-2,0) & (-2,-0.6) & (2,0) & (-2,0.6) \\ (-1,-0.7) & (0.5,2) & (-2,0.7) & (-1,4) \end{bmatrix}$$

Plane 1:

$$y_{0:3,0:3,1} = \begin{bmatrix} (2,-2) & (-1,1) & (0.7,-2) & (-3,-2) \\ (2,2) & (-2,-1) & (-0.5,3) & (0.04,0.5) \\ (-0.4,3) & (-0.009,-3) & (0.9,0.1) & (-1,-0.2) \\ (-2,-2) & (-2,-1) & (-0.5,2) & (0.1,0.005) \end{bmatrix}$$

Plane 2:

$$y_{0:3,0:3,2} = \begin{bmatrix} (3,0) & (0.3,0.5) & (0.1,0) & (0.3,-0.5) \\ (-0.3,-2) & (1,-3) & (2,3) & (-7,3) \\ (2,0) & (2,-1) & (1,0) & (2,1) \\ (-0.3,2) & (-0.7,-3) & (2,-3) & (1,3) \end{bmatrix}$$

Plane 3:

$$y_{0:3,0:3,3} = \begin{bmatrix} (2,2) & (-3,2) & (0.7,2) & (-1,-1) \\ (-2,2) & (1,-0.005) & (-0.5,-2) & (-0.2,1) \\ (-0.4,-3) & (-1,0.2) & (0.9,-0.1) & (-0.009,3) \\ (2,-2) & (0.04,-0.5) & (-0.5,-3) & (-2,1) \end{bmatrix}$$

Because zero-based indexing is used, $y_{0,0,0} = (30,0)$, $y_{2,1,1} = (-0.009,-3)$, and $y_{3,1,3} = (0.04,-0.5)$.

In this example, the real part of $y_{0,0,0}$ is 30, the real part of $y_{0,0,2}$ is 3, and their imaginary parts are zero. For the FFT-packed storage mode, the imaginary parts at these particular positions are not stored. Therefore, the element stored at position $\boldsymbol{Y}_{0,0,0}$ is (30,3), which represents the contents of both $y_{0,0,0}$ and $y_{0,0,2}$.

The element $y_{0,0,1}$ is stored in the global matrix $\boldsymbol{Y}_{1,0,0}$ position.

The real part of $y_{0,2,0}$ is–0.3, the real part of $y_{0,2,2}$ is 0.1, and their imaginary parts are zero. For the FFT-packed storage mode, the imaginary parts at these particular positions are not stored. Therefore, the element stored at position $\boldsymbol{Y}_{2,0,0}$ is (–0.3,0.1), which represents the contents of both $y_{0,2,0}$ and $y_{0,2,2}$.

The element $y_{0,2,1}$ is stored in the global matrix $\boldsymbol{Y}_{3,0,0}$ position.

The real part of $y_{2,0,0}$ is–2, the real part of $y_{2,0,2}$ is 2, and their imaginary parts are zero. For the FFT-packed storage mode, the imaginary parts at these particular positions are not stored. Therefore, the element stored at position $\boldsymbol{Y}_{0,2,0}$ is (–2,2), which represents the contents of both $y_{2,0,0}$ and $y_{2,0,2}$.

The element $y_{2,0,1}$ is stored in the global matrix $\boldsymbol{Y}_{1,2,0}$ position.

The real part of $y_{2,2,0}$ is 2, the real part of $y_{2,2,2}$ is 1, and their imaginary parts are zero. For the FFT-packed storage mode, the imaginary parts at these particular positions are not stored. Therefore, the element stored at position $\boldsymbol{Y}_{2,2,0}$ is (2,1), which represents the contents of both $y_{2,2,0}$ and $y_{2,2,2}$.

The element $y_{2,2,1}$ is stored in the global matrix $\boldsymbol{Y}_{3,2,0}$ position.

The rows $y_{0,1,0:3}$ are stored in columns $\boldsymbol{Y}_{0:3,1,0}$. The rows $y_{2,1,0:3}$ are stored in columns $\boldsymbol{Y}_{0:3,3,0}$. The plane $y_{1,0:3,0:3}$ is stored in plane $\boldsymbol{Y}_{0:3,0:3,1}$. For FFT-packed storage mode, the remaining elements do not need to be stored due to symmetry.

Following is the global matrix $\boldsymbol{Y}$ in FFT-packed storage mode:

Plane 0:

```
B,D                            0
     ┌                                                         ┐
     |    (30,  3)     (2,  -3)       (-2, 2)     (-2, -0.6)  |
     |      (2, -2)     (-1,  1)     (-0.4,3)   (-0.009,-3)   |
 0   | (-0.3, 0.1)   (0.3, 0.5)       (2,  1)        (2,  1)  |
     |   (0.7, -2)      (-3, 2)    (0.9, 0.1)     (-1, 0.2)   |
     └                                                         ┘
```

Plane 1:

```
B,D                            1
     ┌                                                          ┐
     |  (-1,0.7)        (-1, -4)   (-2, -0.7)      (0.5, -2)   |
     |     (2,2)         (-2,-1)    (-0.5, 3)    (0.04, 0.5)   |
 0   | (-0.3,-2)         (1, -3)       (2,  3)     (-0.7, 3)   |
     |   (-2, 2)  (-0.1, -0.005)   (-0.5, -2)      (-0.2, 1)   |
     └                                                          ┘
```

Following is a 1 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}

After the data has been distributed over the process grid, the following local arrays for ***Y*** are stored in FFT-packed storage mode:

```
p,q  |                    0                        |                        1
-----|---------------------------------------------|-----------------------------------------------
     |    (30,  3)   (2,  -3)    (-2, 2)  (-2, -0.6)  |   (-1,0.7)       (-1, -4) (-2, -0.7)   (0.5, -2)
     |     (2, -2)   (-1,  1)    (-0.4,3) (-0.009,-3) |      (2,2)        (-2,-1)  (-0.5, 3) (0.04, 0.5)
 0   | (-0.3, 0.1) (0.3, 0.5)    (2,  1)     (2,  1)  |  (-0.3,-2)        (1, -3)    (2,  3)   (-0.7, 3)
     |   (0.7, -2)    (-3, 2) (0.9, 0.1)   (-1, 0.2)  |    (-2, 2) (-0.1, -0.005) (-0.5, -2)   (-0.2, 1)
```

Example: The following example shows how to pack data from a three-dimensional array X into a global array XG, whose planes could then be block distributed among *q* processes. Array X must contain complex-conjugate even symmetric data.

Each of the *q* processes would get LOCq(*n*) consecutive planes of array XG. Array X is stored as *n1* rows by *n2* columns by *n3* planes. Array XG is stored as *n3* rows by *n2* columns by *n1/2* planes. This is the transposed form required by PSCRFT3 and PDCRFT3 for the input array. *n1*, *n2*, and *n3* are divisible by 2*q*, as required by PSCRFT3 and PDCRFT3.

```
PROGRAM PACK3D
IMPLICIT NONE
INTEGER*4 N1,N2,N3
INTEGER*4 IINDEX,JINDEX,KINDEX
PARAMETER(N1 = 64, N2 = 32, N3 = 48)
COMPLEX*16 XG(0:N3-1,0:N2-1,0:N1/2-1)
COMPLEX*16 X(0:N1-1,0:N2-1,0:N3-1)
XG(0,0,0)    = ( REAL(X(0,0,0))    , REAL(X(0,0,N3/2))    )
XG(N3/2,0,0) = ( REAL(X(0,N2/2,0)) , REAL(X(0,N2/2,N3/2)) )
XG(0,N2/2,0) = ( REAL(X(N1/2,0,0)) , REAL(X(N1/2,0,N3/2)) )
XG(N3/2,N2/2,0) = (REAL(X(N1/2,N2/2,0)),REAL(X(N1/2,N2/2,N3/2)))
DO IINDEX = 1 , N3/2-1
   XG(IINDEX,0,0) = X(0,0,IINDEX)
   XG(N3/2+IINDEX,0,0) = X(0,N2/2,IINDEX)
   XG(IINDEX,N2/2,0) = X(N1/2,0,IINDEX)
   XG(N3/2+IINDEX,N2/2,0) = X(N1/2,N2/2,IINDEX)
ENDDO
DO KINDEX = 0,N3-1
 DO JINDEX = 1,N2/2-1
   XG(KINDEX,JINDEX,0) = X(0,JINDEX,KINDEX)
   XG(KINDEX,N2/2+JINDEX,0) = X(N1/2,JINDEX,KINDEX)
 ENDDO
ENDDO
DO KINDEX = 0,N3-1
 DO JINDEX = 0,N2-1
  DO  IINDEX = 1,N1/2-1
     XG(KINDEX,JINDEX,IINDEX) = X(IINDEX,JINDEX,KINDEX)
  ENDDO
 ENDDO
ENDDO
STOP
END
```

Distributing Data in an HPF Program

You must distribute your data before calling Parallel ESSL from your HPF program. This section shows how to use the HPF data distribution directives to distribute your data. An example of the data distribution techniques are also shown in a sample HPF program in Appendix B.

All the Parallel ESSL HPF subroutines, except the Banded Linear Algebraic Equations, Fourier Transforms, and the Random Number Generation subroutine, support block-cyclic distribution. The Banded Linear Algebraic Equations, Fourier Transforms, and the Random Number Generation subroutine only support block distribution.

This section shows examples of how you could distribute your data over one- or two-dimensional process grids:

- "Vectors"
- "Matrices" on page 80
- "Sequences (Fourier Transform)" on page 81

Notes:

1. The XL HPF default to map processes into a process grid is column-major order. For an example, see Table 13 on page 18.
2. XL HPF supports only CYCLIC(1) in the DISTRIBUTE directive, unless it appears in the interface block of a local procedure. For more information, see "Using Extrinsic Procedures—The Parallel ESSL Subroutines" on page 105.
3. All the examples in this section use CYCLIC (which has the same meaning as CYCLIC(1)) and ALIGN in the DISTRIBUTE directives. To find out where to code distribution directives in your HPF program, see "Sample HPF Programs" on page 107.

   The examples show real data structures, but the techniques apply equally to complex data structures.

Vectors

Parallel ESSL supports block-cyclic distribution for vectors over one- or two-dimensional process grids, except for URNG. For URNG, vectors are distributed over a one-dimensional process grid using block distribution, where the length *n* of the vector *x* must be evenly divisible by the available processes *np* multiplied by the block size *nb*. In other words, *n*/((*np*)(*nb*)) must be an integer.

Distributed over One-Dimensional Process Grids

Block-Cyclically Distributing a Vector: This example shows how to distribute a vector of length 24 block-cyclically over a column-oriented, one-dimensional process grid with four processes.

```
      REAL(KIND(1.0D0)), DIMENSION(:,:) :: A
      REAL(KIND(1.0D0)), DIMENSION(24) :: X
!HPF$ PROCESSORS PROC(4)
!HPF$ ALIGN      X(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC,*) ONTO PROC :: A
```

This example shows how to distribute the same vector block-cyclically over a row-oriented, one-dimensional process grid with four processes.

```
      REAL(KIND(1.0D0)), DIMENSION(:,:) :: A
      REAL(KIND(1.0D0)), DIMENSION(24) :: X
!HPF$ PROCESSORS PROC(4)
!HPF$ ALIGN      X(:) WITH A(*,:)
!HPF$ DISTRIBUTE (*,CYCLIC) ONTO PROC :: A
```

Block Distributing a Vector: For URNG, this example shows how to distribute a vector of length 18 block-column over a row-oriented, one-dimensional process grid with six processes.

```
      REAL(KIND(1.0D0)), DIMENSION(18,18) :: A
      REAL(KIND(1.0D0)), DIMENSION(18) :: X
!HPF$ PROCESSORS PROC(6)
!HPF$ ALIGN      X(:) WITH A(1,:)
!HPF$ DISTRIBUTE (*,BLOCK) ONTO PROC :: A
```

Note: For URNG, the length *n* of the vector *x* must be evenly divisible by the number of available processes *np* multiplied by the block size *nb*. For this example, 18 = (6)(3).

Block-Cyclic Distribution over Two-Dimensional Process Grids

This example shows how to distribute a vector of length 18 block-cyclically over the third column of a two-dimensional process grid.

```
      REAL(KIND(1.0D0)), DIMENSION(:,:) :: A
      REAL(KIND(1.0D0)), DIMENSION(18) :: X
!HPF$ PROCESSORS PROC(2,3)
!HPF$ ALIGN      X(:) WITH A(:,3)
!HPF$ DISTRIBUTE (CYCLIC,CYCLIC) ONTO PROC :: A
```

This example shows how to distribute the same vector block-cyclically over the second row of a two-dimensional process grid:

```
      REAL(KIND(1.0D0)), DIMENSION(:,:) :: A
      REAL(KIND(1.0D0)), DIMENSION(18) :: X
!HPF$ PROCESSORS PROC(2,3)
!HPF$ ALIGN      X(:) WITH A(2,:)
!HPF$ DISTRIBUTE (CYCLIC,CYCLIC) ONTO PROC :: A
```

Matrices

Parallel ESSL, except for the Banded Linear Algebraic Equations, supports block-cyclic distribution for matrices over one- or two-dimensional process grids. The Banded Linear Algebraic Equations supports only block distribution for matrices over one-dimensional process grids.

Distributed over One-Dimensional Process Grids

Block-Cyclically Distributing a Matrix: This example shows how a 6 × 8 matrix ***A*** is distributed block-cyclically over a column-oriented, one-dimensional process grid with three processes.

```
      REAL(KIND(1.0D0)), DIMENSION(6,8) :: A
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (CYCLIC,*) ONTO PROC :: A
```

This example shows how the matrix ***A*** is distributed block-cyclically over a row-oriented, one-dimensional process grid with four processes.

```
      REAL(KIND(1.0D0)), DIMENSION(6,8) :: A
!HPF$ PROCESSORS PROC(4)
!HPF$ DISTRIBUTE (*,CYCLIC) ONTO PROC :: A
```

Block Distributing a Matrix: This example shows how a 5 × 5 matrix ***A*** is distributed over a row-oriented, one-dimensional process grid with four processes, using block-column distribution.

```
      REAL(KIND(1.0D0)), DIMENSION(5,5) :: A
!HPF$ PROCESSORS PROC(4)
!HPF$ DISTRIBUTE (*,BLOCK) ONTO PROC :: A
```

This example shows how a 5 × 5 matrix ***A*** is distributed over a column-oriented, one-dimensional process grid with four processes, using block-row distribution.

```
      REAL(KIND(1.0D0)), DIMENSION(5,5) :: A
!HPF$ PROCESSORS PROC(4)
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: A
```

This example shows how a 100 × 100 matrix ***A*** is distributed over a row-oriented, one-dimensional process grid with five processes, using block-column distribution.

```
      REAL(KIND(1.0D0)), DIMENSION(100,100) :: A
!HPF$ PROCESSORS PROC(5)
!HPF$ DISTRIBUTE (*,BLOCK(40)) ONTO PROC :: A
```

Note: The first two processes receive 40 columns of data, the third process receives 20 columns of data, and the remaining processes receive no data.

Block-Cyclic Distribution over Two-Dimensional Process Grids

This example shows how a 9 × 26 matrix ***A*** is distributed block-cyclically over a 2 × 3, two-dimensional process grid.

```
      REAL(KIND(1.0D0)), DIMENSION(9,26) :: A
!HPF$ PROCESSORS PROC(2,3)
!HPF$ DISTRIBUTE (CYCLIC,CYCLIC) ONTO PROC :: A
```

Sequences (Fourier Transform)

Parallel ESSL supports block data distribution for two- or three-dimensional sequences over a one-dimensional process grid. (For details on FFT-packed storage mode, see “Two-Dimensional Sequence” on page 66 and “Three-Dimensional Sequences” on page 71.)

Two-Dimensional Sequence

This example shows how an 8 × 12, two-dimensional sequence is distributed over a one-dimensional process grid with four processes, using block-column distribution. The sequence is distributed with a block size of 3.

```
      REAL(KIND(1.0D0)), DIMENSION(8, 12) :: Y
!HPF$ PROCESSORS PROC(4)
!HPF$ DISTRIBUTE (*, BLOCK) ONTO PROC :: Y
```

Three-Dimensional Sequences

This example shows how a three-dimensional sequence, with four planes that are each of size 2 × 2, is distributed over a one-dimensional process grid with two processes, using block plane distribution. The sequence is distributed with a block size of 2:

```
      REAL(KIND(1.0D0)), DIMENSION(2, 2, 4) :: X
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, *, BLOCK) ONTO PROC :: X
```

Chapter 3. Coding and Running Your Program

This chapter explains the Parallel ESSL-specific procedures to follow when coding and running your program.

Coding Tips for Optimizing Parallel Performance

Performance has been the primary objective in the design of the Parallel ESSL subroutines. To achieve this performance goal, the Parallel ESSL subroutines use "state-of-the-art" algorithms tailored to specific operational characteristics of the hardware. In addition, Parallel ESSL will leverage the high performance provided by ESSL for AIX for processor computations.

Choosing a Parallel ESSL Library

The Parallel ESSL library you may use depends on:

1. Your choice of MPI library.
2. The type of nodes you are running on.

The MPI library you choose, is dependent upon the following:

- If you are using the Parallel ESSL message passing subroutines, you may use either the MPI signal handling library or the MPI threaded library.
- If you are an HPF user, you may only use the MPI signal handling library.
- If you are using LAPI, you may only use the MPI threaded library.

The Parallel ESSL library choices for use with the MPI signal handling library are specified in Table 29.

Table 29. Parallel ESSL Libraries used with the MPI Signal Handling Library

Node Type	**Parallel ESSL Library**
SMP	Parallel ESSL POWER Library PE 2.4 users may specify multiple User Space PE tasks per SMP node. For example, you could specify the number of User Space Tasks per adapter equal to the number of CPUs in your SMP node. PE 2.3 users may only have one User Space PE task per SMP node.
POWER2	Parallel ESSL POWER2 Library♦
POWER	Parallel ESSL POWER Library
♦All nodes must be POWER2 nodes.	

The Parallel ESSL library choices for use with the MPI threaded library are specified in Table 30.

Table 30 (Page 1 of 2). Parallel ESSL Libraries used with the MPI Threaded Library

Node Type	**Parallel ESSL Library**
SMP	Parallel ESSL SMP LibraryΔ

Table 30 (Page 2 of 2). Parallel ESSL Libraries used with the MPI Threaded Library

Node Type	**Parallel ESSL Library**
POWER2	Parallel ESSL Thread-Tolerant POWER2 Library♦
POWER	Parallel ESSL SMP Library

∆If you choose to spawn multiple user space tasks per SMP node, you may consider explicitly setting the number of threads used by the Parallel ESSL SMP Library, by setting the environment variable **XLSMPOPTS**. For further details, see the XLF or C for AIX manuals.

♦All nodes must be POWER2 nodes.

XL HPF and Parallel ESSL

XL HPF allows you to easily develop parallel software using the SPMD programming model. The XL HPF compiler, guided by HPF directives in your source code, handles the distribution of data and communication between programs on multiple processes. The HPF directives make developing an HPF program that calls Parallel ESSL easier than developing a message passing program that calls Parallel ESSL. However, the performance obtained when using a Parallel ESSL HPF subroutine is less than that obtained when using a Parallel ESSL message passing subroutine because there is a certain amount of overhead involved in supporting the extrinsic hpf_local interface.

Because the XL HPF compiler only supports CYCLIC(N) in the interface blocks for extrinsic hpf_local subroutines, a redistribution of data occurs whenever a Level 3 PBLAS, Dense Linear Algebraic Equations, Eigensystem Analysis or Singular Value Analysis subroutine is called. Also, data may be copied locally because the extrinsic hpf_local subroutines require the use of assumed-shape arrays while the Parallel ESSL message passing subroutines use assumed-size arrays.

Parallel ESSL Techniques

The following techniques are used by most subroutines to optimize performance:

- Minimizing the impact of communications by exchanging larger blocks of data
- Blocking data to match the processor cache size

The following items also impact performance. They generally depend on the specific parallel routine being called. See the subroutine description in the reference section for any exceptions to these rules.

- | Number and types of processors (such as, POWER Thin, POWER2 Thin,
  | POWER3 Wide, POWER Wide)

  | Choosing the number of processors depends primarily on the problem size. It is
  | reasonable to increase the number of processors, if the global problem size
  | increases sufficiently to keep the amount of local data per process at a
  | reasonable size. If, however, using more processes, such as 17 rather than 16,
  | causes you to use a one-dimensional grid rather than a two-dimensional grid,
  | performance may be degraded. See the next item.

- Shape of process grid

  For most subroutines, using a two dimensional (square or as close to square as possible) grid is suggested. For example, if sixteen processors were used, define a 4 by 4 process grid. For exceptions to this rule, see the subroutine descriptions in the reference section.

- Block size(s) in the Message Passing subroutines

  See the following table for suggested block sizes in your message passing program. The optimal block size depends on the underlying node computations, load balancing, communications, system buffering requirements, problem size, and dimension and shape of the process grid. To achieve optimal performance, generally requires experimentation, but the values specified in Table 31 should provide good performance for most cases. For exceptions to these rules, see the subroutine descriptions in the reference section.

Table 31. Suggested Block Sizes

Area	POWER Nodes	POWER2 Nodes	SMP Nodes
Level 2 PBLAS	24	24 (All subroutines, except PDTRSV and PZTRSV) 64 (PDTRSV and PZTRSV)	24 (All subroutines, except PDTRSV and PZTRSV) 48 (PDTRSV and PZTRSV)
Level 3 PBLAS	40	70 (Real subroutines) 30 (Complex subroutines)	100 (Real subroutines) 50 (Complex subroutines)
Dense Linear Algebraic Equations	40	70 (Real subroutines) 30 (Complex subroutines)	100 (Real subroutines) 50 (Complex subroutines)
Eigensystems Analysis and Singular Value Analysis	24	24	24
Random Number Generation	.5 (data cache size)	.5 (data cache size)	.5 (data cache size)

Note: The data cache size can be obtained with this command: `lsattr -E -H -l sys0`

- PESSL_HPF module for the HPF subroutines

  For all HPF subroutines, except GEBRD, data directives are included in the interface module PESSL_HPF; therefore, you can specify any data distribution for your vectors, matrices, and sequences, because the XL HPF compiler will, if necessary, redistribute the data prior to calling the HPF Parallel ESSL subroutine. Data directives for GEBRD cannot be included in the PESSL_HPF module, because the alignment requirements for some of the vectors depend on the size of the matrix. For details, see "GEBRD—Reduce a General Matrix to Bidiagonal Form" on page 957.

  When using cyclic distribution in your HPF program, you can only specify CYCLIC(1) data distributions. However, the performance of the Level 3 PBLAS, Dense Linear Algebraic Equations, and Eigensystems Analysis and Singular Value Analysis subroutines is improved if a CYCLIC(N) data distribution is used. To accomplish this, PESSL_HPF contains CYCLIC(N) data directives for a two-dimensional process grid in the interface for these subroutines. The block sizes specified in the PESSL_HPF module are listed in Table 32 on page 86.

  There are CYCLIC(1) directives for a two-dimensional process grid in the interfaces in PESSL_HPF for the Level 2 PBLAS.

  There are BLOCK data directives for a one-dimensional process grid in the interfaces in PESSL_HPF for the Banded Linear Algebraic Equations, Fourier transform and Random Number Generation subroutines.

For more information about the PESSL_HPF module, see "Using Extrinsic Procedures—The Parallel ESSL Subroutines" on page 105.

Table 32. Block Sizes Specified in the PESSL_HPF Module

Area	**Block Size**
Level 2 PBLAS	1
Level 3 PBLAS	70 (Real) 30 (Complex)
Dense Linear Algebraic Equations	70 (Real) 30 (Complex)
Eigensystems Analysis and Singular Value Analysis	24

- If you are using the MPI threaded library, for a single message-passing thread, specify MP_SINGLE_THREAD=yes to minimize thread overhead.
- You should be able to improve performance of production-level code by using the PESSL_ERROR_SYNC environment variable to disable error synchronization. For details, see "PESSL_ERROR_SYNC Environment Variable" on page 117.

Avoiding Conflicts with Parallel ESSL and ESSL for AIX Routine Names

Do not use names for your own subroutines, functions, and global variables that are the same as any of the ESSL for AIX or Parallel ESSL routine names. All internal routine names begin with the prefix "ESV" or "HPF_P," so you should avoid using these prefixes for your own routines.

Coding Your Message Passing Program

This section contains Parallel ESSL-specific application program coding requirements and considerations for message passing programs—that is, programs coded in Fortran, C, and C++. To make a Parallel ESSL call in a parallel application program:

1. Call the BLACS initialization subroutines (BLACS_GET followed by a call to either BLACS_GRIDINIT or BLACS_GRIDMAP), to initialize the process grid. For details on how to do this, see "Initializing the BLACS" on page 87.
2. Ensure your data has been distributed across your process grid, according to the particular input distribution specified by the Parallel ESSL subroutine. For details on how to do this, see Chapter 2 on page 17.
3. Call the Parallel ESSL subroutine on all processes in the process grid (defined earlier through the BLACS initialization calls). The Parallel ESSL subroutine call interfaces are documented in Part 2 of this book.
4. When the Parallel ESSL subroutine returns control to the application program, you process the solution data, which is distributed in accordance with the output distribution specified by the Parallel ESSL subroutine.

To look at an application program outline, see the following:

- “Application Program Outline” on page 96 (For all subroutines, except the sparse linear algebraic equation subroutines.)
- “Application Program Outline for the Fortran 90 Sparse Linear Algebraic Equations and Their Utilities” on page 97
- “Application Program Outline for the Fortran 77 Sparse Linear Algebraic Equations and Their Utilities” on page 99

For an example of the use of Parallel ESSL in a sample message passing Fortran 90 application program solving a thermal diffusion problem, see Appendix B on page 999.

The *ESSL Version 3 Guide and Reference* manual contains additional information about coding ESSL for AIX subroutine calls in Fortran, C, and C++ programs. That information also applies to Parallel ESSL and is not repeated in this book. The specific topics you may want to reference, that apply to Parallel ESSL, are:

- Coding the calling sequences
- Passing arguments
- Setting up scalar data
- Setting up complex data in C and C++ programs
- Setting up arrays

Initializing the BLACS

A parallel machine with k processes is often thought of as a one-dimensional linear array of processes labeled 0, 1, ..., k–1. For performance reasons, it is sometimes useful to map this one-dimensional array into a logical two-dimensional rectangular grid, which is also referred to as process grid, of processes. The process grid can have p process rows and q process columns, where $p \times q = k$. A process can now be indexed by row and column, (i,j), where $0 \le i < p$ and $0 \le j < q$. (This logical rectangular grid may not necessarily be reflected by the underlying hardware—that is, processor nodes. In most cases k is less than or equal to the number of SP processor nodes that your job is running on. In special cases, however, the number of processes can be greater than the number of SP processor nodes. This is subject to restrictions imposed by PE. For more details refer to the appropriate *Parallel Environment: Operation and Use* manual.)

Before calling the Parallel ESSL subroutines, you need to call BLACS_GET, followed by a call to either BLACS_GRIDINIT or BLACS_GRIDMAP to define the size and dimensions of your process grid. This identifies what processes are involved in the communication. You can reinitialize the BLACS, as needed, at various points in your application program to redefine the process grid.

When you initialize the BLACS, you must specify the (total) size k of the grid to be less than or equal to the value set in the MP_PROCS PE environment variable or its associated command-line flag **-procs**. If argument values are not valid, an error message is issued and the program is terminated.

An example of initializing the BLACS in a Fortran 90 program is shown in Appendix B on page 999. See the subroutine initialize_scale in “Module Scale (Message Passing)” on page 1025.

BLACS_PINFO

You call the BLACS_PINFO routine when you want to determine how many processes are available. You can use this information as input into other BLACS routines that set up your process grid.

Syntax

Language	Call Statement
Fortran	CALL `BLACS_PINFO` (*mypnum, nprocs*)
C	`blacs_pinfo` (*&mypnum, &nprocs*);

C++	extern "FORTRAN" void blacs_pinfo(const int &, const int &);
	`blacs_pinfo` (*mypnum, nprocs*);

On Return

mypnum
: is the local process rank (for example, PE task number) that your program is currently running on.

  Returned as: a fullword integer value, where: 0 ≤ *mypnum* ≤ (*nprocs* − 1).

nprocs
: is the number of processes available for the BLACS to use.

  Returned as: a fullword integer value.

BLACS_GET

You call the BLACS_GET routine when you want the values the BLACS are using for internal defaults. The most common use is in retrieving a default system context for input into BLACS_GRIDINIT or BLACS_GRIDMAP.

Syntax

Language	Call Statement
Fortran	CALL `BLACS_GET` (*icontxt, what, val*)
C	`blacs_get` (*&icontxt, &what, &val*);

C++ and *what* = 0, 2, 10	extern "FORTRAN" void blacs_get(const int &, const int &, const int &);
	`blacs_get` (*icontxt, what, val*);

On Entry

icontxt
: has the following meaning:

  If *what* = 0 or 2, *icontxt* is ignored.

  If *what* = 10, *icontxt* is the integer handle indicating the BLACS context.

  Specified as: a fullword integer value.

what
: indicates the BLACS internal to be returned in *val*. For a description of the values of *what*, see Table 33 on page 89.

Table 33. Input and Output for BLACS_GET

Value of *what*	**BLACS Internals That are Returned in** *val*
0	Handle indicating the default system context
2	BLACS debug level
10	Handle indicating the system context used to define the BLACS context whose handle is *icontxt*. You can redefine the shape of your process grid by calling BLACS_GET with *what*=10. For examples on how to do this, see the “Notes” section in “BLACS_GRIDINIT” on page 89 or “BLACS_GRIDMAP” on page 92.

Specified as: a fullword integer value 0, 2, 10.

On Return

val is the value of the BLACS internal, as defined for each value of *what* in Table 33.

Returned as: a fullword integer value.

BLACS_GRIDINIT

You call the BLACS_GRIDINIT routine when you want to map the processes sequentially in row-major order or column-major order into the process grid. You must specify the same input argument values in the calls to BLACS_GRIDINIT on every process.

Syntax

Language	**Call Statement**
Fortran	CALL `BLACS_GRIDINIT` (*icontxt, order, nprow, npcol*)
C	`blacs_gridinit` (*&icontxt, &order, &nprow, &npcol*);
C++	extern “FORTRAN” void blacs_gridinit(const int &, char *, const int &, const int &); `blacs_gridinit` (*icontxt, order, nprow, npcol*);

On Entry

icontxt

is the system context to be used in creating the BLACS context. For examples on obtaining a default system context and reshaping your process grid, see the “Notes” section.

Specified as: a fullword integer value.

order

indicates how to map processes into the process grid, where:

If *order* = `'R'`, row-major natural ordering is used. This is the default.

If *order* = `'C'`, column-major natural ordering is used.

Specified as: a single character; *order* = `'R'` or `'C'`.

nprow
is the number of rows in this process grid.

Specified as: a fullword integer where: 1 ≤ *nprow* ≤ *p*.

npcol
is the number of columns in this process grid.

Specified as: a fullword integer value where: 1 ≤ *npcol* ≤ *q*.

On Return

icontxt
is the integer handle to the BLACS context, which is a mechanism for partitioning communication space. A defining property of a context is that a message in a context cannot be sent or received in another context. The BLACS context includes the definition of a grid, and each processor's coordinates in the grid.

Returned as: a fullword integer value.

Notes

1. You may obtain a default system context by calling BLACS_GET as follows:

   ```
   CALL BLACS_GET(0, 0, icontxt)
   ```

2. You can redefine the shape of your process grid by calling BLACS_GET with *what*=10 and then calling BLACS_GRIDINIT. The following example shows how to create a 1 × 4 process grid, using the context from a 2 × 2 process grid:

   ```
   *
   * Define the 1 × 4 process grid
   *
   CALL BLACS_GET(0, 0, icontxt)
   CALL BLACS_GRIDINIT(icontxt, 'R' 2, 2)
             .
             .
             .
   *
   * Redefine the shape to a 1 × 4 process grid
   *
   CALL BLACS_GET(icontxt, 10, newcontxt)
   CALL BLACS_GRIDINIT(newcontxt, 'R', 1, 4)
   ```

3. Suppose you specified a total of fifteen processes in your MP_PROCS environment variable, referred to as t_0 through t_{14}. You then call BLACS_GRIDINIT in your Fortran program, as follows:

   ```
      CALL BLACS_GRIDINIT (icontxt,'R',3,4)
   ```

   The processes would be mapped sequentially in row major order into a 3 by 4 process grid as follows:

Table 34. A 3 by 4 process grid

$P_{p,q}$	**0**	**1**	**2**	**3**
0	t_0	t_1	t_2	t_3
1	t_4	t_5	t_6	t_7
2	t_8	t_9	t_{10}	t_{11}

Note: In this example, the process grid is 3 by 4. You must execute a call to Parallel ESSL on all processes whose process row and column index satisfy $0 \le i < 3$ and $0 \le j < 4$, respectively.

BLACS_GRIDINFO

You call the BLACS_GRIDINFO routine to obtain the process row and column index.

Syntax

Language	**Call Statement**
Fortran	CALL `BLACS_GRIDINFO` (*icontxt, nprow, npcol, myrow, mycol*)
C	`blacs_gridinfo` (*&icontxt, &nprow, &npcol, &myrow, &mycol*);
C++	extern “FORTRAN” void blacs_gridinfo(const int &, const int &, const int &, const int &, const int &); `blacs_gridinfo` (*icontxt, nprow, npcol, myrow, mycol*);

On Entry

icontxt
is the integer handle to the BLACS context which is a mechanism for partitioning communication space. A defining property of a context is that a message in a context cannot be sent or received in another context. The BLACS context include the definition of a grid, and each process coordinates in the grid.

Specified as: a fullword integer value, returned by BLACS_GRIDINIT or BLACS_GRIDMAP.

On Return

nprow
is the number of rows in this process grid.

Specified as: a fullword integer where: $1 \le nprow \le p$.

npcol
is the number of columns in this process grid.

Specified as: a fullword integer value where: $1 \le npcol \le q$.

myrow
is the process grid row index.

Returned as: a fullword integer value where: $0 \le myrow < p$.

mycol
is the process grid column index.

Returned as: a fullword integer value where: $0 \le mycol < q$.

BLACS_GRIDMAP

You call the BLACS_GRIDMAP routine when you want to map the processes in a specific manner into a process grid. You pass in a two-dimensional array containing the process numbers, which is mapped into your new process grid. You must specify the same input argument values in the calls to BLACS_GRIDMAP on every process.

Syntax

Language	Call Statement
Fortran	CALL `BLACS_GRIDMAP` (*icontxt, usermap, ldumap, nprow, npcol*)
C	`blacs_gridmap` (*&icontxt, usermap, &ldumap, &nprow, &npcol*);
C++	extern “FORTRAN” void blacs_gridmap(const int &, int *, const int &, const int &, const int &); `blacs_gridmap` (*icontxt, usermap, ldumap, nprow, npcol*);

On Entry

icontxt
: is the system context to be used in creating the BLACS context. For examples on obtaining a default system context and reshaping your process grid, see the “Notes” section.

  Specified as: a fullword integer value.

usermap
: specifies the process-to-grid mapping. `USERMAP`(i,j) contains the number of the process to be mapped to the process grid, location (i,j).

  Specified as: a two dimensional integer array of size *ldumap* by *npcol*.

ldumap
: is the leading dimension of the integer array `USERMAP`.

  Specified as: an integer where: $ldumap \geq nprow$

nprow
: is the number of rows in this process grid.

  Specified as: a fullword integer where: $1 \leq nprow \leq p$.

npcol
: is the number of columns in this process grid.

  Specified as: a fullword integer value where: $1 \leq npcol \leq q$.

On Return

icontxt
: is the integer handle to the BLACS context which is a mechanism for partitioning communication space. A defining property of a context is that a message in a context cannot be sent or received in another context. The BLACS context include the definition of a grid, and each process coordinates in the grid.

  Returned as: a fullword integer value.

Notes

1. You may obtain a default system context by calling BLACS_GET as follows:

   ```
   CALL BLACS_GET(0, 0, icontxt)
   ```

2. You can redefine the shape of your process grid by calling BLACS_GET with *what*=10 and then calling BLACS_GRIDMAP. The following example shows how to create a 1 × 4 process grid, using the context from a 2 × 2 process grid:

   ```
   *
   * Define the 1 × 4 process grid
   *
   CALL BLACS_GET(0, 0, icontxt)
   CALL BLACS_GRIDMAP(icontxt, usermap, 2, 2, 2)
             .
             .
             .
   *
   * Redefine the shape of your 2 × 2 process grid
   * to a 1 × 4 process grid
   *
   CALL BLACS_GET(icontxt, 10, newcontxt)
   CALL BLACS_GRIDMAP(newcontxt, usermap, 2, 1, 4)
   ```

3. Suppose you specified a total of 15 processes in your MP_PROCS environment variable, referred to as t_0 through t_{14}. You then called BLACS_GRIDMAP in your Fortran program, as follows:

   ```
      CALL BLACS_GRIDMAP (icontxt1,USERMAP,5,3,4)
   ```

   Where array USERMAP1 contained the following integer values:

   $$USERMAP1 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 8 & 9 & 10 & 11 \\ 4 & 5 & 6 & 7 \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

   then, the processes would be mapped into a 3 by 4 process grid as follows:

   Table 35. 3 by 4 process grid

$P_{p,q}$	**0**	**1**	**2**	**3**
0	t_0	t_1	t_2	t_3
1	t_8	t_9	t_{10}	t_{11}
2	t_4	t_5	t_6	t_7

   BLACS_GRIDMAP sets *icontxt1*. Use the value of *icontxt1* in any subsequent calls to Parallel ESSL to use this process grid.

   While the above process grid is active, another overlapping process grid can be defined. Suppose you then called BLACS_GRIDMAP in your Fortran program as follows:

   ```
      CALL BLACS_GRIDMAP(icontxt2, USERMAP2, 2, 2, 2)
   ```

   where USERMAP contains the following values:

```
           ┌         ┐
USERMAP2 = │  1   2  │
           │  10  11 │
           └         ┘
```

Then the processes would be mapped into a 2 by 2 process grid as follows:

Table 36. 2 by 2 process grid

$P_{p,q}$	**0**	**1**
0	t_1	t_2
1	t_{10}	t_{11}

BLACS_GRIDMAP will set *icontxt2*. Use the value of *icontxt2* in any subsequent calls to Parallel ESSL to use this process grid.

Notes:

a. In this example, process t_1 is mapped to P_{01} in the first grid and to P_{00} in the second grid.

b. Both grids can simultaneously be used in your program.

BLACS_GRIDEXIT

You call the BLACS_GRIDEXIT routine to release a BLACS context.

Syntax

Language	Call Statement
Fortran	CALL `BLACS_GRIDEXIT` (*icontxt*)
C	`blacs_gridexit` (&*icontxt*);

C++	extern “FORTRAN” void blacs_gridexit(const int &);
	`blacs_gridexit` (*icontxt*);

On Entry

icontxt

is the integer handle to the BLACS context indicating the BLACS context to be released.

Specified as: a fullword integer value, returned by BLACS_GRIDINIT or BLACS_GRIDMAP.

BLACS_EXIT

You call the BLACS_EXIT routine to release all the BLACS context and the memory allocated by the BLACS.

Syntax

Language	Call Statement
Fortran	CALL BLACS_EXIT (*continue*)
C	`blacs_exit` (*&continue*);

C++	extern "FORTRAN" void blacs_exit(const int &);
	`blacs_exit` (*continue*);

On Entry

continue

has the following meaning:

If *continue* = 0, all the BLACS context and memory allocated by the BLACS are released. In addition, Parallel ESSL calls MPI_Finalize to exit from MPI. There can only be one call to MPI_Finalize in your program. Therefore, at the end of your program, you should call BLACS_EXIT with *continue* = 0 or call MPI_Finalize directly.

If *continue* ≠ 0, the BLACS contexts and memory allocated by the BLACS are released, however, you can continue using MPI. When you are finished using MPI, you need to remember to call MPI_Finalize directly.

Specified as: a fullword integer.

Using Extrinsic Procedures—The Fortran 90 Sparse Linear Algebraic Equation Subroutines

In Fortran 90 programs, the Parallel ESSL sparse linear algebraic equation subroutines are invoked with the CALL statement, using the features of Fortran 90—generic interfaces, optional and keyword arguments, assumed-shape arrays, and modules.

The Fortran 90 sparse linear algebraic equation subroutines require that an explicit interface be provided for each extrinsic procedure entry in the scope where it is called, using an interface block. The interface blocks for the Parallel ESSL subroutines are provided for you in the module F90SPARSE, so you do not have to code the interface blocks yourself. In the beginning of your program, before any other specification statements, you must code the statement:

```
use f90sparse
```

This gives the XL Fortran compiler access to the interface blocks. For examples of where to code this statement in your program, see "Application Program Outline for the Fortran 90 Sparse Linear Algebraic Equations and Their Utilities" on page 97.

For further details on coding the CALL statement and other related aspects of Fortran 90 programs, see the Fortran manuals.

Setting Up the Parallel ESSL Header File for C and C++

Before you can call the Parallel ESSL subroutines from your C or C++ program, you must have the Parallel ESSL header file installed on your system. The Parallel ESSL header file allows you to code your function calls as described in Part 2 of this book. The Parallel ESSL header file is named `pessl.h`. You should check with your system support group to verify that the appropriate Parallel ESSL header file is installed.

C Programs

In the beginning of your C program, before you call any of the Parallel ESSL subroutines, **you must code the following statement for the Parallel ESSL header file:**

```
   #include <pessl.h>
```

C++ Programs

In the beginning of your C++ program, before you call any of the Parallel ESSL subroutines, **you must code the following statement for the Parallel ESSL header file:**

```
   #include <pessl.h>
```

Application Program Outline

For the Level 2 and 3 PBLAS, dense and banded linear algebraic equations, and eigensystem analysis and singular value analysis subroutines, this application program outline shows how you can use the BLACS to define a process grid, set up a Type-1 array descriptor, call a Parallel ESSL subroutine, and exit the BLACS. For a complete example, see Appendix B on page 999.

```
              .
              .
              .
*
* Determine my process number and the total number of available
* processes
*
      CALL BLACS_PINFO(IAM, NNODES)
*
* Define a process grid that is as close to square as possible

*
      NPROW = INT(SQRT(REAL(NNODES)))
      NPCOL = NNODES/NPROW
*
* Get the default system context
* Define the process grid
* Determine my process row and column index
*
      CALL BLACS_GET(0, 0, ICONTXT)
      CALL BLACS_GRIDINIT(ICONTXT, 'R', NPROW, NPCOL)
      CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

*
* Only call the Parallel ESSL subroutine if I am in the process grid
*
      IF (MYROW .LT. NPROW .AND. MYCOL .LT. NPCOL) THEN
*
* Setup input arrays, scalars, array descriptors, etc.
*
              .
              .
*
```

```
* NUMROC can be used to return the size of local arrays
* For example, here is one way to setup the descriptor vector for A
*
      DESC_A(1) = DTYPE_A
      DESC_A(2) = ICONTXT
      DESC_A(3) = M_A
      DESC_A(4) = N_A
      DESC_A(5) = MB_A
      DESC_A(6) = NB_A
      DESC_A(7) = RSRC_A
      DESC_A(8) = CSRC_A
      DESC_A(9) = MAX (1, NUMROC(DESC_A(3), DESC_A(5), MYROW, DESC_A(7), NPROW))

              .
              .
              .
*
*  CALL Parallel ESSL subroutine
*
       CALL PDTRAN(M, N, ALPHA, A, IA, JA, DESC_A, BETA, C, IC, JC, DESC_C)
*
* Process output arrays, scalars etc.

*
              .
              .
              .
*
* When finished with this process grid, release the process grid.
*
      CALL BLACS_GRIDEXIT(ICONTXT)
              .
      ENDIF

              .
              .
              .
              .
*
* At the end of the program, exit from the BLACS and MPI
*
      CALL BLACS_EXIT(0)
              .
              .
              .
      END
```

Application Program Outline for the Fortran 90 Sparse Linear Algebraic Equations and Their Utilities

The following is an outline for a application program that is calling the Fortran 90 sparse linear algebraic equation subroutines and their utilities. For a more complete example, see “Example—Using the Fortran 90 Sparse Subroutines” on page 649 or “Fortran 90 Sample Sparse Program” on page 1056.

```
USE F90SPARSE
      .
      .
      .
!User-defined subroutine
INTERFACE PARTS
   SUBROUTINE PARTS(...)
     INTEGER  GLOBAL_INDEX, N, NP

     INTEGER  NV
     INTEGER  PV(*)
   END SUBROUTINE PARTS
END INTERFACE
      .
      .
      .
!Define the process grid
CALL BLACS_GET (...)
CALL BLACS_GRIDINIT(...)
CALL BLACS_GRIDINFO(...)

      .
      .
      .
!Allocate space for and initialize array descriptor desc_a.
CALL PADALL(...)

!Allocate space and initialize some values
!for sparse matrix A.
CALL PSPALL(...)


!Allocate and build vectors b and x.
CALL PGEALL(...)

!Build the sparse matrix A with multiple calls to PSPINS.
!Each process has to call PSPINS as many times as
!necessary to insert the local rows it owns.
!Update array descriptor desc_a.
do
  CALL PSPINS(...)
enddo

!Build vectors b and x with multiple calls to PGEINS.
!Each process has to call PGEINS as many times as
!necessary to insert the local elements it owns.
do
   CALL PGEINS(...)
enddo

!Finalize the sparse matrix A and array descriptor desc_a
CALL PSPASB(...)

!Finalize the vectors b and x.
!Matrix A and array descriptor desc_a
!must be finalized before calling PGEASB.
CALL PGEASB(...)

!Prepare preconditioner
CALL PSPGPR(...)
```

```
!Call solver
CALL PSPGIS(...)

!Cleanup and exit.
!Deallocate vectors b and x
!Deallocate matrix A and the preconditioner data structure PRC
CALL PGEFREE(...)
CALL PSPFREE(...)

!Deallocate the array descriptor desc_a only after
!vectors b and x, and matrix A are deallocated.
CALL PADFREE(...)
      .
      .
      .
CALL BLACS_GRIDEXIT(...)
CALL BLACS_EXIT(...)
```

Application Program Outline for the Fortran 77 Sparse Linear Algebraic Equations and Their Utilities

The following is an outline for a application program that is calling the Fortran 77 sparse linear algebraic equation subroutines and their utilities. For a complete example, see "Example—Using the Fortran 77 Sparse Subroutines" on page 685 or "Fortran 77 Sample Sparse Program" on page 1066.

```
      .
      .
      .
EXTERNAL PARTS
      .
      .
      .
!Define the process grid
CALL BLACS_GET (...)
CALL BLACS_GRIDINIT(...)
CALL BLACS_GRIDINFO(...)
      .
      .
      .
!Initialize array descriptor desc_a.
CALL PADINIT(...)

!Initialize some values
!for sparse matrix A.
CALL PDSPINIT(...)

!Build the sparse matrix A with multiple calls to PDSPINS.
!Each process has to call PDSPINS as many times as
!necessary to insert the local rows it owns.
!Update array descriptor desc_a.
do
  CALL PDSPINS(...)
enddo
```

```
!Build vectors b and x with multiple calls to PDGEINS.
!Each process has to call PDGEINS as many times as
!necessary to insert the local elements it owns.
do
   CALL PDGEINS(...)
enddo

!Finalize the sparse matrix A and array descriptor desc_a
CALL PDSPASB(...)


!Finalize the vectors b and x.
CALL PDGEASB(...)

!Prepare preconditioner
CALL PDSPGPR(...)

!Call solver
CALL PDSPGIS(...)

      .
      .
      .
CALL BLACS_GRIDEXIT(...)
CALL BLACS_EXIT(...)
```

Running Your Message Passing Program

This section describes **both the Parallel ESSL-specific and ESSL-specific changes** you need to make to your PE job procedures for compiling, linking, and running your message passing program. For details on general PE job procedures, see the appropriate *Parallel Environment: Operation and Use* manual.

You can use any procedures you are currently using to compile, link, and run your Fortran, C, and C++ programs, as long as you make the necessary modifications required by Parallel ESSL.

Notes:

1. The default search path for the Parallel ESSL and ESSL libraries is: `/usr/lib`. (Note that /lib is a symbolic link to /usr/lib.)

   If the libraries are installed somewhere else, add the path name of that directory to the beginning of the **LIBPATH** environment variable, being careful to keep /usr/lib in the path. The correct **LIBPATH** setting is needed both for linking and executing the program.

   For example, if you are using POWER2 nodes only, and you installed the Parallel ESSL libraries in /home/me/lib you would issue `ksh` commands similar to the following in order to compile and link a program:

   ```
   LIBPATH=/home/me/lib:/usr/lib
   export LIBPATH
   mpxlf -o myprog myprog.f -lesslp2 -lpesslp2 -lblacsp2
   ```

   After setting the **LIBPATH** command, the /home/me/lib directory is the directory that gets searched first for the necessary libraries. This same search criteria is used at both compile and link time and run time.

2. The ESSL and Parallel ESSL libraries are shared libraries and must be used in conjunction with each other. Equivalent subroutines with the same names in other libraries (such as, libblas.a) will not be used even if they are specified on the command line in place of the ESSL library.
3. In your job procedures, you must use only the allowable compilers and libraries listed in Table 1 on page 7 for AIX.

Dynamic Linking Versus Static Linking

Only dynamic linking is supported for programs using Parallel ESSL. For details about how to do this, see the appropriate *Parallel Environment: Operation and Use* manual.

Fortran Program Procedures

You do not need to modify your existing Fortran compilation procedures when using Parallel ESSL. For example, you can use:

ESSL Library Name	**Command**
SMP or Thread-Tolerant	`mpxlf_r -c` *xyz*`.f`
POWER2 or POWER	`mpxlf -c` *xyz*`.f`

where *xyz*`.f` is the name of your Fortran program.

When linking and running your program, you need to modify your existing PE job procedures for Parallel ESSL, to set up the necessary libraries. If you are accessing Parallel ESSL from a Fortran program, you can compile and link using the following command:

ESSL Library Name	**Command**
SMP	`mpxlf_r -O` *xyz*`.f -lesslsmp -lpesslsmp -lblacssmp`
Thread-Tolerant	`mpxlf_r -O` *xyz*`.f -lesslp2_r -lpesslp2_t -lblacsp2_t`
POWER2	`mpxlf -O` *xyz*`.f -lesslp2 -lpesslp2 -lblacsp2`
POWER	`mpxlf -O` *xyz*`.f -lessl -lpessl -lblacs`

where *xyz*`.f` is the name of your Fortran program.

If you want to compile and link your Fortran program in separate steps, you can use the following commands:

ESSL Library Name	**Command**

SMP	`mpxlf_r -O -c` *xyz*`.f`
	`mpxlf_r` *xyz*`.o -lesslsmp -lpesslsmp -lblacssmp`

Thread-Tolerant	`mpxlf_r -O -c` *xyz*`.f`
	`mpxlf_r` *xyz*`.o -lesslp2_r -lpesslp2_t -lblacsp2_t`

ESSL Library Name	Command

POWER2	`mpxlf  -O  -c xyz.f`
	`mpxlf  xyz.o  -lesslp2 -lpesslp2 -lblacsp2`

POWER	`mpxlf  -O  -c xyz.f`
	`mpxlf  xyz.o  -lessl -lpessl -lblacs`

where *xyz*.f is the name of your Fortran program, and *xyz*.o is the name of your object file.

Parallel ESSL supports the XL Fortran compile-time option **-qextname**. For details, see the Fortran manuals.

An example of a makefile is shown in "Makefile (Message Passing)" on page 1097.

C Program Procedures

The Parallel ESSL header file `pessl.h`, used for C and C++ programs, is installed in the `/usr/include` directory. You do not need to modify your existing C compilation procedures when using Parallel ESSL, unless you want to specify your own definitions for complex data.

If you want to specify your own definitions for short- and long-precision complex data, add `-D_CMPLX` and `-D_DCMPLX`, respectively, to your compile command, as shown here:

ESSL Library Name	Command
SMP or Thread-Tolerant	`mpcc_r  -c  -D_CMPLX  -D_DCMPLX  xyz.c`
POWER2 or POWER	`mpcc  -c  -D_CMPLX  -D_DCMPLX  xyz.c`

where *xyz*.c is the name of your C program. Otherwise, you automatically use the definitions of short- and long-precision complex data provided in the Parallel ESSL header file.

When linking and running your program, you need to modify your existing PE job procedures for Parallel ESSL, to set up the necessary libraries. If you are accessing Parallel ESSL from a C program, you can compile and link using the following command:

ESSL Library Name	Command
SMP	`mpcc_r  -O  xyz.c  -lesslsmp -lpesslsmp  -lblacssmp`
Thread-Tolerant	`mpcc_r  -O xyz.c  -lesslp2_r -lpesslp2_t -lblacsp2_t`
POWER2	`mpcc -O  xyz.c  -lesslp2 -lpesslp2 -lblacsp2`
POWER	`mpcc  -O  xyz.c  -lessl -lpessl -lblacs`

where *xyz*.c is the name of your C program.

If you want to compile and link your C program in separate steps, use the following commands:

ESSL Library Name	Command

SMP	`mpcc_r -O -c xyz.c`
	`mpcc_r xyz.o -lesslsmp -lpesslsmp -lblacssmp`

Thread-Tolerant	`mpcc_r -O -c xyz.c`
	`mpcc_r xyz.o -lesslp2_r -lpesslp2_t -lblacsp2_t`

POWER2	`mpcc -O -c xyz.c`
	`mpcc xyz.o -lesslp2 -lpesslp2 -lblacsp2`

POWER	`mpcc -O -c xyz.c`
	`mpcc xyz.o -lessl -lpessl -lblacs`

where *xyz*.c is the name of your C program and *xyz*.o is the name of your object file.

In the above cases, you automatically use the definitions of short- and long-precision complex data provided in the Parallel ESSL header file. If you prefer to specify your own definitions for short- and long-precision complex data, add -D_CMPLX and -D_DCMPLX, respectively, to your commands.

ESSL Library Name	Command
SMP	`mpcc_r -O -D_CMPLX -D_DCMPLX xyz.c -lesslsmp -lpesslsmp -lblacssmp`
Thread-Tolerant	`mpcc_r -O -D_CMPLX -D_DCMPLX xyz.c -lesslp2_r -lpesslp2_t -lblacsp2_t`
POWER2	`mpcc -O -D_CMPLX -D_DCMPLX xyz.c -lesslp2 -lpesslp2 -lblacsp2`
POWER	`mpcc -O -D_CMPLX -D_DCMPLX xyz.c -lessl -lpessl -lblacs`

where *xyz*.c is the name of your C program.

C++ Program Procedures

The Parallel ESSL header file pessl.h, used for C and C++ programs, is installed in the /usr/include directory. You do not need to modify your existing C++ compilation procedures when using Parallel ESSL, unless you want to specify your own definitions for complex data.

If a C++ application containing calls to Parallel ESSL subroutines is compiled using IBM C++ for AIX, Version 3.6.4 or later, the compiler option **-qnocinc=/usr/include/pessl** must be specified.

If you want to specify your own definition for short-precision complex data, add -D_CMPLX to your command, as shown here:

ESSL Library Name	Command
SMP or Thread-Tolerant	`mpCC_r -c -D_CMPLX -D_DCMPLX xyz.C -qnocinc=/usr/include/pessl`
POWER2 or POWER	`mpCC -c -D_CMPLX -D_DCMPLX xyz.C -qnocinc=/usr/include/pessl`

where *xyz*.C is the name of your C++ program. Otherwise, you automatically use the definition of short-precision complex data provided in the Parallel ESSL header file.

When linking and running your program, you need to modify your existing PE job procedures for Parallel ESSL, to set up the necessary libraries. If you are accessing Parallel ESSL from a C++ program, you can compile and link using the following command:

ESSL Library Name	**Command**
SMP	`mpCC_r -O xyz.C -lesslsmp -lpesslsmp -lblacssmp -qnocinc=/usr/include/pessl`
Thread-Tolerant	`mpCC_r -O xyz.C -lesslp2_r -lpesslp2_t -lblacsp2_t -qnocinc=/usr/include/pessl`
POWER2	`mpCC -O xyz.C -lesslp2 -lpesslp2 -lblacsp2 -qnocinc=/usr/include/pessl`
POWER	`mpCC -O xyz.C -lessl -lpessl -lblacs -qnocinc=/usr/include/pessl`

where *xyz*.C is the name of your C++ program.

If you want to compile and link your C++ program in separate steps, you can use the following commands:

ESSL Library Name	**Command**

SMP	`mpCC_r -O -c xyz.C -qnocinc=/usr/include/pessl`
	`mpCC_r xyz.o -lesslsmp -lpesslsmp -lblacssmp`

Thread-Tolerant	`mpCC_r -O -c xyz.C -qnocinc=/usr/include/pessl`
	`mpCC_r xyz.o -lesslp2_r -lpesslp2_t -lblacsp2_t`

POWER2	`mpCC -O -c xyz.C -qnocinc=/usr/include/pessl`
	`mpCC xyz.o -lesslp2 -lpesslp2 -lblacsp2`

POWER	`mpCC -O -c xyz.C -qnocinc=/usr/include/pessl`
	`mpCC xyz.o -lessl -lpessl -lblacs`

where *xyz*.C is the name of your C++ program, and *xyz*.o is the name of your object file.

In the above cases, you automatically use the definition of short-precision complex data provided in the Parallel ESSL header file. If you prefer to specify your own definition for short-precision complex data, add `-D_CMPLX` to your commands.

ESSL Library Name	**Command**
SMP	`mpCC_r -O -D_CMPLX xyz.C -lesslsmp -lpesslsmp -lblacssmp -qnocinc=/usr/include/pessl`
Thread-Tolerant	`mpCC_r -O -D_CMPLX xyz.C -lesslp2_r -lpesslp2_t -lblacsp2_t -qnocinc=/usr/include/pessl`
POWER2	`mpCC -O -D_CMPLX xyz.C -lesslp2 -lpesslp2 -lblacsp2 -qnocinc=/usr/include/pessl`
POWER	`mpCC -O -D_CMPLX xyz.C -lessl -lpessl -lblacs -qnocinc=/usr/include/pessl`

where *xyz*.C is the name of your C++ program.

Coding Your HPF Program

This section contains Parallel ESSL-specific application program coding requirements and considerations for HPF programs, as illustrated in the sample HPF programs in “Sample HPF Programs” on page 107. For an example of the use of Parallel ESSL in a sample HPF application program solving a thermal diffusion problem, see Appendix B on page 999.

For further details on coding the CALL statement and other related aspects of HPF programs, see references [11], [17], [30], [31], [41], [44], and [45], as well as the HPF manuals.

Coding the Calling Sequences

In HPF programs, the Parallel ESSL subroutines are invoked with the CALL statement, using the features of Fortran 90—generic interfaces, optional and keyword arguments, assumed-shape arrays, and modules. This section explains how these are used.

Using Extrinsic Procedures—The Parallel ESSL Subroutines

The Parallel ESSL subroutines are HPF_LOCAL extrinsic procedures, conforming to all restrictions imposed by HPF.

HPF requires that an explicit interface be provided for each extrinsic procedure entry in the scope where it is called, using an interface block. The interface blocks for the Parallel ESSL subroutines are provided for you in the module PESSL_HPF, so you do not have to code the interface blocks yourself. In the beginning of your HPF program, before any other specification statements, you must code the statement:

```
use pessl_hpf
```

This gives the XL HPF compiler access to the interface blocks. It also enables the XL HPF compiler to detect, at compile time, certain errors in the calls to the Parallel ESSL subroutines. For examples of where to code this statement in your HPF program, see Figure 9 on page 108 and Figure 10 on page 109.

To understand the redistribution of data that occurs when using the Parallel ESSL HPF subroutines and the performance implications, see “Coding Tips for Optimizing Parallel Performance” on page 83.

Generic Interfaces

The Parallel ESSL subroutines can be accessed through generic interfaces, where a single name covers one or more specific subroutines, whose arguments may differ in data type, precision, or rank (vector or matrix). The XL HPF compiler chooses the specific Parallel ESSL subroutine whose dummy arguments exactly match the characteristics of the actual arguments in the generic call statement. The use of generic interfaces for Parallel ESSL is dependent upon your coding the `use` statement described in “Using Extrinsic Procedures—The Parallel ESSL Subroutines.”

The generic name used for each Parallel ESSL HPF subroutine is indicated in the “Syntax” section for each subroutine in Part 3 of this book.

For examples of how to specify a generic name in a Parallel ESSL call statement, see the sample programs in Figure 9 on page 108 and Figure 10 on page 109.

Optional and Keyword Arguments

As with standard Fortran 90, Parallel ESSL supports both optional and keyword arguments in its calling sequences.

Optional Arguments: Optional arguments do not have to be specified in the calling sequence when the Parallel ESSL subroutine is called. Optional arguments are indicated in the “Syntax” section for the subroutine in Part 3 of this book. Where optional arguments apply, the syntax shows two call statements in succession—the first lists only the required arguments, and the second lists both the required and optional arguments. For example, in the syntax for FFT, shown in “FFT—Fourier Transforms in Two Dimensions” on page 968, there are two call statements for case 1. The first call statement includes only the required argument *x*, and the second statement includes *x*, plus the optional arguments *y*, *transpose*, *isign*, and *scale*.

Keyword Arguments: Keywords are dummy argument names. You must **not** specify positional arguments after you specify the first keyword argument. Keyword arguments can be arranged in any order, following the positional arguments in the calling sequence. You may follow a (possibly empty) positional argument list by a keyword argument list. The dummy argument names are listed in the calling sequences in the “Syntax” section for each subroutine in Part 3 of this book. For example, in the syntax for case 1 in FFT, shown in “FFT—Fourier Transforms in Two Dimensions” on page 968, the dummy argument names are *x*, *y*, *transpose*, *isign*, *scale*.

Applying these rules, you could code any of the following call statements, and more, for the call statement shown in Figure 10 on page 109.

```
call fft(xc1, yc1, isign=isignc)
call fft(xc1, y=yc1, isign=isignc)
call fft(xc1, isign=isignc, y=yc1)
call fft(x=xc1, y=yc1, isign=isignc)
call fft(isign=isignc, y=yc1, x=xc1)
call fft(scale=scalec, isign=isignc, y=yc1, x=xc1)
call fft(xc1, yc1, transpose='T', isign=isignc)
call fft(xc1, yc1, 'T', isign=isignc)
call fft(xc1, yc1, 'T', isignc, scalec)
call fft(xc1, yc1, 'T', isignc)
call fft(xc1(1:n1,1:n2), yc1(1:n2,1:n1), isign=isignc)
```

Figure 8. Call Statements for FFT

Assumed-Shape Arrays and Problem Sizes

All array arguments in the Parallel ESSL HPF subroutines are assumed-shape arrays of one dimension (shape(:)), two dimensions (shape(:,:)), or three dimensions (shape(:,:,:)). Depending on the subroutine, you have several ways to specify, in the calling sequence, the vectors, matrices, and sequences that you want Parallel ESSL to use.

The assumed-shape arrays must have the exact shape required for the problem. For example, SYMM computes a matrix multiplication using three matrices ***A***, ***B***, and ***C***. The required shape of these assumed-shape arrays depends on the value of the *side* argument. When *side* = `'L'`, Parallel ESSL computes ***C*** = α***AB***+β***C***. For this case, the sizes of the assumed-shape arrays must satisfy the following:

- size(*a*,1)=size(*a*,2)=size(*b*,1)=size(*c*,1)
- size(*b*,2)=size(*c*,2)

These required sizes of the assumed-shape arrays are described in the “Notes” section for each subroutine in Part 3 of this book.

You have two ways to code the vectors, matrices, and sequences in the calling sequences:

- If you want to solve a problem **using the entire assumed-shape array**, specify only the vector, matrix, or sequence argument in the calling sequence, such as shown here for *a*, *b*, and *c*:

  ```
  call symm(alpha, a, b, beta, c, uplo='U', side='L')
  ```

  Most of the examples in Part 3 of this book use this technique—for example, see “Example 1” on page 825.

- If you want to solve a problem **using a submatrix**, specify the vector, matrix, or sequence in the calling sequence using Fortran 90 array sections—for example, `A(IA:IA+M-1,JA:JA+N-1)` or `X(IX:IX+N-1)`, such as shown here for *a*, *b*, and *c*:

```
call symm(alpha, a(1:10,1:10), b(11:20,1:20), beta, c(1:10,1:20), uplo='U', side='L')
```

  Some of the examples in Part 3 of this book use this technique—for example, see “Example 3” on page 818 and “Example 4” on page 819.

Coding the Directives for Distributing Your Data

Parallel ESSL supports the same data distribution techniques as HPF:

- Block-cyclic—used for the PBLAS, Dense Linear Algebraic Equations, and Eigensystems Analysis subroutines. An example of coding the directives for block-cyclic distribution is shown in Figure 9 on page 108.
- Block—used for the Fourier Transforms, Banded Linear Algebraic Equations, and Random Number Generation subroutines. An example of coding the directives for block distribution is shown in Figure 10 on page 109.

For a description of the various ways to code your directives for block-cyclic and block distribution, see “Distributing Data in an HPF Program” on page 79.

Note: XL HPF only supports CYCLIC(*n*) distributions for n > 1 in HPF_LOCAL procedures, such as the Parallel ESSL HPF subroutines. The module PESSL_HPF contains the CYCLIC(*n*) data distribution directives corresponding to the block sizes listed in Table 32 on page 86.

Sample HPF Programs

Following is a sample program calling a PBLAS subroutine, which uses block-cyclic distribution. It corresponds to “Example 1” on page 825 for computing equation 2 (***C*** $\leftarrow$ α***BA***+β***C***) for SYMM on a 2 × 2 process grid. See the example for details about the input and output data for this sample program.

An example of the usage of a PBLAS subroutine is also shown in Appendix B. See “Program Main (HPF)” on page 1036.

```
 program example_symm
      use pessl_hpf
      implicit none
      real(kind(1d0)), dimension(8,8)          :: a
      real(kind(1d0)), dimension(16,8)         :: b
      real(kind(1d0)), dimension(16,8)         :: c
      integer                                  :: i
      integer                                  :: j
! ...
! ... Initialize a and b with the data
! ... shown in Example 1 for SYMM
! ...(Steps not shown here)
! ...
! ... Block Cyclic Distribution on a 2d Process Grid
! ...
!hpf$ processors proc(2,2)
!hpf$ distribute(cyclic, cyclic) onto proc     :: a, b, c
! ...
! ... Call Parallel ESSL HPF Subroutine
! ...
      call symm(alpha=1.0d0, a=a, b=b, beta=0.0d0, c=c,            &
     &                 uplo='u', side='r')
 end program example_symm
```

Figure 9. PBLAS Sample Program Using Block-Cyclic Distribution

Following is a sample program calling a Fourier transform subroutine, which uses block distribution. It corresponds to “Example 1” on page 973 for the two-dimensional complex-to-complex Fourier transform computation for FFT.

An example of the usage of a Fourier transform subroutine is also shown in Appendix B. See “Module Fourier (HPF)” on page 1046.

```
 program xpfft
      use pessl_hpf
      implicit none
      integer, parameter                       :: isignc = -1
      integer, parameter                       :: n1 = 8
      integer, parameter                       :: n2 = 6
      complex(kind(1d0)), dimension(n1,n2)     :: xc1
      complex(kind(1d0)), dimension(n2,n1)     :: yc1
      real(kind(1d0))                          :: scalec
      integer                                  :: i
      integer                                  :: j
!hpf$ processors p1(number_of_processors())
!hpf$ distribute (*, block) onto p1           :: xc1
!hpf$ distribute (*, block) onto p1           :: yc1
! ...
! ... Setup input
! ...
      xc1      = cmplx(0.0d0,0.0d0)
      xc1(1,1) = cmplx(float(n1*n2),0.0d0)
      scalec = 1.0d0/(float(n1*n2))
! ...
! ... Call Parallel ESSL HPF Subroutine
! ...
      call fft(xc1, yc1, isign=isignc, scale=scalec)
! ...
! ... Check the answers
! ...
      do j = 1, n1
       do i = 1, n2
       if (yc1(i,j) .ne. (1.0d0,0.0d0)) write(6,*) 'ERROR:', i, j, yc1(i,j)
       enddo
      endo
 end program xpfft
```

Figure 10. Fourier Transform Sample Program Using Block Distribution

Running Your HPF Program

This section describes **both the Parallel ESSL-specific and ESSL for AIX-specific changes** you need to make to your HPF job procedures for compiling, linking, and running your HPF program. For details on general HPF procedures, see the HPF manuals.

You can use any procedures you are currently using to compile, link, and run your HPF programs, as long as you make the necessary modifications required by Parallel ESSL.

Dynamic Linking Versus Static Linking

When using an `xlhpf` or `xlhpf90` command, only dynamic linking is supported for programs using Parallel ESSL. For details about how to do this, see the appropriate *Parallel Environment: Operation and Use* and HPF manuals.

HPF Program Procedures

You do not need to modify your existing XL HPF compilation procedures when using Parallel ESSL. For example, you can use:

ESSL Library Name	**Command**
POWER2 or POWER	`xlhpf  -c  xyz.f`

where *xyz*.f is the name of your HPF program.

Note: You can use either `xlhpf` or `xlhpf90`, as needed, in the commands shown below.

When linking and running your program, you need to modify your existing HPF job procedures for Parallel ESSL, to set up the necessary libraries. If you are accessing Parallel ESSL from an HPF program, you can compile and link using the following command:

ESSL Library Name	**Command**
POWER2	`xlhpf   -O  xyz.f  -lesslp2  -lpesslp2  -lblacsp2  -lpesslhpfp2`
POWER	`xlhpf   -O  xyz.f -lessl -lpessl  -lblacs  -lpesslhpf`

where *xyz*.f is the name of your Fortran program.

If you want to compile and link your HPF program in separate steps, you can use the following commands:

ESSL Library Name	**Command**

POWER2	`xlhpf  -O  -c  xyz.f`
	`xlhpf  xyz.o  -lesslp2  -lpesslp2  -lblacsp2  -lpesslhpfp2`

POWER	`xlhpf  -O  -c  xyz.f`
	`xlhpf  xyz.o  -lessl  -lpessl  -lblacs  -lpesslhpf`

where *xyz*.f is the name of your HPF program, and *xyz*.o is the name of your object file.

Parallel ESSL supports the XL HPF compile-time option **-qextname**. For details, see the XL HPF manuals.

Notes:

1. The default search path for the Parallel ESSL and ESSL for AIX libraries is: `/usr/lib`. (Note that /lib is a symbolic link to /usr/lib.)

   If the libraries are installed somewhere else, add the path name of that directory to the beginning of the **LIBPATH** environment variable, being careful to keep /usr/lib in the path. The correct **LIBPATH** setting is needed both for linking and executing the program.

For example, if you are using POWER2 nodes only, and you installed the Parallel ESSL libraries in /home/me/lib you would issue `ksh` commands similar to the following in order to compile and link a program:

```
LIBPATH=/home/me/lib:/usr/lib
export LIBPATH
xlhpf -o myprog myprog.f -lesslp2 -lpesslp2 -lblacsp2 -lpesslhpfp2
```

After setting the **LIBPATH** command, the /home/me/lib directory is the directory that gets searched first for the necessary libraries. This same search criteria is used at both compile and link time and run time.

2. **When you specify** `-lesslp2`**,** `-lpesslp2`**,** `-lblacsp2`**, and** `-lpesslhpfp2` **in your commands, then all nodes that Parallel ESSL is running on must be POWER2 nodes.** If you are running on both POWER and POWER2 nodes, you must specify `-lessl`, `-lpessl`, `-lblacs`, and `-lpesslhpf`.
3. The ESSL for AIX and Parallel ESSL libraries are shared libraries and must be used in conjunction with each other. Equivalent subroutines with the same names in other libraries (such as, libblas.a) will not be used even if they are specified on the command line in place of the ESSL for AIX library.
4. In your job procedures, you must use only the allowable compilers and libraries listed in Table 1 on page 7 for AIX.

An example of a makefile for an HPF program is shown in Appendix B. See “Makefile (Message Passing)” on page 1097.

Chapter 4. Migrating Your Programs

This chapter explains many aspects of migrating your application programs.

Migrating to Parallel ESSL Version 2 Release 1.2

The format of the output from PDDTTRF and DTTRF has changed. Therefore, the factorization and solve must be performed using Parallel ESSL Version 2 Release 1.2.

Banded Linear Algebraic Equations subroutines PDPBSV, PDGTSV, PDDTSV and PDPTSV have been modified for the case where N is greater than zero and NRHS is zero so that the matrix is factored. Previously, this was a quick return condition and the matrix was not factored. For all other subroutines, no changes to your application programs are required if you are migrating from Parallel ESSL Version 2 Release 1.1 to Parallel ESSL Version 2 Release 1.2.

Migrating to Parallel ESSL Version 2 Release 1.1

No changes to your application programs are required if you are migrating from Parallel ESSL Version 2 Release 1 to Parallel ESSL Version 2 Release 1.1.

Migrating to Parallel ESSL Version 2.1

This section explains how to update your message passing application programs when migrating from an earlier release to Parallel ESSL Version 2.1.

All application programs previously migrated to accommodate the new array descriptor, can run unchanged with Parallel ESSL Version 2.1. However, if you were dependent upon the PESSL_DESC_TYPE environment variable, you must change the array descriptors as described in “Array Descriptor Considerations” on page 114.

Subroutines with the option of dynamic allocation have been updated to be consistent with ScaLAPACK 1.5. You do not need to update your application programs unless you choose to exploit the new capability.

The message-passing and HPF tridiagonal subroutines have been updated to be consistent with ScaLAPACK 1.5. If Parallel ESSL detects a computational error, the value returned in *info* is the process number where the error occurred. Previously, the index of the pivot where the matrix failed was returned in *info*. For the message-passing tridiagonal subroutines, the scope of *info* is now global. You do not have to make any modifications to your existing programs that call these subroutines. See the subroutines descriptions for specific details.

HPF application programs that run using Parallel ESSL Version 1.2 can be run unchanged using Parallel ESSL Version 2.1.

Array Descriptor Considerations

When using Parallel ESSL, you must code your new application programs using the array descriptors described in Chapter 2 on page 17. Also, if you were dependent upon the PESSL_DESC_TYPE environment variable, you must update any existing application programs to use the array descriptors described in Chapter 2 on page 17.

For more details on the array descriptors and how they are used for data distribution, see “Specifying and Distributing Data in a Message Passing Program” on page 23.

Type-1 Array Descriptor

A field DTYPE_ is at location 1, and the CTXT_ field, previously at location 7, is moved to location 2. This causes all other fields to move down one or more locations in the array descriptor. The format of the array descriptor is shown in Table 16 on page 25.

Type-501 and -502 Array Descriptors

The field at location 1 is renamed DTYPE_, and the CTXT_ field, previously at location 5, is moved to location 2. This causes some of the fields to move down one location in the array descriptor. The format of the array descriptors is shown in Table 21 on page 29 and Table 22 on page 29.

Future Migration Considerations for Array Descriptors

To minimize coding changes in the future, due to changes in the array descriptors, consider referencing the fields in the array descriptors symbolically in your program. For an example of this technique, see the Message Passing sample program in Appendix B on page 999.

Migrating from ScaLAPACK 1.5 to Parallel ESSL Version 2.1

If you are currently using the ScaLAPACK 1.5 offerings from the Oak Ridge National Laboratory, Parallel ESSL Version 2.1 uses compatible calling sequences with this version of ScaLAPACK.

Chapter 5. Using Error Handling

This chapter provides the following information for your use in dealing with errors:

- How to obtain IBM support.
- What to do about NLS (National Language Support) problems.
- A description of the different types of errors that can occur in Parallel ESSL. It explains what happens when an error occurs and, in some instances, how you can use error handling to obtain further information.
- All of the Parallel ESSL error messages are categorized into the different error types. There is also a description of the error message format.

Where to Find More Information About Errors

Information about errors and how to handle them can be found in the following places:

- Specific errors associated with each Parallel ESSL subroutine are listed under “Error Conditions” in each subroutine description in Part 2 and 3 of this book.
- Diagnostic procedures for errors associated with ESSL are provided in the *ESSL Version 3 Guide and Reference* manual.

Getting Help from IBM Support

Should you require help from IBM in resolving a Parallel ESSL problem, report it and provide the following information, if available and appropriate.

1. Your customer number
2. The Parallel ESSL program number, 5765-C41
3. The version of the AIX operating system that you are running on. To get this information, enter the following command:

   ```
   oslevel
   ```

4. The names and versions of key products being run. To get this information, enter the following command:

   ```
   lslpp  -h product
   ```

   where:

Table 37 (Page 1 of 2). Product Names when Using AIX Version 4

Product	Descriptive Name
essl.*	ESSL for AIX
pessl.*	Parallel ESSL for AIX
ppe.poe	Parallel Operating Environment
xlfrte	XL Fortran Run-Time Environment
xlfcmp	XL Fortran Compiler
xlhpf.rte	XL HPF Run-Time Environment

Table 37 (Page 2 of 2). Product Names when Using AIX Version 4

Product	Descriptive Name
xlhpf.cmp	XL HPF Compiler
vac.C	C for AIX Compiler
ibmcxx.cmp	IBM C, C++ Version 3.6 Compiler

5. The message that is returned when an error is detected.
6. Any error message relating to core dumps.
7. The compiler listings, including compiler options in effect, and any run-time listings produced
8. Program changes made in comparison with a previous successful run
9. A small test case demonstrating the problem using the minimum number of statements and variables, including input data

Consult your IBM Service representative for more assistance.

National Language Support

For National Language Support (NLS), all Parallel ESSL subroutines display messages located in externalized message catalogs. English versions of the message catalogs are shipped with the Parallel ESSL product, but your site may be using its own translated message catalogs. The AIX environment variable **NLSPATH** is used by the various Parallel ESSL subroutines to find the appropriate message catalog. **NLSPATH** specifies a list of directories to search for message catalogs. The directories are searched, in the order listed, to locate the message catalog. In resolving the path to the message catalog, **NLSPATH** is affected by the value of the environment variables **LC_MESSAGES** and **LANG**. If you get an error saying that a message catalog is not found and want the default message catalog, enter the following:

```
export NLSPATH=/usr/lib/nls/msg/%L/%N

export LANG=C
```

The Parallel ESSL message catalogs are in English, and are located in the following directories:

```
/usr/lib/nls/msg/C
/usr/lib/nls/msg/En_US
/usr/lib/nls/msg/en_US
```

If your site is using its own translations of the message catalogs, consult your system administrator for the appropriate value of **NLSPATH** or **LANG**. For additional information on NLS and message catalogs, see *IBM AIX for RS/6000 General Programming Concepts*.

If Parallel ESSL cannot successfully find a message, Parallel ESSL returns message 799, indicating which message could not be located. Message 799 is described in “Miscellaneous Error Messages (700-799)” on page 153.

PESSL_ERROR_SYNC Environment Variable

The PESSL_ERROR_SYNC environment variable allows you to enable and disable error synchronization. If error synchronization is disabled, the first process containing input-argument error(s) to finish computing, issues its error message(s) and terminates Parallel ESSL processing on all processes. Therefore, you should only disable error synchronization when your application program is debugged.

```
PESSL_ERROR_SYNC=no
        -or-
PESSL_ERROR_SYNC=NO

export PESSL_ERROR_SYNC
```

This causes Parallel ESSL to disable error synchronization in all calls to the Parallel ESSL subroutines.

If you do not set the environment variable or you set something other than `'no'` or `'NO'`, Parallel ESSL uses error synchronization in all calls to the Parallel ESSL subroutines.

Dealing with Errors

At run time, you can encounter a number of different types of errors that are specifically related to the use of the Parallel ESSL subroutines:

- Program exceptions
- Input-argument errors (001-299, 800-999)
- Computational errors (300-399)
- Resource errors (400-499)
- Communication errors (500-599)
- Miscellaneous errors (700-799)

This section explains what causes these errors, what happens when they occur (all are terminating, except computational errors), and what you can do to fix them.

This section also explains what to do when you receive informational and attention messages (600-699).

Program Exceptions

The program exceptions you can encounter in Parallel ESSL are described in the RS/6000 architecture manuals. For details, see:

- The *ANSI/IEEE Standard for Binary Floating-Point Arithmetic, ANSI/IEEE Standard 754-1985*
- The *IBM RS/6000 POWERstation and POWERserver Hardware Technical Reference Information—General Architectures* manual

Input-Argument Errors

This section describes how Parallel ESSL implements input-argument error checking when error synchronization is enabled. For more information on the PESSL_ERROR_SYNC environment variable, which allows you to enable or disable error synchronization, see “PESSL_ERROR_SYNC Environment Variable.”

Two types of input-argument error checking may be performed:

- First, on each participating process, Parallel ESSL checks the validity of most input-arguments in multiple stages.

  When all the input-arguments in one stage are valid, Parallel ESSL checks the validity of the input-arguments in the next stage, and so on. (The number of errors and stages that can occur for each subroutine are listed under its “Error Conditions” section, which is in Part 2 and 3 of this book.)

  When a message passing input argument is not valid on all participating processes in the parallel environment, a single comprehensive error message is issued, rather than one for each process. (This is indicated in the error message by Process(−1,−1).) Otherwise, an error message is issued from each process where the discrepancy occurred. When an HPF input-argument is not valid, an error message is issued from one or more processes.

  Parallel ESSL then terminates your program on all processes, and any arguments in the stages that follow are not checked. When this occurs, you should use standard programming techniques to diagnose and fix the errors.

- Next, Linear Algebraic Equations and Eigensystem Analysis subroutines check to ensure that global scalar arguments are the same on all participating processes.

  If the value of the global scalar argument on all processes except P_{00} does not match the value of the argument at process P_{00}, a single error message is issued. (This is indicated in the error message by Process(−1,−1).) Otherwise, an error message is issued from each process where the discrepancy occurred. Parallel ESSL then terminates your program on all processes, and you should use standard programming techniques to diagnose and fix the errors.

  For all other Parallel ESSL subroutines, the global scalar arguments are not checked to ensure they are the same for all processes.

How This Differs from ESSL for AIX:

The capabilities of ERRSET, ERRSAV, and ERRSTR, supported in ESSL for AIX, are not provided in Parallel ESSL.

Using the capabilities of ERRSET, ERRSAV, and ERRSTR with your ESSL for AIX subroutines does not affect the Parallel ESSL subroutines.

For the Fourier transform subroutines, an invalid transform length is not recoverable, as in ESSL for AIX. Parallel ESSL checks the validity of the transform length you provide to the Fourier transform subroutine. If it is not an acceptable value, a Parallel ESSL input-argument error message is issued, containing the next larger acceptable transform length required for successful computing of a Fourier transform. See the appropriate subroutine for additional constraints on valid transform lengths. Your program is then terminated on all processes. You should correct the value and rerun your program.

Computational Errors

Parallel ESSL computational errors are errors that occur in the computational data, such as in your vectors and matrices, during a computation—for example, the detection of a singular system during a factorization. (The computational errors that can occur for each subroutine, are listed under “Computational Errors.”) When a computational error occurs, Parallel ESSL issues an error message containing information key to the diagnosis of the error—such as the location in the input matrix where the singularity occurred. Any subroutine that issues a computational error has an *info* argument in its calling sequence. For all the Parallel ESSL subroutines, *info* is a global argument containing fullword integers, except in the tridiagonal subroutines. For these tridiagonal subroutines, *info* is a local argument containing fullword integers.

For message passing programs, when a computational error occurs, your program continues to execute. After each call where a computational error can occur, you should check the *info* output argument to see if an error occurred and take the appropriate action. When a computational error occurs, you should assume that the results are unpredictable. The result of the computation is valid only if no errors have occurred.

For HPF programs, when a computational error occurs and if the *info* argument is **present**, your program continues to execute. After each call where a computational error can occur, you should check the *info* output argument to see if an error occurred and take the appropriate action. When a computational error occurs, you should assume that the results are unpredictable. The result of the computation is valid only if no errors have occurred. If the *info* argument is **not present** and a computational error occurs, Parallel ESSL issues an additional error message containing the value of *info* and the application program is terminated.

How This Differs from ESSL for AIX:

The way you handle computational errors for Parallel ESSL differs from how you handle them for ESSL for AIX. This is because the capabilities of ERRSET, ERRSAV, and ERRSTR, supported in ESSL for AIX for recoverable computational errors, are **not supported** in Parallel ESSL. This results in the following differences:

- For message passing programs, you do not have the option of Parallel ESSL terminating your program when a computational error occurs in an Parallel ESSL subroutine. Control always returns to your program.

  For HPF programs, if you choose not to specify the optional *info* argument and a computational error occurs, Parallel ESSL issues a computational error message and the application program is terminated.
- The information about the error is returned to your program through the *info* argument (if present for HPF), rather than through a subsequent call to the EINFO subroutine.

Using the capabilities of ERRSET, ERRSAV, and ERRSTR with your ESSL for AIX subroutines does not affect the Parallel ESSL subroutines.

Resource Errors

A resource error occurs when a buffer storage allocation request fails in a Parallel ESSL subroutine. In general, the Parallel ESSL subroutines allocate internal auxiliary storage dynamically as needed. Without sufficient storage, the subroutine cannot complete the computation.

When a buffer storage allocation request fails, a resource error message is issued, and the application program is terminated. You need to reduce the memory constraint on the system or increase the amount of memory available before rerunning the application program.

Ways to Reduce Memory Constraints:

The following ways may reduce memory constraints:

- If you are using a one-dimensional process grid, change to a two-dimensional process grid, if possible. (Keep the shape of the two-dimensional process grid as close to a square as possible.)
- If you are using a two-dimensional process grid, change the shape of the process grid to be a square or as close to a square as possible.
- Increase the number of nodes. As you increase the number of nodes, keep the process grid as square as possible. For example, if using more processes, such as 17 rather than 16, causes you to use a one-dimensional grid rather than a two-dimensional grid, performance may be degraded.
- For message passing subroutines, reduce the block sizes.
- If your application terminated because you did not have enough storage and you received resource error 400 issued by the internal Parallel ESSL subroutine emergency_buff, consider running your PE application in user space (US) mode.
- For message passing subroutines, set the leading dimension equal to the number of rows in the local matrix.
- Investigate the load of your process and run in a more dedicated environment.
- Increase your node's paging space.
- Select nodes with more available memory.
- Select nodes that are not being used by other programs.

Communication Errors

Communication errors are errors that occur when Parallel ESSL encounters problems in communicating between processes—sending and receiving data or synchronizing operations. When a communication error occurs, at least one communication message is issued and the application program is terminated. This is because communication errors usually indicate a serious problem, where it is not feasible to continue.

Be aware that, due to the nature of communication errors, some error messages, including communications error messages from various processes, may be lost.

Informational and Attention Messages

When you receive an informational or attention message, check your application to determine why the condition was detected. You may decide to change your application so you do not receive the message. For example, if your application called a BLACS routine to send data from one process to the same process, you would receive an attention message.

Parallel ESSL does not terminate your application program, but performance may be degraded.

Miscellaneous Errors

A miscellaneous error is an error that does not fall under any of the other categories. Miscellaneous errors are checked in stages along with input-argument errors.

If no errors are detected in the first stage, Parallel ESSL checks the next stage, and so on. (The number of errors and stages that can occur for each subroutine are listed under its “Error Conditions” section.)

When Parallel ESSL detects a miscellaneous error, you receive an error message with information on how to correct the problem, your application program is terminated, and any arguments in the stages that follow are not checked.

ESSL for AIX Error Messages

For problems relating directly to ESSL for AIX, see the *ESSL Version 3 Guide and Reference* manual. If the ESSL for AIX error resulted from a Parallel ESSL subroutine, see “Getting Help from IBM Support” on page 115 to find out how to report the problem.

MPI Error Messages

If you receive an MPI error message while calling a BLACS routine, the cause is most likely one of the following:

- The BLACS have not been initialized.
- The context passed to the BLACS routine is not the same as the context obtained from a call to the BLACS_GET, BLACS_GRIDINIT, or BLACS_GRIDMAP routine.

Messages

This section describes the message conventions and lists all messages for input-argument errors, computational errors, resource errors, communication errors, informational and attention messages, and miscellaneous errors.

Message Conventions

About Upper- and Lowercase:

The literals, such as `'N'`, `'T'`, `'U'`, and so forth, appear in the messages in this book in uppercase; however, they may be specified in your Parallel ESSL calling sequence in either upper- or lowercase, for example, `'n'`, `'t'`, and `'u'`.

Message Format:

The Parallel ESSL messages are issued in your output in the following format:

```
rtn-name : 0040-nnn Context(l) Task(k) Process(p,q) Grid P × Q
message-text
```

Figure 11. Message Format

The parts of the Parallel ESSL message are as follows:

rtn-name	gives the name of the PE subroutine that encountered the error.
0040	is the Parallel ESSL component identification number.
nnn	is the message identification number:

001–299	Input-argument error messages
300–399	Computational error messages
400–499	Resource-allocation error messages
500–599	Communications error messages
600–699	Informational and attention messages
700–799	Miscellaneous error messages
800–999	Input-argument error messages

Context — *l* is the communication context number defined for this process grid, where *l* is an integer. If $l = -1$, then the context is invalid; in addition, the process and grid coordinates are set to −1.

Task(*k*) — is the PE task identification number.

Process(*p,q*) — are the process grid coordinates, indicating the process where the error occurred.

If $p = q = -1$ and the context is valid, then the same error occurred on all the processes, but is only reported on P_{00}.

Grid $P \times Q$ — gives the dimensions of the process grid.

message-text — describes the nature of the error. The possible unique parts are:

- For the message passing error messages, the argument number of each argument involved in the error is included in the message description as (ARG NO. _).

  For the HPF error messages, the name of each argument involved in the error is included in the message description.

- Additional information about the error is included in the message. The placement of this information is shown in the messages as (_)

Input-Argument Error Messages (001-299)

```
RTN_NAME : 0040-001 Context(_) Task(_) Process(_,_) Grid _ x _
The SCOPE (ARG NO. _) of a broadcast must be 'R', 'C', or 'A'

RTN_NAME : 0040-002 Context(_) Task(_) Process(_,_) Grid _ x _
UPLO (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is upper or lower, must be 'U' or 'L'.

RTN_NAME : 0040-003 Context(_) Task(_) Process(_,_) Grid _ x _
DIAG (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is unit, must be 'U' or 'N'.

RTN_NAME : 0040-004 Context(_) Task(_) Process(_,_) Grid _ x _
The process row (ARG NO. _) must be greater than or equal to zero and
less than the total number of processes in a row.

RTN_NAME : 0040-005 Context(_) Task(_) Process(_,_) Grid _ x _
The process column (ARG NO. _) must be greater than or equal to zero and
less than the total number of processes in a column.

RTN_NAME : 0040-006 Context(_) Task(_) Process(_,_) Grid _ x _
The SCOPE is specified by (ARG NO. _); therefore, the index of the source
process (ARG NO. _) must be equal to (_).

RTN_NAME : 0040-007 Context(_) Task(_) Process(_,_) Grid _ x _
The TOPOLOGY parameter (ARG NO. _) is invalid.

RTN_NAME : 0040-008 Context(_) Task(_) Process(_,_) Grid _ x _
The requested number of processes (_) is greater than the available number
of processes (_).

RTN_NAME : 0040-009 Context(_) Task(_) Process(_,_) Grid _ x _
The requested number of process rows (_) and process columns (_)
must be positive.

RTN_NAME : 0040-010 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows (ARG NO. _) in a matrix must be greater than or equal
to zero.

RTN_NAME : 0040-011 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns (ARG NO. _) in a matrix must be greater than or
equal to zero.

RTN_NAME : 0040-012 Context(_) Task(_) Process(_,_) Grid _ x _
The block size (ARG NO. _) must be greater than zero.

RTN_NAME : 0040-013 Context(_) Task(_) Process(_,_) Grid _ x _
The offset (ARG NO. _) must be greater than or equal to zero and less
than the block size (ARG NO. _).

RTN_NAME : 0040-014 Context(_) Task(_) Process(_,_) Grid _ x _
The stride (ARG NO. _) for a vector must be positive.

RTN_NAME : 0040-015 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be a double precision odd whole number greater than
or equal to 1.0 and less than 2**48.

RTN_NAME : 0040-016 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be zero or one.
```

```
RTN_NAME : 0040-017 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 0040-018 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be greater than zero.

RTN_NAME : 0040-019 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows (ARG NO. _) must be less than or equal to the block
size (ARG NO. _).

RTN_NAME : 0040-020 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns (ARG NO. _) must be less than or equal to the block
size (ARG NO. _).

RTN_NAME : 0040-021 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows (ARG NO. _) must be less than or equal to the size of
the leading dimension (ARG NO. _) of its array.

RTN_NAME : 0040-022 Context(_) Task(_) Process(_,_) Grid _ x _
The order of a matrix (ARG NO. _) must be less than or equal to the block
size (ARG NO. _).

RTN_NAME : 0040-023 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be a multiple of the product of (ARG NO. _) and
the number of processes (_).

RTN_NAME : 0040-024 Context(_) Task(_) Process(_,_) Grid _ x _
The size of the leading dimension (ARG NO. _) of the local array must be
greater than zero.

RTN_NAME : 0040-025 Context(_) Task(_) Process(_,_) Grid _ x _
The process column (ARG NO. _) that contains matrix (ARG NO. _) must
be equal to the process column (ARG NO. _) that contains matrix (ARG NO. _)

RTN_NAME : 0040-026 Context(_) Task(_) Process(_,_) Grid _ x _
The process row (ARG NO. _) that contains matrix (ARG NO. _) must
be equal to the process row (ARG NO. _) that contains matrix (ARG NO. _)

RTN_NAME : 0040-027 Context(_) Task(_) Process(_,_) Grid _ x _
The order (ARG NO. _) of a matrix must be greater than or equal to zero.

RTN_NAME : 0040-028 Context(_) Task(_) Process(_,_) Grid _ x _
MTXBLK (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is a full block matrix or a single block matrix, must be 'M' or 'B'.

RTN_NAME : 0040-029 Context(_) Task(_) Process(_,_) Grid _ x _
The process row (ARG NO. _) must be greater than or equal to -1 and less
 than the total number of rows in the process grid.

RTN_NAME : 0040-030 Context(_) Task(_) Process(_,_) Grid _ x _
The process column (ARG NO. _) must be greater than or equal to -1 and
less than the total number of columns in the process grid.

RTN_NAME : 0040-031 Context(_) Task(_) Process(_,_) Grid _ x _
The argument which specifies whether a matrix (ARG NO. _) is workspace
must be 'Y' or 'N'.
```

```
RTN_NAME : 0040-032 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies the computation to be performed, must be
'N', 'T', or 'C'.

RTN_NAME : 0040-033 Context(_) Task(_) Process(_,_) Grid _ x _
The size of leading dimension (ARG NO. _) of the local array (ARG NO. _)
must be greater than or equal to (_).

RTN_NAME : 0040-034 Context(_) Task(_) Process(_,_) Grid _ x _
SIDE (ARG NO. _), which specifies whether the input matrix
(ARG NO. _) appears on the left or right of the other input matrix, must
be 'L' or 'R'.

RTN_NAME : 0040-035 Context(_) Task(_) Process(_,_) Grid _ x _
The number of right hand sides (ARG NO. _) must be greater than or equal
to zero.

RTN_NAME : 0040-036 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), specifies whether an input matrix (ARG NO. _),
its transpose, or its conjugate transpose should be used.  TRANS must be
'N', 'T', or 'C'.

RTN_NAME : 0040-037 Context(_) Task(_) Process(_,_) Grid _ x _
Task has issued a receive for its own broadcast.

RTN_NAME : 0040-038 Context(_) Task(_) Process(_,_) Grid _ x _
Minimum message id in message id range (element 1 of ARG NO. 3) must be
less than the maximum message id (element 2 of ARG NO. 3).

RTN_NAME : 0040-039 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context (ARG NO. _) is invalid.

RTN_NAME : 0040-040 Context(_) Task(_) Process(_,_) Grid _ x _
The process row or column (ARG NO. _) must be greater than 0.

RTN_NAME : 0040-041 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_, (element 7 of ARG NO. _) must be greater than
or equal to 0 and less than the total number of rows in the process grid.

RTN_NAME : 0040-042 Context(_) Task(_) Process(_,_) Grid _ x _
The process column, CSRC_, (element 8 of ARG NO. _) must be greater than
or equal to 0 and less than the total number of columns in the process grid.

RTN_NAME : 0040-043 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context, CTXT_, (element 2 of ARG NO. _) of the matrix
(ARG NO. _) must be equal to the communications context
(element 2 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-044 Context(_) Task(_) Process(_,_) Grid _ x _
The size of the leading dimension, LLD_, (element 9 of ARG NO. _) of the
local array (ARG NO. _) must be greater than zero.

RTN_NAME : 0040-045 Context(_) Task(_) Process(_,_) Grid _ x _
The size of leading dimension, LLD_, (element 9 of ARG NO. _) of the
local array (ARG NO. _) must be greater than or equal to (_).
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RTN_NAME : 0040-046 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_, (element 3 of ARG NO. _) in the global matrix
(ARG NO. _) must be greater than zero.

RTN_NAME : 0040-047 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 4 of ARG NO. _) in the global matrix
(ARG NO. _) must be greater than zero.

RTN_NAME : 0040-048 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of matrix (ARG NO. _) must be
greater than 0.

RTN_NAME : 0040-049 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of matrix (ARG NO. _) must be
greater than 0.

RTN_NAME : 0040-050 Context(_) Task(_) Process(_,_) Grid _ x _
The stride (ARG NO. _) for vector (ARG NO. _) is 1, but the row
block size, MB_, (element 5 of ARG NO. _) is not equal to the block
size (element _ of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-051 Context(_) Task(_) Process(_,_) Grid _ x _
The row and column block sizes, MB_ and NB_,
(elements 5 and 6 of ARG NO. _) of the matrix (ARG NO. _) must
be equal.

RTN_NAME : 0040-052 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced is incompatible with the global matrix definition.
The global row index (ARG NO. _) plus the number of rows (ARG NO. _)
of the matrix (ARG NO. _) minus 1 must be less than or equal to the
number of rows, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-053 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced is incompatible with the global matrix definition.
The global column index (ARG NO. _) plus the number of columns (ARG NO. _)
of the matrix (ARG NO. _) minus 1 must be less than or equal to the
number of columns, N_, (element 4 of ARG NO. _).

RTN_NAME : 0040-054 Context(_) Task(_) Process(_,_) Grid _ x _
The stride (ARG NO. _) and global indices (ARG NO. _ and ARG NO. _)
of the vector (ARG NO. _) are inconsistent with the global dimensions,
M_ and N_, (elements 3 and 4 of ARG NO. _) of the matrix.

RTN_NAME : 0040-055 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed but the column block size, NB_,
(element 6 of ARG NO. _) is not equal to the row block size, MB_,
(element 5 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-056 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed but the row block size, MB_,
(element 5 of ARG NO. _) is not equal to the row block size, MB_,
(element 5 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-057 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed, but the block column offset of
the vector is not equal to the block row offset of the matrix (ARG NO. _).
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RTN_NAME : 0040-058 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed but the column block size, NB_,
(element 6 of ARG NO. _) is not equal to the column block size, NB_,
(element 6 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-059 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed, but the block column offset of
the vector is not equal to the block column offset of the matrix (ARG NO. _).

RTN_NAME : 0040-060 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed, but the process column (_),
containing the first element of the vector is not equal to the process
column (_) containing the first column of the submatrix (ARG NO. _).

RTN_NAME : 0040-061 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed but the row block size, MB_,
(element 5 of ARG NO. _) is not equal to the column block size, NB_,
(element 6 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-062 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed, but the block row offset of
the vector is not equal to the block column offset of the matrix
(ARG NO. _).

RTN_NAME : 0040-063 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed, but the block row offset of
the vector is not equal to the block row offset of the matrix (ARG NO. _).

RTN_NAME : 0040-064 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed, but the process row (_),
containing the first element of the vector is not equal to the process row
(_) containing the first row of the submatrix (ARG NO. _).

RTN_NAME : 0040-065 Context(_) Task(_) Process(_,_) Grid _ x _
The stride (ARG NO. _) for vector (ARG NO. _) must be equal to either 1 or
the number of rows, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-066 Context(_) Task(_) Process(_,_) Grid _ x _
The calculated block row offset and block column offset of the submatrix
referenced within the global matrix (ARG NO. _) must be equal.

RTN_NAME : 0040-067 Context(_) Task(_) Process(_,_) Grid _ x _
Matrices (ARG NO. _) and (ARG NO. _) have incompatible block sizes.
The block size (element _ of ARG NO. _) must be equal to the block
size (element _ of ARG NO. _).

RTN_NAME : 0040-068 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) and global column index (ARG NO. _)
of matrix (ARG NO. _) must be equal.

RTN_NAME : 0040-069 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _), which represents a process row or column, must be
greater than or equal to zero and less than (ARG NO. _).

RTN_NAME : 0040-070 Context(_) Task(_) Process(_,_) Grid _ x _
The transform length (ARG NO. _) must be divisible by the number of
tasks (_).
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RTN_NAME : 0040-071 Context(_) Task(_) Process(_,_) Grid _ x _
The transform length (ARG NO. _) divided by the number of tasks must be
an even number.

RTN_NAME : 0040-072 Context(_) Task(_) Process(_,_) Grid _ x _
The scaling parameter (ARG NO. _) must be nonzero.

RTN_NAME : 0040-073 Context(_) Task(_) Process(_,_) Grid _ x _
The transform length (ARG NO. _) is not an allowed value.  The next
higher value is (_).

RTN_NAME : 0040-074 Context(_) Task(_) Process(_,_) Grid _ x _
The output data distribution format (element 2 of ARG NO. _) must be zero
or one.

RTN_NAME : 0040-075 Context(_) Task(_) Process(_,_) Grid _ x _
(Element _ of ARG NO. _) must be either zero or greater than or
equal to the transform dimension (ARG NO. _).

RTN_NAME : 0040-076 Context(_) Task(_) Process(_,_) Grid _ x _
The transform direction parameter (ARG NO. _) must be nonzero.

RTN_NAME : 0040-077 Context(_) Task(_) Process(_,_) Grid _ x _
The transform length (ARG NO. _) must be less than or equal to (_).

RTN_NAME : 0040-078 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be nonzero.

RTN_NAME : 0040-079 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced must be a block row matrix.
The block row offset plus the number of rows (ARG NO. _) of the matrix
(ARG NO. _) must be less than or equal to the row block size
(element 5 of ARG NO. _).

RTN_NAME : 0040-080 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced must be a block column matrix.
The block column offset plus the number of columns (ARG NO. _) of the
matrix (ARG NO. _) must be less than or equal to the column block
size (element 6 of ARG NO. _).

RTN_NAME : 0040-081 Context(_) Task(_) Process(_,_) Grid _ x _
In the process grid, the process row (_), containing the first row of the
submatrix (ARG NO. _) must be equal to the process row (_)
containing the first row of the submatrix (ARG NO. _).

RTN_NAME : 0040-082 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context, CTXT_, (element 2 of ARG NO. _) of the matrix
(ARG NO. _) must be equal to the communications context
(element 2 of ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-083 Context(_) Task(_) Process(_,_) Grid _ x _
In the process grid, the process column (_), containing the first column
of the submatrix (ARG NO. _) must be equal to the process column (_)
containing the first column of the submatrix (ARG NO. _).

RTN_NAME : 0040-084 Context(_) Task(_) Process(_,_) Grid _ x _
The dimension (ARG NO. _) of the matrices must be greater than or equal
to zero.
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RTN_NAME : 0040-085 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrices referenced must be properly aligned.
The block offset for matrix (ARG NO. _) generated by (ARG NO. _) and block
size (element _ of ARG NO. _) must be equal to the block offset for
matrix (ARG NO. _) generated by (ARG NO. _) and block size
(element _ of ARG NO. _).

RTN_NAME : 0040-086 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context (_) is not currently active.

RTN_NAME : 0040-087 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context (_) is invalid.

RTN_NAME : 0040-088 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be defined with the number of rows set to 1.

RTN_NAME : 0040-089 Context(_) Task(_) Process(_,_) Grid _ x _
The vectors referenced must be distributed along the same axis.
Either the stride (ARG NO. _) for vector (ARG NO. _) and the stride
(ARG NO. _) for vector (ARG NO. _) must both be equal to 1 or the
stride for vector (ARG NO. _) must be equal to the number of rows, M_,
(element 3 of ARG NO. _) and the stride for vector (ARG NO. _)
must be equal to the number of rows, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-090 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size, MB_, (element 5 of ARG NO. _) of the matrix (ARG NO. _)
must be greater than zero.

RTN_NAME : 0040-091 Context(_) Task(_) Process(_,_) Grid _ x _
The column block size, NB_, (element 6 of ARG NO. _) of the matrix
(ARG NO. _) must be greater than zero.

RTN_NAME : 0040-092 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced must be aligned on a row block boundary.
(ARG NO. _) minus 1 must be a multiple of the row block size, MB_,
(element 5 of ARG NO. _) of matrix (ARG NO. _).

RTN_NAME : 0040-093 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced must be aligned on a column block boundary.
(ARG NO. _) minus 1 must be a multiple of the column block size, NB_,
(element 6 of ARG NO. _) of matrix (ARG NO. _).

RTN_NAME : 0040-094 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of vector (ARG NO. _) must be greater
than 0 and less than the number of rows, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-095 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of vector (ARG NO. _) must be
greater than 0 and less than or equal to the number of columns, N_,
(element 4 of ARG NO. _).

RTN_NAME : 0040-096 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies the operation to be performed, must be
'T' or 'C'.

RTN_NAME : 0040-097 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of matrix (ARG NO. _) must be
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greater than 0 and less than or equal to the number of rows in the global
matrix, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-098 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of matrix (ARG NO. _) must be
greater than 0 and less than or equal to the number of columns in the
global matrix, N_, (element 4 of ARG NO. _).

RTN_NAME : 0040-099 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_, (element 3 of ARG NO. _) in a null matrix
(ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 0040-100 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 4 of ARG NO. _) in a null matrix
(ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 0040-101 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows (ARG NO. _) of a matrix must be the same for all
processes.

RTN_NAME : 0040-102 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns (ARG NO. _) of a matrix must be the same for all
processes.

RTN_NAME : 0040-103 Context(_) Task(_) Process(_,_) Grid _ x _
The order (ARG NO. _) of a matrix must be the same for all processes.

RTN_NAME : 0040-104 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of the matrix (ARG NO. _) must be
the same for all processes.

RTN_NAME : 0040-105 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of the matrix (ARG NO. _) must
be the same for all processes.

RTN_NAME : 0040-106 Context(_) Task(_) Process(_,_) Grid _ x _
UPLO (ARG NO. _), which specifies whether an input matrix (ARG NO. _)
is upper or lower, must be the same for all processes.

RTN_NAME : 0040-107 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies whether an input matrix,
its transpose, or its conjugate transpose should be used, must be
the same for all processes.

RTN_NAME : 0040-108 Context(_) Task(_) Process(_,_) Grid _ x _
NRHS (ARG NO. _), which specifies the number of right hand sides in the
system to be solved, must be the same for all processes.

RTN_NAME : 0040-109 Context(_) Task(_) Process(_,_) Grid _ x _
ILO (ARG NO. _), which specifies a lower range of rows or columns in a
matrix, must be the same for all processes.

RTN_NAME : 0040-110 Context(_) Task(_) Process(_,_) Grid _ x _
IHI (ARG NO. _), which specifies an upper range of rows or columns in a
matrix, must be the same for all processes.
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RTN_NAME : 0040-111 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_, (element 3 of ARG NO. _) in the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-112 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 4 of ARG NO. _) in the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-113 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size MB_, (element 5 of ARG NO. _) of the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-114 Context(_) Task(_) Process(_,_) Grid _ x _
The column block size NB_, (element 6 of ARG NO. _) of the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-115 Context(_) Task(_) Process(_,_) Grid _ x _
The process row RSRC_, (element 7 of ARG NO. _) must be the same
for all processes.

RTN_NAME : 0040-116 Context(_) Task(_) Process(_,_) Grid _ x _
The process column CSRC_, (element 8 of ARG NO. _) must be the same
for all processes.

RTN_NAME : 0040-117 Context(_) Task(_) Process(_,_) Grid _ x _
The number of elements (ARG NO. _) in a work array (ARG NO. _)
must be zero, to indicate dynamic allocation, minus one, to indicate
workspace query, or greater than or equal to (_) if a work array
is being supplied.

RTN_NAME : 0040-118 Context(_) Task(_) Process(_,_) Grid _ x _
ILO (ARG NO. _), which specifies a lower range of rows or columns in a
matrix, must be greater than or equal to one and less than or equal to
the larger of one and the order (ARG NO. _) of the matrix.

RTN_NAME : 0040-119 Context(_) Task(_) Process(_,_) Grid _ x _
IHI (ARG NO. _), which specifies an upper range of rows or columns in a
matrix, must be greater than or equal to the smaller of ILO (ARG NO. _)
and the order (ARG NO. _) of the matrix and less than or equal to the
order (ARG NO. _) of the matrix.

RTN_NAME : 0040-120 Context(_) Task(_) Process(_,_) Grid _ x _
The row-distributed vector referenced is incompatible with the global
matrix definition.  The global column index (ARG NO. _) plus the number
of columns (ARG NO. _) of the vector (ARG NO. _) minus 1 must
be less than or equal to the number of columns, N_,
(element 4 of ARG NO. _).

RTN_NAME : 0040-121 Context(_) Task(_) Process(_,_) Grid _ x _
The column-distributed vector referenced is incompatible with the global
matrix definition.  The global row index (ARG NO. _) plus the number of
rows (ARG NO. _) of the vector (ARG NO. _) minus 1 must be less
than or equal to the number of rows, M_, (element 3 of ARG NO. _).

RTN_NAME : 0040-122 Context(_) Task(_) Process(_,_) Grid _ x _
JOBZ (ARG NO. _), which specifies whether or not to compute eigenvectors,
must be 'N' or 'V'.
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RTN_NAME : 0040-123 Context(_) Task(_) Process(_,_) Grid _ x _
RANGE (ARG NO. _), which specifies which eigenvalues to find, must be
'A', 'V', or 'I'.

RTN_NAME : 0040-124 Context(_) Task(_) Process(_,_) Grid _ x _
VU (ARG NO. _), which specifies the upper bound of the interval to be
searched for eigenvalues, must be greater than VL (ARG NO. _), which
specifies the lower bound of the interval to be searched for eigenvalues.

RTN_NAME : 0040-125 Context(_) Task(_) Process(_,_) Grid _ x _
IL (ARG NO. _), which specifies the index of the smallest eigenvalue to
be returned, must be greater than or equal to 1.

RTN_NAME : 0040-126 Context(_) Task(_) Process(_,_) Grid _ x _
IU (ARG NO. _), which specifies the index of the largest eigenvalue to be
returned, must be greater than or equal to the smaller of the order
(ARG NO. _) of the matrix (ARG NO. _) and IL (ARG NO. _) and
less than or equal to the order of the matrix.

RTN_NAME : 0040-127 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of the matrix (ARG NO. _) must be 1.

RTN_NAME : 0040-128 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of the matrix (ARG NO. _) must be 1.

RTN_NAME : 0040-129 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of the matrix (ARG NO. _) must be
equal to the global row index (ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-130 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of the matrix (ARG NO. _) must
be equal to the global column index (ARG NO. _) of the matrix (ARG NO. _).

RTN_NAME : 0040-131 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_ (element 3 of ARG NO. _) in the global matrix
(ARG NO. _) must be equal to the number of rows,
M_ (element 3 of ARG NO. _) in the global matrix (ARG NO. _).

RTN_NAME : 0040-132 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_ (element 4 of ARG NO. _) in the global matrix
(ARG NO. _) must be equal to the number of columns, N_
(element 4 of ARG NO. _) in the global matrix (ARG NO. _).

RTN_NAME : 0040-133 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_ (element 7 of ARG NO. _) must be zero.

RTN_NAME : 0040-134 Context(_) Task(_) Process(_,_) Grid _ x _
The process column, CSRC_ (element 8 of ARG NO. _) must be zero.

RTN_NAME : 0040-135 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_ (element 7 of ARG NO. _) must be equal to the
process row, RSRC_ (element 7 of ARG NO. _).

RTN_NAME : 0040-136 Context(_) Task(_) Process(_,_) Grid _ x _
The process column, CSRC_ (element 8 of ARG NO. _) must be equal to
the process column, CSRC_ (element 8 of ARG NO. _).
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RTN_NAME : 0040-137 Context(_) Task(_) Process(_,_) Grid _ x _
ORFAC (ARG NO. _), which specifies which eigenvectors should be
orthogonalized, must be the same for all processes.

RTN_NAME : 0040-138 Context(_) Task(_) Process(_,_) Grid _ x _
JOBZ (ARG NO. _), which specifies whether or not to compute eigenvectors,
must be the same for all processes.

RTN_NAME : 0040-139 Context(_) Task(_) Process(_,_) Grid _ x _
RANGE (ARG NO. _), which specifies which eigenvalues to find, must be the
same for all processes.

RTN_NAME : 0040-140 Context(_) Task(_) Process(_,_) Grid _ x _
VL (ARG NO. _), which specifies the lower bound of the interval to be
searched for eigenvalues, must be the same for all processes.

RTN_NAME : 0040-141 Context(_) Task(_) Process(_,_) Grid _ x _
VU (ARG NO. _), which specifies the upper bound of the interval to be
searched for eigenvalues, must be the same for all processes.

RTN_NAME : 0040-142 Context(_) Task(_) Process(_,_) Grid _ x _
IL (ARG NO. _), which specifies the index of the smallest eigenvalue to
be returned, must be the same for all processes.

RTN_NAME : 0040-143 Context(_) Task(_) Process(_,_) Grid _ x _
IU (ARG NO. _), which specifies the index of the largest eigenvalue to be
returned, must be the same for all processes.

RTN_NAME : 0040-144 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is row-distributed and TRANS (ARG NO. _) is
'T' or 'C', but the process column (_), containing the first element of
the vector is not equal to the process column (_) containing the first
column of the submatrix (ARG NO. _).

RTN_NAME : 0040-145 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (ARG NO. _) is column-distributed and TRANS (ARG NO. _) is
'N', but the process row (_), containing the first element of the
vector is not equal to the process row (_) containing the first row of
the submatrix (ARG NO. _).

RTN_NAME : 0040-146 Context(_) Task(_) Process(_,_) Grid _ x _
ABSTOL (ARG NO. _), which specifies the absolute error tolerance for the
eigenvalues, must be the same for all processes.

RTN_NAME : 0040-147 Context(_) Task(_) Process(_,_) Grid _ x _
No attributes or key is defined for the communicator.  The probable cause
is that the BLACS have not been initialized.

RTN_NAME : 0040-148 Context(_) Task(_) Process(_,_) Grid _ x _
MPI is not initialized.  The probable cause is that the BLACS have not been
initialized.

RTN_NAME : 0040-149 Context(_) Task(_) Process(_,_) Grid _ x _
The Cartesian grid is not defined.  The probable cause is that the BLACS
have not been initialized.
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RTN_NAME : 0040-150 Context(_) Task(_) Process(_,_) Grid _ x _
The Cartesian grid is not defined as two-dimensional.  The probable cause
is that the BLACS have not been initialized.

RTN_NAME : 0040-151 Context(_) Task(_) Process(_,_) Grid _ x _
The rank of the ultimate align target for (_) is (_).  It must be
equal to 1 or 2.

RTN_NAME : 0040-152 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank must be the same for all assumed shape arrays.
The process rank of (_) is (_).
The process rank of (_) is (_).

RTN_NAME : 0040-153 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank must be the same for all assumed shape arrays.
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).

RTN_NAME : 0040-154 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank of (_) is (_).  The rank must be equal to 1 or 2.

RTN_NAME : 0040-155 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank of (_) is (_).  The rank must be 1.

RTN_NAME : 0040-156 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_).
The process grid for (_) is (_).

RTN_NAME : 0040-157 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).

RTN_NAME : 0040-158 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).

RTN_NAME : 0040-159 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).

RTN_NAME : 0040-160 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrices.
The process rank is 1 and one of the following must be true: The first
dimension of the matrices must both be collapsed or the second dimension of
the matrices must both be collapsed.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
```

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RTN_NAME : 0040-161 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution among the matrices.
The process rank is 1 and one of the following must be true: The first
dimension of the matrices must all be collapsed or the second dimension of
the matrices must all be collapsed.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.

RTN_NAME : 0040-162 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrices.
The values for axis_map(1) and axis_map(2) for the matrices must be 1 or 2.
The values for axis_map(1) for both matrices must be equal and the values
for axis_map(2) for both matrices must be equal.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.

RTN_NAME : 0040-163 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution among the matrices.
The values for axis_map(1) and axis_map(2) for the matrices must be 1 or 2.
The values for axis_map(1) for all matrices must be equal and the values
for axis_map(2) for all matrices must be equal.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.

RTN_NAME : 0040-164 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) has incorrect dimensions for the transpose case.  It must have
(_) rows and (_) columns.

RTN_NAME : 0040-165 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) has incorrect dimensions for the normal case.  It must have
(_) rows and (_) columns.

RTN_NAME : 0040-166 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) has incorrect dimensions for the transpose case.  It must have
(_) rows, (_) columns, and (_) planes.

RTN_NAME : 0040-167 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) has incorrect dimensions for the normal case.  It must have
(_) rows, (_) columns, and (_) planes.

RTN_NAME : 0040-168 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) must be (*,BLOCK) distributed.

RTN_NAME : 0040-169 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) must be (*,*,BLOCK) distributed.
```

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RTN_NAME : 0040-170 Context(_) Task(_) Process(_,_) Grid _ x _
Argument (_) must be less than or equal to (_).

RTN_NAME : 0040-171 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank must be the same for all assumed shape arrays.
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).

RTN_NAME : 0040-172 Context(_) Task(_) Process(_,_) Grid _ x _
The process rank must be the same for all assumed shape arrays.
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).
The process rank of (_) is (_).

RTN_NAME : 0040-173 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrix and
vector.
The value for axis_map(1) for the vector must be 1 or 2.
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-174 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrix and
vectors.
The values for axis_map(1) for the vectors must be 1 or 2.
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-175 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution for the vectors.
The value for axis_map(1) for the vectors must be 1.
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-176 Context(_) Task(_) Process(_,_) Grid _ x _
The data distribution for the matrix is unsupported.
The process rank is 1 and one of the following must be true: The first
dimension of the matrix must be collapsed or the second dimension of the
matrix must be collapsed.
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.

RTN_NAME : 0040-177 Context(_) Task(_) Process(_,_) Grid _ x _
The data distribution for the matrix is unsupported.
The process rank is 2 and the following must be true: The values
for axis_map(1) and axis_map(2) for the matrix must be 1 or 2.
the matrix must not be collapsed.
```

```
The values for axis_map(1) and axis_map(2) for matrix (_) are (_) and
(_), respectively.

RTN_NAME : 0040-178 Context(_) Task(_) Process(_,_) Grid _ x _
Vectors (_), (_), (_), (_), and (_) must all be BLOCK distributed.

RTN_NAME : 0040-179 Context(_) Task(_) Process(_,_) Grid _ x _
The vectors must all be not replicated.
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).

RTN_NAME : 0040-180 Context(_) Task(_) Process(_,_) Grid _ x _
The data distribution for matrix (_) is unsupported.
The array must be either a block row, a block column, a single block,
or the first element of the array must be aligned on the block boundary(s)
of the ultimate align target.

RTN_NAME : 0040-181 Context(_) Task(_) Process(_,_) Grid _ x _
SIDE, which specifies whether the input matrix (_) appears on the left
or right of the other input matrix (_), must be 'L' or 'R'.

RTN_NAME : 0040-182 Context(_) Task(_) Process(_,_) Grid _ x _
TRANSA, which specifies whether input matrix A, its transpose, or
its conjugate transpose should be used, must be 'N', 'T', or 'C'.

RTN_NAME : 0040-183 Context(_) Task(_) Process(_,_) Grid _ x _
TRANSB, which specifies whether input matrix B, its transpose, or
its conjugate transpose should be used, must be 'N', 'T', or 'C'.

RTN_NAME : 0040-184 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS, which specifies the computation to be performed, must be 'N', 'T',
or 'C'.

RTN_NAME : 0040-185 Context(_) Task(_) Process(_,_) Grid _ x _
The column block size of the matrix and the block size of the vector are
incompatible.
The column block size of the matrix (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-186 Context(_) Task(_) Process(_,_) Grid _ x _
ILO, which specifies a lower range of rows or columns in a matrix, must be
greater than or equal to one and less than or equal to the larger of one
and the order of the matrix (_).

RTN_NAME : 0040-187 Context(_) Task(_) Process(_,_) Grid _ x _
IHI, which specifies an upper range of rows or columns in a matrix, must
be greater than or equal to the smaller of ILO and the order of the
matrix (_) and less than or equal to the order of the matrix.

RTN_NAME : 0040-188 Context(_) Task(_) Process(_,_) Grid _ x _
The assumed shape array must be evenly distributed among the processes.
The column block size (_) of the assumed shape array (_) must be
equal to the number of columns (_) distributed on this process.
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RTN_NAME : 0040-189 Context(_) Task(_) Process(_,_) Grid _ x _
The assumed shape array must be evenly distributed among the processes.
The plane block size (_) of the assumed shape array (_) must be
equal to the number of planes (_) distributed on this process.

RTN_NAME : 0040-190 Context(_) Task(_) Process(_,_) Grid _ x _
TRANSPOSE, which specifies whether the output data is stored in normal or
transposed format, must be 'N' or 'T'.

RTN_NAME : 0040-191 Context(_) Task(_) Process(_,_) Grid _ x _
The rank of the ultimate align target for (_) is (_).  It must be
equal to 1.

RTN_NAME : 0040-192 Context(_) Task(_) Process(_,_) Grid _ x _
The assumed shape array has an invalid shape.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-193 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 3 assumed shape arrays are incompatible.
The value of (_) is (_).
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements
Array (_) has (_) elements.

RTN_NAME : 0040-194 Context(_) Task(_) Process(_,_) Grid _ x _
The size of the assumed shape array (_) is (_).
It must be a multiple of the product of the block size (_), and the
number of processes (_).

RTN_NAME : 0040-195 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 3 assumed shape arrays are incompatible.
The value of (_) is (_).
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-196 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 3 assumed shape arrays are incompatible.
The value of (_) is (_).
The value of (_) is (_).
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-197 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 2 assumed shape arrays are incompatible.
The value of (_) is (_).
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-198 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 2 assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-199 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 2 assumed shape arrays are incompatible.
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Array (_) has (_) rows, (_) columns, and (_) planes.
Array (_) has (_) rows, (_) columns, and (_) planes.

RTN_NAME : 0040-200 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 3 assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.

RTN_NAME : 0040-201 Context(_) Task(_) Process(_,_) Grid _ x _
The orientation of vectors (_) and (_) must be either both
row-distributed or both column-distributed.

RTN_NAME : 0040-202 Context(_) Task(_) Process(_,_) Grid _ x _
The data distribution for the vector is unsupported.
The vector (_) must not be replicated.

RTN_NAME : 0040-203 Context(_) Task(_) Process(_,_) Grid _ x _
The data distribution for the vector(s) is unsupported.
Vectors (_) and (_) must not be replicated.

RTN_NAME : 0040-204 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be column-distributed.

RTN_NAME : 0040-205 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size of the matrix and the block size of the vector are
incompatible.
The row block size of the matrix (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-206 Context(_) Task(_) Process(_,_) Grid _ x _
The column block size of the matrix and the block sizes of the vectors are
incompatible.
The column block size of the matrix (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-207 Context(_) Task(_) Process(_,_) Grid _ x _
The block sizes of the matrix are incompatible with the block sizes of the
vectors.
The row block size of the matrix (_) is (_) and the column block
size of the matrix is (_).
The block size of the vector (_) is (_) and must be equal to the
row block size of the matrix.
The block size of the vector (_) is (_) and must be equal to the
column block size of the matrix.
The block size of the vector (_) is (_) and must be equal to the
column block size of the matrix.
The block size of the vector (_) is (_) and must be equal to the
row block size of the matrix.

RTN_NAME : 0040-208 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.
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RTN_NAME : 0040-209 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-210 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-211 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-212 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process index for the vector.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-213 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract row process index for the matrix must be equal to the abstract
row process index for the vector.
The abstract row process index for matrix (_) is (_).
The abstract row process index for vector (_) is (_).

RTN_NAME : 0040-214 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract column process index for the matrix must be equal to the
abstract column process index for the vector.
The abstract column process index for matrix (_) is (_).
The abstract column process index for vector (_) is (_).

RTN_NAME : 0040-215 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.

RTN_NAME : 0040-216 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process indices for the vectors.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-217 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract column process index for the matrix must be equal to the
abstract process indices for the vectors.
The abstract column process index for matrix (_) is (_).
The abstract column process index for vector (_) is (_).
The abstract column process index for vector (_) is (_).
The abstract column process index for vector (_) is (_).

RTN_NAME : 0040-218 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process indices for the vectors.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-219 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process indices for the matrix are inconsistent with the
abstract process indices for the vectors.
The abstract process indices for matrix (_) are (_,_).
The abstract process index for vector (_) is (_) and must be
equal to the abstract process row index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process column index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process column index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process row index for the matrix.

RTN_NAME : 0040-220 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).

RTN_NAME : 0040-221 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).

RTN_NAME : 0040-222 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).

RTN_NAME : 0040-223 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).
The process grid for (_) is (_ x _).

RTN_NAME : 0040-224 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be replicated and row-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-225 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be replicated and column-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-226 Context(_) Task(_) Process(_,_) Grid _ x _
The vectors must all be replicated and row-distributed.
The value for axis_map(1) for vector (_) is (_).
The number of copies of the vector is (_).
The value for axis_map(1) for vector (_) is (_).
The number of copies of the vector is (_).
The value for axis_map(1) for vector (_) is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-227 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be replicated and row-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and column-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and row-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and column-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-228 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be replicated and column-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and row-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and row-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).
The vector (_) must be replicated and column-distributed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-229 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process indices for the matrix are inconsistent with the
abstract process indices for the vectors.
The abstract process indices for matrix (_) are (_,_).
The abstract process index for vector (_) is (_) and must be
equal to the abstract process column index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process row index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process column index for the matrix.
The abstract process index for vector (_) is (_) and must be
equal to the abstract process row index for the matrix.
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RTN_NAME : 0040-230 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be defined with either the number of process rows or
the number of process columns set to 1.

RTN_NAME : 0040-231 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 3 of ARG NO. _) in the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-232 Context(_) Task(_) Process(_,_) Grid _ x _
The block size (element 4 of ARG NO. _) must be equal to (_).

RTN_NAME : 0040-233 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_, (element 5 of ARG NO. _) must be equal to (_).

RTN_NAME : 0040-234 Context(_) Task(_) Process(_,_) Grid _ x _
The size of leading dimension, LLD_, (element 6 of ARG NO. _) of the local
array (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 0040-235 Context(_) Task(_) Process(_,_) Grid _ x _
Argument (_) must be greater than or equal to (_).

RTN_NAME : 0040-236 Context(_) Task(_) Process(_,_) Grid _ x _
DTYPE_ (element 1 of ARG NO. _), which specifies the descriptor type,
must be the same for all processes.

RTN_NAME : 0040-237 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context, CTXT_, (element 2 of ARG NO. _), must be the
same for all processes.

RTN_NAME : 0040-238 Context(_) Task(_) Process(_,_) Grid _ x _
The number of elements in the matrix (ARG NO. _) supplied to store the
factor must be greater than or equal to (_).

RTN_NAME : 0040-239 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies the operation to be performed, must be
'N' or 'n'.

RTN_NAME : 0040-240 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process indices for the vectors.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-241 Context(_) Task(_) Process(_,_) Grid _ x _
The block sizes of the matrix are incompatible with the block sizes of the
vectors.
The row block size of the matrix (_) is (_) and the column block
size of the matrix is (_).
The block size of the vector (_) is (_) and must be equal to the
column block size of the matrix.
The block size of the vector (_) is (_) and must be equal to the
row block size of the matrix.
The block size of the vector (_) is (_) and must be equal to the
column block size of the matrix.
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The block size of the vector (_) is (_) and must be equal to the
row block size of the matrix.

RTN_NAME : 0040-242 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_, (element 3 of ARG NO. _) in the global matrix
must be greater than the half-bandwidth, k.

RTN_NAME : 0040-243 Context(_) Task(_) Process(_,_) Grid _ x _
The half bandwidth, k, (ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-244 Context(_) Task(_) Process(_,_) Grid _ x _
The block size, NB_, (element 4 of ARG NO. _) of matrix (ARG NO. _)
must be equal to the least integer that is greater than or equal to the
quotient of the global number of columns in packed format divided by the
number of processes.

RTN_NAME : 0040-245 Context(_) Task(_) Process(_,_) Grid _ x _
The process row or column, (element 5 of ARG NO. _) must be equal to (_).

RTN_NAME : 0040-246 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of matrix (ARG NO. _) must be equal to 1
for a 1-dimensional process grid with 1 column.

RTN_NAME : 0040-247 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of matrix (ARG NO. _) must be
equal to 1 for a 1-dimensional process grid with 1 row.

RTN_NAME : 0040-248 Context(_) Task(_) Process(_,_) Grid _ x _
The half bandwidth (ARG NO. _) of a matrix must be greater than or
equal to zero.

RTN_NAME : 0040-249 Context(_) Task(_) Process(_,_) Grid _ x _
The half bandwidth (ARG NO. _) of the band matrix (ARG NO. _)
must be less than the order of the matrix.

RTN_NAME : 0040-250 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows (ARG NO. _) of matrix (ARG NO. _) must be smaller
than or equal to the product of the number of processors and the block
size (element _ of ARG NO. _) minus the modulus of
(ARG NO. _) minus one with the block size (element _ of ARG NO. _).

RTN_NAME : 0040-251 Context(_) Task(_) Process(_,_) Grid _ x _
It is not possible to determine the type of eigenvalue computation
to perform.
If you want to compute all eigenvalues, then vl, vu, il and iu must not be
present.  If you want to select eigenvalues by specifying a range of
values, then vl and vu must be present and il and iu must not be present.
If you want to select eigenvalues by specifying a range of indices, then
either or both of il and iu must be present and vl and vu must
not be present.

RTN_NAME : 0040-252 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements.
```

```
RTN_NAME : 0040-253 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be replicated and collapsed.
The value for axis_map(1) for the vector is (_).
The number of copies of the vector is (_).

RTN_NAME : 0040-254 Context(_) Task(_) Process(_,_) Grid _ x _
The assumed shape array has an invalid shape.
Array (_) has (_) elements.

RTN_NAME : 0040-255 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-256 Context(_) Task(_) Process(_,_) Grid _ x _
The block sizes of the vectors must all be equal.
The block size of vector (_) is (_).
The block size of vector (_) is (_).
The block size of vector (_) is (_).
The block size of vector (_) is (_).
The block size of vector (_) is (_).

RTN_NAME : 0040-257 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index must be the same for all vectors.
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-258 Context(_) Task(_) Process(_,_) Grid _ x _
Vectors (_), (_), and (_) must all be BLOCK distributed.

RTN_NAME : 0040-259 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be the same for all assumed shape arrays.
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).
The process grid for (_) is (_).

RTN_NAME : 0040-260 Context(_) Task(_) Process(_,_) Grid _ x _
Array (_) must be (BLOCK,*) distributed.

RTN_NAME : 0040-261 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrix and
vectors. The process rank is 1 and the following must be true:
The values for axis_map(1) for the matrix and vectors must be 1.
The value for axis_map(1) for matrix (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-262 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the 6 assumed shape arrays are incompatible.
Array (_) has (_) rows and (_) columns.
Array (_) has (_) elements
Array (_) has (_) elements
Array (_) has (_) elements
Array (_) has (_) elements
Array (_) has (_) elements.

RTN_NAME : 0040-263 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size of the matrix and the block sizes of the vectors are
incompatible.
The row block size of the matrix (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-264 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process indices for the vectors.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-265 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_type argument of HPF library routine global_distribution
indicates that there is an inconsistent data distribution among the
vectors.
The values for axis_type(1) for all of the vectors must be BLOCK.
The value for axis_type(1) for vector (_) is (_).
The value for axis_type(1) for vector (_) is (_).
The value for axis_type(1) for vector (_) is (_).

RTN_NAME : 0040-266 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrix and
vectors. The process rank is 1 and the following must be true:
The values for axis_map(1) for the matrix and vectors must be 1.
The value for axis_map(1) for matrix (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-267 Context(_) Task(_) Process(_,_) Grid _ x _
The vectors must all be not replicated.
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).
The number of copies of the vector (_) is (_).
```

```
RTN_NAME : 0040-268 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size of the matrix and the block sizes of the vectors are
incompatible.
The row block size of the matrix (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-269 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) elements.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-270 Context(_) Task(_) Process(_,_) Grid _ x _
The block sizes of the vectors must all be equal.
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-271 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process indices for the vectors must be equal.
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-272 Context(_) Task(_) Process(_,_) Grid _ x _
The value of UPLO is U.  NB_ (element _ of ARG NO. _) must be greater
than or equal to the half bandwidth, K (ARG NO. _).

RTN_NAME : 0040-273 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be CYCLIC distributed.

RTN_NAME : 0040-274 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution for the vectors.
The value for axis_map(1) for the vectors must be 1.
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-275 Context(_) Task(_) Process(_,_) Grid _ x _
The value for axis_map(1) for the vector must be 1.
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-276 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 4 of ARG NO. _) in the global matrix
must be greater than or equal to the number of right hand sides
(ARG NO. _).

RTN_NAME : 0040-277 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of matrix (ARG NO. _) must be
greater than 0 and less than or equal to the number of rows in the global
matrix, M_, (element _ of ARG NO. _).

RTN_NAME : 0040-278 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index (ARG NO. _) of matrix (ARG NO. _) must be
```

```
greater than 0 and less than or equal to the number of columns in the
global matrix, N_, (element _ of ARG NO. _).

RTN_NAME : 0040-279 Context(_) Task(_) Process(_,_) Grid _ x _
The block size (element 4 of ARG NO. _) must be the same for
all processes.

RTN_NAME : 0040-280 Context(_) Task(_) Process(_,_) Grid _ x _
The process column CSRC_, (element 5 of ARG NO. _) must be the same
for all processes.

RTN_NAME : 0040-281 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 3 of ARG NO. _) in the global matrix
(ARG NO. _) must be greater than zero.

RTN_NAME : 0040-282 Context(_) Task(_) Process(_,_) Grid _ x _
The process row RSRC_, (element 5 of ARG NO. _) must be the same
for all processes.

RTN_NAME : 0040-283 Context(_) Task(_) Process(_,_) Grid _ x _
The block size (element 4 of ARG NO. _) in the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-284 Context(_) Task(_) Process(_,_) Grid _ x _
The (_) block size of array (_) must be equal to the (_) block size
of array (_).

RTN_NAME : 0040-285 Context(_) Task(_) Process(_,_) Grid _ x _
The vector (_) must be BLOCK distributed.

RTN_NAME : 0040-286 Context(_) Task(_) Process(_,_) Grid _ x _
The descriptor type, DTYPE_ (element 1 of ARG NO. _) is invalid.
The valid descriptor type for this routine is _.

RTN_NAME : 0040-287 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced is incompatible with the global matrix definition.
The global column index (ARG NO. _) plus the number of columns (ARG NO. _)
of the matrix (ARG NO. _) minus 1 must be less than or equal to the
number of columns, N_, (element 3 of ARG NO. _).

RTN_NAME : 0040-288 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process indices for the matrices must be equal.
The abstract process index for matrix (_) is (_).
The abstract process index for matrix (_) is (_).

RTN_NAME : 0040-289 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size, MB_, (element 4 of ARG NO. _) of the matrix (ARG NO. _)
must be greater than zero.

RTN_NAME : 0040-290 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element 3 of ARG NO. _) in a null matrix
(ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 0040-291 Context(_) Task(_) Process(_,_) Grid _ x _
The column block size, NB_, (element 4 of ARG NO. _) of the matrix
(ARG NO. _) must be greater than zero.
```

```
RTN_NAME : 0040-292 Context(_) Task(_) Process(_,_) Grid _ x _
The order (ARG NO. _) of matrix (ARG NO. _) must be smaller
than or equal to the product of the number of processors and the block size
(element _ of ARG NO. _) minus the modulus of
(ARG NO. _) minus one with the block size (element _ of ARG NO. _).

RTN_NAME : 0040-293 Context(_) Task(_) Process(_,_) Grid _ x _
The descriptor type, DTYPE_ (element 1 of ARG NO. _) is invalid.
Valid descriptor types for this routine are _ and _.

RTN_NAME : 0040-294 Context(_) Task(_) Process(_,_) Grid _ x _
The descriptor type, DTYPE_ (element 1 of ARG NO. _) is invalid.
Valid descriptor types for this routine are _, _, and _.

RTN_NAME : 0040-297 Context(_) Task(_) Process(_,_) Grid _ x _
End of global input-argument error reporting.  For more information,
refer to Parallel ESSL Guide and Reference.

RTN_NAME : 0040-299 Context(_) Task(_) Process(_,_) Grid _ x _
End of input-argument error reporting.  For more information,
refer to Parallel ESSL Guide and Reference.
```

Note: There are more input-argument error messages listed in "Input-Argument Error Messages (800-999)" on page 154.

Computational Error Messages (300-399)

```
RTN_NAME : 0040-300 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix (ARG NO. _) is singular.  The first diagonal element
found to be exactly 0, was in column (_).

RTN_NAME : 0040-301 Context(_) Task(_) Process(_,_) Grid _ x _
The storage space, specified by (ARG NO. _) is insufficient.
(_) bytes are required.

RTN_NAME : 0040-302 Context(_) Task(_) Process(_,_) Grid _ x _
The matrix (ARG NO. _) is not positive definite.  The leading minor of
order (_) has a nonpositive determinant.

RTN_NAME : 0040-303 Context(_) Task(_) Process(_,_) Grid _ x _
Bisection failed to converge for some eigenvalues. The eigenvalues may not
be as accurate as the absolute and relative tolerances.

RTN_NAME : 0040-304 Context(_) Task(_) Process(_,_) Grid _ x _
The number of eigenvalues computed (ARG NO. _) does not match the
number of eigenvalues requested.

RTN_NAME : 0040-305 Context(_) Task(_) Process(_,_) Grid _ x _
No eigenvalues were computed since the Gershgorin interval initially used
was incorrect.

RTN_NAME : 0040-306 Context(_) Task(_) Process(_,_) Grid _ x _
(_) eigenvectors failed to converge after (_) iterations.
The indices are stored in IFAIL (ARG NO. _).

RTN_NAME : 0040-307 Context(_) Task(_) Process(_,_) Grid _ x _
Eigenvectors corresponding to one or more clusters of eigenvalues could not
```

```
be reorthogonalized because of insufficient workspace. The indices of the
clusters are stored in ICLUSTR (ARG NO. _).

RTN_NAME : 0040-308 Context(_) Task(_) Process(_,_) Grid _ x _
All of the eigenvectors between VL (ARG NO. _) and VU (ARG NO. _)
could not be computed due to insufficient workspace.  The number of
eigenvectors computed is returned in NZ (ARG NO. _).

RTN_NAME : 0040-309 Context(_) Task(_) Process(_,_) Grid _ x _
The number of eigenvalues computed (ARG NO. _) does not equal the number
of eigenvectors computed (ARG NO. _).

RTN_NAME : 0040-310 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is either (nearly) singular or reducible.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) and factored locally is
either (nearly) singular or reducible.  A pivot element
whose magnitude is too small or zero was detected.

RTN_NAME : 0040-311 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is not diagonally dominant.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) and factored
locally is not diagonally dominant.  A pivot element
whose magnitude is too small or zero was detected.

RTN_NAME : 0040-312 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is not positive definite.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) and factored
locally is not positive definite.  A pivot element
whose value is less than or equal to a small positive
number was detected.

RTN_NAME : 0040-313 Context(_) Task(_) Process(_,_) Grid _ x _
The maximum number of specified iterations (_) has been performed
without satisfying the convergence criterion as specified by (_).

RTN_NAME : 0040-314 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner, as specified by argument (_) for sparse
matrix (_) is unstable.

RTN_NAME : 0040-315 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (_) contains duplicated coefficients.

RTN_NAME : 0040-316 Context(_) Task(_) Process(_,_) Grid _ x _
The maximum number of specified iterations (_) has been performed
without satisfying the convergence criterion as specified by (ARG NO. _).

RTN_NAME : 0040-317 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner, as specified by argument (ARG NO. _) for sparse
matrix (ARGS NO. _-_) is unstable.

RTN_NAME : 0040-318 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (ARGS NO. _-_) contains duplicated coefficients.

RTN_NAME : 0040-319 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (_) contains empty row(s).
```

```
RTN_NAME : 0040-320 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (ARGS NO. _-_) contains empty row(s).

RTN_NAME : 0040-321 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is either (nearly) singular or reducible.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) representing interactions with
other processes is either (nearly) singular or reducible.
A pivot element whose magnitude is too small or zero was detected.

RTN_NAME : 0040-322 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is not diagonally dominant.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) representing interactions with
other processes is not diagonally dominant.  A pivot element
whose value is less than or equal to a small positive
number was detected.

RTN_NAME : 0040-323 Context(_) Task(_) Process(_,_) Grid _ x _
The input matrix is not positive definite.
The value of INFO is (_).  The portion of the global
submatrix stored on process (_) representing interactions with
other processes is not positive definite.  A pivot element
whose value is less than or equal to a small positive
number was detected.

RTN_NAME : 0040-399 Context(_) Task(_) Process(_,_) Grid _ x _
Job is terminated.  Optional argument (_) was not specified, but a
computational error occurred.  The value returned is (_).
```

Resource Error Messages (400-499)

```
RTN_NAME : 0040-400 Context(_) Task(_) Process(_,_) Grid _ x _
An internal buffer allocation has failed due to insufficient memory.

RTN_NAME : 0040-401 Context(_) Task(_) Process(_,_) Grid _ x _
Unable to allocate component(s) of derived data type (_) due to
insufficient memory.
```

Communication Error Messages (500-599)

```
RTN_NAME : 0040-500 Context(_) Task(_) Process(_,_) Grid _ x _
Communication error encountered
Job Terminated with rc = _. Refer to IBM AIX Parallel
Environment Parallel Programming Subroutine Reference ( SH26-7230 ).

RTN_NAME : 0040-501 Context(_) Task(_) Process(_,_) Grid _ x _
During process grid synchronization, task(_) has reported an incorrect
grid (_,_).  Actual grid dimension is (_,_).

RTN_NAME : 0040-502 Context(_) Task(_) Process(_,_) Grid _ x _
During process grid synchronization, task(_) has returned an incorrect
value (_) for its own task id.

RTN_NAME : 0040-503 Context(_) Task(_) Process(_,_) Grid _ x _
During process grid synchronization, task(_) has returned an incorrect
message id range (_,_). Expected range is (_,_).
```

Informational and Attention Messages (600-699)

```
RTN_NAME : 0040-600 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: process is sending data to itself.

RTN_NAME : 0040-601 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: process is receiving data from itself.

RTN_NAME : 0040-602 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: process has received data whose length (_) differs from the
expected length (_).

RTN_NAME : 0040-603 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: The message id range for point-to-point communications will be
reused every (_) messages.

RTN_NAME : 0040-604 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: The message id range for scoped communications will be
reused every (_) operations.

RTN_NAME : 0040-605 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: gettimeofday system call returned bad rc (_).

RTN_NAME : 0040-606 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Attempt to change message id range after process grid
definition.  Message id range not changed.

RTN_NAME : 0040-607 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Configuration parameter (ARG NO. _) is invalid.

RTN_NAME : 0040-608 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Application waited for memory allocation.

RTN_NAME : 0040-609 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: getrusage system call returned bad rc (_).

RTN_NAME : 0040-610 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Attempt to define process grid before calling BLACS_GET.

RTN_NAME : 0040-611 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: BLACS system context can only be set by calling BLACS_GET.

RTN_NAME : 0040-612 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: The number of rings cannot be set to zero.

RTN_NAME : 0040-613 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: The number of branches (_) must be greater than zero.

RTN_NAME : 0040-614 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Cannot set BLACS debug level.

RTN_NAME : 0040-615 Context(_) Task(_) Process(_,_) Grid _ x _
Attention: Environment variable PESSL_DESC_TYPE has specified the use
of obsolete descriptor vectors.

RTN_NAME : 0040-616 Context(_) Task(_) Process(_,_) Grid _ x _
Convergence indicator for iterative method (_) at step (_): (_).
```

```
RTN_NAME : 0040-617 Context(_) Task(_) Process(_,_) Grid _ x _
Message buffer space exceeded for error message number (_).
One or more instances of the message was suppressed.
```

```
| RTN_NAME : 0040-618 Context(_) Task(_) Process(_,_) Grid _ x _
| Performance may be degraded due to limited buffer space availability.
```

Miscellaneous Error Messages (700-799)

```
RTN_NAME : 0040-700 Context(_) Task(_) Process(_,_) Grid _ x _
Internal Parallel ESSL error number (_).
Contact your IBM service representative.

RTN_NAME : 0040-701 Context(_) Task(_) Process(_,_) Grid _ x _
Unable to open Parallel ESSL Message Catalog.
See your System Administrator for further assistance.

RTN_NAME : 0040-702 Context(_) Task(_) Process(_,_) Grid _ x _
Internal Parallel ESSL error: message buffer space exceeded for error
message number (_).  Contact your IBM service representative.

RTN_NAME : 0040-703 Context(_) Task(_) Process(_,_) Grid _ x _
Internal Parallel ESSL error: message number requested (_) is outside
of the valid range.  Contact your IBM service representative.

RTN_NAME : 0040-704 Context(_) Task(_) Process(_,_) Grid _ x _
Parallel ESSL has been called from outside the process grid definition.

RTN_NAME : 0040-705 Context(_) Task(_) Process(_,_) Grid _ x _
The communications context (_) is invalid.

RTN_NAME : 0040-706 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be defined with the number of rows set to 1.

RTN_NAME : 0040-707 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be defined with the number of columns set to 1.

RTN_NAME : 0040-708 Context(_) Task(_) Process(_,_) Grid _ x _
The user supplied subroutine (_) has produced an incorrect output.
Argument (_) must be greater than or equal to 1 and less than or equal
to the number of processes in the process grid.

RTN_NAME : 0040-709 Context(_) Task(_) Process(_,_) Grid _ x _
The user supplied subroutine (_) has produced an incorrect output.
Argument (_) must be greater than or equal to 0 and less than the
the number of processes in the process grid.

RTN_NAME : 0040-710 Context(_) Task(_) Process(_,_) Grid _ x _
The user supplied subroutine (ARG NO. _) has produced an incorrect output.
(ARG NO. _) of the user supplied subroutine must be greater than
or equal to 1 and less than or equal to the number of processes in the
process grid.

RTN_NAME : 0040-711 Context(_) Task(_) Process(_,_) Grid _ x _
The user supplied subroutine (ARG NO. _) has produced an incorrect output.
(ARG NO. _) of the user supplied subroutine must be greater than
or equal to 0 and less than the number of processes in the process grid.
```

```
RTN_NAME : 0040-712 Context(_) Task(_) Process(_,_) Grid _ x _
The size of input array (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 0040-713 Context(_) Task(_) Process(_,_) Grid _ x _
Environment variable PESSL_DESC_TYPE has specified the use of obsolete
descriptor vectors.  You must update the descriptor vectors in your program
as described in the Parallel ESSL Guide and Reference.
```

Input-Argument Error Messages (800-999)

```
RTN_NAME : 0040-800 Context(_) Task(_) Process(_,_) Grid _ x _
DTYPE_ (element _ of ARG NO. _) for matrix (ARG NO. _) is _.
The process grid for matrix (ARG NO. _) must be defined with the number
of rows set to 1.

RTN_NAME : 0040-801 Context(_) Task(_) Process(_,_) Grid _ x _
DTYPE_ (element _ of ARG NO. _) for matrix (ARG NO. _) is _.
The process grid for matrix (ARG NO. _) must be defined with the number
of columns set to 1.

RTN_NAME : 0040-802 Context(_) Task(_) Process(_,_) Grid _ x _
The global column index, (ARG NO. _) must be equal to the global row
index (ARG NO. _).

RTN_NAME : 0040-803 Context(_) Task(_) Process(_,_) Grid _ x _
The submatrix referenced is incompatible with the global matrix definition.
The global row index (ARG NO. _) plus the number of rows (ARG NO. _)
of the matrix (ARG NO. _) minus 1 must be less than or equal to the
number of rows, N_, (element _ of ARG NO. _).

RTN_NAME : 0040-804 Context(_) Task(_) Process(_,_) Grid _ x _
The number of rows, M_, (element _ of ARG NO. _) in the global matrix
(ARG NO. _) must be equal to one.

RTN_NAME : 0040-805 Context(_) Task(_) Process(_,_) Grid _ x _
The number of columns, N_, (element _ of ARG NO. _) in the global matrix
(ARG NO. _) must be equal to one.

RTN_NAME : 0040-806 Context(_) Task(_) Process(_,_) Grid _ x _
DTYPE_ (element _ of ARG NO. _) is one.  At least one of the following
must be true: For the global matrix (ARG NO. _), the number of rows, M_,
(element _ of ARG NO. _) must be equal to one or the number of
columns, N_, (element _ of ARG NO. _) must be equal to one.

RTN_NAME : 0040-807 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size MB_, (element _ of ARG NO. _) of the global matrix
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-808 Context(_) Task(_) Process(_,_) Grid _ x _
Vectors (_) and (_) must be BLOCK distributed.

RTN_NAME : 0040-809 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution for the vectors.
The value for axis_map(1) for the vectors must be 1.
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).
```

```
RTN_NAME : 0040-810 Context(_) Task(_) Process(_,_) Grid _ x _
The shapes of the assumed shape arrays are incompatible.
Array (_) has (_) elements.
Array (_) has (_) elements.

RTN_NAME : 0040-811 Context(_) Task(_) Process(_,_) Grid _ x _
The block sizes of the vectors must be equal.
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-812 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process indices for the vectors must be equal.
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-813 Context(_) Task(_) Process(_,_) Grid _ x _
The axis_map argument of HPF library routine global_alignment indicates
that there is an inconsistent data distribution between the matrix and
vectors. The process rank is 1 and the following must be true:
The values for axis_map(1) for the matrix and vectors must be 1.
The value for axis_map(1) for matrix (_) is (_).
The value for axis_map(1) for vector (_) is (_).
The value for axis_map(1) for vector (_) is (_).

RTN_NAME : 0040-814 Context(_) Task(_) Process(_,_) Grid _ x _
The row block size of the matrix and the block sizes of the vectors are
incompatible.
The row block size of the matrix (_) is (_).
The block size of the vector (_) is (_).
The block size of the vector (_) is (_).

RTN_NAME : 0040-815 Context(_) Task(_) Process(_,_) Grid _ x _
The abstract process index for the matrix must be equal to the abstract
process indices for the vectors.
The abstract process index for matrix (_) is (_).
The abstract process index for vector (_) is (_).
The abstract process index for vector (_) is (_).

RTN_NAME : 0040-816 Context(_) Task(_) Process(_,_) Grid _ x _
The process grid must be defined with the number of columns set to 1.

RTN_NAME : 0040-817 Context(_) Task(_) Process(_,_) Grid _ x _
The size of array (ARG NO. _) must be greater than or equal to (_).

RTN_NAME : 0040-818 Context(_) Task(_) Process(_,_) Grid _ x _
The array descriptor (_) has not been initialized.
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-819 Context(_) Task(_) Process(_,_) Grid _ x _
The array descriptor (_) contains invalid component(s).
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-820 Context(_) Task(_) Process(_,_) Grid _ x _
The array descriptor (_) contains invalid component(s).

RTN_NAME : 0040-821 Context(_) Task(_) Process(_,_) Grid _ x _
The pointer(s) specified by argument (_) are not associated and
therefore cannot be freed.
```

```
RTN_NAME : 0040-822 Context(_) Task(_) Process(_,_) Grid _ x _
The size of array (_) must be greater than or equal to (_).
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-823 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (_) is invalid.
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-824 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (_) was not initialized properly.
Some local row(s) are missing.
Additional calls to (_) may be required.

RTN_NAME : 0040-825 Context(_) Task(_) Process(_,_) Grid _ x _
The value of argument (_) is (_); therefore argument (_) is required.

RTN_NAME : 0040-826 Context(_) Task(_) Process(_,_) Grid _ x _
The storage format (_) specified for sparse matrix (_)
must be (_).

RTN_NAME : 0040-827 Context(_) Task(_) Process(_,_) Grid _ x _
Argument (_) must be equal to (_).

RTN_NAME : 0040-828 Context(_) Task(_) Process(_,_) Grid _ x _
The storage format for sparse matrix (_) must be (_).
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-829 Context(_) Task(_) Process(_,_) Grid _ x _
The contents of array descriptor (ARG NO. _) are invalid.

RTN_NAME : 0040-830 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (ARGS NO. _-_) is invalid.
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-831 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be greater than or equal to (_) and less than
or equal to (_).

RTN_NAME : 0040-832 Context(_) Task(_) Process(_,_) Grid _ x _
The storage format of sparse matrix (ARGS NO. _-_) is invalid.

RTN_NAME : 0040-833 Context(_) Task(_) Process(_,_) Grid _ x _
One or more of the rows requested for insertion with (_) does not belong
to this process.

RTN_NAME : 0040-834 Context(_) Task(_) Process(_,_) Grid _ x _
One or more of the rows requested for insertion with (ARG NO. _)
does not belong to this process.

RTN_NAME : 0040-835 Context(_) Task(_) Process(_,_) Grid _ x _
Argument (_) must be the same for all processes.

RTN_NAME : 0040-836 Context(_) Task(_) Process(_,_) Grid _ x _
Argument (_) must be greater than or equal to (_) and less than
or equal to (_).
```

```
RTN_NAME : 0040-837 Context(_) Task(_) Process(_,_) Grid _ x _
The sparse matrix (ARGS NO. _-_) was not initialized properly.
Some local row(s) are missing.
Additional calls to (_) may be required.

RTN_NAME : 0040-838 Context(_) Task(_) Process(_,_) Grid _ x _
The storage format (ARG NO. _) specified for sparse matrix (ARGS NO. _-_)
must be (_).

RTN_NAME : 0040-839 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be equal to (_).

RTN_NAME : 0040-840 Context(_) Task(_) Process(_,_) Grid _ x _
(ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-841 Context(_) Task(_) Process(_,_) Grid _ x _
Element (_) of array (ARG NO. _) must be equal to (_).

RTN_NAME : 0040-842 Context(_) Task(_) Process(_,_) Grid _ x _
Element (_) of array (ARG NO. _) must be greater than or equal to
(_) and less than or equal to (_).

RTN_NAME : 0040-843 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_, (element _ of ARG NO. _) must be greater
than or equal to 0 and less than the total number of rows in the process
grid.

RTN_NAME : 0040-844 Context(_) Task(_) Process(_,_) Grid _ x _
The process column, CSRC_, (element _ of ARG NO. _) must be greater
than or equal to 0 and less than the total number of columns in the process
grid.

RTN_NAME : 0040-845 Context(_) Task(_) Process(_,_) Grid _ x _
The process column, CSRC_, (element _ of ARG NO. _) must be equal
to the process row, RSRC_, (element _ of ARG NO. _).

RTN_NAME : 0040-846 Context(_) Task(_) Process(_,_) Grid _ x _
The workspace size (ARG NO. _) has been specified as minus one for
a subset of the processes and therefore it must be specified as minus one
for all processes.

RTN_NAME : 0040-847 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner (_) contains invalid components.
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-848 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner (ARG NO. _) contains invalid components.
Routine (_) must be called prior to this routine.

RTN_NAME : 0040-849 Context(_) Task(_) Process(_,_) Grid _ x _
The size of array (_) must be greater than or equal to (_) and less
than or equal to (_).

RTN_NAME : 0040-850 Context(_) Task(_) Process(_,_) Grid _ x _
The matrix type (_) specified for sparse matrix (_) must be (_).
```

```
RTN_NAME : 0040-851 Context(_) Task(_) Process(_,_) Grid _ x _
The matrix type (ARG NO. _) specified for sparse matrix (ARGS NO. _-_)
must be (_).

RTN_NAME : 0040-852 Context(_) Task(_) Process(_,_) Grid _ x _
Element (_) of vector (ARG NO. _) must be greater than or equal to zero.

RTN_NAME : 0040-853 Context(_) Task(_) Process(_,_) Grid _ x _
Element (_) of vector (ARG NO. _) must be the same for all processes.

RTN_NAME : 0040-854 Context(_) Task(_) Process(_,_) Grid _ x _
LWORK (ARG NO. _), which specifies the size of the local work
array, must be the same for all processes.

RTN_NAME : 0040-855 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner data structure (_) must be passed unchanged to the
solver subroutine.

RTN_NAME : 0040-856 Context(_) Task(_) Process(_,_) Grid _ x _
The preconditioner data structure (ARG NO. _) must be passed unchanged to the
solver subroutine.

RTN_NAME : 0040-857 Context(_) Task(_) Process(_,_) Grid _ x _
The size of array (_) must be greater than or equal to (_).

RTN_NAME : 0040-858 Context(_) Task(_) Process(_,_) Grid _ x _
The global row index (ARG NO. _) of matrix (ARG NO. _) must be
greater than 0 and less than or equal to the number of columns in the global
matrix, N_, (element _ of ARG NO. _).

RTN_NAME : 0040-859 Context(_) Task(_) Process(_,_) Grid _ x _
The process row, RSRC_, (element _ of ARG NO. _) must be equal
to the process row, RSRC_, (element _ of ARG NO. _).

RTN_NAME : 0040-860 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies the computation to be performed, must be
'N' or 'T'.

RTN_NAME : 0040-861 Context(_) Task(_) Process(_,_) Grid _ x _
TRANS (ARG NO. _), which specifies the computation to be performed, must be
'N' or 'C'.
```

Part 2. Reference Information (Message Passing)

This part of the book is organized into seven areas, providing reference information for coding the Parallel ESSL message passing subroutines. It is organized as follows:

- Level 2 PBLAS
- Level 3 PBLAS
- Linear Algebraic Equations
- Eigensystem Analysis and Singular Value Analysis
- Fourier Transforms
- Random Number Generation
- Utilities

Chapter 6. Level 2 PBLAS (Message Passing)

This chapter describes the Level 2 PBLAS subroutines.

Overview of the Level 2 PBLAS Subroutines

The Level 2 PBLAS include a subset of the standard set of distributed memory parallel versions of the Level 2 BLAS.

Note: These subroutines are designed in accordance with the proposed Level 2 PBLAS standard. (See references [14], [15], and [17].) If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 38. List of Level 2 PBLAS (Message Passing)

Descriptive Name	Long-Precision Subprogram	Page
Matrix-Vector Product for a General Matrix or Its Transpose	PDGEMV PZGEMV	163
Matrix-Vector Product for a Real Symmetric or a Complex Hermitian Matrix	PDSYMV PZHEMV	189
Rank-One Update of a General Matrix	PDGER PZGERC PZGERU	204
Rank-One Update of a Real Symmetric or a Complex Hermitian Matrix	PDSYR PZHER	224
Rank-Two Update of a Real Symmetric or a Complex Hermitian Matrix	PDSYR2 PZHER2	236
Matrix-Vector Product for a Triangular Matrix or Its Transpose	PDTRMV PZTRMV	252
Solution of Triangular System of Equations with a Single Right-Hand Side	PDTRSV PZTRSV	265

Level 2 PBLAS Subroutines

This section contains the Level 2 PBLAS subroutine descriptions.

PDGEMV and PZGEMV—Matrix-Vector Product for a General Matrix or Its Transpose

PDGEMV computes one of the following matrix-vector products:

$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}\boldsymbol{x}+\beta \boldsymbol{y}$
$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}+\beta \boldsymbol{y}$

PZGEMV computes one of the following matrix-vector products:

$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}\boldsymbol{x}+\beta \boldsymbol{y}$
$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}+\beta \boldsymbol{y}$
$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}^{\mathrm{H}}\boldsymbol{x}+\beta \boldsymbol{y}$

where, in the formulas above:

A represents the global general submatrix $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$.
x represents the global vector:
- For *transa* = `'N'`:
  - For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
  - For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.
- For *transa* = `'T'` or `'C'`:
  - For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+m-1}$.
  - For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+m-1,\ jx:jx}$.

y represents the global vector:
- For *transa* = `'N'`:
  - For *incy* = M_Y, it is $\boldsymbol{Y}_{iy:iy,\ jy:jy+m-1}$.
  - For *incy* = 1 and *incy* ≠ M_Y, it is $\boldsymbol{Y}_{iy:iy+m-1,\ jy:jy}$.
- For *transa* = `'T'` or `'C'`:
  - For *incy* = M_Y, it is $\boldsymbol{Y}_{iy:iy,\ jy:jy+n-1}$.
  - For *incy* = 1 and *incy* ≠ M_Y, it is $\boldsymbol{Y}_{iy:iy+n-1,\ jy:jy}$.

α and β are scalars.

Note: No data should be moved to form $\boldsymbol{A}^{\mathrm{T}}$ or $\boldsymbol{A}^{\mathrm{H}}$; that is, the matrix ***A*** should always be stored in its untransposed form.

In the following three cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $m = 0$
- $n = 0$
- α is zero and β is one.

See references [14] and [15].

Table 39. Data Types

α, β, ***A***, ***x***, ***y***	**Subprogram**
Long-precision real	PDGEMV
Long-precision complex	PZGEMV

Syntax

Fortran	CALL PDGEMV | PZGEMV (*transa*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*, *beta*, *y*, *iy*, *jy*, *desc_y*, *incy*)

C and C++	pdgemv | pzgemv (*transa*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*, *beta*, *y*, *iy*, *jy*, *desc_y*, *incy*);

On Entry

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = 'N', ***A*** is used in the computation.

If *transa* = 'T', $\boldsymbol{A}^T$ is used in the computation.

If *transa* = 'C', $\boldsymbol{A}^H$ is used in the computation.

Scope: **global**

Specified as: a single character; *transa* = 'N', 'T', or 'C'.

m

is the number of rows in submatrix ***A*** used in the computation, and:

If *transa* = 'N', it is the number of elements in vector ***y***.

If *transa* = 'T' or 'C', it is the number of elements in vector ***x***.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrix ***A*** used in the computation, and:

If *transa* = 'N', it is the number of elements in vector ***x***.

If *transa* = 'T' or 'C', it is the number of elements in vector ***y***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 39 on page 163.

a

is the local part of the global general matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*m*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*m*–1 by *ja*+*n*–1 part of the global matrix.

Note: No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 39 on page 163. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+m-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore, assuming the following:

If *transa* = 'N', *numx* = *n*

| If *transa* = 'T' or 'C', *numx* = *m*

the following must be true:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*numx*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*numx*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*numx*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*numx*–1 by *jx* part of the global matrix.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 39 on page 163. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X, and if *incx* = 1 and *incx* ≠ M_X, then:

If *transa* = `'N'`, then *ix*+*n*–1 ≤ M_X.

| If *transa* = `'T'` or `'C'`, then *ix*+*m*–1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jx* ≤ N_X, and if *incx* = M_X, then:

If *transa* = `'N'`, then *jx*+*n*–1 ≤ N_X.

| If *transa* = `'T'` or `'C'`, then *jx*+*m*–1 ≤ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If *transa* = `'N'` and *n* = 0: M_X ≥ 0 If *transa* = `'T'` and *m* = 0: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If *transa* = `'N'` and *n* = 0: N_X ≥ 0 If *transa* = `'T'` and *m* = 0: N_X ≥ 0 Otherwise: N_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global

desc_x	**Name**	**Description**	**Limits**	**Scope**
7	RSRC_X	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_X $< p$	Global
8	CSRC_X	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_X $< q$	Global
9	LLD_X	The leading dimension of the local array	LLD_X $\ge$ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 39 on page 163.

y

is the local part of the global matrix ***Y***. This identifies the **first element** of the local array Y. This subroutine computes the location of the first element of the local subarray used, based on *iy*, *jy*, *desc_y*, *p*, *q*, *myrow*, and *mycol*; therefore, assuming the following:

If *transa* = 'N', *numy* = *m*

| If *transa* = 'T' or 'C', *numy* = *n*

the following must be true:

- If *incy* = M_Y, the leading LOCp(*iy*) by LOCq(*jy*+*numy*–1) part of the local array Y must contain the local pieces of the leading *iy* by *jy*+*numy*–1 part of the global matrix.
- If *incy* = 1 and *incy* ≠ M_Y, the leading LOCp(*iy*+*numy*–1) by LOCq(*jy*) part of the local array Y must contain the local pieces of the leading *iy*+*numy*–1 by *jy* part of the global matrix.

When β is zero, ***y*** need not be set on input.

Scope: **local**

Specified as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 39 on page 163. Details about the block-cyclic data distribution of the global matrix ***Y*** are stored in *desc_y*.

iy

has the following meaning:

If *incy* = M_Y, it indicates which row of global matrix ***Y*** is used for vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it is the row index of global matrix ***Y***, identifying the first element of vector ***y***.

Scope: **global**

Specified as: a fullword integer; $1 \le iy \le$ M_Y, and if *incy* = 1 and *incy* ≠ M_Y, then:

If *transa* = 'N', then *iy*+*m*–1 ≤ M_Y.

| If *transa* = 'T' or 'C', then *iy*+*n*–1 ≤ M_Y.

jy

has the following meaning:

If *incy* = M_Y, it is the column index of global matrix ***Y***, identifying the first element of vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it indicates which column of global matrix ***Y*** is used for vector ***y***.

Scope: **global**

Specified as: a fullword integer; $1 \le jy \le$ N_Y, and if *incy* = M_Y, then:

If *transa* = 'N', then *jy*+*m*–1 ≤ N_Y.

| If *transa* = 'T' or 'C', then *jy*+*n*–1 ≤ N_Y.

desc_y

is the array descriptor for global matrix ***Y***, described in the following table:

desc_y	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_Y	Descriptor type	DTYPE_Y=1	Global
2	CTXT_Y	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_Y	Number of rows in the global matrix	If *transa* = 'N' and *m* = 0: M_Y ≥ 0 If *transa* = 'T' and *n* = 0: M_Y ≥ 0 Otherwise: M_Y ≥ 1	Global
4	N_Y	Number of columns in the global matrix	If *transa* = 'N' and *m* = 0: N_Y ≥ 0 If *transa* = 'T' and *n* = 0: N_Y ≥ 0 Otherwise: N_Y ≥ 1	Global
5	MB_Y	Row block size	MB_Y ≥ 1	Global
6	NB_Y	Column block size	NB_Y ≥ 1	Global
7	RSRC_Y	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_Y < *p*	Global
8	CSRC_Y	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_Y < *q*	Global

desc_y	**Name**	**Description**	**Limits**	**Scope**
9	LLD_Y	The leading dimension of the local array	LLD_Y ≥ max(1,LOCp(M_Y))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incy

is the stride for global vector ***y***.

Scope: **global**

Specified as: a fullword integer; *incy* = 1 or *incy* = M_Y, where:

If *incy* = M_Y, then ***y*** is a row-distributed vector.

If *incy* = 1 and *incy* ≠ M_Y, then ***y*** is a column-distributed vector.

On Return

y

is the updated local part of the global matrix ***Y***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 39 on page 163.

Notes and Coding Rules

|
1. These subroutines accept lowercase letters for the *transa* argument.
2. For PDGEMV, if you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
3. The matrix and vectors must have no common elements; otherwise, results are unpredictable.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
5. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
6. The following values must be equal: CTXT_A = CTXT_X = CTXT_Y.
7. The following coding rules depend upon the values specified for *transa* and *incx*:
   - If *transa* = `'N'` and *incx* = M_X:
     - The following block sizes must be equal: NB_A = NB_X.
     - In the process grid, the process column containing the first column of the submatrix ***X*** must also contain the first column of the submatrix ***A***; that is, *iacol* = *ixcol*, where:

       iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
       ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)

    - The block column offset of ***x*** must be equal to the block column offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ja*–1, NB_A).
  - If *transa* = `'N'` and *incx* = 1(≠ M_X):
    - The following block sizes must be equal: NB_A = MB_X.
    - The block row offset of ***x*** must be equal to the block column offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ja*–1, NB_A).
  - If *transa* = `'T'` or `'C'` and *incx* = M_X:
    - The following block sizes must be equal: MB_A = NB_X.
    - The block column offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ia*–1, MB_A).
  - If *transa* = `'T'` or `'C'` and *incx* = 1(≠ M_X):
    - The following block sizes must be equal: MB_A = MB_X.
    - In the process grid, the process row containing the first row of the submatrix ***X*** must also contain the first row of the submatrix ***A***; that is, *iarow* = *ixrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    - The block row offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A).

8. The following coding rules depend upon the values specified for *transa* and *incy*:
  - If *transa* = `'N'` and *incy* = M_Y:
    - The following block sizes must be equal: MB_A = NB_Y.
    - The block column offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*jy*–1, NB_Y) = mod(*ia*–1, MB_A).
  - If *transa* = `'N'` and *incy* = 1(≠ M_Y):
    - The following block sizes must be equal: MB_A = MB_Y.
    - In the process grid, the process row containing the first row of the submatrix ***Y*** must also contain the first row of the submatrix ***A***; that is, *iarow* = *iyrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      iyrow = mod((((*iy*–1)/MB_Y)+RSRC_Y), *p*)
    - The block row offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*iy*–1, MB_Y) = mod(*ia*–1, MB_A).
  - If *transa* = `'T'` or `'C'` and *incy* = M_Y:
    - The following block sizes must be equal: NB_A = NB_Y.
    - In the process grid, the process column containing the first column of the submatrix ***Y*** must also contain the first column of the submatrix ***A***; that is, *iacol* = *iycol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)

  - The block column offset of ***y*** must be equal to the block column offset of ***A***; that is, mod(*jy*–1, NB_Y) = mod(*ja*–1, NB_A).
- If *transa* = `'T'` or `'C'` and *incy* = 1(≠ M_Y):
  - The following block sizes must be equal: NB_A = MB_Y.
  - The block row offset of ***y*** must be equal to the block column offset of ***A***; that is, mod(*iy*–1, MB_Y) = mod(*ja*–1, NB_A).

9. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See "Program Main (Message Passing)" on page 1006.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.
3. DTYPE_Y is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *transa* ≠ `'N'`, `'T'`, or `'C'`
2. *m* < 0
3. *n* < 0
4. M_A < 0 and (*m* = 0 or *n* = 0); M_A < 1 otherwise
5. N_A < 0 and (*m* = 0 or *n* = 0); N_A < 1 otherwise
6. MB_A < 1
7. NB_A < 1
8. RSRC_A < 0 or RSRC_A ≥ *p*
9. CSRC_A < 0 or CSRC_A ≥ *q*
10. *ia* < 1
11. *ja* < 1

If (*n* = 0 and *transa* = `'N'`) or (*m* = 0 and *transa* = `'T'` or `'C'`):
12. M_X < 0
13. N_X < 0

Otherwise:
14. M_X < 1
15. N_X < 1

In all cases:
16. MB_X < 1

17. NB_X < 1
18. RSRC_X < 0 or RSRC_X $\geq p$
19. CSRC_X < 0 or CSRC_X $\geq q$
20. CTXT_A $\neq$ CTXT_X
21. $ix < 1$
22. $jx < 1$

| If ($m = 0$ and $transa$ = 'N') or ($n = 0$ and $transa$ = 'T' or 'C'):
23. M_Y < 0
24. N_Y < 0

Otherwise:
25. M_Y < 1
26. N_Y < 1

In all cases:
27. MB_Y < 1
28. NB_Y < 1
29. RSRC_Y < 0 or RSRC_Y $\geq p$
30. CSRC_Y < 0 or CSRC_Y $\geq q$
31. CTXT_A $\neq$ CTXT_Y
32. $iy < 1$
33. $jy < 1$

Stage 5

If $m \neq 0$ and $n \neq 0$:
1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+m-1 >$ M_A
4. $ja+n-1 >$ N_A

| If ($n \neq 0$ and $transa$ = 'N') or ($m \neq 0$ and $transa$ = 'T' or 'C'):
5. $ix >$ M_X
6. $jx >$ N_X

| If ($m \neq 0$ and $transa$ = 'N') or ($n \neq 0$ and $transa$ = 'T' or 'C'):
7. $iy >$ M_Y
8. $jy >$ N_Y

If $incx$ = M_X and $transa$ = 'N':
9. NB_X $\neq$ NB_A
10. mod($jx-1$, NB_X) $\neq$ mod($ja-1$, NB_A)
11. $n \neq 0$ and $jx+n-1 \leq$ N_X

| If $incx$ = M_X and $transa$ = 'T' or 'C':
12. NB_X $\neq$ MB_A
13. mod($jx-1$, NB_X) $\neq$ mod($ia-1$, MB_A)
14. $m \neq 0$ and $jx+m-1 \leq$ N_X

If $incx$ = 1($\neq$ M_X) and $transa$ = 'N':
15. MB_X $\neq$ NB_A
16. mod($ix-1$, MB_X) $\neq$ mod($ja-1$, NB_A)
17. $n \neq 0$ and $ix+n-1 \leq$ M_X

| If $incx$ = 1($\neq$ M_X) and $transa$ = 'T' or 'C':
18. MB_X $\neq$ MB_A
19. mod($ix-1$, MB_X) $\neq$ mod($ia-1$, MB_A)
20. $m \neq 0$ and $ix+m-1 \leq$ M_X

In all cases:
21. *incx* ≠ M_X and *incx* ≠ 1

If *incy* = M_Y and *transa* = 'N':
22. NB_Y ≠ MB_A
23. mod(*jy*–1, NB_Y) ≠ mod(*ia*–1, MB_A)
24. *m* ≠ 0 and *jy*+*m*–1 ≤ N_Y

| If *incy* = M_Y and *transa* = 'T' or 'C':
25. NB_Y ≠ NB_A
26. mod(*jy*–1, NB_Y) ≠ mod(*ja*–1, NB_A)
27. *n* ≠ 0 and *jy*+*n*–1 ≤ N_Y

If *incy* = 1(≠ M_Y) and *transa* = 'N':
28. MB_Y ≠ MB_A
29. mod(*iy*–1, MB_Y) ≠ mod(*ia*–1, MB_A)
30. *m* ≠ 0 and *iy*+*m*–1 ≤ M_Y

| If *incy* = 1(≠ M_Y) and *transa* = 'T' or 'C':
31. MB_Y ≠ NB_A
32. mod(*iy*–1, MB_Y) ≠ mod(*ja*–1, NB_A)
33. *n* ≠ 0 and *iy*+*n*–1 ≤ M_Y

In all cases:
34. *incy* ≠ M_Y and *incy* ≠ 1

Stage 6

If *transa* = 'N':
1. If *incx* = M_X, then (in the process grid) the process column containing the first column of the submatrix ***X*** does not contain the first column of the submatrix ***A***; that is, *iacol* ≠ *ixcol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
2. If *incy* = 1(≠ M_Y), then (in the process grid) the process row containing the first row of the submatrix ***Y*** does not contain the first row of the submatrix ***A***; that is, *iarow* ≠ *iyrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   iyrow = mod((((*iy*–1)/MB_Y)+RSRC_Y), *p*)

| If *transa* = 'T' or 'C':
3. If *incx* = 1(≠ M_X), then (in the process grid) the process row containing the first row of the submatrix ***X*** does not contain the first row of the submatrix ***A***; that is, *iarow* ≠ *ixrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
4. If *incy* = M_Y, then (in the process grid) the process column containing the first column of the submatrix ***Y*** does not contain the first column of the submatrix ***A***; that is, *iacol* ≠ *iycol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)

In all cases:
5. LLD_A < max(1, LOCp(M_A))
6. LLD_X < max(1, LOCp(M_X))
7. LLD_Y < max(1, LOCp(M_Y))

Example 1: This example computes $\boldsymbol{y} = \alpha \boldsymbol{Ax} + \beta \boldsymbol{y}$ using a 2 × 2 process grid. The input matrices ***A***, ***X***, and ***Y***, used here, are the same as ***A***, ***B***, and ***C***, used in "Example 1" on page 292 for PDGEMM. The updated portion of ***Y*** is the same as for ***C*** in PDGEMM, as this computation is equivalent to a portion of the PDGEMM computation.

This example uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 3 and *ja* = 1. It uses vectors ***x*** and ***y***, which are column-distributed vectors within a column of ***X*** and ***Y***, respectively, by specifying *incx* = 1, *ix* = 1, and *jx* = 2 for ***x*** and *incy* = 1, *iy* = 3, and *jy* = 2 for ***y***.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             TRANSA M   N    ALPHA    A  IA  JA    DESC_A   X  IX  JX
               |    |   |      |      |   |   |      |      |   |   |
CALL PDGEMV( 'N' , 4 , 5  , 1.0D0  , A , 3 , 1 ,  DESC_A , X , 1 , 2 ,

             DESC_X  INCX  BETA    Y  IY  JY   DESC_Y   INCY
               |      |     |      |   |   |     |       |
             DESC_X , 1 ,  2.0D0 , Y , 3 , 2 , DESC_Y ,  1 )
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*1	*icontxt*1	*icontxt*1
M_	6	5	6
N_	5	4	4
MB_	3	2	3
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below2	See below2	See below2

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = LLD_Y = 3 on all processes, LLD_X = 3 on P_{00} and P_{01}, and LLD_X = 2 on P_{10} and P_{11}.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 4 × 5 submatrix ***A***, starting at row 3 and column 1 in global general 6 × 5 matrix ***A*** with block size 3 × 2:

```
B,D       0            1         2
     ┌                                  ┐
     │   .    .   │    .    .   │    .  │
 0   │   .    .   │    .    .   │    .  │
     │  1.0 -1.0  │  -1.0  1.0  │   2.0 │
     │ -----------|-------------|------ │
     │ -3.0  2.0  │   2.0  2.0  │   0.0 │
 1   │  4.0  0.0  │  -2.0  1.0  │  -1.0 │
     │ -1.0 -1.0  │   1.0 -3.0  │   2.0 │
     └                                  ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │       0         │      1
-----|-----------------|------------
     │   .    .    .   │    .    .
 0   │   .    .    .   │    .    .
     │  1.0 -1.0  2.0  │  -1.0  1.0
-----|-----------------|------------
     │ -3.0  2.0  0.0  │   2.0  2.0
 1   │  4.0  0.0 -1.0  │  -2.0  1.0
     │ -1.0 -1.0  2.0  │   1.0 -3.0
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 5 × 1, starting at row 1 and column 2 in 5 × 4 global matrix ***X*** with block size 2 × 2:

```
B,D       0            1
     ┌                       ┐
 0   │   .  -1.0  │    .    .│
     │   .   2.0  │    .    .│
     │ -----------|----------│
 1   │   .   0.0  │    .    .│
     │   .  -1.0  │    .    .│
     │ -----------|----------│
 2   │   .   2.0  │    .    .│
     └                       ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |     0      |     1
-----|------------|-----------
     |  .  -1.0   |    .    .
 0   |  .   2.0   |    .    .
     |  .   2.0   |    .    .
-----|------------|-----------
 1   |  .   0.0   |    .    .
     |  .  -1.0   |    .    .
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 4 × 1, starting at row 3 and column 2 in 6 × 4 global matrix ***Y*** with block size 3 × 2:

```
B,D       0           1
     ┌                      ┐
     |  .    .   |    .    .|
 0   |  .    .   |    .    .|
     |  .   0.5  |    .    .|
     | -----------|---------- |
     |  .   0.5  |    .    .|
 1   |  .   0.5  |    .    .|
     |  .   0.5  |    .    .|
     └                      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |     0      |     1
-----|------------|-----------
     |  .    .    |    .    .
 0   |  .    .    |    .    .
     |  .   0.5   |    .    .
-----|------------|-----------
     |  .   0.5   |    .    .
 1   |  .   0.5   |    .    .
     |  .   0.5   |    .    .
```

Output:

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 4 × 1, starting at row 3 and column 2 in 6 × 4 global matrix ***Y*** with block size 3 × 2:

```
B,D       0          1
     ┌                     ┐
     |  .    .  |   .    . |
 0   |  .    .  |   .    . |
     |  .  1.0  |   .    . |
     | -----------|---------- |
     |  .  6.0  |   .    . |
 1   |  . -6.0  |   .    . |
     |  .  7.0  |   .    . |
     └                     ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |     0      |     1
-----|------------|-----------
     |  .    .    |   .    .
 0   |  .    .    |   .    .
     |  .  1.0    |   .    .
-----|------------|-----------
     |  .  6.0    |   .    .
 1   |  . -6.0    |   .    .
     |  .  7.0    |   .    .
```

Example 2: This example computes ***y*** = α***Ax***+β***y*** using a 2 × 2 process grid. The input matrices ***A***, ***X***, and ***Y***, used here, are the same as ***A***, ***B***, and ***C***, used in "Example 1" on page 292 for PDGEMM.

This example uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2. It uses vector ***x***, which is a row-distributed vector within a row of ***X***, by specifying *incx* = M_X = 5, *ix* = 4, and *jx* = 2. It uses vector ***y***, which is a column-distributed vector within a column of ***Y***, by specifying *incy* = 1, *iy* = 2, and *jy* = 3.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            TRANSA M   N    ALPHA    A  IA  JA    DESC_A   X  IX  JX
              |    |   |      |      |   |   |      |      |   |   |
CALL PDGEMV( 'N' , 4 , 3  , 1.0D0  , A , 2 , 2 ,  DESC_A , X , 4 , 2 ,

            DESC_X INCX   BETA    Y  IY  JY   DESC_Y   INCY
              |     |     |       |   |   |     |        |
            DESC_X , 5 ,  2.0D0 , Y , 2 , 3 , DESC_Y ,  1 )
```

	Desc_A	Desc_X	Desc_Y
DTYPE_	1	1	1
CTXT_	*icontxt*1	*icontxt*1	*icontxt*1
M_	6	5	6
N_	5	4	4
MB_	3	2	3
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below2	See below2	See below2

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = LLD_Y = 3 on all processes, LLD_X = 3 on P_{00} and P_{01}, and LLD_X = 2 on P_{10} and P_{11}.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 4 × 3 submatrix ***A***, starting at row 2 and column 2 in global general 6 × 5 matrix ***A*** with block size 3 × 2:

```
B,D       0           1         2
     ┌                              ┐
     |   .    .  |    .    .  |    . |
 0   |   .   0.0 |   1.0  1.0 |    . |
     |   .  -1.0 |  -1.0  1.0 |    . |
     | ----------|------------|----- |
     |   .   2.0 |   2.0  2.0 |    . |
 1   |   .   0.0 |  -2.0  1.0 |    . |
     |   .    .  |    .    .  |    . |
     └                              ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |       0         |      1
-----|-----------------|------------
     |   .    .    .   |    .    .
 0   |   .   0.0   .   |   1.0  1.0
     |   .  -1.0   .   |  -1.0  1.0
-----|-----------------|------------
     |   .   2.0   .   |   2.0  2.0
 1   |   .   0.0   .   |  -2.0  1.0
     |   .    .    .   |    .    .
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a row-distributed vector. Following is the global vector ***x*** of size 1 × 3, starting at row 4 and column 2 in 5 × 4 global matrix ***X*** with block size 2 × 2:

```
B,D        0             1
         ┌                        ┐
 0       |   .    .   |    .    . |
         |   .    .   |    .    . |
         | -----------|-----------|
 1       |   .    .   |    .    . |
         |   .  -1.0  |   1.0 -1.0|
         | -----------|-----------|
 2       |   .    .   |    .    . |
         └                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |      0      |      1
-----|-------------|------------
     |   .    .    |    .    .
 0   |   .    .    |    .    .
     |   .    .    |    .    .
-----|-------------|------------
 1   |   .    .    |    .    .
     |   .  -1.0   |   1.0 -1.0
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 4 × 1, starting at row 2 and column 3 in 6 × 4 global matrix ***Y*** with block size 3 × 2:

```
B,D       0            1
         ┌                     ┐
         |   .    .  |    .   .|
 0       |   .    .  |  0.5   .|
         |   .    .  |  0.5   .|
         | ----------|---------|
         |   .    .  |  0.5   .|
 1       |   .    .  |  0.5   .|
         |   .    .  |    .   .|
         └                     ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |     0     |     1
-----|-----------|-----------
     |   .    .  |     .    .
 0   |   .    .  |   0.5    .
     |   .    .  |   0.5    .
-----|-----------|-----------
     |   .    .  |   0.5    .
 1   |   .    .  |   0.5    .
     |   .    .  |     .    .
```

Output:

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 4 × 1, starting at row 2 and column 3 in 6 × 4 global matrix ***Y*** with block size 3 × 2:

```
B,D      0           1
     ┌                       ┐
     |   .    .  |     .    . |
 0   |   .    .  |   1.0    . |
     |   .    .  |   0.0    . |
     | ----------|---------- |
     |   .    .  |  -1.0    . |
 1   |   .    .  |  -2.0    . |
     |   .    .  |     .    . |
     └                       ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |     0     |     1
-----|-----------|-----------
     |   .    .  |     .    .
 0   |   .    .  |   1.0    .
     |   .    .  |   0.0    .
-----|-----------|-----------
     |   .    .  |  -1.0    .
 1   |   .    .  |  -2.0    .
     |   .    .  |     .    .
```

| ***Example 3:*** This example computes ***y*** = α***Ax***+β***y*** using a 2 × 2 process grid. The input matrices ***A***, ***X***, and ***Y***, used here, are the same as ***A***, ***B***, and ***C***, used in "Example 2" on page 295 for PZGEMM. The updated portion of ***Y*** is the same as for ***C*** in PZGEMM, as this computation is equivalent to a portion of the PZGEMM computation.

| This example uses vectors ***x*** and ***y***, which are column-distributed vectors within a column of ***X*** and ***Y***, respectively, by specifying *incx* = 1, *ix* = 1, and *jx* = 2 for ***x*** and *incy* = 1, *iy* = 1, and *jy* = 2 for ***y***.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            TRANSA M   N    ALPHA    A  IA  JA    DESC_A   X  IX  JX
              |    |   |      |      |   |   |      |      |   |   |
CALL PZGEMV( 'N' , 6 , 3  , ALPHA  , A , 1 , 1 ,  DESC_A , X , 1 , 2 ,

            DESC_X  INCX  BETA    Y  IY  JY   DESC_Y   INCY
              |      |     |      |   |   |     |        |
            DESC_X , 1 ,  BETA  , Y , 1 , 2 , DESC_Y ,  1 )

            ALPHA = (1.0,0.0)

            BETA  = (2.0,0.0)
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	6	3	6
N_	3	2	2
MB_	2	2	2
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example:

LLD_A = LLD_Y = 4 on P_{00} and P_{01}
LLD_X = 2 on P_{00} and P_{01}
LLD_A = LLD_Y = 2 on P_{10} and P_{11}
LLD_X = 1 on P_{10} and P_{11}

Global general 6 × 3 matrix ***A*** with block size 2 × 2:

```
B,D           0                1
     ┌                                 ┐
 0   |  (1.0,5.0)  (9.0,2.0)  |   (1.0,9.0) |
     |  (2.0,4.0)  (8.0,3.0)  |   (1.0,8.0) |
     | -----------------------|------------ |
 1   |  (3.0,3.0)  (7.0,5.0)  |   (1.0,7.0) |
     |  (4.0,2.0)  (4.0,7.0)  |   (1.0,5.0) |
     | -----------------------|------------ |
 2   |  (5.0,1.0)  (5.0,1.0)  |   (1.0,6.0) |
     |  (6.0,6.0)  (3.0,6.0)  |   (1.0,4.0) |
     └                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |          0             |      1
-----|------------------------|-------------
     |  (1.0,5.0)  (9.0,2.0)  |   (1.0,9.0)
     |  (2.0,4.0)  (8.0,3.0)  |   (1.0,8.0)
 0   |  (5.0,1.0)  (5.0,1.0)  |   (1.0,6.0)
     |  (6.0,6.0)  (3.0,6.0)  |   (1.0,4.0)
-----|------------------------|-------------
 1   |  (3.0,3.0)  (7.0,5.0)  |   (1.0,7.0)
     |  (4.0,2.0)  (4.0,7.0)  |   (1.0,5.0)
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 3 × 1, starting at row 1 and column 2 in 3 × 2 global matrix ***X*** with block size 2 × 2:

```
B,D           0
     ┌                      ┐
 0   |      .     (2.0,7.0) |
     |      .     (6.0,8.0) |
     | -------------------- |
 1   |      .     (4.0,5.0) |
     └                      ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |          0
-----|-----------------------
 0   |      .     (2.0,7.0)
     |      .     (6.0,8.0)
-----|-----------------------
 1   |      .     (4.0,5.0)
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 6 × 1, starting at row 1 and column 2 in 6 × 2 global matrix ***Y*** with block size 2 × 2:

```
B,D             0
     ┌                    ┐
 0   |      .     (0.5,0.0) |
     |      .     (0.5,0.0) |
     | -------------------- |
 1   |      .     (0.5,0.0) |
     |      .     (0.5,0.0) |
     | -------------------- |
 2   |      .     (0.5,0.0) |
     |      .     (0.5,0.0) |
     └                    ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |          0
-----|-----------------------
     |      .     (0.5,0.0)
     |      .     (0.5,0.0)
 0   |      .     (0.5,0.0)
     |      .     (0.5,0.0)
-----|-----------------------
 1   |      .     (0.5,0.0)
     |      .     (0.5,0.0)
```

Output:

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 6 × 1, starting at row 1 and column 2 in 6 × 2 global matrix ***Y*** with block size 2 × 2:

```
B,D                0
     ┌                             ┐
 0   │       .      (-35.0.142.0) │
     │       .      (-35.0.141.0) │
     │ ---------------------------- │
 1   │       .      (-43.0.146.0) │
     │       .      (-58.0.131.0) │
     │ ---------------------------- │
 2   │       .        (0.0.112.0) │
     │       .      (-75.0.135.0) │
     └                             ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  │              0
-----│------------------------------
     │       .      (-35.0,142.0)
     │       .      (-35.0,141.0)
 0   │       .      (  0.0,112.0)
     │       .      (-75.0,135.0)
-----│------------------------------
 1   │       .      (-43.0,146.0)
     │       .      (-58.0,131.0)
```

Example 4: This example computes $\boldsymbol{y} = \alpha \boldsymbol{Ax}+\beta \boldsymbol{y}$ using a 2 × 2 process grid. The input matrices ***A***, ***X***, and ***Y***, used here, are the same as ***A***, ***B***, and ***C***, used in “Example 2” on page 295 for PZGEMM.

This example uses vector ***x***, which is a row-distributed vector within a row of ***X***, by specifying *incx* = M_X = 3, *ix* = 1, and *jx* = 1. It uses vector ***y***, which is a column-distributed vector within a column of ***Y***, by specifying *incy* = 1, *iy* = 1, and *jy* = 1.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            TRANSA M   N    ALPHA    A  IA  JA    DESC_A   X  IX  JX
              |    |   |      |      |   |   |      |      |   |   |
CALL PZGEMV( 'N' , 6 , 2  , ALPHA  , A , 1 , 1 ,  DESC_A , X , 1 , 1 ,

            DESC_X  INCX  BETA    Y  IY  JY   DESC_Y   INCY
              |      |     |      |   |   |     |        |
            DESC_X , 3 ,  BETA  , Y , 1 , 1 , DESC_Y ,  1 )

            ALPHA = (1.0,0.0)

            BETA  = (2.0,0.0)
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	6	3	6
N_	3	2	2
MB_	2	2	2
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example:

LLD_A = LLD_Y = 4 on P_{00} and P_{01}
LLD_X = 2 on P_{00} and P_{01}
LLD_A = LLD_Y = 2 on P_{10} and P_{11}
LLD_X = 1 on P_{10} and P_{11}

Global general 6 × 3 matrix ***A*** with block size 2 × 2:

```
B,D             0                 1
     ┌                                   ┐
 0   │  (1.0,5.0)  (9.0,2.0)  │   (1.0,9.0) │
     │  (2.0,4.0)  (8.0,3.0)  │   (1.0,8.0) │
     │ -----------------------|------------ │
 1   │  (3.0,3.0)  (7.0,5.0)  │   (1.0,7.0) │
     │  (4.0,2.0)  (4.0,7.0)  │   (1.0,5.0) │
     │ -----------------------|------------ │
 2   │  (5.0,1.0)  (5.0,1.0)  │   (1.0,6.0) │
     │  (6.0,6.0)  (3.0,6.0)  │   (1.0,4.0) │
     └                                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │           0            │      1
-----|------------------------|-------------
     │  (1.0,5.0)  (9.0,2.0)  │   (1.0,9.0)
     │  (2.0,4.0)  (8.0,3.0)  │   (1.0,8.0)
 0   │  (5.0,1.0)  (5.0,1.0)  │   (1.0,6.0)
     │  (6.0,6.0)  (3.0,6.0)  │   (1.0,4.0)
-----|------------------------|-------------
 1   │  (3.0,3.0)  (7.0,5.0)  │   (1.0,7.0)
     │  (4.0,2.0)  (4.0,7.0)  │   (1.0,5.0)
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a row-distributed vector. Following is the global vector ***x*** of size 1 × 2, starting at row 1 and column 1 in 3 × 2 global matrix ***X*** with block size 2 × 2:

```
B,D             0
     ┌                        ┐
 0   │  (1.0,8.0)  (2.0,7.0) │
     │      .          .     │
     │ --------------------- │
 1   │      .          .     │
     └                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |          0
-----|-----------------------
 0   |  (1.0,8.0)  (2.0,7.0)
     |      .          .
-----|-----------------------
 1   |      .          .
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 6 × 1, starting at row 1 and column 1 in 6 × 2 global matrix ***Y*** with block size 2 × 2:

```
B,D             0
     ┌                    ┐
 0   |  (0.5,0.0)     .   |
     |  (0.5,0.0)     .   |
     | ------------------ |
 1   |  (0.5,0.0)     .   |
     |  (0.5,0.0)     .   |
     | ------------------ |
 2   |  (0.5,0.0)     .   |
     |  (0.5,0.0)     .   |
     └                    ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |          0
-----|-----------------------
     |  (0.5,0.0)     .
     |  (0.5,0.0)     .
 0   |  (0.5,0.0)     .
     |  (0.5,0.0)     .
-----|-----------------------
 1   |  (0.5,0.0)     .
     |  (0.5,0.0)     .
```

Output:

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a column-distributed vector. Following is the global vector ***y*** of size 6 × 1, starting at row 1 and column 1 in 6 × 2 global matrix ***Y*** with block size 2 × 2:

```
B,D                 0
        ┌                             ┐
 0      │  (-34.0, 80.0)       .      │
        │  (-34.0, 82.0)       .      │
        │ ---------------------------- │
 1      │  (-41.0, 86.0)       .      │
        │  (-52.0, 76.0)       .      │
        │ ---------------------------- │
 2      │  (  1.0, 78.0)       .      │
        │  (-77.0, 87.0)       .      │
        └                             ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |              0
-----|------------------------------
     |  (-34.0, 80.0)       .
     |  (-34.0, 82.0)       .
 0   |  (  1.0, 78.0)       .
     |  (-77.0, 87.0)       .
-----|------------------------------
 1   |  (-41.0, 86.0)       .
     |  (-52.0, 76.0)       .
```

PDSYMV and PZHEMV—Matrix-Vector Product for a Real Symmetric or a Complex Hermitian Matrix

These subroutines compute the matrix-vector product:

$$\boldsymbol{y} \leftarrow \alpha \boldsymbol{A}\boldsymbol{x}+\beta \boldsymbol{y}$$

where, in the formula above:

$\boldsymbol{A}$ represents the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
$\boldsymbol{x}$ represents the global vector:
- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.

$\boldsymbol{y}$ represents the global vector:
- For *incy* = M_Y, it is $\boldsymbol{Y}_{iy:iy,\ jy:jy+n-1}$.
- For *incy* = 1 and *incy* ≠ M_Y, it is $\boldsymbol{Y}_{iy:iy+n-1,\ jy:jy}$.

α and β are scalars.

and:

- For PDSYMV, submatrix $\boldsymbol{A}$ is real symmetric.
- For PZHEMV, submatrix $\boldsymbol{A}$ is complex Hermitian.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $n = 0$
- α is zero and β is one.

See references [14] and [15].

Table 40. Data Types

α, β, $\boldsymbol{A}$, $\boldsymbol{x}$, $\boldsymbol{y}$	**Subprogram**
Long-precision real	PDSYMV
Long-precision complex	PZHEMV

Syntax

Fortran	CALL PDSYMV | PZHEMV (*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*, *beta*, *y*, *iy*, *jy*, *desc_y*, *incy*)
C and C++	pdsymv | pzhemv (*uplo*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*, *beta*, *y*, *iy*, *jy*, *desc_y*, *incy*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global submatrix $\boldsymbol{A}$ is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the number of rows and columns in submatrix ***A*** and the number of elements in vectors ***x*** and ***y*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 40 on page 189.

a

| is the local part of the global real symmetric or complex Hermitian matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 40 on page 189. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A $\geq$ 0 Otherwise: M_A $\geq$ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ p	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ q	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*n*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*n*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*n*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*n*–1 by *jx* part of the global matrix.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 40 on page 189. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then *ix*+*n*–1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; $1 \le jx \le$ N_X and:

If *incx* = M_X, then $jx+n-1 \le$ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If $n = 0$: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If $n = 0$: N_X ≥ 0 Otherwise: N_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global
7	RSRC_X	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_X < p	Global
8	CSRC_X	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_X < q	Global
9	LLD_X	The leading dimension of the local array	LLD_X ≥ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 40 on page 189.

y

is the local part of the global matrix ***Y***. This identifies the **first element** of the local array Y. This subroutine computes the location of the first element of the local subarray used, based on *iy*, *jy*, *desc_y*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incy* = M_Y, the leading LOCp(*iy*) by LOCq(*jy*+*n*–1) part of the local array Y must contain the local pieces of the leading *iy* by *jy*+*n*–1 part of the global matrix.
- If *incy* = 1 and *incy* ≠ M_Y, the leading LOCp(*iy*+*n*–1) by LOCq(*jy*) part of the local array Y must contain the local pieces of the leading *iy*+*n*–1 by *jy* part of the global matrix.

When β is zero, ***y*** need not be set on input.

Scope: **local**

Specified as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 40 on page 189. Details about the block-cyclic data distribution of the global matrix ***Y*** are stored in *desc_y*.

iy

has the following meaning:

If *incy* = M_Y, it indicates which row of global matrix ***Y*** is used for vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it is the row index of global matrix ***Y***, identifying the first element of vector ***y***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *iy* ≤ M_Y and:

If *incy* = 1 and *incy* ≠ M_Y, then *iy*+*n*–1 ≤ M_Y.

jy

has the following meaning:

If *incy* = M_Y, it is the column index of global matrix ***Y***, identifying the first element of vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it indicates which column of global matrix ***Y*** is used for vector ***y***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jy* ≤ N_Y and:

If *incy* = M_Y, then *jy*+*n*–1 ≤ N_Y.

desc_y

is the array descriptor for global matrix ***Y***, described in the following table:

desc_y	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_Y	Descriptor type	DTYPE_Y=1	Global
2	CTXT_Y	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_Y	Number of rows in the global matrix	If *n* = 0: M_Y ≥ 0 Otherwise: M_Y ≥ 1	Global

desc_y	**Name**	**Description**	**Limits**	**Scope**
4	N_Y	Number of columns in the global matrix	If $n = 0$: N_Y ≥ 0 Otherwise: N_Y ≥ 1	Global
5	MB_Y	Row block size	MB_Y ≥ 1	Global
6	NB_Y	Column block size	NB_Y ≥ 1	Global
7	RSRC_Y	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_Y < p	Global
8	CSRC_Y	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_Y < q	Global
9	LLD_Y	The leading dimension of the local array	LLD_Y ≥ max(1,LOCp(M_Y))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incy

is the stride for global vector ***y***.

Scope: **global**

Specified as: a fullword integer; *incy* = 1 or *incy* = M_X, where:

If *incy* = M_Y, then ***y*** is a row-distributed vector.

If *incy* = 1 and *incy* ≠ M_Y, then ***y*** is a column-distributed vector.

On Return

y

is the updated local part of the global matrix ***Y***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 40 on page 189.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo* argument.
2. The matrix and vectors must have no common elements; otherwise, results are unpredictable.
3. | The imaginary parts of the diagonal elements of a complex Hermitian matrix ***A*** | are assumed to be zero, so you do not have to set these values.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
5. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.

6. The following values must be equal: CTXT_A = CTXT_X = CTXT_Y.
7. The global matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
8. The block row and block column offsets of the global matrix ***A*** must be equal; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
9. The vectors ***x*** and ***y*** must be distributed along the same axis—that is, they must both be row distributed or column distributed, where:
   - *incx* = M_X and *incy* = M_Y for row distribution
   - *incx* = 1($\neq$ M_X) and *incy* = 1($\neq$ M_Y) for column distribution
10. If *incx* = M_X and *incy* = M_Y, then (in the process grid) the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrices ***X*** and ***Y***; that is:

    iacol = *ixcol*
    iacol = *iycol*
    where:
    iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
    ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
    iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)
11. If *incx* = 1($\neq$ M_X) and *incy* = 1($\neq$ M_Y), then (in the process grid) the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrices ***X*** and ***Y***; that is:

    iarow = *ixrow*
    iarow = *iyrow*
    where:
    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    iyrow = mod((((*iy*–1)/MB_Y)+RSRC_Y), *p*)
12. If *incx* = M_X:
    - The block column offset of ***x*** must be equal to the block column offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ja*–1, NB_A).
    - The following block sizes must be equal: NB_X = NB_A.
13. If *incx* = 1($\neq$ M_X):
    - The block row offset of ***x*** must be equal to the block column offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ja*–1, NB_A).
    - The following block sizes must be equal: MB_X = NB_A.
14. If *incy* = M_Y:
    - The block column offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*jy*–1, NB_Y) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: NB_Y = MB_A.
15. If *incy* = 1($\neq$ M_Y):
    - The block row offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*iy*–1, MB_Y) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: MB_Y = MB_A.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.
3. DTYPE_Y is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ 'U' or 'L'
2. $n < 0$
3. MB_X < 1
4. NB_X < 1
5. M_X < 0 and $n = 0$; M_X < 1 otherwise
6. N_X < 0 and $n = 0$; N_X < 1 otherwise
7. RSRC_X < 0 or RSRC_X ≥ p
8. CSRC_X < 0 or CSRC_X ≥ q
9. CTXT_A ≠ CTXT_X
10. $ix < 1$
11. $jx < 1$
12. MB_Y < 1
13. NB_Y < 1
14. M_Y < 0 and $n = 0$; M_Y < 1 otherwise
15. N_Y < 0 and $n = 0$; N_Y < 1 otherwise
16. RSRC_Y < 0 or RSRC_Y ≥ p
17. CSRC_Y < 0 or CSRC_Y ≥ q
18. CTXT_A ≠ CTXT_Y
19. $iy < 1$
20. $jy < 1$
21. RSRC_A < 0 or RSRC_A ≥ p
22. CSRC_A < 0 or CSRC_A ≥ q
23. M_A < 0 and ($m = 0$ and $n = 0$); M_A < 1 otherwise
24. N_A < 0 and ($m = 0$ and $n = 0$); N_A < 1 otherwise
25. NB_A < 1
26. MB_A < 1
27. $ja < 1$
28. $ia < 1$

Stage 5

1. MB_A ≠ NB_A

If $n \neq 0$:

2. $ix >$ M_X
3. $jx >$ N_X

4. iy > M_Y
5. jy > N_Y
6. $ia+n-1$ > M_A
7. $ja+n-1$ > N_A

If $incx$ = M_X and $incy$ = M_Y:

8. NB_A ≠ NB_X
9. MB_A ≠ NB_Y
10. mod(jx–1, NB_X) ≠ mod(ja–1, NB_A)
11. mod(jy–1, NB_Y) ≠ mod(ia–1, MB_A)
12. $n \neq 0$ and $jx+n-1$ > N_X
13. $n \neq 0$ and $jy+n-1$ > N_Y

If $incx$ = 1(≠ M_X) and $incy$ = 1(≠ M_Y):

14. NB_A ≠ MB_X
15. MB_A ≠ MB_Y
16. mod(ix–1, MB_X) ≠ mod(ja–1, NB_A)
17. mod(iy–1, MB_Y) ≠ mod(ia–1, MB_A)
18. $n \neq 0$ and $ix+n-1$ > M_X
19. $n \neq 0$ and $iy+n-1$ > M_Y

Otherwise:

20. $incx$ ≠ M_X and $incx$ ≠ 1
21. $incy$ ≠ M_Y and $incy$ ≠ 1

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_X < max(1, LOCp(M_X))
3. LLD_Y < max(1, LOCp(M_Y))
4. mod(ia–1, MB_A) ≠ mod(ja–1, NB_A)
5. If $incx$ = M_X and $incy$ = M_Y, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrices ***X*** and ***Y***; that is:
   ixcol ≠ *iacol*
   iycol ≠ *iacol*
   where:
      iacol = mod((((ja–1)/NB_A)+CSRC_A), q)
      ixcol = mod((((jx–1)/NB_X)+CSRC_X), q)
      iycol = mod((((jy–1)/NB_Y)+CSRC_Y), q)
6. If $incx$ = 1(≠ M_X) and $incy$ = 1(≠ M_Y), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrices ***X*** and ***Y***; that is:
   ixrow ≠ *iarow*
   iyrow ≠ *iarow*
   where:
      iarow = mod((((ia–1)/MB_A)+RSRC_A), p)
      ixrow = mod((((ix–1)/MB_X)+RSRC_X), p)
      iyrow = mod((((iy–1)/MB_Y)+RSRC_Y), p)

Example 1: This example computes $\boldsymbol{y} = \alpha \boldsymbol{Ax} + \beta \boldsymbol{y}$ using a 2×2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO   N    ALPHA    A  IA  JA    DESC_A   X   IX   JX
             |     |      |      |   |   |      |      |    |    |
CALL PDSYMV( 'U'  , 8 ,  1.0D0  , A , 1 , 1 ,  DESC_A , X ,  1 ,  1 ,

           DESC_X   INCX    BETA    Y   IY   JY   DESC_Y   INCY
             |        |       |     |    |    |     |        |
           DESC_X ,   1   , 0.0D0  , Y ,  1 ,  1 , DESC_Y ,  1 )
```

	Desc_A	Desc_X	Desc_Y
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	8	8
N_	8	1	1
MB_	2	2	2
NB_	2	1	1
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = LLD_X = LLD_Y = 4 on all processes.

Global real symmetric matrix ***A*** of order 8 with block size 2 × 2:

```
B,D       0            1            2            3
     ┌                                                    ┐
 0   │  0.0 -1.0  │  -1.0  0.0  │   0.0  0.0  │   0.0  0.0 │
     │   .   1.0  │   0.0  1.0  │   0.0  1.0  │   0.0  1.0 │
     │ -----------|-------------|-------------|----------- │
 1   │   .    .   │  -1.0 -1.0  │   0.0  0.0  │   1.0  0.0 │
     │   .    .   │    .  -1.0  │   1.0  1.0  │   0.0  1.0 │
     │ -----------|-------------|-------------|----------- │
 2   │   .    .   │    .    .   │  -1.0  0.0  │   0.0  0.0 │
     │   .    .   │    .    .   │    .   1.0  │   0.0  0.0 │
     │ -----------|-------------|-------------|----------- │
 3   │   .    .   │    .    .   │    .    .   │   0.0  0.0 │
     │   .    .   │    .    .   │    .    .   │    .   0.0 │
     └                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |          0          |          1
-----|---------------------|---------------------
     |  0.0 -1.0  0.0  0.0 |  -1.0  0.0  0.0  0.0
     |   .   1.0  0.0  1.0 |   0.0  1.0  0.0  1.0
 0   |   .    .  -1.0  0.0 |    .    .   0.0  0.0
     |   .    .    .   1.0 |    .    .   0.0  0.0
-----|---------------------|---------------------
     |   .    .   0.0  0.0 |  -1.0 -1.0  1.0  0.0
     |   .    .   1.0  1.0 |    .  -1.0  0.0  1.0
 1   |   .    .    .    .  |    .    .   0.0  0.0
     |   .    .    .    .  |    .    .    .   0.0
```

Global vector ***x*** of size 8 × 1 with block size 2:

```
B,D     0
      ┌      ┐
 0    |  1.0 |
      |  1.0 |
      | ---- |
 1    |  1.0 |
      |  1.0 |
      | ---- |
 2    |  1.0 |
      |  1.0 |
      | ---- |
 3    |  1.0 |
      |  1.0 |
      └      ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------
     |  1.0
     |  1.0
 0   |  1.0
     |  1.0
-----|------
     |  1.0
     |  1.0
 1   |  1.0
     |  1.0
```

Output:

Global vector ***y*** of size 8 × 1 with block size 2:

```
B,D     0
     ┌      ┐
 0   | -2.0 |
     |  3.0 |
     | ---- |
 1   | -2.0 |
     |  2.0 |
     | ---- |
 2   |  0.0 |
     |  3.0 |
     | ---- |
 3   |  1.0 |
     |  2.0 |
     └      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |  0
-----|------
     | -2.0
     |  3.0
 0   |  0.0
     |  3.0
-----|------
     | -2.0
     |  2.0
 1   |  1.0
     |  2.0
```

| ***Example 2:*** This example computes ***y*** = α***Ax***+β***y*** using a 2 × 2 process grid.

| **Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix
| are assumed to be zero, so you do not have to set these values.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              UPLO   N    ALPHA    A  IA  JA    DESC_A   X   IX   JX
               |     |      |      |   |   |      |      |    |    |
CALL PZHEMV(  'U'  , 6 ,  ALPHA  , A , 1 , 1 ,  DESC_A , X ,  1 ,  1 ,

             DESC_X   INCX    BETA    Y   IY   JY   DESC_Y   INCY
               |       |       |      |    |    |     |       |
             DESC_X ,   1  ,  BETA  , Y ,  1 ,  1 , DESC_Y ,  1 )

            ALPHA = (1.0,0.0)

            BETA  = (0.0,0.0)
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	6	6	6
N_	6	1	1
MB_	2	2	2
NB_	2	1	1
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = LLD_X = LLD_Y = 4 on P_{00} and P_{01} and LLD_A = LLD_X = LLD_Y = 2 on P_{10} and P_{11}.

Global complex Hermitian matrix ***A*** of order 6 with block size 2 × 2:

```
B,D             0                         1                         2
     ┌                                                                            ┐
 0   │ ( 0.0, 0.0) (-1.0, 1.0) │ (-1.0,-2.0) ( 0.0, 3.0) │ ( 2.0, 1.0) ( 1.0, 0.0) │
     │      .      ( 2.0, 0.0) │ ( 5.0, 4.0) ( 2.0, 0.0) │ (-1.0,-1.0) ( 0.0, 2.0) │
     │ ------------------------│-------------------------│-------------------------│
 1   │      .           .      │ ( 0.0, 0.0) (-1.0, 0.0) │ ( 0.0, 0.0) ( 1.0, 1.0) │
     │      .           .      │      .      ( 4.0, 0.0) │ ( 2.0,-1.0) (-1.0,-1.0) │
     │ ------------------------│-------------------------│-------------------------│
 2   │      .           .      │      .           .      │ (-1.0, 0.0) ( 0.0, 2.0) │
     │      .           .      │      .           .      │      .      ( 1.0, 0.0) │
     └                                                                            ┘
```

The following is the 2 × 2 process grid for ***A***:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                      0                       |           1
-----|----------------------------------------------|------------------------
     | ( 0.0,  . ) (-1.0, 1.0) ( 2.0, 1.0) ( 1.0, 0.0) | (-1.0,-2.0) ( 0.0, 3.0)
     |      .      ( 2.0,  . ) (-1.0,-1.0) ( 0.0, 2.0) | ( 5.0, 4.0) ( 2.0, 0.0)
 0   |      .           .      (-1.0,  . ) ( 0.0, 2.0) |
     |      .           .           .      ( 1.0,  . ) |
-----|----------------------------------------------|------------------------
     |      .           .      ( 0.0, 0.0) ( 1.0, 1.0) | ( 0.0,  . ) (-1.0,  . )
 1   |      .           .      ( 2.0,-1.0) (-1.0,-1.0) |      .      ( 4.0,  . )
```

Global vector ***x*** of size 6 × 1 with block size 2:

```
B,D        0
     ┌             ┐
 0   | (-1.0, 1.0) |
     | ( 2.0, 1.0) |
     | ----------- |
 1   | ( 1.0, 2.0) |
     | (-2.0,-3.0) |
     | ----------- |
 2   | ( 0.0, 1.0) |
     | ( 1.0, 0.0) |
     └             ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |      0
-----|-------------
     | (-1.0, 1.0)
     | ( 2.0, 1.0)
 0   | ( 0.0, 1.0)
     | ( 1.0, 0.0)
-----|-------------
     | ( 1.0, 2.0)
 1   | (-2.0,-3.0)
```

Output:

Global vector ***y*** of size 6 × 1 with block size 2:

```
|                 B,D        0
|                     ┌               ┐
|                  0  | (  9.0, -7.0) |
|                     | (  0.0, 11.0) |
|                     | ------------- |
|                  1  | ( 16.0, -2.0) |
|                     | ( -2.0, -8.0) |
|                     | ------------- |
|                  2  | ( -5.0, -3.0) |
|                     | ( 12.0, -1.0) |
|                     └               ┘
```

| The following is the 2 × 2 process grid:

B,D	**0**	**—**
0	P_{00}	P_{01}
2		
1	P_{10}	P_{11}

| Local arrays for ***y***:

```
|                 p,q  |       0
|                 -----|---------------
|                      | (  9.0, -7.0)
|                      | (  0.0, 11.0)
|                  0   | ( -5.0, -3.0)
|                      | ( 12.0, -1.0)
|                 -----|---------------
|                      | ( 12.0, -2.0)
|                  1   | ( -2.0, -8.0)
```

PDGER, PZGERC, and PZGERU—Rank-One Update of a General Matrix

| PDGER and PZGERU compute the following rank-one update:

| $\boldsymbol{A} \leftarrow \alpha \boldsymbol{x}\boldsymbol{y}^{\mathrm{T}}+\boldsymbol{A}$

| PZGERC computes the following rank-one update:

| $\boldsymbol{A} \leftarrow \alpha \boldsymbol{x}\boldsymbol{y}^{\mathrm{H}}+\boldsymbol{A}$

where, in the formula above:

$\boldsymbol{A}$ represents the global general submatrix $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$.
$\boldsymbol{x}$ represents the global vector:
- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+m-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+m-1,\ jx:jx}$.

$\boldsymbol{y}$ represents the global vector:
- For *incy* = M_Y, it is $\boldsymbol{Y}_{iy:iy,\ jy:jy+n-1}$.
- For *incy* = 1 and *incy* ≠ M_Y, it is $\boldsymbol{Y}_{iy:iy+n-1,\ jy:jy}$.

α is a scalar.

| **Note:** No data should be moved to form $\boldsymbol{y}^{\mathrm{T}}$ or $\boldsymbol{y}^{\mathrm{H}}$; that is, the vector $\boldsymbol{y}$ should
| always be stored in its untransposed form.

In the following three cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $m = 0$
- $n = 0$
- α is zero.

See references [14] and [15].

Table 41. Data Types

α, $\boldsymbol{A}$, $\boldsymbol{x}$, $\boldsymbol{y}$	**Subprogram**
Long-precision real	PDGER
Long-precision complex	PZGERC and PZGERU

Syntax

Fortran	CALL PDGER | PZGERC | PZGERU (*m*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *y*, *iy*, *jy*, *desc_y*, *incy*, *a*, *ia*, *ja*, *desc_a*)
C and C++	pdger | pzgerc | pzgeru (*m*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *y*, *iy*, *jy*, *desc_y*, *incy*, *a*, *ia*, *ja*, *desc_a*);

On Entry

m

is the number of rows in submatrix $\boldsymbol{A}$ and the number of elements in vector $\boldsymbol{x}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrix $\boldsymbol{A}$ and the number of elements in vector $\boldsymbol{y}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 41 on page 204.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*m*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*m*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*m*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*m*–1 by *jx* part of the global matrix.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 41 on page 204. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then *ix*+*m*–1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jx* ≤ N_X and:

If *incx* = M_X, then *jx*+*m*–1 ≤ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global

desc_x	**Name**	**Description**	**Limits**	**Scope**
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If $m = 0$: M_X $\geq$ 0 Otherwise: M_X $\geq$ 1	Global
4	N_X	Number of columns in the global matrix	If $m = 0$: N_X $\geq$ 0 Otherwise: N_X $\geq$ 1	Global
5	MB_X	Row block size	MB_X $\geq$ 1	Global
6	NB_X	Column block size	NB_X $\geq$ 1	Global
7	RSRC_X	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_X $<$ p	Global
8	CSRC_X	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_X $<$ q	Global
9	LLD_X	The leading dimension of the local array	LLD_X $\geq$ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

y

is the local part of the global matrix ***Y***. This identifies the **first element** of the local array Y. This subroutine computes the location of the first element of the local subarray used, based on *iy*, *jy*, *desc_y*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incy* = M_Y, the leading LOCp(*iy*) by LOCq(*jy*+*n*–1) part of the local array Y must contain the local pieces of the leading *iy* by *jy*+*n*–1 part of the global matrix.
- If *incy* = 1 and *incy* ≠ M_Y, the leading LOCp(*iy*+*n*–1) by LOCq(*jy*) part of the local array Y must contain the local pieces of the leading *iy*+*n*–1 by *jy* part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{y}^T$ or $\boldsymbol{y}^H$; that is, the vector ***y*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 41 on page 204. Details about the block-cyclic data distribution of the global matrix ***Y*** are stored in *desc_y*.

iy

has the following meaning:

If *incy* = M_Y, it indicates which row of global matrix ***Y*** is used for vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it is the row index of global matrix ***Y***, identifying the first element of vector ***y***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *iy* ≤ M_Y and:

If *incy* = 1 and *incy* ≠ M_Y, then *iy*+*n*−1 ≤ M_Y.

jy

has the following meaning:

If *incy* = M_Y, it is the column index of global matrix ***Y***, identifying the first element of vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it indicates which column of global matrix ***Y*** is used for vector ***y***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jy* ≤ N_Y and:

If *incy* = M_Y, then *jy*+*n*−1 ≤ N_Y.

desc_y

is the array descriptor for global matrix ***Y***, described in the following table:

desc_y	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_Y	Descriptor type	DTYPE_Y=1	Global
2	CTXT_Y	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_Y	Number of rows in the global matrix	If *n* = 0: M_Y ≥ 0 Otherwise: M_Y ≥ 1	Global
4	N_Y	Number of columns in the global matrix	If *n* = 0: N_Y ≥ 0 Otherwise: N_Y ≥ 1	Global
5	MB_Y	Row block size	MB_Y ≥ 1	Global
6	NB_Y	Column block size	NB_Y ≥ 1	Global
7	RSRC_Y	The process row of the *p* × *q* grid over which the first row of the global matrix is distributed	0 ≤ RSRC_Y < *p*	Global
8	CSRC_Y	The process column of the *p* × *q* grid over which the first column of the global matrix is distributed	0 ≤ CSRC_Y < *q*	Global
9	LLD_Y	The leading dimension of the local array	LLD_Y ≥ max(1,LOCp(M_Y))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incy

is the stride for global vector ***y***.

Scope: **global**

Specified as: a fullword integer; *incy* = 1 or *incy* = M_X, where:

If *incy* = M_Y, then ***y*** is a row-distributed vector.

If *incy* = 1 and *incy* ≠ M_Y, then ***y*** is a column-distributed vector.

a

is the local part of the global general matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*m*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*m*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 41 on page 204. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ia* ≤ M_A and *ia*+*m*–1 ≤ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ja* ≤ N_A and *ja*+*n*–1 ≤ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If *m* = 0 or *n* = 0: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If *m* = 0 or *n* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 41 on page 204.

Notes and Coding Rules

1. The matrix and vectors must have no common elements; otherwise, results are unpredictable.
2. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
3. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
4. The following values must be equal: CTXT_A = CTXT_X = CTXT_Y.
5. If *incx* = M_X:
   - The block column offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ia*–1, MB_A).
   - The following block sizes must be equal: NB_X = MB_A.
6. If *incx* = 1($\ne$ M_X):
   - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***X***; that is, *iarow* = *ixrow*, where:

     iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
     ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
   - The block row offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A).
   - The following block sizes must be equal: MB_X = MB_A.

7. If *incy* = M_Y:
   - In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***Y***; that is, *iacol* = *iycol*, where:

     iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
     iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)

   - The block column offset of ***y*** must be equal to the block column offset of ***A***; that is, mod(*jy*–1, NB_Y) = mod(*ja*–1, NB_A).
   - The following block sizes must be equal: NB_Y = NB_A.
8. If *incy* = 1(≠ M_Y):
   - The block row offset of ***y*** must be equal to the block column offset of ***A***; that is, mod(*iy*–1, MB_Y) = mod(*ja*–1, NB_A).
   - The following block sizes must be equal: MB_Y = NB_A.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.
3. DTYPE_Y is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *m* < 0
2. *n* < 0
3. M_X < 0 and *m* = 0; M_X < 1 otherwise
4. N_X < 0 and *m* = 0; N_X < 1 otherwise
5. MB_X < 1
6. NB_X < 1
7. RSRC_X < 0 or RSRC_X ≥ *p*
8. CSRC_X < 0 or CSRC_X ≥ *q*
9. CTXT_A ≠ CTXT_X
10. *ix* < 1
11. *jx* < 1
12. M_Y < 0 and *n* = 0; M_Y < 1 otherwise
13. N_Y < 0 and *n* = 0; N_Y < 1 otherwise
14. MB_Y < 1
15. NB_Y < 1

16. RSRC_Y $<$ 0 or RSRC_Y $\geq$ *p*
17. CSRC_Y $<$ 0 or CSRC_Y $\geq$ *q*
18. CTXT_A $\neq$ CTXT_Y
19. *iy* $<$ 1
20. *jy* $<$ 1
21. M_A $<$ 0 and (*m* $=$ 0 or *n* $=$ 0); M_A $<$ 1 otherwise
22. N_A $<$ 0 and (*m* $=$ 0 or *n* $=$ 0); N_A $<$ 1 otherwise
23. MB_A $<$ 1
24. NB_A $<$ 1
25. RSRC_A $<$ 0 or RSRC_A $\geq$ *p*
26. CSRC_A $<$ 0 or CSRC_A $\geq$ *q*
27. *ia* $<$ 1
28. *ja* $<$ 1

Stage 5

If *m* $\neq$ 0 and *n* $\neq$ 0:
1. *ia* $>$ M_A
2. *ja* $>$ N_A
3. *ia*+*m*–1 $>$ M_A
4. *ja*+*n*–1 $>$ N_A

If *m* $\neq$ 0:
5. *ix* $>$ M_X
6. *jx* $>$ N_X

If *n* $\neq$ 0:
7. *iy* $>$ M_Y
8. *jy* $>$ N_Y

If *incx* $=$ M_X:
9. NB_X $\neq$ MB_A
10. mod(*jx*–1, NB_X) $\neq$ mod(*ia*–1, MB_A)
11. *m* $\neq$ 0 and *jx*+*m*–1 $>$ N_X

If *incx* $=$ 1($\neq$ M_X):
12. MB_X $\neq$ MB_A
13. mod(*ix*–1, MB_X) $\neq$ mod(*ia*–1, MB_A)
14. *m* $\neq$ 0 and *ix*+*m*–1 $>$ M_X

Otherwise:
15. *incx* $\neq$ M_X and *incx* $\neq$ 1

If *incy* $=$ M_Y:
16. NB_Y $\neq$ NB_A
17. mod(*jy*–1, NB_Y) $\neq$ mod(*ja*–1, NB_A)
18. *n* $\neq$ 0 and *jy*+*n*–1 $>$ N_Y

If *incy* $=$ 1($\neq$ M_Y):
19. MB_Y $\neq$ NB_A
20. mod(*iy*–1, MB_Y) $\neq$ mod(*ja*–1, NB_A)
21. *n* $\neq$ 0 and *iy*+*n*–1 $>$ M_Y

Otherwise:
22. *incy* $\neq$ M_Y and *incy* $\neq$ 1

Stage 6

1. If *incx* = 1(≠ M_X), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***X***; that is, *iarow* ≠ *ixrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
2. If *incy* = M_Y, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***Y***; that is, *iacol* ≠ *iycol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)
3. LLD_A < max(1, LOCp(M_A))
4. LLD_X < max(1, LOCp(M_X))
5. LLD_Y < max(1, LOCp(M_Y))

Example 1: This example computes ***A*** = α***xy***^{T}+***A*** using a 2 × 2 process grid. It uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2. It uses vector ***x***, which is a column-distributed vector within a column of global matrix ***X***, by specifying *incx* = 1, *ix* = 2, and *jx* = 1. It uses vector ***y***, which is a row-distributed vector within a row of global matrix ***Y***, by specifying *incy* = M_Y = 1, *iy* = 1, and *jy* = 2.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             M   N    ALPHA    X  IX  JX    DESC_X   INCX   Y   IY   JY
             |   |      |      |   |   |      |        |     |    |    |
CALL PDGER(  9 , 9 ,  1.0D0  , X , 2 , 1 ,  DESC_X ,  1   , Y ,  1 ,  2 ,

             DESC_Y   INCY   A   IA   JA    DESC_A
               |        |    |    |    |      |
             DESC_Y ,   1  , A ,  2 ,  2 ,  DESC_A  )
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	10	11	1
N_	10	1	11
MB_	4	4	1
NB_	4	1	4
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

	Desc_A	Desc_X	Desc_Y

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = 6 on P_{00} and P_{01}, LLD_A = 4 on P_{10} and P_{11}, LLD_X = 7 on P_{00}, LLD_X = 4 on P_{10}, LLD_Y = 1 on P_{00} and P_{01}.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 9 × 9 submatrix ***A***, starting at row 2 and column 2 in global general 10 × 10 matrix ***A*** with block size 4 × 4:

```
B,D           0                           1                     2
     ┌                                                                ┐
     |    .    .    .    .   |    .    .    .    .   |    .    .  |
     |    .  12.0 22.0 32.0  |  42.0 52.0 62.0 72.0   |  82.0 92.0  |
 0   |    .  13.0 23.0 33.0  |  43.0 53.0 63.0 73.0   |  83.0 93.0  |
     |    .  14.0 24.0 34.0  |  44.0 54.0 64.0 74.0   |  84.0 94.0  |
     | ----------------------|------------------------|-------------  |
     |    .  15.0 25.0 35.0  |  45.0 55.0 65.0 75.0   |  85.0 95.0  |
     |    .  16.0 26.0 36.0  |  46.0 56.0 66.0 76.0   |  86.0 96.0  |
 1   |    .  17.0 27.0 37.0  |  47.0 57.0 67.0 77.0   |  87.0 97.0  |
     |    .  18.0 28.0 38.0  |  48.0 58.0 68.0 78.0   |  88.0 98.0  |
     | ----------------------|------------------------|-------------  |
 2   |    .  19.0 29.0 39.0  |  49.0 59.0 69.0 79.0   |  89.0 99.0  |
     |    .  20.0 30.0 40.0  |  50.0 60.0 70.0 80.0   |  90.0 100.0 |
     └                                                                ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                  0                   |            1
-----|--------------------------------------|-------------------------
     |    .    .    .    .    .    .        |    .    .    .    .
     |    .  12.0 22.0 32.0 82.0  92.0      |  42.0 52.0 62.0 72.0
     |    .  13.0 23.0 33.0 83.0  93.0      |  43.0 53.0 63.0 73.0
 0   |    .  14.0 24.0 34.0 84.0  94.0      |  44.0 54.0 64.0 74.0
     |    .  19.0 29.0 39.0 89.0  99.0      |  49.0 59.0 69.0 79.0
     |    .  20.0 30.0 40.0 90.0 100.0      |  50.0 60.0 70.0 80.0
-----|--------------------------------------|-------------------------
     |    .  15.0 25.0 35.0 85.0  95.0      |  45.0 55.0 65.0 75.0
     |    .  16.0 26.0 36.0 86.0  96.0      |  46.0 56.0 66.0 76.0
 1   |    .  17.0 27.0 37.0 87.0  97.0      |  47.0 57.0 67.0 77.0
     |    .  18.0 28.0 38.0 88.0  98.0      |  48.0 58.0 68.0 78.0
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 9 × 1, starting at row 2 and column 1 in 11 × 1 global matrix ***X*** with block size 4 × 1:

```
B,D     0
      ┌      ┐
      |   .  |
      |  1.0 |
 0    |  1.0 |
      |  1.0 |
      | ---- |
      |  1.0 |
      |  1.0 |
 1    |  1.0 |
      |  1.0 |
      | ---- |
      |  1.0 |
 2    |  1.0 |
      |   .  |
      └      ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------
     |   .
     |  1.0
     |  1.0
 0   |  1.0
     |  1.0
     |  1.0
     |   .
-----|------
     |  1.0
     |  1.0
 1   |  1.0
     |  1.0
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a row-distributed vector. Following is the global vector ***y*** of size 1 × 9, starting at row 1 and column 2 in 1 × 11 global matrix ***Y*** with block size 1 × 4:

```
B,D           0                    1                 2
      ┌                                                       ┐
 0    |   .   2.0  3.0  4.0 |  5.0  6.0  7.0  8.0 |  9.0 10.0   . |
      └                                                       ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
—	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |                  0                  |          1
-----|-------------------------------------|----------------------
 0   |   .   2.0  3.0  4.0  9.0  10.0   .  |   5.0  6.0  7.0  8.0
```

Output:

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 9 × 9 submatrix ***A***, starting at row 2 and column 2 in global general 10 × 10 matrix ***A*** with block size 4 × 4:

```
B,D              0                           1                     2
      ┌                                                                      ┐
      |    .    .    .    .   |    .    .    .    .   |    .    .   |
      |    .  14.0  25.0  36.0 |  47.0  58.0  69.0  80.0 |  91.0 102.0 |
 0    |    .  15.0  26.0  37.0 |  48.0  59.0  70.0  81.0 |  92.0 103.0 |
      |    .  16.0  27.0  38.0 |  49.0  60.0  71.0  82.0 |  93.0 104.0 |
      | -----------------------|-------------------------|------------- |
      |    .  17.0  28.0  39.0 |  50.0  61.0  72.0  83.0 |  94.0 105.0 |
      |    .  18.0  29.0  40.0 |  51.0  62.0  73.0  84.0 |  95.0 106.0 |
 1    |    .  19.0  30.0  41.0 |  52.0  63.0  74.0  85.0 |  96.0 107.0 |
      |    .  20.0  31.0  42.0 |  53.0  64.0  75.0  86.0 |  97.0 108.0 |
      | -----------------------|-------------------------|------------- |
 2    |    .  21.0  32.0  43.0 |  54.0  65.0  76.0  87.0 |  98.0 109.0 |
      |    .  22.0  33.0  44.0 |  55.0  66.0  77.0  88.0 |  99.0 110.0 |
      └                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                  0                   |              1
-----|--------------------------------------|-------------------------
     |   .    .     .     .     .      .    |    .     .     .     .
     |   .  14.0  25.0  36.0  91.0  102.0   |  47.0  58.0  69.0  80.0
     |   .  15.0  26.0  37.0  92.0  103.0   |  48.0  59.0  70.0  81.0
 0   |   .  16.0  27.0  38.0  93.0  104.0   |  49.0  60.0  71.0  82.0
     |   .  21.0  32.0  43.0  98.0  109.0   |  54.0  65.0  76.0  87.0
     |   .  22.0  33.0  44.0  99.0  110.0   |  55.0  66.0  77.0  88.0
-----|--------------------------------------|-------------------------
     |   .  17.0  28.0  39.0  94.0  105.0   |  50.0  61.0  72.0  83.0
     |   .  18.0  29.0  40.0  95.0  106.0   |  51.0  62.0  73.0  84.0
 1   |   .  19.0  30.0  41.0  96.0  107.0   |  52.0  63.0  74.0  85.0
     |   .  20.0  31.0  42.0  97.0  108.0   |  53.0  64.0  75.0  86.0
```

|　***Example 2:*** This example computes $\boldsymbol{A} = \alpha \boldsymbol{x}\boldsymbol{y}^{H}+\boldsymbol{A}$ using a 2 × 2 process grid. It
|　uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2.
|　It uses vector ***x***, which is a column-distributed vector within a column of global
|　matrix ***X***, by specifying *incx* = 1, *ix* = 2, and *jx* = 1. It uses vector ***y***, which is a
|　row-distributed vector within a row of global matrix ***Y***, by specifying
|　*incy* = M_Y = 1, *iy* = 1, and *jy* = 2.

|　*Call Statements and Input*

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET (0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|                M   N    ALPHA    X  IX  JX    DESC_X   INCX   Y   IY   JY
|                |   |      |      |   |   |      |        |     |    |    |
| CALL PZGERC(  9 , 9 ,  ALPHA  , X , 2 , 1 ,  DESC_X ,  1   , Y ,  1 ,  2 ,

|              DESC_Y   INCY   A   IA   JA    DESC_A
|                |        |    |    |    |      |
|              DESC_Y ,   1  , A ,  2 ,  2 ,  DESC_A  )

|              ALPHA = (1.0, -1.0)
```

	Desc_A	Desc_X	Desc_Y
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	10	11	1
N_	10	1	11
MB_	4	4	1
NB_	4	1	4
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = 6 on P_{00} and P_{01}, LLD_A = 4 on P_{10} and P_{11}, LLD_X = 7 on P_{00}, LLD_X = 4 on P_{10}, LLD_Y = 1 on P_{00} and P_{01}.

|　After the global matrix ***A*** is distributed over the process grid, only a portion of the
|　global data structure is used—that is, global submatrix ***A***. Following is the global
|　9 × 9 submatrix ***A***, starting at row 2 and column 2 in global general 10 × 10 matrix
|　***A*** with block size 4 × 4:

```
B,D                0                                           1                                     2
      ┌                                                                                                            ┐
      |    .        .           .           .      |      .           .           .           .      |      .           .       |
      |    .   (12.0, 2.0) (22.0, 1.0) (32.0, 0.0) | (42.0,-1.0) (52.0,-2.0) (62.0, 2.0) (72.0, 1.0) | (82.0, 0.0) ( 92.0,-1.0) |
 0    |    .   (13.0, 2.0) (23.0, 1.0) (33.0, 0.0) | (43.0,-1.0) (53.0,-2.0) (63.0, 2.0) (73.0, 1.0) | (83.0, 0.0) ( 93.0,-1.0) |
      |    .   (14.0, 2.0) (24.0, 1.0) (34.0, 0.0) | (44.0,-1.0) (54.0,-2.0) (64.0, 2.0) (74.0, 1.0) | (84.0, 0.0) ( 94.0,-1.0) |
      | -------------------------------------------|-----------------------------------------------|------------------------- |
      |    .   (15.0, 2.0) (25.0, 1.0) (35.0, 0.0) | (45.0,-1.0) (55.0,-2.0) (65.0, 2.0) (75.0, 1.0) | (85.0, 0.0) ( 95.0,-1.0) |
      |    .   (16.0, 2.0) (26.0, 1.0) (36.0, 0.0) | (46.0,-1.0) (56.0,-2.0) (66.0, 2.0) (76.0, 1.0) | (86.0, 0.0) ( 96.0,-1.0) |
 1    |    .   (17.0, 2.0) (27.0, 1.0) (37.0, 0.0) | (47.0,-1.0) (57.0,-2.0) (67.0, 2.0) (77.0, 1.0) | (87.0, 0.0) ( 97.0,-1.0) |
      |    .   (18.0, 2.0) (28.0, 1.0) (38.0, 0.0) | (48.0,-1.0) (58.0,-2.0) (68.0, 2.0) (78.0, 1.0) | (88.0, 0.0) ( 98.0,-1.0) |
      | -------------------------------------------|-----------------------------------------------|------------------------- |
 2    |    .   (19.0, 2.0) (29.0, 1.0) (39.0, 0.0) | (49.0,-1.0) (59.0,-2.0) (69.0, 2.0) (79.0, 1.0) | (89.0, 0.0) ( 99.0,-1.0) |
      |    .   (20.0, 2.0) (30.0, 1.0) (40.0, 0.0) | (50.0,-1.0) (60.0,-2.0) (70.0, 2.0) (80.0, 1.0) | (90.0, 0.0) (100.0,-1.0) |
      └                                                                                                            ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
 p,q  |                               0                                   |                  1
 -----|-------------------------------------------------------------------|------------------------------------------------
      |    .        .           .           .           .           .     |      .           .           .           .
      |    .   (12.0, 2.0) (22.0, 1.0) (32.0, 0.0) (82.0, 0.0) ( 92.0,-1.0) | (42.0,-1.0) (52.0,-2.0) (62.0, 2.0) (72.0, 1.0)
      |    .   (13.0, 2.0) (23.0, 1.0) (33.0, 0.0) (83.0, 0.0) ( 93.0,-1.0) | (43.0,-1.0) (53.0,-2.0) (63.0, 2.0) (73.0, 1.0)
  0   |    .   (14.0, 2.0) (24.0, 1.0) (34.0, 0.0) (84.0, 0.0) ( 94.0,-1.0) | (44.0,-1.0) (54.0,-2.0) (64.0, 2.0) (74.0, 1.0)
      |    .   (19.0, 2.0) (29.0, 1.0) (39.0, 0.0) (89.0, 0.0) ( 99.0,-1.0) | (49.0,-1.0) (59.0,-2.0) (69.0, 2.0) (79.0, 1.0)
      |    .   (20.0, 2.0) (30.0, 1.0) (40.0, 0.0) (90.0, 0.0) (100.0,-1.0) | (50.0,-1.0) (60.0,-2.0) (70.0, 2.0) (80.0, 1.0)
 -----|-------------------------------------------------------------------|------------------------------------------------
      |    .   (15.0, 2.0) (25.0, 1.0) (35.0, 0.0) (85.0, 0.0) ( 95.0,-1.0) | (45.0,-1.0) (55.0,-2.0) (65.0, 2.0) (75.0, 1.0)
      |    .   (16.0, 2.0) (26.0, 1.0) (36.0, 0.0) (86.0, 0.0) ( 96.0,-1.0) | (46.0,-1.0) (56.0,-2.0) (66.0, 2.0) (76.0, 1.0)
  1   |    .   (17.0, 2.0) (27.0, 1.0) (37.0, 0.0) (87.0, 0.0) ( 97.0,-1.0) | (47.0,-1.0) (57.0,-2.0) (67.0, 2.0) (77.0, 1.0)
      |    .   (18.0, 2.0) (28.0, 1.0) (38.0, 0.0) (88.0, 0.0) ( 98.0,-1.0) | (48.0,-1.0) (58.0,-2.0) (68.0, 2.0) (78.0, 1.0)
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 9 × 1, starting at row 2 and column 1 in 11 × 1 global matrix ***X*** with block size 4 × 1:

```
B,D     0
     ┌            ┐
     |     .      |
     | (1.0, 4.0) |
 0   | (1.0, 3.0) |
     | (1.0, 2.0) |
     | ---------- |
     | (1.0, 1.0) |
     | (1.0, 0.0) |
 1   | (1.0,-1.0) |
     | (1.0,-2.0) |
     | ---------- |
     | (1.0,-3.0) |
 2   | (1.0,-4.0) |
     |     .      |
     └            ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------------
     |     .
     | (1.0, 4.0)
     | (1.0, 3.0)
 0   | (1.0, 2.0)
     | (1.0,-3.0)
     | (1.0,-4.0)
     |     .
-----|------------
     | (1.0, 1.0)
     | (1.0, 0.0)
 1   | (1.0,-1.0)
     | (1.0,-2.0)
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a row-distributed vector. Following is the global vector ***y*** of size 1 × 9, starting at row 1 and column 2 in 1 × 11 global matrix ***Y*** with block size 1 × 4:

```
B,D                 0                                        1                                2
     ┌                                                                                                       ┐
 0   │  .   (2.0, 1.0) (3.0, 1.0) (4.0, 1.0) | (5.0, 1.0) (6.0, 1.0) (7.0, 1.0) (8.0, 1.0) | (9.0, 1.0) (10.0, 1.0)   . │
     └                                                                                                       ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0	P_{00}	P_{01}
—	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |                          0                                  |                    1
-----|-------------------------------------------------------------|------------------------------------------
 0   |  .   (2.0, 1.0) (3.0, 1.0) (4.0, 1.0) (9.0, 1.0) (10.0, 1.0)   .  | (5.0, 1.0) (6.0, 1.0) (7.0, 1.0) (8.0, 1.0)
```

Output:

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 9 × 9 submatrix ***A***, starting at row 2 and column 2 in global general 10 × 10 matrix ***A*** with block size 4 × 4:

```
B,D                        0                                                     1                                         2
     ┌                                                                                                                                 ┐
     |    .        .         .         .      |     .         .          .           .       |      .         .     |
     |    .   (25.0,  3.0) (40.0,  5.0) (55.0,  7.0) | (70.0,  9.0) (85.0, 11.0) (100.0, 18.0) (115.0, 20.0) | (130.0, 22.0) (145.0, 24.0) |
 0   |    .   (23.0,  2.0) (37.0,  3.0) (51.0,  4.0) | (65.0,  5.0) (79.0,  6.0) ( 93.0, 12.0) (107.0, 13.0) | (121.0, 14.0) (135.0, 15.0) |
     |    .   (21.0,  1.0) (34.0,  1.0) (47.0,  1.0) | (60.0,  1.0) (73.0,  1.0) ( 86.0,  6.0) ( 99.0,  6.0) | (112.0,  6.0) (125.0,  6.0) |
     | -------------------------------------------- |---------------------------------------------------- |---------------------------- |
     |    .   (19.0,  0.0) (31.0, -1.0) (43.0, -2.0) | (55.0, -3.0) (67.0, -4.0) ( 79.0,  0.0) ( 91.0, -1.0) | (103.0, -2.0) (115.0, -3.0) |
     |    .   (17.0, -1.0) (28.0, -3.0) (39.0, -5.0) | (50.0, -7.0) (61.0, -9.0) ( 72.0, -6.0) ( 83.0, -8.0) | ( 94.0,-10.0) (105.0,-12.0) |
 1   |    .   (15.0, -2.0) (25.0, -5.0) (35.0, -8.0) | (45.0,-11.0) (55.0,-14.0) ( 65.0,-12.0) ( 75.0,-15.0) | ( 85.0,-18.0) ( 95.0,-21.0) |
     |    .   (13.0, -3.0) (22.0, -7.0) (31.0,-11.0) | (40.0,-15.0) (49.0,-19.0) ( 58.0,-18.0) ( 67.0,-22.0) | ( 76.0,-26.0) ( 85.0,-30.0) |
     | -------------------------------------------- |---------------------------------------------------- |---------------------------- |
 2   |    .   (11.0, -4.0) (19.0, -9.0) (27.0,-14.0) | (35.0,-19.0) (43.0,-24.0) ( 51.0,-24.0) ( 59.0,-29.0) | ( 67.0,-34.0) ( 75.0,-39.0) |
     |    .   ( 9.0, -5.0) (16.0,-11.0) (23.0,-17.0) | (30.0,-23.0) (37.0,-29.0) ( 44.0,-30.0) ( 51.0,-36.0) | ( 58.0,-42.0) ( 65.0,-48.0) |
     └                                                                                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                                   0                                    |                          1
-----|------------------------------------------------------------------------|---------------------------------------------------
     |  .      .         .         .          .            .                  |     .          .          .           .
     |  .  (25.0,  3.0) (40.0,  5.0) (55.0,  7.0) (130.0, 22.0) (145.0, 24.0) | (70.0,  9.0) (85.0, 11.0) (100.0, 18.0) (115.0, 20.0)
     |  .  (23.0,  2.0) (37.0,  3.0) (51.0,  4.0) (121.0, 14.0) (135.0, 15.0) | (65.0,  5.0) (79.0,  6.0) ( 93.0, 12.0) (107.0, 13.0)
 0   |  .  (21.0,  1.0) (34.0,  1.0) (47.0,  1.0) (112.0,  6.0) (125.0,  6.0) | (60.0,  1.0) (73.0,  1.0) ( 86.0,  6.0) ( 99.0,  6.0)
     |  .  (11.0, -4.0) (19.0, -9.0) (27.0,-14.0) ( 67.0,-34.0) ( 75.0,-39.0) | (35.0,-19.0) (43.0,-24.0) ( 51.0,-24.0) ( 59.0,-29.0)
     |  .  ( 9.0, -5.0) (16.0,-11.0) (23.0,-17.0) ( 58.0,-42.0) ( 65.0,-48.0) | (30.0,-23.0) (37.0,-29.0) ( 44.0,-30.0) ( 51.0,-36.0)
-----|------------------------------------------------------------------------|---------------------------------------------------
     |  .  (19.0,  0.0) (31.0, -1.0) (43.0, -2.0) (103.0, -2.0) (115.0, -3.0) | (55.0, -3.0) (67.0, -4.0) ( 79.0,  0.0) ( 91.0, -1.0)
     |  .  (17.0, -1.0) (28.0, -3.0) (39.0, -5.0) ( 94.0,-10.0) (105.0,-12.0) | (50.0, -7.0) (61.0, -9.0) ( 72.0, -6.0) ( 83.0, -8.0)
 1   |  .  (15.0, -2.0) (25.0, -5.0) (35.0, -8.0) ( 85.0,-18.0) ( 95.0,-21.0) | (45.0,-11.0) (55.0,-14.0) ( 65.0,-12.0) ( 75.0,-15.0)
     |  .  (13.0, -3.0) (22.0, -7.0) (31.0,-11.0) ( 76.0,-26.0) ( 85.0,-30.0) | (40.0,-15.0) (49.0,-19.0) ( 58.0,-18.0) ( 67.0,-22.0)
```

Example 3: This example computes ***A*** = α***xy***^{T}+***A*** using a 2 × 2 process grid. It uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2. It uses vector ***x***, which is a column-distributed vector within a column of global matrix ***X***, by specifying *incx* = 1, *ix* = 2, and *jx* = 1. It uses vector ***y***, which is a row-distributed vector within a row of global matrix ***Y***, by specifying *incy* = M_Y = 1, *iy* = 1, and *jy* = 2.

Call Statements and Input

```
|   ORDER = 'R'
|   NPROW = 2
|   NPCOL = 2
|   CALL BLACS_GET (0, 0, ICONTXT)
|   CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
|   CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|                M   N    ALPHA    X  IX  JX    DESC_X   INCX   Y   IY   JY
|                |   |      |      |  |   |       |       |     |   |    |
|   CALL PZGERU(  9 , 9 ,  ALPHA  , X , 2 , 1 ,  DESC_X ,  1   , Y ,  1 ,  2 ,

|              DESC_Y   INCY   A   IA   JA    DESC_A
|                |        |    |    |    |      |
|              DESC_Y ,   1  , A ,  2 ,  2 ,  DESC_A  )

|              ALPHA = (1.0,-1.0)
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	10	11	1
N_	10	1	11
MB_	4	4	1
NB_	4	1	4
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = 6 on P_{00} and P_{01}, LLD_A = 4 on P_{10} and P_{11}, LLD_X = 7 on P_{00}, LLD_X = 4 on P_{10}, LLD_Y = 1 on P_{00} and P_{01}.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 9×9 submatrix ***A***, starting at row 2 and column 2 in global general 10×10 matrix ***A*** with block size 4×4:

```
B,D                       0                                           1                                       2
      ┌                                                                                                                        ┐
      |    .         .          .          .      |      .          .          .          .      |      .          .      |
      |    .  (12.0, 2.0) (22.0, 1.0) (32.0, 0.0) | (42.0,-1.0) (52.0,-2.0) (62.0, 2.0) (72.0, 1.0) | (82.0, 0.0) ( 92.0,-1.0) |
 0    |    .  (13.0, 2.0) (23.0, 1.0) (33.0, 0.0) | (43.0,-1.0) (53.0,-2.0) (63.0, 2.0) (73.0, 1.0) | (83.0, 0.0) ( 93.0,-1.0) |
      |    .  (14.0, 2.0) (24.0, 1.0) (34.0, 0.0) | (44.0,-1.0) (54.0,-2.0) (64.0, 2.0) (74.0, 1.0) | (84.0, 0.0) ( 94.0,-1.0) |
      | ------------------------------------------|-----------------------------------------------|------------------------  |
      |    .  (15.0, 2.0) (25.0, 1.0) (35.0, 0.0) | (45.0,-1.0) (55.0,-2.0) (65.0, 2.0) (75.0, 1.0) | (85.0, 0.0) ( 95.0,-1.0) |
      |    .  (16.0, 2.0) (26.0, 1.0) (36.0, 0.0) | (46.0,-1.0) (56.0,-2.0) (66.0, 2.0) (76.0, 1.0) | (86.0, 0.0) ( 96.0,-1.0) |
 1    |    .  (17.0, 2.0) (27.0, 1.0) (37.0, 0.0) | (47.0,-1.0) (57.0,-2.0) (67.0, 2.0) (77.0, 1.0) | (87.0, 0.0) ( 97.0,-1.0) |
      |    .  (18.0, 2.0) (28.0, 1.0) (38.0, 0.0) | (48.0,-1.0) (58.0,-2.0) (68.0, 2.0) (78.0, 1.0) | (88.0, 0.0) ( 98.0,-1.0) |
      | ------------------------------------------|-----------------------------------------------|------------------------  |
 2    |    .  (19.0, 2.0) (29.0, 1.0) (39.0, 0.0) | (49.0,-1.0) (59.0,-2.0) (69.0, 2.0) (79.0, 1.0) | (89.0, 0.0) ( 99.0,-1.0) |
      |    .  (20.0, 2.0) (30.0, 1.0) (40.0, 0.0) | (50.0,-1.0) (60.0,-2.0) (70.0, 2.0) (80.0, 1.0) | (90.0, 0.0) (100.0,-1.0) |
      └                                                                                                                        ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                                  0                                  |                     1
-----|---------------------------------------------------------------------|------------------------------------------------
     |    .         .          .          .          .           .       |      .          .          .          .
     |    .  (12.0, 2.0) (22.0, 1.0) (32.0, 0.0) (82.0, 0.0) ( 92.0,-1.0) | (42.0,-1.0) (52.0,-2.0) (62.0, 2.0) (72.0, 1.0)
     |    .  (13.0, 2.0) (23.0, 1.0) (33.0, 0.0) (83.0, 0.0) ( 93.0,-1.0) | (43.0,-1.0) (53.0,-2.0) (63.0, 2.0) (73.0, 1.0)
 0   |    .  (14.0, 2.0) (24.0, 1.0) (34.0, 0.0) (84.0, 0.0) ( 94.0,-1.0) | (44.0,-1.0) (54.0,-2.0) (64.0, 2.0) (74.0, 1.0)
     |    .  (19.0, 2.0) (29.0, 1.0) (39.0, 0.0) (89.0, 0.0) ( 99.0,-1.0) | (49.0,-1.0) (59.0,-2.0) (69.0, 2.0) (79.0, 1.0)
     |    .  (20.0, 2.0) (30.0, 1.0) (40.0, 0.0) (90.0, 0.0) (100.0,-1.0) | (50.0,-1.0) (60.0,-2.0) (70.0, 2.0) (80.0, 1.0)
-----|---------------------------------------------------------------------|------------------------------------------------
     |    .  (15.0, 2.0) (25.0, 1.0) (35.0, 0.0) (85.0, 0.0) ( 95.0,-1.0) | (45.0,-1.0) (55.0,-2.0) (65.0, 2.0) (75.0, 1.0)
     |    .  (16.0, 2.0) (26.0, 1.0) (36.0, 0.0) (86.0, 0.0) ( 96.0,-1.0) | (46.0,-1.0) (56.0,-2.0) (66.0, 2.0) (76.0, 1.0)
 1   |    .  (17.0, 2.0) (27.0, 1.0) (37.0, 0.0) (87.0, 0.0) ( 97.0,-1.0) | (47.0,-1.0) (57.0,-2.0) (67.0, 2.0) (77.0, 1.0)
     |    .  (18.0, 2.0) (28.0, 1.0) (38.0, 0.0) (88.0, 0.0) ( 98.0,-1.0) | (48.0,-1.0) (58.0,-2.0) (68.0, 2.0) (78.0, 1.0)
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 9 × 1, starting at row 2 and column 1 in 11 × 1 global matrix ***X*** with block size 4 × 1:

```
B,D     0
      ┌              ┐
      |      .       |
      | (1.0, 4.0)   |
 0    | (1.0, 3.0)   |
      | (1.0, 2.0)   |
      | ----------   |
      | (1.0, 1.0)   |
      | (1.0, 0.0)   |
 1    | (1.0,-1.0)   |
      | (1.0,-2.0)   |
      | ----------   |
      | (1.0,-3.0)   |
 2    | (1.0,-4.0)   |
      |      .       |
      └              ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------------
     |     .
     | (1.0, 4.0)
     | (1.0, 3.0)
 0   | (1.0, 2.0)
     | (1.0,-3.0)
     | (1.0,-4.0)
     |     .
-----|------------
     | (1.0, 1.0)
     | (1.0, 0.0)
 1   | (1.0,-1.0)
     | (1.0,-2.0)
```

After the global matrix ***Y*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***y***, which is a row-distributed vector. Following is the global vector ***y*** of size 1 × 9, starting at row 1 and column 2 in 1 × 11 global matrix ***Y*** with block size 1 × 4:

```
B,D                0                                        1                               2
      ┌                                                                                                             ┐
 0    │  .   (2.0, 1.0) (3.0, 1.0) (4.0, 1.0) | (5.0, 1.0) (6.0, 1.0) (7.0, 1.0) (8.0, 1.0) | (9.0, 1.0) (10.0, 1.0)   . │
      └                                                                                                             ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
—	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |                             0                             |                    1
-----|-----------------------------------------------------------|----------------------------------------
 0   |  .   (2.0, 1.0) (3.0, 1.0) (4.0, 1.0) (9.0, 1.0) (10.0, 1.0)   .  | (5.0, 1.0) (6.0, 1.0) (7.0, 1.0) (8.0, 1.0)
```

Output:

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global 9 × 9 submatrix ***A***, starting at row 2 and column 2 in global general 10 × 10 matrix ***A*** with block size 4 × 4:

```
B,D                                   0                                                  1                                              2
      ┌                                                                                                                                         ┐
      |    .      .          .          .      |      .           .          .          .        |      .           .      |
      |    .  (19.0, 13.0) (34.0, 15.0) (49.0, 17.0) | (64.0, 19.0) (79.0, 21.0) (94.0, 28.0) (109.0, 30.0) | (124.0, 32.0) (139.0, 34.0) |
 0    |    .  (19.0, 10.0) (33.0, 11.0) (47.0, 12.0) | (61.0, 13.0) (75.0, 14.0) (89.0, 20.0) (103.0, 21.0) | (117.0, 22.0) (131.0, 23.0) |
      |    .  (19.0,  7.0) (32.0,  7.0) (45.0,  7.0) | (58.0,  7.0) (71.0,  7.0) (84.0, 12.0) ( 97.0, 12.0) | (110.0, 12.0) (123.0, 12.0) |
      | --------------------------------------------|----------------------------------------------------|---------------------------- |
      |    .  (19.0,  4.0) (31.0,  3.0) (43.0,  2.0) | (55.0,  1.0) (67.0,  0.0) (79.0,  4.0) ( 91.0,  3.0) | (103.0,  2.0) (115.0,  1.0) |
      |    .  (19.0,  1.0) (30.0, -1.0) (41.0, -3.0) | (52.0, -5.0) (63.0, -7.0) (74.0, -4.0) ( 85.0, -6.0) | ( 96.0, -8.0) (107.0,-10.0) |
 1    |    .  (19.0, -2.0) (29.0, -5.0) (39.0, -8.0) | (49.0,-11.0) (59.0,-14.0) (69.0,-12.0) ( 79.0,-15.0) | ( 89.0,-18.0) ( 99.0,-21.0) |
      |    .  (19.0, -5.0) (28.0, -9.0) (37.0,-13.0) | (46.0,-17.0) (55.0,-21.0) (64.0,-20.0) ( 73.0,-24.0) | ( 82.0,-28.0) ( 91.0,-32.0) |
      | --------------------------------------------|----------------------------------------------------|---------------------------- |
 2    |    .  (19.0, -8.0) (27.0,-13.0) (35.0,-18.0) | (43.0,-23.0) (51.0,-28.0) (59.0,-28.0) ( 67.0,-33.0) | ( 75.0,-38.0) ( 83.0,-43.0) |
      |    .  (19.0,-11.0) (26.0,-17.0) (33.0,-23.0) | (40.0,-29.0) (47.0,-35.0) (54.0,-36.0) ( 61.0,-42.0) | ( 68.0,-48.0) ( 75.0,-54.0) |
      └                                                                                                                                         ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                                     0                                     |                         1
-----|---------------------------------------------------------------------------|---------------------------------------------------
     |  .     .          .          .           .             .                 |      .          .          .           .
     |  .  (19.0, 13.0) (34.0, 15.0) (49.0, 17.0) (124.0, 32.0) (139.0, 34.0) | (64.0, 19.0) (79.0, 21.0) (94.0, 28.0) (109.0, 30.0)
     |  .  (19.0, 10.0) (33.0, 11.0) (47.0, 12.0) (117.0, 22.0) (131.0, 23.0) | (61.0, 13.0) (75.0, 14.0) (89.0, 20.0) (103.0, 21.0)
 0   |  .  (19.0,  7.0) (32.0,  7.0) (45.0,  7.0) (110.0, 12.0) (123.0, 12.0) | (58.0,  7.0) (71.0,  7.0) (84.0, 12.0) ( 97.0, 12.0)
     |  .  (19.0, -8.0) (27.0,-13.0) (35.0,-18.0) ( 75.0,-38.0) ( 83.0,-43.0) | (43.0,-23.0) (51.0,-28.0) (59.0,-28.0) ( 67.0,-33.0)
     |  .  (19.0,-11.0) (26.0,-17.0) (33.0,-23.0) ( 68.0,-48.0) ( 75.0,-54.0) | (40.0,-29.0) (47.0,-35.0) (54.0,-36.0) ( 61.0,-42.0)
-----|---------------------------------------------------------------------------|---------------------------------------------------
     |  .  (19.0,  4.0) (31.0,  3.0) (43.0,  2.0) (103.0,  2.0) (115.0,  1.0) | (55.0,  1.0) (67.0,  0.0) (79.0,  4.0) ( 91.0,  3.0)
     |  .  (19.0,  1.0) (30.0, -1.0) (41.0, -3.0) ( 96.0, -8.0) (107.0,-10.0) | (52.0, -5.0) (63.0, -7.0) (74.0, -4.0) ( 85.0, -6.0)
 1   |  .  (19.0, -2.0) (29.0, -5.0) (39.0, -8.0) ( 89.0,-18.0) ( 99.0,-21.0) | (49.0,-11.0) (59.0,-14.0) (69.0,-12.0) ( 79.0,-15.0)
     |  .  (19.0, -5.0) (28.0, -9.0) (37.0,-13.0) ( 82.0,-28.0) ( 91.0,-32.0) | (46.0,-17.0) (55.0,-21.0) (64.0,-20.0) ( 73.0,-24.0)
```

PDSYR and PZHER—Rank-One Update of a Real Symmetric or a Complex Hermitian Matrix

PDSYR computes the following rank-one update:

$$\boldsymbol{A} \leftarrow \alpha \boldsymbol{x}\boldsymbol{x}^{T}+\boldsymbol{A}$$

| PZHER computes the following rank-one update:

| $$\boldsymbol{A} \leftarrow \alpha \boldsymbol{x}\boldsymbol{x}^{H}+\boldsymbol{A}$$

where, in the formula above:

A represents the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
x represents the global vector:
- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.

α is a scalar.

and:

| • For PDSYR, submatrix ***A*** is real symmetric.
| • For PZHER, submatrix ***A*** is complex Hermitian.

| **Note:** No data should be moved to form $\boldsymbol{x}^{T}$ or $\boldsymbol{x}^{H}$; that is, the vector ***x*** should
| always be stored in its untransposed form.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- *n* = 0
- α is zero.

See references [14] and [15].

| *Table 42. Data Types*

A, ***x***	α	**Subprogram**
Long-precision real	Long-precision real	PDSYR
Long-precision complex	Long-precision real	PZHER

Syntax

Fortran	CALL PDSYR | PZHER (*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *a*, *ia*, *ja*, *desc_a*)
C and C++	pdsyr | pzher (*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *a*, *ia*, *ja*, *desc_a*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the number of rows and columns in submatrix ***A*** and the number of elements in vector ***x*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 42 on page 224.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*n*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*n*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*n*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*n*–1 by *jx* part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{x}^T$ or $\boldsymbol{x}^H$; that is, the vector ***x*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 42 on page 224. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ix \leq$ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then $ix+n-1 \leq$ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; $1 \leq jx \leq$ N_X and:

If *incx* = M_X, then $jx+n-1 \leq$ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If $n = 0$: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If $n = 0$: N_X ≥ 0 Otherwise: N_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global
7	RSRC_X	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_X < p	Global
8	CSRC_X	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_X < q	Global
9	LLD_X	The leading dimension of the local array	LLD_X ≥ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

a

| is the local part of the global real symmetric or complex Hermitian matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*−1) by LOCq(*ja*+*n*−1) part of the local array A must contain the local pieces of the leading *ia*+*n*−1 by *ja*+*n*−1 part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 42 on page 224. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 42 on page 224.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo* argument.
2. The matrix and vector must have no common elements; otherwise, results are unpredictable.
3. The imaginary parts of the diagonal elements of the complex Hermitian matrix are assumed to be zero, so you do not have to set these values. On output, they are set to zero except when N is zero or α is zero, in which case no computation is performed.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
5. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
6. The following values must be equal: CTXT_A = CTXT_X.
7. The global matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
8. The block row and block column offsets of the global matrix ***A*** must be equal; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
9. If *incx* = M_X:
   - In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***X***; that is, *iacol* = *ixcol*, where:

     iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
     ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
   - The block column offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ia*–1, MB_A).
   - The following block sizes must be equal: NB_X = NB_A.
10. If *incx* = 1($\neq$ M_X):
    - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***X***; that is, *iarow* = *ixrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    - The block row offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: MB_X = MB_A.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ 'U' or 'L'
2. $n < 0$
3. M_X < 0 and $n = 0$; M_X < 1 otherwise
4. N_X < 0 and $n = 0$; N_X < 1 otherwise
5. MB_X < 1
6. NB_X < 1
7. RSRC_X < 0 or RSRC_X ≥ p
8. CSRC_X < 0 or CSRC_X ≥ q
9. CTXT_A ≠ CTXT_X
10. $ix < 1$
11. $jx < 1$
12. M_A < 0 and $n = 0$; M_A < 1 otherwise
13. N_A < 0 and $n = 0$; N_A < 1 otherwise
14. MB_A < 1
15. NB_A < 1
16. RSRC_A < 0 or RSRC_A ≥ p
17. CSRC_A < 0 or CSRC_A ≥ q
18. $ia < 1$
19. $ja < 1$

Stage 5

1. NB_A ≠ MB_A

If $n \neq 0$:

2. ia > M_A
3. ja > N_A
4. $ia+n-1$ > M_A
5. $ja+n-1$ > N_A
6. ix > M_X
7. jx > N_X

If $incx$ = M_X:

8. NB_X ≠ NB_A
9. mod($jx-1$, NB_X) ≠ mod($ia-1$, MB_A)
10. $n \neq 0$ and $jx+n-1$ > N_X

If $incx$ = 1(≠ M_X):

11. MB_X ≠ MB_A
12. mod($ix-1$, MB_X) ≠ mod($ia-1$, MB_A)

13. $n \neq 0$ and $ix+n-1 > \text{M\_X}$

Otherwise:
14. $incx \neq \text{M\_X}$ and $incx \neq 1$

Stage 6

1. mod(*ja*–1, NB_A) ≠ mod(*ia*–1, MB_A)
2. If *incx* = M_X, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***X***; that is, *iacol* ≠ *ixcol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
3. If *incx* = 1(≠ M_X), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***X***; that is, *iarow* ≠ *ixrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
4. LLD_A < max(1, LOCp(M_A))
5. LLD_X < max(1, LOCp(M_X))

Example 1: This example computes ***A*** = $\alpha \boldsymbol{x}\boldsymbol{x}^T + \boldsymbol{A}$ using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO  N   ALPHA   X  IX  JX   DESC_X  INCX  A  IA  JA   DESC_A
             |    |     |     |   |   |     |      |    |   |   |     |
CALL PDSYR( 'L' , 9 , 1.0D0 , X , 1 , 1 , DESC_X , 1  , A , 1 , 1 , DESC_A)
```

	Desc_A	Desc_X
DTYPE_	1	1
CTXT_	*icontxt*1	*icontxt*1
M_	9	9
N_	9	1
MB_	4	4
NB_	4	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below2	See below2

	Desc_A	**Desc_X**

[1] *icontxt* is the output of the BLACS_GRIDINIT call.
[2] Each process should set the LLD_ as follows:
`LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))`
`LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))`
In this example, LLD_A = 5 on P_{00} and P_{01}, LLD_A = 4 on P_{10} and P_{11}, LLD_X = 5 on P_{00}, and LLD_X = 4 on P_{10}.		

Global real symmetric matrix ***A*** of order 9 with block size 4 × 4:

```
B,D             0                            1               2
      ┌                                                        ┐
      │   1.0    .     .     .  │     .     .     .     .  │     .  │
      │   2.0  12.0    .     .  │     .     .     .     .  │     .  │
 0    │   3.0  13.0  23.0    .  │     .     .     .     .  │     .  │
      │   4.0  14.0  24.0  34.0 │     .     .     .     .  │     .  │
      │ ------------------------│--------------------------│------- │
      │   5.0  15.0  25.0  35.0 │   45.0    .     .     .  │     .  │
      │   6.0  16.0  26.0  36.0 │   46.0  56.0    .     .  │     .  │
 1    │   7.0  17.0  27.0  37.0 │   47.0  57.0  67.0    .  │     .  │
      │   8.0  18.0  28.0  38.0 │   48.0  58.0  68.0  78.0 │     .  │
      │ ------------------------│--------------------------│------- │
 2    │   9.0  19.0  29.0  39.0 │   49.0  59.0  69.0  79.0 │   89.0 │
      └                                                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │               0                │            1
-----│--------------------------------│-------------------------
     │   1.0    .     .     .     .   │     .     .     .     .
     │   2.0  12.0    .     .     .   │     .     .     .     .
 0   │   3.0  13.0  23.0    .     .   │     .     .     .     .
     │   4.0  14.0  24.0  34.0    .   │     .     .     .     .
     │   9.0  19.0  29.0  39.0  89.0  │   49.0  59.0  69.0  79.0
-----│--------------------------------│-------------------------
     │   5.0  15.0  25.0  35.0    .   │   45.0    .     .     .
     │   6.0  16.0  26.0  36.0    .   │   46.0  56.0    .     .
 1   │   7.0  17.0  27.0  37.0    .   │   47.0  57.0  67.0    .
     │   8.0  18.0  28.0  38.0    .   │   48.0  58.0  68.0  78.0
```

Global vector ***x*** of size 9 × 1 with block size 4:

```
B,D     0
      ┌      ┐
      │  1.0 │
      │  1.0 │
 0    │  1.0 │
      │  1.0 │
      │ ---- │
      │  1.0 │
      │  1.0 │
 1    │  1.0 │
      │  1.0 │
      │ ---- │
 2    │  1.0 │
      └      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  │  0
-----│------
     │  1.0
     │  1.0
 0   │  1.0
     │  1.0
     │  1.0
-----│------
     │  1.0
     │  1.0
 1   │  1.0
     │  1.0
```

Output:

Global real symmetric matrix ***A*** of order 9 with block size 4 × 4:

```
B,D              0                           1                  2
      ┌                                                                  ┐
      │   2.0    .     .     .   │     .     .     .     .   │      .  │
      │   3.0  13.0    .     .   │     .     .     .     .   │      .  │
 0    │   4.0  14.0  24.0    .   │     .     .     .     .   │      .  │
      │   5.0  15.0  25.0  35.0  │     .     .     .     .   │      .  │
      │ -------------------------│---------------------------│------- │
      │   6.0  16.0  26.0  36.0  │   46.0    .     .     .   │      .  │
      │   7.0  17.0  27.0  37.0  │   47.0  57.0    .     .   │      .  │
 1    │   8.0  18.0  28.0  38.0  │   48.0  58.0  68.0    .   │      .  │
      │   9.0  19.0  29.0  39.0  │   49.0  59.0  69.0  79.0  │      .  │
      │ -------------------------│---------------------------│------- │
 2    │  10.0  20.0  30.0  40.0  │   50.0  60.0  70.0  80.0  │   90.0 │
      └                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |               0                |            1
-----|--------------------------------|-------------------------
     |   2.0    .     .     .     .   |     .     .     .     .
     |   3.0  13.0    .     .     .   |     .     .     .     .
 0   |   4.0  14.0  24.0    .     .   |     .     .     .     .
     |   5.0  15.0  25.0  35.0    .   |     .     .     .     .
     |  10.0  20.0  30.0  40.0  90.0  |   50.0  60.0  70.0  80.0
-----|--------------------------------|-------------------------
     |   6.0  16.0  26.0  36.0    .   |   46.0    .     .     .
     |   7.0  17.0  27.0  37.0    .   |   47.0  57.0    .     .
 1   |   8.0  18.0  28.0  38.0    .   |   48.0  58.0  68.0    .
     |   9.0  19.0  29.0  39.0    .   |   49.0  59.0  69.0  79.0
```

| ***Example 2:*** This example computes $\boldsymbol{A} = \alpha \boldsymbol{x}\boldsymbol{x}^{H}+\boldsymbol{A}$ using a 2×2 process grid.

| **Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix
| are assumed to be zero, so you do not have to set these values. On output,
| they are set to zero except when N is zero or α is zero.

| *Call Statements and Input*

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET (0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|              UPLO  N   ALPHA   X  IX  JX   DESC_X  INCX  A  IA  JA   DESC_A
|               |    |     |     |   |   |     |      |    |   |   |     |
| CALL PZHER( 'L' , 3 , 1.0D0 , X , 1 , 1 , DESC_X , 1  , A , 1 , 1 , DESC_A)
```

	Desc_A	Desc_X
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	3	3
N_	3	1
MB_	2	2
NB_	2	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

	Desc_A	Desc_X

[1] *icontxt* is the output of the BLACS_GRIDINIT call.
[2] Each process should set the LLD_ as follows:
`LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))`
`LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))`
In this example, LLD_A = 2 on P_{00} and P_{01}, LLD_A = 1 on P_{10} and P_{11}, LLD_X = 2 on P_{00}, and LLD_X = 1 on P_{10}.		

Global complex Hermitian matrix ***A*** of order 3 with block size 2 × 2:

```
B,D              0                  1
      ┌                                      ┐
      │  ( 1.0, 0.0)      .       │     .     │
 0    │  ( 3.0,-5.0) ( 7.0, 0.0)  │     .     │
      │ ------------------------- │---------- │
 1    │  ( 2.0, 3.0) ( 4.0, 8.0)  │ ( 6.0, 0.0) │
      └                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │            0            │            1
-----│-------------------------│--------------------------
     │ ( 1.0,  . )      .      │            .
 0   │ ( 3.0,-5.0) ( 7.0,  . ) │            .
-----│-------------------------│--------------------------
 1   │ ( 2.0, 3.0) ( 4.0, 8.0) │ ( 6.0,  . )      .
```

Global vector ***x*** of size 3 × 1 with block size 2:

```
B,D     0
      ┌           ┐
      │ (1.0,2.0) │
 0    │ (4.0,0.0) │
      │ --------- │
 1    │ (3.0,4.0) │
      └           ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |     0
-----|-----------
     | (1.0,2.0)
 0   | (4.0,0.0)
-----|-----------
 1   | (3.0,4.0)
```

Output:

Global complex Hermitian matrix ***A*** of order 3 with block size 2 × 2:

```
B,D               0                    1
     ┌                                        ┐
     | ( 6.0,  0.0)      .      |      .      |
 0   | ( 7.0,-13.0) (23.0, 0.0) |      .      |
     | -------------------------|-------------|
 1   | (13.0,  1.0) (16.0,24.0) | (31.0, 0.0) |
     └                                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |             0            |      1
-----|--------------------------|-------------
     | ( 6.0,  0.0)      .      |      .
 0   | ( 7.0,-13.0) (23.0, 0.0) |      .
-----|--------------------------|-------------
 1   | (13.0,  1.0) (16.0,24.0) | (31.0, 0.0)
```

PDSYR2 and PZHER2—Rank-Two Update of a Real Symmetric or a Complex Hermitian Matrix

PDSYR2 computes the following rank-two update:

$$\boldsymbol{A} \leftarrow \alpha \boldsymbol{x}\boldsymbol{y}^T + \alpha \boldsymbol{y}\boldsymbol{x}^T + \boldsymbol{A}$$

| PZHER2 computes the following rank-two update:

$$\mathbf{A} \leftarrow \alpha \mathbf{x}\mathbf{y}^H + \bar{\alpha}\mathbf{y}\mathbf{x}^H + \mathbf{A}$$

where, in the formula above:

$\boldsymbol{A}$ represents the global submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
$\boldsymbol{x}$ represents the global vector:
- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.

$\boldsymbol{y}$ represents the global vector:
- For *incy* = M_Y, it is $\boldsymbol{Y}_{iy:iy,\ jy:jy+n-1}$.
- For *incy* = 1 and *incy* ≠ M_Y, it is $\boldsymbol{Y}_{iy:iy+n-1,\ jy:jy}$.

α is a scalar.

and:

- | For PDSYR2, submatrix ***A*** is real symmetric.
- | For PZHER2, submatrix ***A*** is complex Hermitian.

| **Note:** No data should be moved to form $\boldsymbol{x}^T$, $\boldsymbol{x}^H$, $\boldsymbol{y}^T$, or $\boldsymbol{y}^H$; that is, the vectors ***x***
| and ***y*** should always be stored in their untransposed form.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- *n* = 0
- α is zero.

See references [14] and [15].

Table 43. Data Types

A, ***x***, ***y***, α	**Subprogram**
Long-precision real	PDSYR2
Long-precision complex	PZHER2

Syntax

Fortran	CALL PDSYR2 | PZHER2 (*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *y*, *iy*, *jy*, *desc_y*, *incy*, *a*, *ia*, *ja*, *desc_a*)
C and C++	pdsyr2 | pzher2 (*uplo*, *n*, *alpha*, *x*, *ix*, *jx*, *desc_x*, *incx*, *y*, *iy*, *jy*, *desc_y*, *incy*, *a*, *ia*, *ja*, *desc_a*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global symmetric submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the number of rows and columns in submatrix ***A*** and the number of elements in vectors ***x*** and ***y*** used in the computation.

Scope: **global**

Specified as: a fullword integer; *n* ≥ 0.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 43 on page 236.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array `X`. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*n*–1) part of the local array `X` must contain the local pieces of the leading *ix* by *jx*+*n*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*n*–1) by LOCq(*jx*) part of the local array `X` must contain the local pieces of the leading *ix*+*n*–1 by *jx* part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{x}^T$ or $\boldsymbol{x}^H$; that is, the vector ***x*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 43 on page 236. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then *ix*+*n*–1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; $1 \le jx \le$ N_X and:

If *incx* = M_X, then $jx+n-1 \le$ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If $n = 0$: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If $n = 0$: N_X ≥ 0 Otherwise: N_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global
7	RSRC_X	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_X $< p$	Global
8	CSRC_X	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_X $< q$	Global
9	LLD_X	The leading dimension of the local array	LLD_X $\ge$ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

y

is the local part of the global matrix ***Y***. This identifies the **first element** of the local array Y. This subroutine computes the location of the first element of the local subarray used, based on *iy*, *jy*, *desc_y*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incy* = M_Y, the leading LOCp(*iy*) by LOCq(*jy*+*n*–1) part of the local array Y must contain the local pieces of the leading *iy* by *jy*+*n*–1 part of the global matrix.
- If *incy* = 1 and *incy* ≠ M_Y, the leading LOCp(*iy*+*n*–1) by LOCq(*jy*) part of the local array Y must contain the local pieces of the leading *iy*+*n*–1 by *jy* part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{y}^T$ or $\boldsymbol{y}^H$; that is, the vector ***x*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_Y by (at least) LOCq(N_Y) array, containing numbers of the data type indicated in Table 43 on page 236. Details about the block-cyclic data distribution of the global matrix ***Y*** are stored in *desc_y*.

iy

has the following meaning:

If *incy* = M_Y, it indicates which row of global matrix ***Y*** is used for vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it is the row index of global matrix ***Y***, identifying the first element of vector ***y***.

Scope: **global**

Specified as: a fullword integer; $1 \le iy \le$ M_Y and:

If *incy* = 1 and *incy* ≠ M_Y, then $iy+n-1 \le$ M_Y.

jy

has the following meaning:

If *incy* = M_Y, it is the column index of global matrix ***Y***, identifying the first element of vector ***y***.

If *incy* = 1 and *incy* ≠ M_Y, it indicates which column of global matrix ***Y*** is used for vector ***y***.

Scope: **global**

Specified as: a fullword integer; $1 \le jy \le$ N_Y and:

If *incy* = M_Y, then $jy+n-1 \le$ N_Y.

desc_y

is the array descriptor for global matrix ***Y***, described in the following table:

desc_y	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_Y	Descriptor type	DTYPE_Y=1	Global
2	CTXT_Y	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_Y	Number of rows in the global matrix	If $n = 0$: M_Y ≥ 0 Otherwise: M_Y ≥ 1	Global
4	N_Y	Number of columns in the global matrix	If $n = 0$: N_Y ≥ 0 Otherwise: N_Y ≥ 1	Global
5	MB_Y	Row block size	MB_Y ≥ 1	Global
6	NB_Y	Column block size	NB_Y ≥ 1	Global
7	RSRC_Y	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_Y < p	Global

desc_y	**Name**	**Description**	**Limits**	**Scope**
8	CSRC_Y	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_Y $< q$	Global
9	LLD_Y	The leading dimension of the local array	LLD_Y $\ge$ max(1,LOCp(M_Y))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incy

is the stride for global vector ***y***.

Scope: **global**

Specified as: a fullword integer; *incy* = 1 or *incy* = M_X, where:

If *incy* = M_Y, then ***y*** is a row-distributed vector.

If *incy* = 1 and *incy* ≠ M_Y, then ***y*** is a column-distributed vector.

a

| is the local part of the global real symmetric or complex Hermitian matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 43 on page 236. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A $\geq$ 0 Otherwise: M_A $\geq$ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ p	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ q	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 43 on page 236.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo* argument.
2. The matrix and vectors must have no common elements; otherwise, results are unpredictable.
3. The imaginary parts of the diagonal elements of the complex Hermitian matrix are assumed to be zero, so you do not have to set these values. On output, they are set to zero except when N is zero or α is zero, in which case no computation is performed.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.

5. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
6. The following values must be equal: CTXT_A = CTXT_X = CTXT_Y.
7. The vectors ***x*** and ***y*** must be distributed along the same axis—that is, they must both be row distributed or column distributed, where:
   - *incx* = M_X and *incy* = M_Y for row distribution
   - *incx* = 1(≠ M_X) and *incy* = 1(≠ M_Y) for column distribution
8. The global symmetric matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
9. The block row and block column offsets of the global symmetric matrix ***A*** must be equal; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
10. If *incx* = M_X:
    - In the process grid, the process column containing the first column of the submatrix ***X*** must also contain the first column of the submatrix ***A***; that is, *iacol* = *ixcol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
    - The block column offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: NB_X = NB_A.
11. If *incx* = 1(≠ M_X):
    - In the process grid, the process row containing the first row of the submatrix ***X*** must also contain the first row of the submatrix ***A***; that is, *iarow* = *ixrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    - The block row offset of ***x*** must be equal to the block row offset of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: MB_X = MB_A.
12. If *incy* = M_Y:
    - In the process grid, the process column containing the first column of the submatrix ***Y*** must also contain the first column of the submatrix ***A***; that is, *iacol* = *iycol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)
    - The block column offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*jy*–1, NB_Y) = mod(*ia*–1, MB_A).
    - The following block sizes must be equal: NB_Y = NB_A.
13. If *incy* = 1(≠ M_Y):
    - In the process grid, the process row containing the first row of the submatrix ***Y*** must also contain the first row of the submatrix ***A***; that is, *iarow* = *iyrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)

$iyrow = \text{mod}((((iy-1)/\text{MB\_Y})+\text{RSRC\_Y}), p)$

- The block row offset of ***y*** must be equal to the block row offset of ***A***; that is, mod(*iy*–1, MB_Y) = mod(*ia*–1, MB_A).
- The following block sizes must be equal: MB_Y = MB_A.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.
3. DTYPE_Y is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ `'U'` or `'L'`
2. $n < 0$
3. M_X < 0 and $n = 0$; M_X < 1 otherwise
4. N_X < 0 and $n = 0$; N_X < 1 otherwise
5. MB_X < 1
6. NB_X < 1
7. RSRC_X < 0 or RSRC_X ≥ p
8. CSRC_X < 0 or CSRC_X ≥ q
9. CTXT_A ≠ CTXT_X
10. $ix < 1$
11. $jx < 1$
12. M_Y < 0 and $n = 0$; M_Y < 1 otherwise
13. N_Y < 0 and $n = 0$; N_Y < 1 otherwise
14. MB_Y < 1
15. NB_Y < 1
16. RSRC_Y < 0 or RSRC_Y ≥ p
17. CSRC_Y < 0 or CSRC_Y ≥ q
18. CTXT_A ≠ CTXT_Y
19. $iy < 1$
20. $jy < 1$
21. M_A < 0 and $n = 0$; M_A < 1 otherwise
22. N_A < 0 and $n = 0$; N_A < 1 otherwise
23. MB_A < 1
24. NB_A < 1
25. RSRC_A < 0 or RSRC_A ≥ p
26. CSRC_A < 0 or CSRC_A ≥ q
27. $ia < 1$

28. $ja < 1$

Stage 5

1. NB_A ≠ MB_A

If $n \neq 0$:

2. $ia >$ M_A
3. $ja >$ N_A
4. $ia+n-1 >$ M_A
5. $ja+n-1 >$ N_A
6. $ix >$ M_X
7. $jx >$ N_X
8. $iy >$ M_Y
9. $jy >$ N_Y

If $incx$ = M_X:

10. NB_X ≠ NB_A
11. mod(jx–1, NB_X) ≠ mod(ia–1, MB_A)
12. $n \neq 0$ and $jx+n-1 >$ N_X

If $incx$ = 1(≠ M_X):

13. MB_X ≠ MB_A
14. mod(ix–1, MB_X) ≠ mod(ia–1, MB_A)
15. $n \neq 0$ and $ix+n-1 >$ M_X

Otherwise:

16. $incx$ ≠ M_X and $incx \neq 1$

If $incy$ = M_Y:

17. NB_Y ≠ NB_A
18. mod(jy–1, NB_Y) ≠ mod(ia–1, MB_A)
19. $n \neq 0$ and $jy+n-1 >$ N_Y

If $incy$ = 1(≠ M_Y):

20. MB_Y ≠ MB_A
21. mod(iy–1, MB_Y) ≠ mod(ia–1, MB_A)
22. $n \neq 0$ and $iy+n-1 >$ M_Y

Otherwise:

23. $incy$ ≠ M_Y and $incy \neq 1$

Stage 6

1. mod(ja–1, NB_A) ≠ mod(ia–1, MB_A)
2. If $incx$ = M_X, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***X***; that is, $iacol \neq ixcol$, where:
   $iacol$ = mod((((ja–1)/NB_A)+CSRC_A), q)
   $ixcol$ = mod((((jx–1)/NB_X)+CSRC_X), q)
3. If $incx$ = 1(≠ M_X), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***X***; that is, $iarow \neq ixrow$, where:
   $iarow$ = mod((((ia–1)/MB_A)+RSRC_A), p)
   $ixrow$ = mod((((ix–1)/MB_X)+RSRC_X), p)
4. If $incy$ = M_Y, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***Y***; that is, $iacol \neq iycol$, where:
   $iacol$ = mod((((ja–1)/NB_A)+CSRC_A), q)

iycol = mod((((*jy*–1)/NB_Y)+CSRC_Y), *q*)

5. If *incy* = 1($\neq$ M_Y), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***Y***; that is, *iarow* $\neq$ *iyrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   iyrow = mod((((*iy*–1)/MB_Y)+RSRC_Y), *p*)
6. LLD_A < max(1, LOCp(M_A))
7. LLD_X < max(1, LOCp(M_X))
8. LLD_Y < max(1, LOCp(M_Y))

Example 1: This example computes ***A*** = $\alpha \boldsymbol{x}\boldsymbol{y}^T + \alpha \boldsymbol{y}\boldsymbol{x}^T + \boldsymbol{A}$ using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             UPLO   N    ALPHA    X  IX  JX    DESC_X   INCX   Y  IY  JY
              |     |      |      |   |   |      |       |     |   |   |
CALL PDSYR2( 'L'  , 9 ,  1.0D0  , X , 1 , 1 ,  DESC_X ,  1   ,  Y , 1 , 1 ,

              DESC_Y   INCY   A  IA  JA   DESC_A
                |       |     |   |   |     |
              DESC_Y ,  1   , A , 1 , 1 , DESC_A  )
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*1	*icontxt*1	*icontxt*1
M_	9	9	9
N_	9	1	1
MB_	4	4	4
NB_	4	1	1
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below2	See below2	See below2

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = 5 on P_{00} and P_{01}, LLD_A = 4 on P_{10} and P_{11}, LLD_X = LLD_Y = 5 on P_{00}, and LLD_X = LLD_Y = 4 on P_{10}.

Global real symmetric matrix ***A*** of order 9 with block size 4 × 4:

```
B,D            0                           1                  2
     ┌                                                           ┐
     |   1.0    .     .     .   |    .     .     .     .   |     .  |
     |   2.0  12.0    .     .   |    .     .     .     .   |     .  |
 0   |   3.0  13.0  23.0    .   |    .     .     .     .   |     .  |
     |   4.0  14.0  24.0  34.0  |    .     .     .     .   |     .  |
     | -------------------------|--------------------------|------- |
     |   5.0  15.0  25.0  35.0  |  45.0    .     .     .   |     .  |
     |   6.0  16.0  26.0  36.0  |  46.0  56.0    .     .   |     .  |
 1   |   7.0  17.0  27.0  37.0  |  47.0  57.0  67.0    .   |     .  |
     |   8.0  18.0  28.0  38.0  |  48.0  58.0  68.0  78.0  |     .  |
     | -------------------------|--------------------------|------- |
 2   |   9.0  19.0  29.0  39.0  |  49.0  59.0  69.0  79.0  |   89.0 |
     └                                                           ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                0               |             1
-----|--------------------------------|--------------------------
     |   1.0    .     .     .     .   |     .     .     .     .
     |   2.0  12.0    .     .     .   |     .     .     .     .
 0   |   3.0  13.0  23.0    .     .   |     .     .     .     .
     |   4.0  14.0  24.0  34.0    .   |     .     .     .     .
     |   9.0  19.0  29.0  39.0  89.0  |   49.0  59.0  69.0  79.0
-----|--------------------------------|--------------------------
     |   5.0  15.0  25.0  35.0    .   |   45.0    .     .     .
     |   6.0  16.0  26.0  36.0    .   |   46.0  56.0    .     .
 1   |   7.0  17.0  27.0  37.0    .   |   47.0  57.0  67.0    .
     |   8.0  18.0  28.0  38.0    .   |   48.0  58.0  68.0  78.0
```

Global vector ***x*** of size 9 × 1 with block size 4:

```
B,D     0
     ┌      ┐
     |  1.0 |
     |  1.0 |
 0   |  1.0 |
     |  1.0 |
     | ---- |
     |  1.0 |
     |  1.0 |
 1   |  1.0 |
     |  1.0 |
     | ---- |
 2   |  1.0 |
     └      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------
     |  1.0
     |  1.0
 0   |  1.0
     |  1.0
     |  1.0
-----|------
     |  1.0
     |  1.0
 1   |  1.0
     |  1.0
```

Global vector ***y*** of size 9 × 1 with block size 4:

```
B,D     0
      ┌      ┐
      |  2.0 |
      |  2.0 |
 0    |  2.0 |
      |  2.0 |
      | ---- |
      |  2.0 |
      |  2.0 |
 1    |  2.0 |
      |  2.0 |
      | ---- |
 2    |  2.0 |
      └      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |  0
-----|------
     |  2.0
     |  2.0
 0   |  2.0
     |  2.0
     |  2.0
-----|------
     |  2.0
     |  2.0
 1   |  2.0
     |  2.0
```

Output:

Global real symmetric matrix ***A*** of order 9 with block size 4 × 4:

```
B,D            0                          1                2
     ┌                                                             ┐
     |   5.0    .     .     .  |    .     .     .     .  |      .  |
     |   6.0  16.0    .     .  |    .     .     .     .  |      .  |
 0   |   7.0  17.0  27.0    .  |    .     .     .     .  |      .  |
     |   8.0  18.0  28.0  38.0 |    .     .     .     .  |      .  |
     |  -----------------------|-------------------------|-------  |
     |   9.0  19.0  29.0  39.0 |  49.0    .     .     .  |      .  |
     |  10.0  20.0  30.0  40.0 |  50.0  60.0    .     .  |      .  |
 1   |  11.0  21.0  31.0  41.0 |  51.0  61.0  71.0    .  |      .  |
     |  12.0  22.0  32.0  42.0 |  52.0  62.0  72.0  82.0 |      .  |
     |  -----------------------|-------------------------|-------  |
 2   |  13.0  23.0  33.0  43.0 |  53.0  63.0  73.0  83.0 |   93.0  |
     └                                                             ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |              0                |            1
-----|-------------------------------|--------------------------
     |   5.0    .     .     .     .  |    .     .     .     .
     |   6.0  16.0    .     .     .  |    .     .     .     .
 0   |   7.0  17.0  27.0    .     .  |    .     .     .     .
     |   8.0  18.0  28.0  38.0    .  |    .     .     .     .
     |  13.0  23.0  33.0  43.0  93.0 |  53.0  63.0  73.0  83.0
-----|-------------------------------|--------------------------
     |   9.0  19.0  29.0  39.0    .  |  49.0    .     .     .
     |  10.0  20.0  30.0  40.0    .  |  50.0  60.0    .     .
 1   |  11.0  21.0  31.0  41.0    .  |  51.0  61.0  71.0    .
     |  12.0  22.0  32.0  42.0    .  |  52.0  62.0  72.0  82.0
```

Example 2: This example computes:

$$A \leftarrow A + \alpha x y^{H} + \bar{\alpha} y x^{H}$$

using a 2 × 2 process grid.

Note: The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not have to set these values. On output, they are set to zero except when N is zero or α is zero.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             UPLO   N    ALPHA    X  IX  JX    DESC_X   INCX   Y  IY  JY
              |     |      |      |   |   |      |       |     |   |   |
CALL PZHER2( 'L'  , 3 ,  ALPHA  , X , 1 , 1 ,  DESC_X ,  1  ,  Y , 1 , 1 ,

              DESC_Y   INCY   A  IA  JA   DESC_A
                |       |     |   |   |     |
              DESC_Y ,  1   , A , 1 , 1 , DESC_A  )

              ALPHA = (1.0,0.0)
```

	Desc_A	**Desc_X**	**Desc_Y**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	3	3	3
N_	3	1	1
MB_	2	2	2
NB_	2	1	1
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
LLD_Y = MAX(1,NUMROC(M_Y, MB_Y, MYROW, RSRC_Y, NPROW))
```

In this example, LLD_A = 2 on P_{00} and P_{01}, LLD_A = 1 on P_{10} and P_{11}, LLD_X = LLD_Y = 2 on P_{00}, and LLD_X = LLD_Y = 1 on P_{10}.

Global complex Hermitian matrix ***A*** of order 3 with block size 2 × 2:

```
B,D               0                    1
     ┌                                          ┐
     | ( 1.0, 0.0)      .       |      .      |
 0   | ( 3.0,-5.0) ( 7.0, 0.0) |      .      |
     | ------------------------|------------ |
 1   | ( 2.0, 3.0) ( 4.0, 8.0) | ( 6.0, 0.0) |
     └                                          ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |           0             |      1
-----|-------------------------|-------------
     | ( 1.0,  . )      .      |      .
 0   | ( 3.0,-5.0) ( 7.0,  . ) |      .
-----|-------------------------|-------------
 1   | ( 2.0, 3.0) ( 4.0, 8.0) | ( 6.0,  . )
```

Global vector ***x*** of size 3 × 1 with block size 2:

```
B,D     0
     ┌             ┐
     | ( 1.0, 2.0) |
 0   | ( 4.0, 0.0) |
     | ----------- |
 1   | ( 3.0, 4.0) |
     └             ┘
```

The following is the 2 × 1 process grid:

B,D	**0**	**—**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |      0
-----|-------------
     | ( 1.0, 2.0)
 0   | ( 4.0, 0.0)
-----|-------------
 1   | ( 3.0, 4.0)
```

Global vector ***y*** of size 3 × 1 with block size 2:

```
B,D        0
     ┌             ┐
     | ( 1.0, 0.0) |
 0   | ( 2.0,-1.0) |
     | ----------- |
 1   | ( 2.0, 1.0) |
     └             ┘
```

The following is the 2 × 1 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***y***:

```
p,q  |      0
-----|-------------
     | ( 1.0, 0.0)
 0   | ( 2.0,-1.0)
-----|-------------
 1   | ( 2.0, 1.0)
```

Output:

Global complex Hermitian matrix ***A*** of order 3 with block size 2 × 2:

```
B,D                0                        1
     ┌                                              ┐
     | (  3.0,  0.0)       .        |       .       |
 0   | (  7.0,-10.0) ( 23.0,  0.0)  |       .       |
     | ---------------------------- |-------------- |
 1   | (  9.0,  4.0) ( 14.0, 23.0)  | ( 26.0,  0.0) |
     └                                              ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |              0               |       1
-----|------------------------------|---------------
     | (  3.0,  0.0)       .        |       .
 0   | (  7.0,-10.0) ( 23.0,  0.0)  |       .
-----|------------------------------|---------------
 1   | (  9.0,  4.0) ( 14.0, 23.0)  | ( 26.0,  0.0)
```

PDTRMV and PZTRMV—Matrix-Vector Product for a Triangular Matrix or Its Transpose

PDTRMV computes one of the following matrix-vector products:

1. $\boldsymbol{x} \leftarrow \boldsymbol{Ax}$
2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$

| PZTRMV computes one of the following matrix-vector products:

| 1. $\boldsymbol{x} \leftarrow \boldsymbol{Ax}$
| 2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{T}\boldsymbol{x}$
| 3. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{H}\boldsymbol{x}$

where, in the formulas above:

A represents the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
x represents the global vector:
- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.

| **Note:** No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

If *n* = 0, no computation is performed and the subroutine returns after doing some parameter checking. See references [14] and [15].

Table 44. Data Types

A, ***x***	**Subprogram**
Long-precision real	PDTRMV
Long-precision complex	PZTRMV

Syntax

Fortran	CALL PDTRMV | PZTRMV (*uplo*, *transa*, *diag*, *n*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*)
C and C++	pdtrmv | pztrmv (*uplo*, *transa*, *diag*, *n*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global triangular submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

transa
indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^{T}$ is used in the computation.

| If *transa* = `'C'`, $\boldsymbol{A}^{H}$ is used in the computation.

Scope: **global**

| Specified as: a single character; *transa* = 'N', 'T', or 'C'.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = 'U', ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Scope: **global**

Specified as: a single character; *diag* = 'U' or 'N'.

n

is the order of global triangular submatrix ***A*** and the length of global vector ***x***.

Scope: **global**

Specified as: a fullword integer; $n \ge 0$.

a

is the local part of the global triangular matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ia+n-1$) by LOCq($ja+n-1$) part of the local array A must contain the local pieces of the leading $ia+n-1$ by $ja+n-1$ part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading lower triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

| **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 44 on page 252. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia$ and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja$ and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A $\ge$ 0 Otherwise: M_A $\ge$ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\ge$ 0 Otherwise: M_A $\ge$ 1	Global
5	MB_A	Row block size	MB_A $\ge$ 1	Global
6	NB_A	Column block size	NB_A $\ge$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

x

is the local part of the global matrix ***X***. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*n*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*n*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*n*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*n*–1 by *jx* part of the global matrix.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 44 on page 252. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then *ix*+*n*−1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jx* ≤ N_X and:

If *incx* = M_X, then *jx*+*n*−1 ≤ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If *n* = 0: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If *n* = 0: N_X ≥ 0 Otherwise: M_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global
7	RSRC_X	The process row of the *p* × *q* grid over which the first row of the global matrix is distributed	0 ≤ RSRC_X < *p*	Global
8	CSRC_X	The process column of the *p* × *q* grid over which the first column of the global matrix is distributed	0 ≤ CSRC_X < *q*	Global
9	LLD_X	The leading dimension of the local array	LLD_X ≥ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

On Return

x

is the updated local part of the global matrix ***X***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 44 on page 252.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo*, *transa*, and *diag* arguments.
2. For PDTRMV, if you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
3. The matrix and vector must have no common elements; otherwise, results are unpredictable.
4. PDTRMV and PZTRMV assume certain values in your array for parts of a triangular matrix. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the strictly lower or upper triangular part, respectively, are assumed to be zero. As a result, you do not have to set these values.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
6. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
7. The following values must be equal: CTXT_A = CTXT_X.
8. The global triangular matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
9. The block row and block column offsets of the global triangular matrix ***A*** must be equal; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
10. If *incx* = M_X:
    - The following block sizes must be equal: NB_X = MB_A = NB_A
    - In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***X***; that is, *iacol* = *ixcol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
    - The block column offset of ***x*** must be equal to the block row and block column offsets of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ja*–1, NB_A) = mod(*ia*–1, MB_A).
11. If *incx* = 1(≠ M_X):
    - The following block sizes must be equal: MB_X = MB_A = NB_A

- In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***X***; that is, *iarow* = *ixrow*, where:

  iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
  ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)

- The block row offset of ***x*** must be equal to the block row and block column offsets of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ `'U'` or `'L'`
2. *transa* ≠ `'N'`, `'T'`, or `'C'`
3. *diag* ≠ `'N'` or `'U'`
4. *n* < 0
5. M_A < 0 and *n* = 0; M_A < 1 otherwise
6. N_A < 0 and *n* = 0; N_A < 1 otherwise
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A ≥ *p*
10. CSRC_A < 0 or CSRC_A ≥ *q*
11. CTXT_A ≠ CTXT_X
12. M_X < 0 and *n* = 0; M_X < 1 otherwise
13. N_X < 0 and *n* = 0; N_X < 1 otherwise
14. MB_X < 1
15. NB_X < 1
16. RSRC_X < 0 or RSRC_X ≥ *p*
17. CSRC_X < 0 or CSRC_X ≥ *q*

Stage 5

1. MB_A = NB_A
2. mod(*ia*–1, MB_A) ≠ mod(*ja*–1, NB_A)

If *n* ≠ 0:

3. *ix* > M_X
4. *jx* > N_X
5. *ia* > M_A
6. *ja* > N_A
7. *ia*+*n*–1 > M_A
8. *ja*+*n*–1 > N_A

If *incx* = M_X:

9. NB_A ≠ NB_X
10. mod(*jx*–1, NB_X) ≠ mod(*ja*–1, NB_A)
11. *n* ≠ 0 and *jx*+*n*–1 > N_X

If *incx* = 1(≠ M_X):

12. MB_A ≠ MB_X
13. mod(*ix*–1, MB_X) ≠ mod(*ia*–1, MB_A)
14. *n* ≠ 0 and *ix*+*n*–1 > M_X

Otherwise:

15. *incx* ≠ 1 and *incx* ≠ M_X

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_X < max(1, LOCp(M_X))
3. If *incx* = M_X, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***X***; that is, *iacol* ≠ *ixcol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
4. If *incx* = 1(≠ M_X), then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***X***; that is, *iarow* ≠ *ixrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)

Example 1: This example computes ***x*** = ***Ax*** using a 2 × 2 process grid. It uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2. It uses vector ***x***, which is a column-distributed vector within a column of ***X***, by specifying *incx* = 1, *ix* = 2, and *jx* = 1.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                UPLO  TRANSA DIAG  N    A  IA  JA   DESC_A   X  IX  JX
                  |     |     |    |    |   |   |     |      |   |   |
CALL PDTRMV( 'U' , 'N' , 'N' , 12 , A , 2 , 2 , DESC_A , X , 2 , 1 ,

                   DESC_X INCX
                     |     |
                   DESC_X , 1 )
```

	Desc_A	**Desc_X**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	13	13
N_	13	1
MB_	3	3
NB_	3	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
```

In this example, LLD_A = 7 on P_{00} and P_{01}, LLD_A = 6 on P_{10} and P_{11}, LLD_X = 7 on P_{00}, and LLD_X = 6 on P_{10}.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global submatrix ***A*** of order 12, starting at row 2 and column 2 in global triangular matrix ***A*** of order 13 with block size 3 × 3:

```
B,D         0                   1                   2                   3              4
     ┌                                                                                     ┐
     │   .    .    .   │   .    .    .   │   .    .    .   │   .    .    .   │    .  │
 0   │   .   1.0  2.0  │  1.0  2.0  1.0  │  1.0  3.0  1.0  │  1.0  2.0  3.0  │   2.0 │
     │   .    .   3.0  │  2.0  3.0  1.0  │  2.0  3.0  1.0  │  1.0  2.0  3.0  │   3.0 │
     │ ----------------|-----------------|-----------------|-----------------|------ │
     │   .    .    .   │  3.0  1.0  3.0  │  2.0  1.0  2.0  │  1.0  2.0  3.0  │   1.0 │
 1   │   .    .    .   │   .   1.0  2.0  │  2.0  1.0  1.0  │  1.0  2.0  3.0  │   2.0 │
     │   .    .    .   │   .    .   2.0  │  1.0  2.0  2.0  │  1.0  2.0  3.0  │   3.0 │
     │ ----------------|-----------------|-----------------|-----------------|------ │
     │   .    .    .   │   .    .    .   │  1.0  2.0  1.0  │  1.0  2.0  3.0  │   1.0 │
 2   │   .    .    .   │   .    .    .   │   .   2.0  1.0  │  1.0  2.0  3.0  │   2.0 │
     │   .    .    .   │   .    .    .   │   .    .   2.0  │  1.0  2.0  3.0  │   3.0 │
     │ ----------------|-----------------|-----------------|-----------------|------ │
     │   .    .    .   │   .    .    .   │   .    .    .   │  3.0  1.0  3.0  │   1.0 │
 3   │   .    .    .   │   .    .    .   │   .    .    .   │   .   2.0  2.0  │   2.0 │
     │   .    .    .   │   .    .    .   │   .    .    .   │   .    .   1.0  │   3.0 │
     │ ----------------|-----------------|-----------------|-----------------|------ │
 4   │   .    .    .   │   .    .    .   │   .    .    .   │   .    .    .   │   1.0 │
     └                                                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2 4**	**1 3**
0 **2** **4**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                 0                  |                1
-----|------------------------------------|-------------------------------
     |   .    .    .    .    .    .    .  |    .    .    .    .    .    .
     |   .   1.0  2.0  1.0  3.0  1.0  2.0 |   1.0  2.0  1.0  1.0  2.0  3.0
     |   .    .   3.0  2.0  3.0  1.0  3.0 |   2.0  3.0  1.0  1.0  2.0  3.0
 0   |   .    .    .   1.0  2.0  1.0  1.0 |    .    .    .   1.0  2.0  3.0
     |   .    .    .    .   2.0  1.0  2.0 |    .    .    .   1.0  2.0  3.0
     |   .    .    .    .    .   2.0  3.0 |    .    .    .   1.0  2.0  3.0
     |   .    .    .    .    .    .   1.0 |    .    .    .    .    .    .
-----|------------------------------------|-------------------------------
     |   .    .    .   2.0  1.0  2.0  1.0 |   3.0  1.0  3.0  1.0  2.0  3.0
     |   .    .    .   2.0  1.0  1.0  2.0 |    .   1.0  2.0  1.0  2.0  3.0
     |   .    .    .   1.0  2.0  2.0  3.0 |    .    .   2.0  1.0  2.0  3.0
 1   |   .    .    .    .    .    .   1.0 |    .    .    .   3.0  1.0  3.0
     |   .    .    .    .    .    .   2.0 |    .    .    .    .   2.0  2.0
     |   .    .    .    .    .    .   3.0 |    .    .    .    .    .   1.0
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 12 × 1, starting at row 2 in 13 × 1 global matrix ***X*** with block size 3 × 1:

```
B,D     0
      ┌      ┐
      |   .  |
 0    |  2.0 |
      |  3.0 |
      | ---- |
      |  1.0 |
 1    |  2.0 |
      |  3.0 |
      | ---- |
      |  1.0 |
 2    |  2.0 |
      |  3.0 |
      | ---- |
      |  1.0 |
 3    |  2.0 |
      |  3.0 |
      | ---- |
 4    |  1.0 |
      └      ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2 4	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |  0
-----|------
     |   .
     |  2.0
     |  3.0
 0   |  1.0
     |  2.0
     |  3.0
     |  1.0
-----|------
     |  1.0
     |  2.0
     |  3.0
 1   |  1.0
     |  2.0
     |  3.0
```

Output:

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a column-distributed vector. Following is the global vector ***x*** of size 12 × 1, starting at row 2 in 13 × 1 global matrix ***X*** with block size 3 × 1:

```
B,D      0
     ┌        ┐
     |     .  |
 0   |  42.0  |
     |  48.0  |
     |  ----- |
     |  39.0  |
 1   |  31.0  |
     |  34.0  |
     |  ----- |
     |  23.0  |
 2   |  23.0  |
     |  23.0  |
     |  ----- |
     |  15.0  |
 3   |  12.0  |
     |   6.0  |
     |  ----- |
 4   |   1.0  |
     └        ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2** **4**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |   0
-----|-------
     |    .
     |  42.0
     |  48.0
 0   |  23.0
     |  23.0
     |  23.0
     |   1.0
-----|-------
     |  39.0
     |  31.0
     |  34.0
 1   |  15.0
     |  12.0
     |   6.0
```

Example 2: This example computes ***x*** = ***Ax*** using a 2 × 2 process grid.

Note: For unit triangular matrices, the elements of the diagonal are assumed to be one, so you do not have to set these values.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO  TRANSA DIAG  N   A  IA  JA   DESC_A   X  IX  JX
             |     |     |     |   |   |   |     |      |   |   |
CALL PZTRMV( 'L' , 'N' , 'U' , 4 , A , 1 , 1 , DESC_A , X , 1 , 1 ,

              DESC_X INCX
                |     |
              DESC_X , 1 )
```

	Desc_A	**Desc_X**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	4	4
N_	4	1

	Desc_A	Desc_X
MB_	2	2
NB_	2	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
```

In this example, LLD_A = 2 on P_{00} and P_{01}, LLD_A = 2 on P_{10} and P_{11}, LLD_X = 2 on P_{00}, and LLD_X = 2 on P_{10}.

Global triangular matrix ***A*** of order 4 with block size 2 × 2:

```
B,D            0                   1

       ┌                                        ┐
       | (1.0,0.0)     .     |     .         .    |
 0     | (1.0,1.0) (1.0,0.0) |     .         .    |
       | --------------------|--------------------|
       | (1.0,1.0) (3.0,3.0) | (1.0,0.0)     .    |
 1     | (3.0,3.0) (4.0,4.0) | (3.0,3.0) (1.0,0.0) |
       └                                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |          0          |          1
-----|---------------------|--------------------
     |     .         .     |     .         .
 0   | (1.0,1.0)     .     |     .         .
-----|---------------------|---------------------
     | (1.0,1.0) (3.0,3.0) |     .         .
 1   | (3.0,3.0) (4.0,4.0) | (3.0,3.0)     .
```

Global vector ***x*** of size 4 × 1 with block size 2:

```
B,D        0

       ┌           ┐
       | (1.0,1.0) |
 0     | (2.0,2.0) |
       | --------- |
       | (3.0,3.0) |
 1     | (4.0,4.0) |
       └           ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |     0
-----|-----------
     | (1.0,1.0)
 0   | (2.0,2.0)
-----|-----------
     | (3.0,3.0)
 1   | (4.0,4.0)
```

Output:

Global vector ***x*** of size 4 × 1 with block size 2:

```
B,D        0
     ┌             ┐
     | ( 1.0, 1.0) |
 0   | ( 2.0, 4.0) |
     | ----------- |
     | ( 3.0,17.0) |
 1   | ( 4.0,44.0) |
     └             ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |      0
-----|-------------
     | ( 1.0, 1.0)
 0   | ( 2.0, 4.0)
-----|-------------
     | ( 3.0,17.0)
 1   | ( 4.0,44.0)
```

PDTRSV and PZTRSV—Solution of Triangular System of Equations with a Single Right-Hand Side

PDTRSV performs one of the following solves for a triangular system of equations with a single right-hand side:

Solution	Equation
1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$	$\boldsymbol{Ax} = \boldsymbol{b}$
2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$	$\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$

| PZTRSV performs one of the following solves for a triangular system of equations
| with a single right-hand side:

Solution	Equation
1. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$	$\boldsymbol{Ax} = \boldsymbol{b}$
2. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-T}\boldsymbol{x}$	$\boldsymbol{A}^{T}\boldsymbol{x} = \boldsymbol{b}$
3. $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-H}\boldsymbol{x}$	$\boldsymbol{A}^{H}\boldsymbol{x} = \boldsymbol{b}$

where, in the formulas above:

A represents the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
x represents the global vector:

- For *incx* = M_X, it is $\boldsymbol{X}_{ix:ix,\ jx:jx+n-1}$.
- For *incx* = 1 and *incx* ≠ M_X, it is $\boldsymbol{X}_{ix:ix+n-1,\ jx:jx}$.

Notes:

1. The term ***b*** used in the systems of equations listed above represents the right-hand side of the system. It is important to note that in these subroutines the right-hand side of the equation is actually provided in the input-output argument *x*.
2. No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix ***A*** should always be stored in its untransposed form.

If *n* = 0, no computation is performed and the subroutine returns after doing some parameter checking. See references [14] and [15].

Table 45. Data Types

A, ***x***	**Subprogram**
Long-precision real	PDTRSV
Long-precision complex	PZTRSV

Syntax

Fortran	CALL PDTRSV | PZTRSV (*uplo*, *transa*, *diag*, *n*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*)
C and C++	pdtrsv | pztrsv (*uplo*, *transa*, *diag*, *n*, *a*, *ia*, *ja*, *desc_a*, *x*, *ix*, *jx*, *desc_x*, *incx*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global triangular submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

transa

indicates the form of matrix ***A*** used in the system of equations, where:

If *transa* = `'N'`, ***A*** is used in the system of equations.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the system of equations.

| If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the system of equations.

Scope: **global**

| Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Scope: **global**

Specified as: a single character; *diag* = `'U'` or `'N'`.

n

is the order of global triangular submatrix ***A*** and the length of global vector ***x***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

a

is the local part of the global triangular matrix ***A***, used in the system of equations. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*−1) by LOCq(*ja*+*n*−1) part of the local array `A` must contain the local pieces of the leading *ia*+*n*−1 by *ja*+*n*−1 part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading lower triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

| **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 45 on page 265. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia$ and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja$ and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A $\ge$ 0 Otherwise: M_A $\ge$ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\ge$ 0 Otherwise: M_A $\ge$ 1	Global
5	MB_A	Row block size	MB_A $\ge$ 1	Global
6	NB_A	Column block size	NB_A $\ge$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

x

is the local part of the global matrix ***X***, containing the right-hand side of the triangular system to be solved. This identifies the **first element** of the local array X. This subroutine computes the location of the first element of the local subarray used, based on *ix*, *jx*, *desc_x*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *incx* = M_X, the leading LOCp(*ix*) by LOCq(*jx*+*n*–1) part of the local array X must contain the local pieces of the leading *ix* by *jx*+*n*–1 part of the global matrix.
- If *incx* = 1 and *incx* ≠ M_X, the leading LOCp(*ix*+*n*–1) by LOCq(*jx*) part of the local array X must contain the local pieces of the leading *ix*+*n*–1 by *jx* part of the global matrix.

Scope: **local**

Specified as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 45 on page 265. Details about the block-cyclic data distribution of the global matrix ***X*** are stored in *desc_x*.

ix

has the following meaning:

If *incx* = M_X, it indicates which row of global matrix ***X*** is used for vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it is the row index of global matrix ***X***, identifying the first element of vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ix* ≤ M_X and:

If *incx* = 1 and *incx* ≠ M_X, then *ix*+*n*–1 ≤ M_X.

jx

has the following meaning:

If *incx* = M_X, it is the column index of global matrix ***X***, identifying the first element of vector ***x***.

If *incx* = 1 and *incx* ≠ M_X, it indicates which column of global matrix ***X*** is used for vector ***x***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *jx* ≤ N_X and:

If *incx* = M_X, then *jx*+*n*–1 ≤ N_X.

desc_x

is the array descriptor for global matrix ***X***, described in the following table:

desc_x	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_X	Descriptor type	DTYPE_X=1	Global
2	CTXT_X	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_X	Number of rows in the global matrix	If *n* = 0: M_X ≥ 0 Otherwise: M_X ≥ 1	Global
4	N_X	Number of columns in the global matrix	If *n* = 0: N_X ≥ 0 Otherwise: M_X ≥ 1	Global
5	MB_X	Row block size	MB_X ≥ 1	Global
6	NB_X	Column block size	NB_X ≥ 1	Global
7	RSRC_X	The process row of the *p* × *q* grid over which the first row of the global matrix is distributed	0 ≤ RSRC_X < *p*	Global
8	CSRC_X	The process column of the *p* × *q* grid over which the first column of the global matrix is distributed	0 ≤ CSRC_X < *q*	Global
9	LLD_X	The leading dimension of the local array	LLD_X ≥ max(1,LOCp(M_X))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

incx

is the stride for global vector ***x***.

Scope: **global**

Specified as: a fullword integer; *incx* = 1 or *incx* = M_X, where:

If *incx* = M_X, then ***x*** is a row-distributed vector.

If *incx* = 1 and *incx* ≠ M_X, then ***x*** is a column-distributed vector.

On Return

x

is the updated local part of the global matrix ***X***, containing the solution vector.

Scope: **local**

Returned as: an LLD_X by (at least) LOCq(N_X) array, containing numbers of the data type indicated in Table 45 on page 265.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo*, *transa*, and *diag* arguments.
2. For PDTRSV, if you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
3. The matrix and vector must have no common elements; otherwise, results are unpredictable.
4. PDTRSV and PZTRSV assume certain values in your array for parts of a triangular matrix. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the strictly lower or upper triangular part, respectively, are assumed to be zero. As a result, you do not have to set these values.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
6. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
7. The following values must be equal: CTXT_A = CTXT_X.
8. The global triangular matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
9. The block row and block column offsets of the global triangular matrix ***A*** must be equal; that is, mod(*ia*–1,MB_A) = mod(*ja*–1,NB_A).
10. If *incx* = M_X:
    - The following block sizes must be equal: NB_X = MB_A = NB_A
    - If *transa* = `'T'`, then (in the process grid) the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***X***; that is, *iacol* = *ixcol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)

- The block column offset of ***x*** must be equal to the block row and block column offsets of ***A***; that is, mod(*jx*–1, NB_X) = mod(*ja*–1, NB_A) = mod(*ia*–1, MB_A).

11. If *incx* = 1(≠ M_X):
    - The following block sizes must be equal: MB_X = MB_A = NB_A
    - If *transa* = `'N'`, then (in the process grid) the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***X***; that is, *iarow* = *ixrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    - The block row offset of ***x*** must be equal to the block row and block column offsets of ***A***; that is, mod(*ix*–1, MB_X) = mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_X is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ `'U'` or `'L'`
2. *transa* ≠ `'N'`, `'T'`, or `'C'`
3. *diag* ≠ `'N'` or `'U'`
4. $n < 0$
5. M_A < 0 and *n* = 0; M_A < 1 otherwise
6. N_A < 0 and *n* = 0; N_A < 1 otherwise
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A ≥ *p*
10. CSRC_A < 0 or CSRC_A ≥ *q*
11. CTXT_A ≠ CTXT_X
12. M_X < 0 and *n* = 0; M_X < 1 otherwise
13. N_X < 0 and *n* = 0; N_X < 1 otherwise
14. MB_X < 1
15. NB_X < 1
16. RSRC_X < 0 or RSRC_X ≥ *p*
17. CSRC_X < 0 or CSRC_X ≥ *q*

Stage 5

1. MB_A = NB_A
2. mod(*ia*–1, MB_A) ≠ mod(*ja*–1, NB_A)

If $n \neq 0$:

3. *ix* > M_X
4. *jx* > N_X
5. *ia* > M_A
6. *ja* > N_A
7. *ia*+*n*–1 > M_A
8. *ja*+*n*–1 > N_A

If *incx* = M_X:

9. NB_A ≠ NB_X
10. mod(*jx*–1, NB_X) ≠ mod(*ja*–1, NB_A)
11. *n* ≠ 0 and *jx*+*n*–1 > N_X

If *incx* = 1(≠ M_X):

12. MB_A ≠ MB_X
13. mod(*ix*–1, MB_X) ≠ mod(*ia*–1, MB_A)
14. *n* ≠ 0 and *ix*+*n*–1 > M_X

Otherwise:

15. *incx* ≠ 1 and *incx* ≠ M_X

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_X < max(1, LOCp(M_X))
3. If *incx* = M_X and *transa* = `'T'`, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***X***; that is, *iacol* ≠ *ixcol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)
4. If *incx* = 1(≠ M_X) and *transa* = `'N'`, then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***X***; that is, *iarow* ≠ *ixrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)

Example 1: This example solves $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ using a 2 × 2 process grid, where ***A*** is a unit triangular matrix. It uses a global submatrix ***A*** within a global matrix ***A*** by specifying *ia* = 2 and *ja* = 2. It uses vector ***x***, which is a row-distributed vector within a row of global matrix ***X***, by specifying *incx* = M_X = 1, *ix* = 1, and *jx* = 2.

Note: Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. This subroutine assumes a value of 1.0 for the diagonal elements.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO  TRANSA DIAG   N    A  IA  JA   DESC_A   X  IX  JX
              |     |     |     |    |   |   |     |      |   |   |
CALL PDTRSV( 'L' , 'N'  , 'U' , 12 , A , 2 , 2 , DESC_A , X , 1 , 2 ,

            DESC_X INCX
               |     |
            DESC_X , 1 )
```

	Desc_A	Desc_X
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	13	1
N_	13	13
MB_	3	1
NB_	3	3
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
```

In this example, LLD_A = 7 on P_{00} and P_{01}, LLD_A = 6 on P_{10} and P_{11}, and LLD_X = 1 on all processes.

After the global matrix ***A*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***A***. Following is the global submatrix ***A*** of order 12, starting at row 2 and column 2 in global triangular matrix ***A*** of order 13 with block size 3 × 3:

```
B,D        0              1              2              3          4
    ┌                                                                 ┐
    │ .   .    .   │  .    .    .  │  .    .    .  │  .    .    .  │   .  │
 0  │ .  1.0   .   │  .    .    .  │  .    .    .  │  .    .    .  │   .  │
    │ .  2.0  1.0  │  .    .    .  │  .    .    .  │  .    .    .  │   .  │
    │ -------------|---------------|---------------|---------------|----- │
    │ .  3.0  2.0  │ 1.0   .    .  │  .    .    .  │  .    .    .  │   .  │
 1  │ .  1.0  3.0  │ 2.0  1.0   .  │  .    .    .  │  .    .    .  │   .  │
    │ .  2.0  1.0  │ 3.0  2.0  1.0 │  .    .    .  │  .    .    .  │   .  │
    │ -------------|---------------|---------------|---------------|----- │
    │ .  3.0  2.0  │ 1.0  3.0  2.0 │ 1.0   .    .  │  .    .    .  │   .  │
 2  │ .  1.0  3.0  │ 2.0  1.0  3.0 │ 2.0  1.0   .  │  .    .    .  │   .  │
    │ .  2.0  1.0  │ 3.0  2.0  1.0 │ 3.0  2.0  1.0 │  .    .    .  │   .  │
    │ -------------|---------------|---------------|---------------|----- │
    │ .  3.0  2.0  │ 1.0  3.0  2.0 │ 1.0  3.0  2.0 │ 1.0   .    .  │   .  │
 3  │ .  1.0  3.0  │ 2.0  1.0  3.0 │ 2.0  1.0  3.0 │ 2.0  1.0   .  │   .  │
    │ .  2.0  1.0  │ 3.0  2.0  1.0 │ 3.0  2.0  1.0 │ 3.0  2.0  1.0 │   .  │
    │ -------------|---------------|---------------|---------------|----- │
 4  │ .  3.0  2.0  │ 1.0  3.0  2.0 │ 1.0  3.0  2.0 │ 1.0  3.0  2.0 │  1.0 │
    └                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0 2 4	1 3
0 2 4	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                 0                  |                1
-----|------------------------------------|-------------------------------
     |  .   .    .    .    .    .    .    |  .    .    .    .    .    .
     |  .   .    .    .    .    .    .    |  .    .    .    .    .    .
     |  .  2.0   .    .    .    .    .    |  .    .    .    .    .    .
 0   |  .  3.0  2.0   .    .    .    .    | 1.0  3.0  2.0   .    .    .
     |  .  1.0  3.0  2.0   .    .    .    | 2.0  1.0  3.0   .    .    .
     |  .  2.0  1.0  3.0  2.0   .    .    | 3.0  2.0  1.0   .    .    .
     |  .  3.0  2.0  1.0  3.0  2.0   .    | 1.0  3.0  2.0  1.0  3.0  2.0
-----|------------------------------------|-------------------------------
     |  .  3.0  2.0   .    .    .    .    |  .    .    .    .    .    .
     |  .  1.0  3.0   .    .    .    .    | 2.0   .    .    .    .    .
     |  .  2.0  1.0   .    .    .    .    | 3.0  2.0   .    .    .    .
 1   |  .  3.0  2.0  1.0  3.0  2.0   .    | 1.0  3.0  2.0   .    .    .
     |  .  1.0  3.0  2.0  1.0  3.0   .    | 2.0  1.0  3.0  2.0   .    .
     |  .  2.0  1.0  3.0  2.0  1.0   .    | 3.0  2.0  1.0  3.0  2.0   .
```

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a row-distributed vector. Following is the global vector ***x*** of size 1 × 12, starting at row 1 and column 2 in 1 × 13 global matrix ***X*** with block size 1 × 3:

```
B,D        0                1                2                3          4
       ┌                                                                      ┐
 0     │  .   2.0  7.0 │ 13.0 15.0 17.0 │ 26.0 28.0 27.0 │ 39.0 41.0 37.0 │ 52.0 │
       └                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2 4	1 3
0	P_{00}	P_{01}
—	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  │                0                  │                 1
-----|-----------------------------------|-------------------------------
 0   │  .   2.0  7.0 26.0 28.0 27.0 52.0 │  13.0 15.0 17.0 39.0 41.0 37.0
```

Output:

After the global matrix ***X*** is distributed over the process grid, only a portion of the global data structure is used—that is, global vector ***x***, which is a row-distributed vector. Following is the global vector ***x*** of size 1 × 12, starting at row 1 and column 2 in 1 × 13 global matrix ***X*** with block size 1 × 3:

```
B,D        0                1                2                3          4
       ┌                                                                      ┐
 0     │  .   2.0  3.0 │  1.0  2.0  3.0 │  1.0  2.0  3.0 │  1.0  2.0  3.0 │  1.0 │
       └                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2 4	1 3
0	P_{00}	P_{01}
—	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  │                0                  │                 1
-----|-----------------------------------|-------------------------------
 0   │  .   2.0  3.0  1.0  2.0  3.0  1.0 │  1.0  2.0  3.0  1.0  2.0  3.0
```

| ***Example 2:*** This example solves $\boldsymbol{x} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{x}$ using a 2 × 2 process grid, where ***A*** is a unit triangular matrix.

| **Note:** Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. This subroutine assumes a value of (1.0,0.0) for the diagonal elements.

| *Call Statements and Input*

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                UPLO  TRANSA DIAG   N   A  IA  JA   DESC_A   X  IX  JX
                  |      |      |    |   |   |   |     |      |   |   |
CALL PZTRSV( 'L' , 'N'  , 'U' , 4 , A , 1 , 1 , DESC_A , X , 1 , 1 ,

                DESC_X INCX
                   |     |
                DESC_X , 1 )
```

	Desc_A	Desc_X
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	4	4
N_	4	1
MB_	2	2
NB_	2	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_X = MAX(1,NUMROC(M_X, MB_X, MYROW, RSRC_X, NPROW))
```

In this example, LLD_A = 2 on P_{00} and P_{01}, LLD_A = 2 on P_{10} and P_{11}, and LLD_X = 2 on P_{00} and P_{10}.

Global triangular matrix ***A*** of order 4 with block size 2 × 2:

```
B,D           0                   1
      ┌                                          ┐
      | (1.0,0.0)     .     |     .         .    |
 0    | (1.0,1.0) (1.0,0.0) |     .         .    |
      | --------------------|------------------- |
      | (1.0,1.0) (3.0,3.0) | (1.0,0.0)     .    |
 1    | (3.0,3.0) (4.0,4.0) | (3.0,3.0) (1.0,0.0)|
      └                                          ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |         0          |        1
-----|--------------------|-------------------
     |     .        .     |     .        .
 0   | (1.0,1.0)    .     |     .        .
-----|--------------------|--------------------
     | (1.0,1.0) (3.0,3.0) |     .        .
 1   | (3.0,3.0) (4.0,4.0) | (3.0,3.0)    .
```

Global vector ***x*** of size 4 × 1 with block size 2:

```
B,D        0
     ┌             ┐
     | ( 1.0, 1.0) |
 0   | ( 2.0, 4.0) |
     | ----------- |
     | ( 3.0,17.0) |
 1   | ( 4.0,44.0) |
     └             ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
p,q  |      0
-----|-------------
     | ( 1.0, 1.0)
 0   | ( 2.0, 4.0)
-----|-------------
     | ( 3.0,17.0)
 1   | ( 4.0,44.0)
```

Output:

Global vector ***x*** of size 4 × 1 with block size 2:

```
B,D       0
     ┌           ┐
     | (1.0,1.0) |
 0   | (2.0,2.0) |
     | --------- |
     | (3.0,3.0) |
 1   | (4.0,4.0) |
     └           ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***x***:

```
|                          p,q  |     0
|                          -----|-----------
|                               | (1.0,1.0)
|                           0   | (2.0,2.0)
|                          -----|-----------
|                               | (3.0,3.0)
|                           1   | (4.0,4.0)
```

Chapter 7. Level 3 PBLAS (Message Passing)

This chapter describes the Level 3 PBLAS subroutines.

Overview of the Level 3 PBLAS Subroutines

The Level 3 PBLAS include a subset of the standard set of distributed memory parallel versions of the Level 3 BLAS.

Note: These subroutines are designed in accordance with the proposed Level 3 PBLAS standard. (See references [14], [15], and [17].) If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 46. List of Level 3 PBLAS (Message Passing)

Descriptive Name	Long-Precision Subprogram	Page
Matrix-Matrix Product for a General Matrix, Its Transpose, or Its Conjugate Transpose	PDGEMM PZGEMM	281
Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian	PDSYMM PZSYMM PZHEMM	299
Triangular Matrix-Matrix Product	PDTRMM PZTRMM	321
Solution of Triangular System of Equations with Multiple Right-Hand Sides	PDTRSM PZTRSM	335
Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix	PDSYRK PZSYRK PZHERK	349
Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix	PDSYR2K PZSYR2K PZHER2K	365
Matrix Transpose for a General Matrix	PDTRAN PZTRANC PZTRANU	386

Level 3 PBLAS Subroutines

This section contains the Level 3 PBLAS subroutine descriptions.

PDGEMM and PZGEMM—Matrix-Matrix Product for a General Matrix, Its Transpose, or Its Conjugate Transpose

PDGEMM performs any one of the following combined matrix computations:

$\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}^{T}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}^{T}+\beta\boldsymbol{C}$

PZGEMM performs any one of the following combined matrix computations:

$\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}^{T}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}^{T}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{H}\boldsymbol{B}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{H}\boldsymbol{B}^{T}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}^{H}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}^{H}+\beta\boldsymbol{C}$
$\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{H}\boldsymbol{B}^{H}+\beta\boldsymbol{C}$

where, in the PDGEMM and PZGEMM formulas above:

A represents the global general submatrix:
- For *transa* = 'N', it is $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+k-1}$.
- For *transa* = 'T' or 'C', it is $\boldsymbol{A}_{ia:ia+k-1,\ ja:ja+m-1}$.

B represents the global general submatrix:
- For *transb* = 'N', it is $\boldsymbol{B}_{ib:ib+k-1,\ jb:jb+n-1}$.
- For *transb* = 'T' or 'C', it is $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+k-1}$.

C represents the global general submatrix $\boldsymbol{C}_{ic:ic+m-1,\ jc:jc+n-1}$.
α and β are scalars.

Note: No data should be moved to form $\boldsymbol{A}^{T}$, $\boldsymbol{A}^{H}$, $\boldsymbol{B}^{T}$, or $\boldsymbol{B}^{H}$; that is, the ***A*** and ***B*** matrices should always be stored in their untransposed forms.

In the following four cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $m = 0$
- $n = 0$
- α is zero and β is one.
- $k = 0$ and β is one.

Assuming the above conditions do not exist, if β is not one and k is 0, then $\beta\boldsymbol{C}$ is returned.

See references [14] and [15].

Table 47. Data Types

A, ***B***, ***C***, α, β	**Subroutine**
Long-precision real	PDGEMM
Long-precision complex	PZGEMM

Syntax

Fortran	CALL PDGEMM | PZGEMM (*transa*, *transb*, *m*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*)
C and C++	pdgemm | pzgemm (*transa*, *transb*, *m*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*);

On Entry

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = 'N', ***A*** is used in the computation.

If *transa* = 'T', $\boldsymbol{A}^T$ is used in the computation.

If *transa* = 'C', $\boldsymbol{A}^H$ is used in the computation.

Scope: **global**

Specified as: a single character; *transa* = 'N', 'T', or 'C'

transb

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = 'N', ***B*** is used in the computation.

If *transb* = 'T', $\boldsymbol{B}^T$ is used in the computation.

If *transb* = 'C', $\boldsymbol{B}^H$ is used in the computation.

Scope: **global**

Specified as: a single character; *transb* = 'N', 'T', or 'C'

m

is the number of rows in submatrix ***C*** used in the computation, and:

If *transa* = 'N', it is the number of rows in submatrix ***A***.

If *transa* = 'T' or 'C', it is the number of columns in submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrix ***C*** used in the computation, and:

If *transb* = 'N', it is the number of columns in submatrix ***B***.

If *transb* = 'T' or 'C', it is the number of rows in submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

k

has the following meaning:

If *transa* = 'N', it is the number of columns in submatrix ***A***.

If *transa* = 'T' or 'C', it is the number of rows in submatrix ***A***.

In addition:

If *transb* = 'N', it is the number of rows in submatrix ***B***.

If *transb* = 'T' or 'C', it is the number of columns in submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $k \geq 0$.

alpha
is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 47 on page 281.

a
is the local part of the global general matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *transa* = 'N', the leading LOCp(*ia*+*m*–1) by LOCq(*ja*+*k*–1) part of the local array A must contain the local pieces of the leading *ia*+*m*–1 by *ja*+*k*–1 part of the global matrix.
- If *transa* = 'T' or 'C', the leading LOCp(*ia*+*k*–1) by LOCq(*ja*+*m*–1) part of the local array A must contain the local pieces of the leading *ia*+*k*–1 by *ja*+*m*–1 part of the global matrix.

Note: No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 47 on page 281. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia
is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ia* ≤ M_A, and:

If *transa* = 'N', then *ia*+*m*–1 ≤ M_A.

If *transa* = 'T' or 'C', then *ia*+*k*–1 ≤ M_A.

ja
is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ja* ≤ N_A, and:

If *transa* = 'N', then *ja*+*k*–1 ≤ N_A.

If *transa* = 'T' or 'C', then *ja*+*m*–1 ≤ N_A.

desc_a
is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
3	M_A	Number of rows in the global matrix	If $m = 0$ or $k = 0$: M_A $\geq$ 0 Otherwise: M_A $\geq$ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ or $k = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ p	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ q	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *transb* = 'N', the leading LOCp(*ib*+*k*–1) by LOCq(*jb*+*n*–1) part of the local array B must contain the local pieces of the leading *ib*+*k*–1 by *jb*+*n*–1 part of the global matrix.
- If *transb* = 'T' or 'C', the leading LOCp(*ib*+*n*–1) by LOCq(*jb*+*k*–1) part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *jb*+*k*–1 part of the global matrix.

Note: No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix ***B*** should always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 47 on page 281. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 $\leq$ *ib* $\leq$ M_B, and:

If *transb* = 'N', then *ib*+*k*–1 $\leq$ M_B.

If *transb* = 'T' or 'C', then *ib*+*n*–1 $\leq$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B, and:

If *transb* = `'N'`, then $jb+n-1 \le$ N_B.

If *transb* = `'T'` or `'C'`, then $jb+k-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $k = 0$ or $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $k = 0$ or $n = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_B < *q*	Global
9	LLD_B	The leading dimension of the local array	LLD_B ≥ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 47 on page 281.

c

is the local part of the global general matrix ***C***. This identifies the **first element** of the local array `C`. This subroutine computes the location of the first element of the local subarray used, based on *ic*, *jc*, *desc_c*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ic+m-1$) by LOCq($jc+n-1$) part of the local array `C` must contain the local pieces of the leading $ic+m-1$ by $jc+n-1$ part of the global matrix.

When β is zero, ***C*** need not be set on input.

Scope: **local**

Specified as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 47 on page 281. Details about the block-cyclic data distribution of global matrix ***C*** are stored in *desc_c*.

ic

is the row index of the global matrix ***C***, identifying the first row of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le ic \le$ M_C and $ic+m-1 \le$ M_C.

jc

is the column index of the global matrix ***C***, identifying the first column of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le jc \le$ N_C and $jc+n-1 \le$ N_C.

desc_c

is the array descriptor for global matrix ***C***, described in the following table:

desc_c	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_C	Descriptor type	DTYPE_C=1	Global
2	CTXT_C	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_C	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_C ≥ 0 Otherwise: M_C ≥ 1	Global
4	N_C	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_C ≥ 0 Otherwise: N_C ≥ 1	Global
5	MB_C	Row block size	MB_C ≥ 1	Global
6	NB_C	Column block size	NB_C ≥ 1	Global
7	RSRC_C	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_C $< p$	Global
8	CSRC_C	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_C $< q$	Global
9	LLD_C	The leading dimension of the local array	LLD_C $\ge$ max(1,LOCp(M_C))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

c

is the updated local part of the global matrix ***C***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 47 on page 281.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *transa* and *transb* arguments.
2. For PDGEMM, if you specify `'C'` for the *transa* or *transb* argument, it is interpreted as though you specified `'T'`.
3. The matrices must have no common elements; otherwise, results are unpredictable.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
5. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
6. The following values must be equal: CTXT_A = CTXT_B = CTXT_C.
7. The coding rules described in this note depend upon which matrix—***A***, ***B***, or ***C***—is used as the reference matrix, which is referred to, in general, as matrix ***X***. For each of the three possible selections for the reference matrix, there is a unique set of coding rules that must be met. These are detailed in Table 48 on page 288 and Table 49 on page 289. Follow these steps to select a reference matrix and determine what coding rules to use:

   Step 1: First, the reference matrix is selected. For optimal performance, the reference matrix is selected based on the arguments *m*, *n*, and *k*, as follows:

   If $k \le \min(m, n)$, then $\boldsymbol{X} = \boldsymbol{C}$
   If $n \le \min(m, k)$, then $\boldsymbol{X} = \boldsymbol{A}$
   If $m \le \min(n, k)$, then $\boldsymbol{X} = \boldsymbol{B}$

   The matrix selected must satisfy **coding rules a and d**, described below, to be a suitable reference matrix. If it does, you go to step 2. If it does not, then it checks to see if either of the other two matrices satisfies **coding rules a, c, and d**, making one of them a suitable reference matrix. If one of them is suitable, then you go to step 2. If neither matrix is suitable, an error condition results.

   Step 2: After a suitable reference matrix is chosen in Step 2, **all remaining coding rules**, described below, are checked. If the rules are satisfied, the subroutine continues normally. If they are not, an error condition results.

   Coding Rules: Following are the coding rules:

   a. The reference matrix must be aligned on a block boundary; that is:

      $ix-1$ must be a multiple of MB_X.
      $jx-1$ must be a multiple of NB_X.

      These indexes are indicated in column 5 of Table 48 on page 288 for each entry for ***X***.

b. The block sizes that must be equal are indicated in column 4 of Table 48 on page 288 for each entry for ***X***. The rules for block sizes depend only upon the values of *transa* and *transb*, and not on the reference matrix selected; however, for your convenience, the rules are repeated in the table for each reference matrix.

c. Given the reference matrix ***X***, additional rules apply to the block row and block column offsets of the two nonreference matrices. These rules are listed in column 7 of Table 48 for each entry for ***X***. These rules must only be met when looping is required—that is, either of the conditions in column 8 is met.

d. The indexes of the nonreference matrices, which need to be on a block boundary, are listed in column 6 of Table 48 for each entry for ***X***.

Table 48 (Page 1 of 2). Coding Rules for the Reference Matrix X

–1– X	–2– *transa*	–3– *transb*	**–4– (b) Equal Block Sizes**	**–5– (a) Block Bndry For X**	**–6– (d) Block Bndry For Other**	**–7– (c) Equal Block Offsets (If Looping is Required)**	**–8– (c) Conditions For Looping**

A	'N'	'N'	MB_A = MB_C
			NB_B = NB_C
			NB_A = MB_B
				ia, ja	*ib, ic*	mod(*jb*–1, NB_B)
						=
						mod(*jc*–1, NB_C)
							n+mod(*jb*–1, NB_B) > NB_B
							–or–
							n+mod(*jc*–1, NB_C) > NB_C

A	'N'	'T'
		or
		'C'
			MB_A = MB_C
			MB_B = NB_C
			NB_A = NB_B
				ia, ja	*jb, ic*	mod(*ib*–1, MB_B)
						=
						mod(*jc*–1, NB_C)
							n+mod(*ib*–1, MB_B) > MB_B
							–or–
							n+mod(*jc*–1, NB_C) > NB_C

A	'T'
	or
	'C'
		'N'	NB_A = MB_C
			NB_B = NB_C
			MB_A = MB_B
				ia, ja	*ib, ic*	mod(*jb*–1, NB_B)
						=
						mod(*jc*–1, NB_C)
							n+mod(*jb*–1, NB_B) > NB_B
							–or–
							n+mod(*jc*–1, NB_C) > NB_C

A	'T'
	or
	'C'
		'T'
		or
		'C'
			NB_A = MB_C
			MB_B = NB_C
			MB_A = NB_B
				ia, ja	*jb, ic*	mod(*ib*–1, MB_B)
						=
						mod(*jc*–1, NB_C)
							n+mod(*ib*–1, MB_B) > MB_B
							–or–
							n+mod(*jc*–1, NB_C) > NB_C

B	'N'	'N'	MB_A = MB_C
			NB_B = NB_C
			NB_A = MB_B
				ib, jb	*ja, jc*	mod(*ia*–1, MB_A)
						=
						mod(*ic*–1, MB_C)
							m+mod(*ia*–1, MB_A) > MB_A
							–or–
							m+mod(*ic*–1, MB_C) > MB_C

B	'N'	'T'
		or
		'C'
			MB_A = MB_C
			MB_B = NB_C
			NB_A = NB_B
				ib, jb	*ja, jc*	mod(*ia*–1, MB_A)
						=
						mod(*ic*–1, MB_C)
							m+mod(*ia*–1, MB_A) > MB_A
							–or–
							m+mod(*ic*–1, MB_C) > MB_C

B	'T'
	or
	'C'
		'N'	NB_A = MB_C
			NB_B = NB_C
			MB_A = MB_B
				ib, jb	*ia, jc*	mod(*ja*–1, NB_A)
						=
						mod(*ic*–1, MB_C)
							m+mod(*ja*–1, NB_A) > NB_A
							–or–
							m+mod(*ic*–1, MB_C) > MB_C

B	'T'
	or
	'C'
		'T'
		or
		'C'
			NB_A = MB_C
			MB_B = NB_C
			MB_A = NB_B
				ib, jb	*ia, jc*	mod(*ja*–1, NB_A)
						=
						mod(*ic*–1, MB_C)
							m+mod(*ja*–1, NB_A) > NB_A
							–or–
							m+mod(*ic*–1, MB_C) > MB_C

C	'N'	'N'	MB_A = MB_C
			NB_B = NB_C
			NB_A = MB_B
				ic, jc	*ia, jb*	mod(*ja*–1, NB_A)
						=
						mod(*ib*–1, MB_B)
							k+mod(*ja*–1, NB_A) > NB_A
							–or–
							k+mod(*ib*–1, MB_B) > MB_B

C	'N'	'T'
		or
		'C'
			MB_A = MB_C
			MB_B = NB_C
			NB_A = NB_B
				ic, jc	*ia, ib*	mod(*ja*–1, NB_A)
						=
						mod(*jb*–1, NB_B)
							k+mod(*ja*–1, NB_A) > NB_A
							–or–
							k+mod(*jb*–1, NB_B) > NB_B

Table 48 (Page 2 of 2). Coding Rules for the Reference Matrix X

–1– X	–2– *transa*	–3– *transb*	–4– (b) Equal Block Sizes	–5– (a) Block Bndry For X	–6– (d) Block Bndry For Other	–7– (c) Equal Block Offsets (If Looping is Required)	–8– (c) Conditions For Looping
C	'T' or 'C'	'N'	NB_A = MB_C NB_B = NB_C MB_A = MB_B	*ic, jc*	*ja, jb*	mod(*ia*–1, MB_A) = mod(*ib*–1, MB_B)	*k*+mod(*ia*–1, MB_A) > MB_A –or– *k*+mod(*ib*–1, MB_B) > MB_B
C	'T' or 'C'	'T' or 'C'	NB_A = MB_C MB_B = NB_C MB_A = NB_B	*ic, jc*	*ja, ib*	mod(*ia*–1, MB_A) = mod(*jb*–1, NB_B)	*k*+mod(*ia*–1, MB_A) > MB_A –or– *k*+mod(*jb*–1, NB_B) > NB_B

e. Additional rules apply to the row and column alignment of the various matrices in the process grid; specifically, the process row or process column containing the first row or column of the reference submatrix ***X***, respectively, must also contain the first row or column of one of the other two nonreference submatrices, as indicated in column 4 of Table 49 for each entry for ***X***. Following is the definition of *ixrow* and *ixcol*, which holds true for ***A***, ***B***, and ***C***:

ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)

Table 49. Coding Rules for the Reference Matrix X

–1– X	–2– *transa*	–3– *transb*	–4– (e) Process Grid Alignment
A	'N'	'N'	*iarow* = *icrow*
A	'N'	'T' or 'C'	*iarow* = *icrow* *ibcol* = *iacol*
A	'T' or 'C'	'N'	*iarow* = *ibrow*
A	'T' or 'C'	'T' or 'C'	(no rules)
B	'N'	'N'	*ibcol* = *iccol*
B	'N'	'T' or 'C'	*ibcol* = *iacol*
B	'T' or 'C'	'N'	*iarow* = *ibrow* *ibcol* = *iccol*
B	'T' or 'C'	'T' or 'C'	(no rules)
C	'N'	'N'	*iarow* = *icrow* *ibcol* = *iccol*
C	'N'	'T' or 'C'	*iarow* = *icrow*
C	'T' or 'C'	'N'	*ibcol* = *iccol*
C	'T' or 'C'	'T' or 'C'	(no rules)

Example: Following is an example of the coding rules necessary for the case where *transa* = 'N' and *transb* = 'N', where the reference matrix selected is

A. Following are the indexes, dimensions, and block sizes used in the computation for the matrices:

```
Indexes:       ic  jc            ia  ja       ib  jb            ic  jc
                |   |             |   |        |   |             |   |
Dimensions:  C ( m , n )  ←  α  A ( m , k )  B ( k , n )  +  β  C ( m , n )
                |   |             |   |        |   |             |   |
Block Sizes:   MB_C NB_C         MB_A NB_A    MB_B NB_B         MB_C NB_C
```

a. ***A*** must be aligned on a block boundary, as indicated in column 5 in Table 48 on page 288:

   ia–1 must be a multiple of MB_A.
   ja–1 must be a multiple of NB_A.

b. The block sizes that correspond to each matrix dimension must be equal, where MB_ represents the row dimension and NB_ represents the column dimension, as indicated in column 4 in Table 48 on page 288:

   MB_A = MB_C
   NB_B = NB_C
   NB_A = MB_B

c. As shown above, *m* and *k* are the dimensions of the reference matrix ***A***; therefore, *n* is used to determine if looping is required; that is, if one of the following is true, as indicated in column 8 in Table 48 on page 288:

   n+mod(*jc*–1, NB_C) > NB_C
   n+mod(*jb*–1, NB_B) > NB_B

   then the following offsets must be equal, as indicated in column 7 in Table 48 on page 288:

   mod(*jb*–1, NB_B) = mod(*jc*–1, NB_C)

d. The other indexes from each of the nonreference matrices—not used in c above—must be aligned on a block boundary, as indicated in column 6 in Table 48 on page 288:

   ic–1 must be a multiple of MB_C.
   ib–1 must be a multiple of MB_B.

e. In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***C***, as indicated in column 4 in Table 49 on page 289; that is, *iarow* = *icrow*, where:

   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.
3. DTYPE_C is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. The subroutine was called from outside the process grid.

Stage 4

1. $transa \neq$ `'N'`, `'T'`, or `'C'`
2. $transb \neq$ `'N'`, `'T'`, or `'C'`
3. $m < 0$
4. $n < 0$
5. $k < 0$
6. M_A < 0 and ($m = 0$ or $k = 0$); M_A < 1 otherwise
7. N_A < 0 and ($m = 0$ or $k = 0$); N_A < 1 otherwise
8. M_B < 0 and ($k = 0$ or $n = 0$); M_B < 1 otherwise
9. N_B < 0 and ($k = 0$ or $n = 0$); N_B < 1 otherwise
10. M_C < 0 and ($m = 0$ or $n = 0$); M_C < 1 otherwise
11. N_C < 0 and ($m = 0$ or $n = 0$); N_C < 1 otherwise
12. $ia < 1$
13. $ib < 1$
14. $ic < 1$
15. $ja < 1$
16. $jb < 1$
17. $jc < 1$
18. MB_A < 1
19. MB_B < 1
20. MB_C < 1
21. NB_A < 1
22. NB_B < 1
23. NB_C < 1
24. RSRC_A < 0 or RSRC_A $\geq p$
25. RSRC_B < 0 or RSRC_B $\geq p$
26. RSRC_C < 0 or RSRC_C $\geq p$
27. CSRC_A < 0 or CSRC_A $\geq q$
28. CSRC_B < 0 or CSRC_B $\geq q$
29. CSRC_C < 0 or CSRC_C $\geq q$
30. CTXT_A $\neq$ CTXT_B
31. CTXT_A $\neq$ CTXT_C

Stage 5

If $m \neq 0$ and $k \neq 0$:
1. $transa =$ `'N'` and $ia+m-1 >$ M_A
2. $transa =$ `'T'` or `'C'` and $ia+k-1 >$ M_A
3. $transa =$ `'N'` and $ja+k-1 >$ N_A
4. $transa =$ `'T'` or `'C'` and $ja+m-1 >$ N_A
5. $ia >$ M_A

6. *ja* > N_A

If $n \neq 0$ and $k \neq 0$:

7. *transb* = 'N' and *ib*+*k*–1 > M_B
8. *transb* = 'T' or 'C' and *ib*+*n*–1 > M_B
9. *transb* = 'N' and *jb*+*n*–1 > N_B
10. *transb* = 'T' or 'C' and *jb*+*k*–1 > N_B
11. *ib* > M_B
12. *jb* > N_B

If $m \neq 0$ and $n \neq 0$:

13. *ic*+*m*–1 > M_C
14. *jc*+*n*–1 > N_C
15. *ic* > M_C
16. *jc* > N_C
17. For the reference matrix (defined in note 7 in "Notes and Coding Rules" on page 287) and the appropriate *transa* and *transb* values, the indexes listed in column 5 of Table 48 are not aligned on a block boundary, where boundary alignment is defined as:
    ix–1 must be a multiple of MB_X.
    jx–1 must be a multiple of NB_X.
18. For the two nonreference matrices (defined in note 7 in "Notes and Coding Rules" on page 287) and the appropriate *transa* and *transb* values, the indexes listed in column 6 of Table 48 are not aligned on a block boundary. Using ***Z*** to represent one of the nonreference matrices, each boundary alignment is expressed as one of the following:
    iz–1 must be a multiple of MB_Z.
    jz–1 must be a multiple of NB_Z.
19. For the reference matrix (defined in note 7 in "Notes and Coding Rules" on page 287) and the appropriate *transa* and *transb* values, if looping occurs—that is, one of the conditions in column 8 of Table 48 is true—then the block offsets indicated in column 7 are not equal.

Stage 6

1. For the appropriate *transa* and *transb* values indicated in Table 48 (where the reference matrix does not matter), some of the block sizes indicated in column 4 are not equal.
2. LLD_A < max(1, LOCp(M_A))
3. LLD_B < max(1, LOCp(M_B))
4. LLD_C < max(1, LOCp(M_C))
5. In the process grid, the process row or process column containing the first row or column of the reference submatrix ***X*** (defined in note 7 in "Notes and Coding Rules" on page 287), respectively, does not contain the first row or column of one of the other two nonreference submatrices, as indicated in column 4 of Table 49. Following is the definition of *ixrow* and *ixcol*, which holds true for ***A***, ***B***, and ***C***:
    ixrow = mod((((*ix*–1)/MB_X)+RSRC_X), *p*)
    ixcol = mod((((*jx*–1)/NB_X)+CSRC_X), *q*)

Example 1: This example computes $\boldsymbol{C} = \beta\boldsymbol{C}+\alpha\boldsymbol{AB}$ using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

           TRANSA TRANSB  M    N    K   ALPHA    A  IA  JA   DESC_A   B  IB  JB
              |      |    |    |    |     |      |   |   |     |      |   |   |
CALL PDGEMM( 'N' ,  'N' , 6  , 4  , 5 , 1.0D0  , A , 1 , 1 , DESC_A , B , 1 , 1 ,

             DESC_B    BETA    C  IC  JC   DESC_C
               |         |     |   |   |     |
             DESC_B ,  2.0D0 , C , 1 , 1 , DESC_C )
```

	Desc_A	**Desc_B**	**Desc_C**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	6	5	6
N_	5	4	4
MB_	3	2	3
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = LLD_C = 3 on all processes, and LLD_B = 3 on P_{10} and P_{01} and LLD_B = 2 on P_{10} and P_{11}.

Global general 6 × 5 matrix ***A*** with block size 3 × 2:

```
B,D       0             1          2
     ┌                                   ┐
     |  1.0  2.0 |  -1.0 -1.0 |   4.0 |
 0   |  2.0  0.0 |   1.0  1.0 |  -1.0 |
     |  1.0 -1.0 |  -1.0  1.0 |   2.0 |
     | -----------|-------------|------ |
     | -3.0  2.0 |   2.0  2.0 |   0.0 |
 1   |  4.0  0.0 |  -2.0  1.0 |  -1.0 |
     | -1.0 -1.0 |   1.0 -3.0 |   2.0 |
     └                                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |        0         |      1
-----|------------------|------------
     |  1.0  2.0  4.0   |  -1.0 -1.0
 0   |  2.0  0.0 -1.0   |   1.0  1.0
     |  1.0 -1.0  2.0   |  -1.0  1.0
-----|------------------|------------
     | -3.0  2.0  0.0   |   2.0  2.0
 1   |  4.0  0.0 -1.0   |  -2.0  1.0
     | -1.0 -1.0  2.0   |   1.0 -3.0
```

Global general 5 × 4 matrix ***B*** with block size 2 × 2:

```
B,D       0            1
     ┌                         ┐
 0   |  1.0 -1.0  |   0.0  2.0 |
     |  2.0  2.0  |  -1.0 -2.0 |
     | -----------|----------- |
 1   |  1.0  0.0  |  -1.0  1.0 |
     | -3.0 -1.0  |   1.0 -1.0 |
     | -----------|----------- |
 2   |  4.0  2.0  |  -1.0  1.0 |
     └                         ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |     0      |      1
-----|------------|------------
     |  1.0 -1.0  |   0.0  2.0
 0   |  2.0  2.0  |  -1.0 -2.0
     |  4.0  2.0  |  -1.0  1.0
-----|------------|------------
 1   |  1.0  0.0  |  -1.0  0.0
     | -3.0 -1.0  |   1.0 -1.0
```

Global general 6 × 4 matrix ***C*** with block size 3 × 2:

```
B,D       0            1
     ┌                         ┐
     |  0.5  0.5  |   0.5  0.5 |
 0   |  0.5  0.5  |   0.5  0.5 |
     |  0.5  0.5  |   0.5  0.5 |
     | -----------|----------- |
     |  0.5  0.5  |   0.5  0.5 |
 1   |  0.5  0.5  |   0.5  0.5 |
     |  0.5  0.5  |   0.5  0.5 |
     └                         ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |     0      |     1
-----|------------|------------
     | 0.5  0.5   |  0.5  0.5
 0   | 0.5  0.5   |  0.5  0.5
     | 0.5  0.5   |  0.5  0.5
-----|------------|------------
     | 0.5  0.5   |  0.5  0.5
 1   | 0.5  0.5   |  0.5  0.5
     | 0.5  0.5   |  0.5  0.5
```

Output:

Global general 6 × 4 matrix ***C*** with block size 3 × 2:

```
B,D        0              1
     ┌                            ┐
     | 24.0  13.0  |   -5.0   3.0 |
 0   | -3.0  -4.0  |    2.0   4.0 |
     |  4.0   1.0  |    2.0   5.0 |
     | ------------|------------- |
     | -2.0   6.0  |   -1.0  -9.0 |
 1   | -4.0  -6.0  |    5.0   5.0 |
     | 16.0   7.0  |   -4.0   7.0 |
     └                            ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |      0       |      1
-----|--------------|--------------
     | 24.0  13.0   |  -5.0   3.0
 0   | -3.0  -4.0   |   2.0   4.0
     |  4.0   1.0   |   2.0   5.0
-----|--------------|--------------
     | -2.0   6.0   |  -1.0  -9.0
 1   | -4.0  -6.0   |   5.0   5.0
     | 16.0   7.0   |  -4.0   7.0
```

Example 2: This example computes ***C*** = β***C***+α***AB*** using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

          TRANSA TRANSB M   N   K       ALPHA       A  IA  JA   DESC_A   B  IB  JB
             |      |   |   |   |         |         |   |   |     |      |   |   |
CALL PZGEMM('N' , 'N' , 6 , 2 , 3 , (1.0D0,0.0D0) , A , 1 , 1 , DESC_A , B , 1 , 1 ,

             DESC_B       BETA        C  IC  JC   DESC_C
               |            |         |   |   |     |
             DESC_B , (2.0D0,0.0D0) , C , 1 , 1 , DESC_C)
```

	Desc_A	**Desc_B**	**Desc_C**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	6	3	6
N_	3	2	2
MB_	2	2	2
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on P_{00} and P_{01} and LLD_A = 2 on P_{10} and P_{11}. LLD_B = 2 on P_{00} and LLD_B = 1 on P_{10}. LLD_C = 4 on P_{00} and LLD_C = 2 on P_{10}.

Global general 6 × 3 matrix ***A*** with block size 2 × 2:

```
B,D           0                 1
     ┌                                  ┐
 0   |  (1.0,5.0)  (9.0,2.0)  |   (1.0,9.0) |
     |  (2.0,4.0)  (8.0,3.0)  |   (1.0,8.0) |
     | -----------------------|------------ |
 1   |  (3.0,3.0)  (7.0,5.0)  |   (1.0,7.0) |
     |  (4.0,2.0)  (4.0,7.0)  |   (1.0,5.0) |
     | -----------------------|------------ |
 2   |  (5.0,1.0)  (5.0,1.0)  |   (1.0,6.0) |
     |  (6.0,6.0)  (3.0,6.0)  |   (1.0,4.0) |
     └                                  ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |           0            |      1
-----|------------------------|-------------
     |  (1.0,5.0)  (9.0,2.0)  |   (1.0,9.0)
     |  (2.0,4.0)  (8.0,3.0)  |   (1.0,8.0)
 0   |  (5.0,1.0)  (5.0,1.0)  |   (1.0,6.0)
     |  (6.0,6.0)  (3.0,6.0)  |   (1.0,4.0)
-----|------------------------|-------------
 1   |  (3.0,3.0)  (7.0,5.0)  |   (1.0,7.0)
     |  (4.0,2.0)  (4.0,7.0)  |   (1.0,5.0)
```

Global general 3 × 2 matrix ***B*** with block size 2 × 2:

```
B,D            0
     ┌                       ┐
 0   |  (1.0,8.0)  (2.0,7.0) |
     |  (4.0,4.0)  (6.0,8.0) |
     | --------------------- |
 1   |  (6.0,2.0)  (4.0,5.0) |
     └                       ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |           0
-----|-----------------------
 0   |  (1.0,8.0)  (2.0,7.0)
     |  (4.0,4.0)  (6.0,8.0)
-----|-----------------------
 1   |  (6.0,2.0)  (4.0,5.0)
```

Global general 6 × 2 matrix ***C*** with block size 2 × 2:

```
B,D            0
     ┌                       ┐
 0   |  (0.5,0.0)  (0.5,0.0) |
     |  (0.5,0.0)  (0.5,0.0) |
     | --------------------- |
 1   |  (0.5,0.0)  (0.5,0.0) |
     |  (0.5,0.0)  (0.5,0.0) |
     | --------------------- |
 2   |  (0.5,0.0)  (0.5,0.0) |
     |  (0.5,0.0)  (0.5,0.0) |
     └                       ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |          0
-----|----------------------
     |  (0.5,0.0)  (0.5,0.0)
     |  (0.5,0.0)  (0.5,0.0)
 0   |  (0.5,0.0)  (0.5,0.0)
     |  (0.5,0.0)  (0.5,0.0)
-----|----------------------
 1   |  (0.5,0.0)  (0.5,0.0)
     |  (0.5,0.0)  (0.5,0.0)
```

Output:

Global general 6 × 2 matrix ***C*** with block size 2 × 2:

```
B,D               0
     ┌                              ┐
 0   |  (-22.0,113.0)  (-35.0.142.0) |
     |  (-19.0,114.0)  (-35.0.141.0) |
     | ----------------------------- |
 1   |  (-20.0,119.0)  (-43.0.146.0) |
     |  (-27.0,110.0)  (-58.0.131.0) |
     | ----------------------------- |
 2   |  (8.0,103.0)    (0.0.112.0)   |
     |  (-55.0,116.0)  (-75.0.135.0) |
     └                              ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |              0
-----|------------------------------
     |  (-22.0,113.0)  (-35.0.142.0)
     |  (-19.0,114.0)  (-35.0.141.0)
 0   |  (8.0,103.0)    (0.0.112.0)
     |  (-55.0,116.0)  (-75.0.135.0)
-----|------------------------------
 1   |  (-20.0,119.0)  (-43.0.146.0)
     |  (-27.0,110.0)  (-58.0.131.0)
```

PDSYMM, PZSYMM, and PZHEMM—Matrix-Matrix Product Where One Matrix is Real or Complex Symmetric or Complex Hermitian

These subroutines compute one of the following matrix-matrix products:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{BA}+\beta \boldsymbol{C}$

where, in the formulas above:

A represents the global submatrix:
- For *side* = `'L'`, it is $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+m-1}$.
- For *side* = `'R'`, it is $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.

B represents the global general submatrix $\boldsymbol{B}_{ib:ib+m-1,\ jb:jb+n-1}$.
C represents the global general submatrix $\boldsymbol{C}_{ic:ic+m-1,\ jc:jc+n-1}$.
α and β are scalars.

and:

| • For PDSYMM, submatrix ***A*** is real symmetric.
| • For PZSYMM, submatrix ***A*** is complex symmetric.
| • For PZHEMM, submatrix ***A*** is complex Hermitian.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- m = 0 or n = 0
- α is zero and β is one.

See references [14] and [15].

Table 50. Data Types

α, β, ***A***, ***B***, ***C***	**Subprogram**
Long-precision real	PDSYMM
Long-precision complex	PZSYMM and PZHEMM

Syntax

Fortran	CALL PDSYMM | PZSYMM | PZHEMM (*side*, *uplo*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*)
C and C++	pdsymm | pzsymm | pzhemm (*side*, *uplo*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*);

On Entry

side

indicates whether ***A*** is located to the left or right of ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in equation 1.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in equation 2.

Scope: **global**

Specified as: a single character; *side* = `'L'` or `'R'`.

uplo

indicates whether the upper or lower triangular part of the global submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

m

is the number of rows in submatrices ***B*** and ***C*** used in the computation, and:

If *side* = `'L'`, it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrices ***B*** and ***C*** used in the computation, and:

If *side* = `'R'`, it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 50 on page 299.

a

| is the local part of the global real symmetric, complex symmetric, or complex
| Hermitian matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, assuming the following:

If *side* = `'L'`, *numa* = *m*

If *side* = `'R'`, *numa* = *n*

the leading LOCp(*ia*+*numa*−1) by LOCq(*ja*+*numa*−1) part of the local array A must contain the local pieces of the leading *ia*+*numa*−1 by *ja*+*numa*−1 part of the global matrix, and:

- If *uplo* = `'U'`, the leading $numa \times numa$ upper triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $numa \times numa$ lower triangular part of the global submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 50 on page 299. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+numa-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+numa-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $m = 0$ and *side* = 'L' or $n = 0$ and *side* = 'R': M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ and *side* = 'L' or $n = 0$ and *side* = 'R': N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ib+m-1$) by LOCq($jb+n-1$) part of the local array B must contain the local pieces of the leading $ib+m-1$ by $jb+n-1$ part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 50 on page 299. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B and $ib+m-1 \le$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B and $jb+n-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_B $< p$	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_B $< q$	Global
9	LLD_B	The leading dimension of the local array	LLD_B $\ge$ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 50 on page 299.

c

is the local part of the global general matrix ***C***. This identifies the **first element** of the local array C. This subroutine computes the location of the first element of the local subarray used, based on *ic*, *jc*, *desc_c*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ic*+*m*–1) by LOCq(*jc*+*n*–1) part of the local array `C` must contain the local pieces of the leading *ic*+*m*–1 by *jc*+*n*–1 part of the global matrix.

When β is zero, ***C*** need not be set on input.

Scope: **local**

Specified as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 50 on page 299. Details about the block-cyclic data distribution of global matrix ***C*** are stored in *desc_c*.

ic

is the row index of the global matrix ***C***, identifying the first row of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le ic \le$ M_C and $ic+m-1 \le$ M_C.

jc

is the column index of the global matrix ***C***, identifying the first column of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le jc \le$ N_C and $jc+n-1 \le$ N_C.

desc_c

is the array descriptor for global matrix ***C***, described in the following table:

desc_c	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_C	Descriptor type	DTYPE_C=1	Global
2	CTXT_C	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_C	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_C ≥ 0 Otherwise: M_C ≥ 1	Global
4	N_C	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_C ≥ 0 Otherwise: N_C ≥ 1	Global
5	MB_C	Row block size	MB_C ≥ 1	Global
6	NB_C	Column block size	NB_C ≥ 1	Global
7	RSRC_C	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_C < p	Global
8	CSRC_C	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_C < q	Global
9	LLD_C	The leading dimension of the local array	LLD_C ≥ max(1,LOCp(M_C))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

c

is the updated local part of the global matrix ***C***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 50 on page 299.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *side* and *uplo* arguments.
2. The matrices must have no common elements; otherwise, results are unpredictable.
3. | The imaginary parts of the diagonal elements of a complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. |
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
5. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
6. The following values must be equal: CTXT_A = CTXT_B = CTXT_C.
7. If *side* = `'L'`:
   - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrices ***B*** and ***C***; that is:

     iarow = *ibrow*
     iarow = *icrow*
     where:
     iarow = mod((((*ia*−1)/MB_A)+RSRC_A), *p*)
     ibrow = mod((((*ib*−1)/MB_B)+RSRC_B), *p*)
     icrow = mod((((*ic*−1)/MB_C)+RSRC_C), *p*)
   - If looping is required—that is, **either** of the following is true:

     n+mod(*jb*−1, NB_B) > NB_B
     n+mod(*jc*−1, NB_C) > NB_C

     then:
     - The following block sizes must be equal: NB_B = NB_C.
     - The block column offset of ***B*** must be equal to the block column offset of ***C***; that is, mod(*jb*−1, NB_B) = mod(*jc*−1, NB_C).
8. If *side* = `'R'`:
   - In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrices ***B*** and ***C***; that is:

     iacol = *ibcol*

iacol = *iccol*
where:
iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)

- If looping is required—that is, **either** of the following is true:

  m+mod(*ib*–1, MB_B) > MB_B
  m+mod(*ic*–1, MB_C) > MB_C

  then:

  – The following block sizes must be equal: MB_B = MB_C.

  – The block row offset of ***B*** must be equal to the block row offset of ***C***; that is, mod(*ib*–1, MB_B) = mod(*ic*–1, MB_C)

9. If all the following are true:

   - ***A*** is contained within a single block, that is:

     numa+mod(*ia*–1, MB_A) ≤ MB_A
     numa+mod(*ja*–1, NB_A) ≤ NB_A
     where:
     If *side* = `'L'`, *numa* = *m*
     If *side* = `'R'`, *numa* = *n*

   - If *side* = `'L'`, then (in the process grid) the process column containing the first column of the submatrix ***B*** must also contain the first column of the submatrix ***C***, that is, *ibcol* = *iccol*, where:

     ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
     iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)

   - If *side* = `'R'`, then (in the process grid) the process row containing the first row of the submatrix ***B*** must also contain the first row of the submatrix ***C***; that is, *ibrow* = *icrow*, where:

     ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
     icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)

   then you must follow these rules:

   - If *side* = `'L'`, then ***B*** and ***C*** must be block row matrices; that is, if *p* > 1:

     m+mod(*ib*–1, MB_B) ≤ MB_B
     m+mod(*ic*–1, MB_C) ≤ MB_C

   - If *side* = `'R'`, then ***B*** and ***C*** must be block column matrices; that is, if *q* > 1:

     n+mod(*jb*–1, NB_B) ≤ NB_B
     n+mod(*jc*–1, NB_C) ≤ NB_C

10. If the following is true:

    - ***A*** is **not** contained within a single block.

    or if all the following are true:

    - ***A*** is contained within a single block.

- If *side* = `'L'`, then (in the process grid) the process column containing the first column of the submatrix ***B*** does not contain the first column of the submatrix ***C***, that is, $ibcol \neq iccol$, where:

  ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
  iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)

- If *side* = `'R'`, then (in the process grid) the process row containing the first row of the submatrix ***B*** does not contain the first row of the submatrix ***C***; that is, $ibrow \neq icrow$, where:

  ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
  icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)

then you must follow these rules:

- The global matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
- The global matrix ***A*** must be aligned on a block boundary, that is:

  ia–1 must be a multiple of MB_A.
  ja–1 must be a multiple of NB_A.

- If *side* = `'L'`:
  - The following block sizes must be equal: MB_B = MB_C = NB_A.
  - The global matrices ***B*** and ***C*** must be aligned on a block row boundary, that is:

    ib–1 must be a multiple of MB_B.
    ic–1 must be a multiple of MB_C.

- If *side* = `'R'`:
  - The following block sizes must be equal: NB_B = NB_C = MB_A.
  - The global matrices ***B*** and ***C*** must be aligned on a block column boundary, that is:

    jb–1 must be a multiple of NB_B.
    jc–1 must be a multiple of NB_C.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.
3. DTYPE_C is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *side* ≠ `'L'` or `'R'`
2. *uplo* ≠ `'U'` or `'L'`
3. $m < 0$
4. $n < 0$
5. M_A < 0 and $m = 0$ and *side* = `'L'`; M_A < 0 and $n = 0$ and *side* = `'R'`; M_A < 1 otherwise
6. N_A < 0 and $m = 0$ and *side* = `'L'`; N_A < 0 and $n = 0$ and *side* = `'R'`; N_A < 1 otherwise
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A ≥ *p*
10. CSRC_A < 0 or CSRC_A ≥ *q*
11. *ia* < 1
12. *ja* < 1
13. M_B < 0 and ($m = 0$ or $n = 0$); M_B < 1 otherwise
14. N_B < 0 and ($m = 0$ or $n = 0$); N_B < 1 otherwise
15. MB_B < 1
16. NB_B < 1
17. RSRC_B < 0 or RSRC_B ≥ *p*
18. CSRC_B < 0 or CSRC_B ≥ *q*
19. *ib* < 1
20. *jb* < 1
21. M_C < 0 and ($m = 0$ or $n = 0$); M_C < 1 otherwise
22. N_C < 0 and ($m = 0$ or $n = 0$); N_C < 1 otherwise
23. MB_C < 1
24. NB_C < 1
25. RSRC_C < 0 or RSRC_C ≥ *p*
26. CSRC_C < 0 or CSRC_C ≥ *q*
27. *ic* < 1
28. *jc* < 1
29. CTXT_A ≠ CTXT_B
30. CTXT_A ≠ CTXT_C

Stage 5: If ($m \neq 0$ or *side* ≠ `'L'`) and ($n \neq 0$ or *side* ≠ `'R'`):

1. *ia* > M_A
2. *ja* > N_A
3. *ia*+*numa*–1 > M_A
4. *ja*+*numa*–1 > N_A

   where *numa* = *m* if *side* = `'L'` and *numa* = *n* if *side* = `'R'`.

If $m \neq 0$ and $n \neq 0$:

1. *ib* > M_B
2. *jb* > N_B
3. *ib*+*m*–1 > M_B
4. *jb*+*n*–1 > N_B
5. *ic* > M_C
6. *jc* > N_C

7. *ic*+*m*−1 > M_C
8. *jc*+*n*−1 > N_C

Stage 6: If ***A*** is contained within a single block, that is:

numa+mod(*ia*−1, MB_A) ≤ MB_A
numa+mod(*ja*−1, NB_A) ≤ NB_A
where:
If *side* = `'L'`, *numa* = *m*
If *side* = `'R'`, *numa* = *n*

and:

- If *side* = `'L'`, then (in the process grid) the process column containing the first column of the submatrix ***B*** must also contain the first column of the submatrix ***C***, that is, *ibcol* = *iccol*, where:
  ibcol = mod((((*jb*−1)/NB_B)+CSRC_B), *q*)
  iccol = mod((((*jc*−1)/NB_C)+CSRC_C), *q*)
- If *side* = `'R'`, then (in the process grid) the process row containing the first row of the submatrix ***B*** must also contain the first row of the submatrix ***C***; that is, *ibrow* = *icrow*, where:
  ibrow = mod((((*ib*−1)/MB_B)+RSRC_B), *p*)
  icrow = mod((((*ic*−1)/MB_C)+RSRC_C), *p*)

then:

- If *side* = `'L'`:
  1. *p* > 1 and *m*+mod(*ib*−1, MB_B) > MB_B
  2. *p* > 1 and *m*+mod(*ic*−1, MB_C) > MB_C
- If *side* = `'R'`:
  1. *q* > 1 and *n*+mod(*jb*−1, NB_B) > NB_B
  2. *q* > 1 and *n*+mod(*jc*−1, NB_C) > NB_C

If ***A*** is **not** contained within a single block, or if ***A*** is contained within a single block and:

- If *side* = `'L'`, then (in the process grid) the process column containing the first column of the submatrix ***B*** does not contain the first column of the submatrix ***C***, that is, *ibcol* ≠ *iccol*, where:
  ibcol = mod((((*jb*−1)/NB_B)+CSRC_B), *q*)
  iccol = mod((((*jc*−1)/NB_C)+CSRC_C), *q*)
- If *side* = `'R'`, then (in the process grid) the process row containing the first row of the submatrix ***B*** does not contain the first row of the submatrix ***C***; that is, *ibrow* ≠ *icrow*, where:
  ibrow = mod((((*ib*−1)/MB_B)+RSRC_B), *p*)
  icrow = mod((((*ic*−1)/MB_C)+RSRC_C), *p*)

then:

1. MB_A ≠ NB_A
2. mod(*ia*−1, MB_A) ≠ 0
3. mod(*ja*−1, NB_A) ≠ 0

If *side* = `'L'`:
4. MB_B ≠ NB_A
5. MB_C ≠ NB_A

6. mod(*ib*–1, MB_B) $\neq$ 0
7. mod(*ic*–1, MB_C) $\neq$ 0

If *side* = `'R'`:

8. NB_B $\neq$ MB_A
9. NB_C $\neq$ MB_A
10. mod(*jb*–1, NB_B) $\neq$ 0
11. mod(*jc*–1, NB_C) $\neq$ 0

In all cases:

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_B $<$ max(1, LOCp(M_B))
3. LLD_C $<$ max(1, LOCp(M_C))

If *side* = `'L'` and looping is required—that is, **either** of the following is true:

n+mod(*jb*–1, NB_B) $>$ NB_B
n+mod(*jc*–1, NB_C) $>$ NB_C

then:

4. NB_B $\neq$ NB_C
5. mod(*jb*–1, NB_B) $\neq$ mod(*jc*–1, NB_C).

If *side* = `'L'`:

6. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* $\neq$ *ibrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
7. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***C***; that is, *iarow* $\neq$ *icrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)

If *side* = `'R'` and looping is required—that is, **either** of the following is true:

m+mod(*ib*–1, MB_B) $>$ MB_B
m+mod(*ic*–1, MB_C) $>$ MB_C

then:

8. MB_B $\neq$ MB_C
9. mod(*ib*–1, MB_B) $\neq$ mod(*ic*–1, MB_C).

If *side* = `'R'`:

10. In the process grid, the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***B***; that is, *iacol* $\neq$ *ibcol*, where:
    iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
    ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
11. In the process grid, the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***C***; that is, *iacol* $\neq$ *iccol*, where:
    iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
    iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)

Example 1: This example computes ***C*** = β***C***+α***BA*** using a 2 $\times$ 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

           SIDE  UPLO   M   N    ALPHA    A  IA  JA   DESC_A   B  IB   JB
             |     |    |   |      |      |   |   |      |     |   |    |
CALL PDSYMM( 'R' , 'U' , 16 , 8 ,  1.0D0  , A , 1 , 1 , DESC_A , B , 1 , 1 ,

           DESC_B   BETA    C  IC  JC   DESC_C
             |        |     |   |   |      |
           DESC_B , 0.0D0 , C , 1 , 1 , DESC_C )
```

	Desc_A	Desc_B	Desc_C
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	16	16
N_	8	8	8
MB_	2	4	4
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on all processes, and LLD_B = LLD_C = 8 on all processes.

Global real symmetric matrix ***A*** of order 8 with block size 2 × 2:

```
B,D      0            1            2            3
     ┌                                                    ┐
 0   |  0.0 -1.0  |  -1.0  0.0  |   0.0  0.0  |   0.0  0.0 |
     |   .   1.0  |   0.0  1.0  |   0.0  1.0  |   0.0  1.0 |
     | -----------|-------------|-------------|----------- |
 1   |   .   .    |  -1.0 -1.0  |   0.0  0.0  |   1.0  0.0 |
     |   .   .    |    .  -1.0  |   1.0  1.0  |   0.0  1.0 |
     | -----------|-------------|-------------|----------- |
 2   |   .    .   |    .    .   |  -1.0  0.0  |   0.0  0.0 |
     |   .    .   |    .    .   |    .   1.0  |   0.0  0.0 |
     | -----------|-------------|-------------|----------- |
 3   |   .    .   |    .    .   |    .    .   |   0.0  0.0 |
     |   .    .   |    .    .   |    .    .   |    .   0.0 |
     └                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |          0           |          1
-----|----------------------|---------------------
     |  0.0 -1.0  0.0  0.0  |  -1.0  0.0  0.0  0.0
     |   .   1.0  0.0  1.0  |   0.0  1.0  0.0  1.0
 0   |   .    .  -1.0  0.0  |    .    .   0.0  0.0
     |   .    .    .   1.0  |    .    .   0.0  0.0
-----|----------------------|---------------------
     |   .    .   0.0  0.0  |  -1.0 -1.0  1.0  0.0
     |   .    .   1.0  1.0  |    .  -1.0  0.0  1.0
 1   |   .    .    .    .   |    .    .   0.0  0.0
     |   .    .    .    .   |    .    .    .   0.0
```

Global general 16 × 8 matrix ***B*** with block size 4 × 2:

```
B,D       0            1             2             3
     ┌                                                     ┐
     | -1.0  0.0 |   1.0 -1.0 |   1.0  1.0 |  -1.0 -1.0 |
     | -1.0 -1.0 |   1.0  0.0 |   1.0 -1.0 |  -1.0  1.0 |
 0   |  1.0  1.0 |  -1.0  0.0 |  -1.0  0.0 |   1.0  0.0 |
     |  0.0 -1.0 |   0.0  0.0 |   0.0  0.0 |   0.0 -1.0 |
     | ----------|------------|------------|----------- |
     |  0.0  1.0 |   0.0  1.0 |   0.0  1.0 |   1.0  0.0 |
     |  0.0  0.0 |   1.0  0.0 |  -1.0 -1.0 |   0.0  0.0 |
 1   |  1.0  1.0 |   0.0  0.0 |   1.0  1.0 |   0.0 -1.0 |
     |  0.0  0.0 |  -1.0  0.0 |   0.0  1.0 |   0.0  1.0 |
     | ----------|------------|------------|----------- |
     |  0.0  0.0 |   0.0 -1.0 |   1.0  1.0 |   0.0  1.0 |
     | -1.0 -1.0 |   1.0  0.0 |   0.0 -1.0 |   0.0  1.0 |
 2   |  0.0  0.0 |   0.0  1.0 |   1.0  0.0 |   0.0  0.0 |
     |  0.0  0.0 |   1.0  1.0 |   0.0 -1.0 |   0.0  0.0 |
     | ----------|------------|------------|----------- |
     |  1.0  1.0 |  -1.0  0.0 |  -1.0 -1.0 |   1.0  1.0 |
     |  0.0  0.0 |   0.0  0.0 |   1.0  0.0 |   0.0 -1.0 |
 3   |  0.0  1.0 |   0.0  0.0 |   0.0  0.0 |   0.0  0.0 |
     | -1.0  0.0 |  -1.0  0.0 |   0.0  1.0 |   1.0  0.0 |
     └                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |          0           |          1
-----|----------------------|----------------------
     | -1.0  0.0  1.0  1.0  |  1.0 -1.0 -1.0 -1.0
     | -1.0 -1.0  1.0 -1.0  |  1.0  0.0 -1.0  1.0
     |  1.0  1.0 -1.0  0.0  | -1.0  0.0  1.0  0.0
     |  0.0 -1.0  0.0  0.0  |  0.0  0.0  0.0 -1.0
 0   |  0.0  0.0  1.0  1.0  |  0.0 -1.0  0.0  1.0
     | -1.0 -1.0  0.0 -1.0  |  1.0  0.0  0.0  1.0
     |  0.0  0.0  1.0  0.0  |  0.0  1.0  0.0  0.0
     |  0.0  0.0  0.0 -1.0  |  1.0  1.0  0.0  0.0
-----|----------------------|----------------------
     |  0.0  1.0  0.0  1.0  |  0.0  1.0  1.0  0.0
     |  0.0  0.0 -1.0 -1.0  |  1.0  0.0  0.0  0.0
     |  1.0  1.0  1.0  1.0  |  0.0  0.0  0.0 -1.0
     |  0.0  0.0  0.0  1.0  | -1.0  0.0  0.0  1.0
 1   |  1.0  1.0 -1.0 -1.0  | -1.0  0.0  1.0  1.0
     |  0.0  0.0  1.0  0.0  |  0.0  0.0  0.0 -1.0
     |  0.0  1.0  0.0  0.0  |  0.0  0.0  0.0  0.0
     | -1.0  0.0  0.0  1.0  | -1.0  0.0  1.0  0.0
```

Output:

Global general 16 × 8 matrix ***C*** with block size 4 × 2:

```
B,D      0            1            2            3
     ┌                                                   ┐
     | -1.0  0.0  |   0.0  1.0  |  -2.0  0.0  |   1.0 -1.0 |
     |  0.0  0.0  |  -1.0 -1.0  |  -1.0 -2.0  |   1.0 -1.0 |
 0   |  0.0  0.0  |   1.0  1.0  |   1.0  1.0  |  -1.0  1.0 |
     |  1.0 -2.0  |   0.0 -2.0  |   0.0 -1.0  |   0.0 -1.0 |
     | -----------|-------------|-------------|----------- |
     | -1.0  3.0  |   0.0  1.0  |   1.0  3.0  |   0.0  2.0 |
     | -1.0 -1.0  |  -1.0 -3.0  |   1.0 -1.0  |   1.0  0.0 |
 1   | -1.0  0.0  |  -1.0  2.0  |  -1.0  2.0  |   0.0  1.0 |
     |  1.0  2.0  |   1.0  3.0  |   0.0  1.0  |  -1.0  0.0 |
     | -----------|-------------|-------------|----------- |
     |  0.0  1.0  |   1.0  4.0  |  -2.0  0.0  |   0.0 -1.0 |
     |  0.0  0.0  |   0.0 -2.0  |   0.0 -2.0  |   1.0 -1.0 |
 2   |  0.0  1.0  |  -1.0  0.0  |   0.0  1.0  |   0.0  1.0 |
     | -1.0  0.0  |  -2.0 -3.0  |   1.0  0.0  |   1.0  1.0 |
     | -----------|-------------|-------------|----------- |
     |  0.0  0.0  |   1.0  1.0  |   1.0  0.0  |  -1.0  1.0 |
     |  0.0 -1.0  |   0.0  0.0  |  -1.0  0.0  |   0.0  0.0 |
 3   | -1.0  1.0  |   0.0  1.0  |   0.0  1.0  |   0.0  1.0 |
     |  1.0  2.0  |   3.0  2.0  |   0.0  1.0  |  -1.0  0.0 |
     └                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |          0          |          1
-----|---------------------|---------------------
     | -1.0  0.0 -2.0  0.0 |  0.0  1.0  1.0 -1.0
     |  0.0  0.0 -1.0 -2.0 | -1.0 -1.0  1.0 -1.0
     |  0.0  0.0  1.0  1.0 |  1.0  1.0 -1.0  1.0
     |  1.0 -2.0  0.0 -1.0 |  0.0 -2.0  0.0 -1.0
 0   |  0.0  1.0 -2.0  0.0 |  1.0  4.0  0.0 -1.0
     |  0.0  0.0  0.0 -2.0 |  0.0 -2.0  1.0 -1.0
     |  0.0  1.0  0.0  1.0 | -1.0  0.0  0.0  1.0
     | -1.0  0.0  1.0  0.0 | -2.0 -3.0  1.0  1.0
-----|---------------------|---------------------
     | -1.0  3.0  1.0  3.0 |  0.0  1.0  0.0  2.0
     | -1.0 -1.0  1.0 -1.0 | -1.0 -3.0  1.0  0.0
     | -1.0  0.0 -1.0  2.0 | -1.0  2.0  0.0  1.0
     |  1.0  2.0  0.0  1.0 |  1.0  3.0 -1.0  0.0
 1   |  0.0  0.0  1.0  0.0 |  1.0  1.0 -1.0  1.0
     |  0.0 -1.0 -1.0  0.0 |  0.0  0.0  0.0  0.0
     | -1.0  1.0  0.0  1.0 |  0.0  1.0  0.0  1.0
     |  1.0  2.0  0.0  1.0 |  3.0  2.0 -1.0  0.0
```

Example 2: This example computes $\boldsymbol{C} = \beta\boldsymbol{C}+\alpha\boldsymbol{BA}$ using a 2×2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                SIDE  UPLO   M   N     ALPHA     A  IA  JA   DESC_A   B  IB   JB
                 |     |     |   |       |       |   |   |    |       |   |    |
CALL PZSYMM( 'R' , 'U' , 16 , 8 ,   ALPHA   , A , 1 , 1 , DESC_A , B , 1 , 1 ,

                DESC_B    BETA     C  IC  JC   DESC_C
                  |        |       |   |   |     |
                DESC_B ,  BETA   , C , 1 , 1 , DESC_C )

                ALPHA  =  (1.0, 2.0)

                BETA   =  (0.0, 0.0)
```

	Desc_A	**Desc_B**	**Desc_C**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	16	16
N_	8	8	8
MB_	2	4	4
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

	Desc_A	Desc_B	Desc_C

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on all processes, and LLD_B = LLD_C = 8 on all processes.

Global complex symmetric matrix ***A*** of order 8 with block size 2 × 2:

```
B,D         0                         1                         2                         3
     ┌                                                                                                  ┐
 0   │ ( 0.0, 1.0) (-1.0, 0.0) │ (-1.0, 0.0) ( 0.0, 1.0) │( 0.0, 1.0) ( 0.0, 1.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │      .      ( 1.0, 2.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │( 0.0, 1.0) ( 1.0, 2.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 1   │      .           .      │ (-1.0, 0.0) (-1.0, 0.0) │( 0.0, 1.0) ( 0.0, 1.0) │ ( 1.0, 2.0) ( 0.0, 1.0) │
     │      .           .      │      .      (-1.0, 0.0) │( 1.0, 2.0) ( 1.0, 2.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 2   │      .           .      │      .           .      │(-1.0, 0.0) ( 0.0, 1.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │      .           .      │      .           .      │     .      ( 1.0, 2.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 3   │      .           .      │      .           .      │     .           .      │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │      .           .      │      .           .      │     .           .      │      .      ( 0.0, 1.0) │
     └                                                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                        0                        |                        1
-----|-------------------------------------------------|---------------------------------------------
     | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
     |      .      ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
 0   |      .           .      (-1.0, 0.0) ( 0.0, 1.0) |      .           .      ( 0.0, 1.0) ( 0.0, 1.0)
     |      .           .           .      ( 1.0, 2.0) |      .           .      ( 0.0, 1.0) ( 0.0, 1.0)
-----|-------------------------------------------------|---------------------------------------------
     |      .           .      ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) (-1.0, 0.0) ( 1.0, 2.0) ( 0.0, 1.0)
     |      .           .      ( 1.0, 2.0) ( 1.0, 2.0) |      .      (-1.0, 0.0) ( 0.0, 1.0) ( 1.0, 2.0)
 1   |      .           .           .           .      |      .           .      ( 0.0, 1.0) ( 0.0, 1.0)
     |      .           .           .           .      |      .           .           .      ( 0.0, 1.0)
```

Global general 16 × 8 matrix ***B*** with block size 4 × 2:

```
 B,D             0                         1                         2                         3
       ┌                                                                                                     ┐
       │ (-1.0,-3.0) ( 0.0,-2.0) │ ( 1.0,-1.0) (-1.0,-3.0) │ ( 1.0,-1.0) ( 1.0,-1.0) │ (-1.0,-3.0) (-1.0,-3.0) │
       │ (-1.0,-3.0) (-1.0,-3.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │ ( 1.0,-1.0) (-1.0,-3.0) │ (-1.0,-3.0) ( 1.0,-1.0) │
  0    │ ( 1.0,-1.0) ( 1.0,-1.0) │ (-1.0,-3.0) ( 0.0,-2.0) │ (-1.0,-3.0) ( 0.0,-2.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │
       │ ( 0.0,-2.0) (-1.0,-3.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) (-1.0,-3.0) │
       │ ------------------------│-------------------------│-------------------------│------------------------ │
       │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │
       │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │ (-1.0,-3.0) (-1.0,-3.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │
  1    │ ( 1.0,-1.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 1.0,-1.0) ( 1.0,-1.0) │ ( 0.0,-2.0) (-1.0,-3.0) │
       │ ( 0.0,-2.0) ( 0.0,-2.0) │ (-1.0,-3.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │
       │ ------------------------│-------------------------│-------------------------│------------------------ │
       │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) (-1.0,-3.0) │ ( 1.0,-1.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │
       │ (-1.0,-3.0) (-1.0,-3.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │ ( 0.0,-2.0) (-1.0,-3.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │
  2    │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │
       │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 1.0,-1.0) ( 1.0,-1.0) │ ( 0.0,-2.0) (-1.0,-3.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │
       │ ------------------------│-------------------------│-------------------------│------------------------ │
       │ ( 1.0,-1.0) ( 1.0,-1.0) │ (-1.0,-3.0) ( 0.0,-2.0) │ (-1.0,-3.0) (-1.0,-3.0) │ ( 1.0,-1.0) ( 1.0,-1.0) │
       │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │ ( 0.0,-2.0) (-1.0,-3.0) │
  3    │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 0.0,-2.0) │
       │ (-1.0,-3.0) ( 0.0,-2.0) │ (-1.0,-3.0) ( 0.0,-2.0) │ ( 0.0,-2.0) ( 1.0,-1.0) │ ( 1.0,-1.0) ( 0.0,-2.0) │
       └                                                                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***B***:

```
 p,q  |                        0                        |                        1
 -----|-------------------------------------------------|---------------------------------------------------
      | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0) (-1.0,-3.0)
      | (-1.0,-3.0) (-1.0,-3.0) ( 1.0,-1.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) (-1.0,-3.0) ( 1.0,-1.0)
      | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
  0   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) (-1.0.-3.0) ( 0.0,-2.0) ( 1.0,-1.0)
      | (-1.0,-3.0) (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
 -----|-------------------------------------------------|---------------------------------------------------
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 1.0,-1.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0)
  1   | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
```

Output:

Global general 16 × 8 matrix ***C*** with block size 4 × 2:

```
 B,D            0                         1                         2                         3
       ┌                                                                                                    ┐
       | (11.0,27.0) (37.0,39.0) | ( 7.0,29.0) (27.0,39.0) | (27.0,29.0) (37.0,39.0) | (21.0,37.0) (35.0,35.0) |
       | ( 7.0,29.0) (37.0,39.0) | (11.0,27.0) (35.0,35.0) | (23.0,31.0) (45.0,35.0) | (21.0,37.0) (35.0,35.0) |
  0    | ( 1.0,27.0) (31.0,37.0) | (-3.0,29.0) (21.0,37.0) | ( 9.0,33.0) (27.0,39.0) | (23.0,31.0) (21.0,37.0) |
       | ( 6.0,32.0) (48.0,36.0) | (10.0,30.0) (42.0,34.0) | (22.0,34.0) (44.0,38.0) | (28.0,36.0) (38.0,36.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | (-4.0,22.0) (10.0,40.0) | (-8.0,24.0) (12.0,34.0) | ( 0.0,30.0) (10.0,40.0) | (10.0,30.0) ( 8.0,36.0) |
       | (11.0,27.0) (41.0,37.0) | (11.0,27.0) (43.0,31.0) | (15.0,35.0) (41.0,37.0) | (21.0,37.0) (31.0,37.0) |
  1    | (-1.0,23.0) (25.0,35.0) | (-1.0,23.0) (11.0,37.0) | (11.0,27.0) (17.0,39.0) | (13.0,31.0) (15.0,35.0) |
       | (-3.0,29.0) (23.0,41.0) | (-3.0,29.0) (13.0,41.0) | (13.0,31.0) (27.0,39.0) | (23.0,31.0) (25.0,35.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | (-2.0,26.0) (24.0,38.0) | (-6.0,28.0) ( 6.0,42.0) | (18.0,26.0) (28.0,36.0) | (16.0,32.0) (26.0,32.0) |
       | ( 7.0,29.0) (37.0,39.0) | ( 7.0,29.0) (39.0,33.0) | (19.0,33.0) (45.0,35.0) | (21.0,37.0) (35.0,35.0) |
  2    | (-2.0,26.0) (24.0,38.0) | ( 2.0,24.0) (22.0,34.0) | (10.0,30.0) (24.0,38.0) | (16.0,32.0) (18.0,36.0) |
       | ( 5.0,25.0) (31.0,37.0) | ( 9.0,23.0) (37.0,29.0) | ( 9.0,33.0) (31.0,37.0) | (15.0,35.0) (21.0,37.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | ( 1.0,27.0) (31.0,37.0) | (-3.0,29.0) (21.0,37.0) | ( 9.0,33.0) (31.0,37.0) | (23.0,31.0) (21.0,37.0) |
       | ( 4.0,28.0) (38.0,36.0) | ( 4.0,28.0) (28.0,36.0) | (20.0,30.0) (34.0,38.0) | (22.0,34.0) (28.0,36.0) |
  3    | ( 5.0,25.0) (27.0,39.0) | ( 1.0,27.0) (21.0,37.0) | (13.0,31.0) (27.0,39.0) | (19.0,33.0) (21.0,37.0) |
       | ( 0.0,30.0) (26.0,42.0) | (-8.0,34.0) (20.0,40.0) | (16.0,32.0) (30.0,40.0) | (26.0,32.0) (28.0,36.0) |
       └                                                                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***C***:

```
 p,q  |                       0                       |                       1
 -----|-----------------------------------------------|-----------------------------------------------
      | (11.0,27.0) (37.0,39.0) (27.0,29.0) (37.0,39.0) | ( 7.0,29.0) (27.0,39.0) (21.0,37.0) (35.0,35.0)
      | ( 7.0,29.0) (37.0,39.0) (23.0,31.0) (45.0,35.0) | (11.0,27.0) (35.0,35.0) (21.0,37.0) (35.0,35.0)
      | ( 1.0,27.0) (31.0,37.0) ( 9.0,33.0) (27.0,39.0) | (-3.0,29.0) (21.0,37.0) (23.0,31.0) (21.0,37.0)
      | ( 6.0,32.0) (48.0,36.0) (22.0,34.0) (44.0,38.0) | (10.0,30.0) (42.0,34.0) (28.0,36.0) (38.0,36.0)
  0   | (-2.0,26.0) (24.0,38.0) (18.0,26.0) (28.0,36.0) | (-6.0,28.0) ( 6.0,42.0) (16.0,32.0) (26.0,32.0)
      | ( 7.0,29.0) (37.0,39.0) (19.0,33.0) (45.0,35.0) | ( 7.0,29.0) (39.0,33.0) (21.0,37.0) (35.0,35.0)
      | (-2.0,26.0) (24.0,38.0) (10.0,30.0) (24.0,38.0) | ( 2.0,24.0) (22.0,34.0) (16.0,32.0) (18.0,36.0)
      | ( 5.0,25.0) (31.0,37.0) ( 9.0,33.0) (31.0,37.0) | ( 9.0,23.0) (37.0,29.0) (15.0,35.0) (21.0,37.0)
 -----|-----------------------------------------------|-----------------------------------------------
      | (-4.0,22.0) (10.0,40.0) ( 0.0,30.0) (10.0,40.0) | (-8.0,24.0) (12.0,34.0) (10.0,30.0) ( 8.0,36.0)
      | (11.0,27.0) (41.0,37.0) (15.0,35.0) (41.0,37.0) | (11.0,27.0) (43.0,31.0) (21.0,37.0) (31.0,37.0)
      | (-1.0,23.0) (25.0,35.0) (11.0,27.0) (17.0,39.0) | (-1.0,23.0) (11.0,37.0) (13.0,31.0) (15.0,35.0)
      | (-3.0,29.0) (23.0,41.0) (13.0,31.0) (27.0,39.0) | (-3.0,29.0) (13.0,41.0) (23.0,31.0) (25.0,35.0)
  1   | ( 1.0,27.0) (31.0,37.0) ( 9.0,33.0) (31.0,37.0) | (-3.0,29.0) (21.0,37.0) (23.0,31.0) (21.0,37.0)
      | ( 4.0,28.0) (38.0,36.0) (20.0,30.0) (34.0,38.0) | ( 4.0,28.0) (28.0,36.0) (22.0,34.0) (28.0,36.0)
      | ( 5.0,25.0) (27.0,39.0) (13.0,31.0) (27.0,39.0) | ( 1.0,27.0) (21.0,37.0) (19.0,33.0) (21.0,37.0)
      | ( 0.0,30.0) (26.0,42.0) (16.0,32.0) (30.0,40.0) | (-8.0,34.0) (20.0,40.0) (26.0,32.0) (28.0,36.0)
```

Example 3: This example computes $\boldsymbol{C} = \beta \boldsymbol{C} + \alpha \boldsymbol{BA}$ using a 2 × 2 process grid.

Note: The imaginary parts of the diagonal elements of a complex Hermitian matrix are assumed to be zero, so you do not have to set these values.

Call Statements and Input

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET(0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|              SIDE  UPLO   M   N     ALPHA     A  IA  JA   DESC_A   B  IB   JB
|                |     |    |   |       |       |   |   |    |       |   |    |
| CALL PZHEMM( 'R' , 'U' , 16 , 8 ,   ALPHA   , A , 1 , 1 , DESC_A , B , 1 , 1 ,

|              DESC_B    BETA     C  IC  JC   DESC_C
|                |         |      |   |   |    |
|              DESC_B ,  BETA   , C , 1 , 1 , DESC_C )

|              ALPHA  =  (1.0, 0.0)

|              BETA   =  (0.0, 0.0)
```

	Desc_A	Desc_B	Desc_C
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	16	16
N_	8	8	8
MB_	2	4	4
NB_	2	2	2
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on all processes, and LLD_B = LLD_C = 8 on all processes.

Global Hermitian matrix ***A*** of order 8 with block size 2×2:

```
B,D            0                         1                         2                         3
     ┌                                                                                                    ┐
 0   │ ( 0.0, 0.0) (-1.0, 0.0) │ (-1.0, 0.0) ( 0.0, 1.0) │( 0.0, 1.0) ( 0.0, 1.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │      .      ( 1.0, 0.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │( 0.0, 1.0) ( 1.0, 2.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 1   │      .          .       │ (-1.0, 0.0) (-1.0, 0.0) │( 0.0, 1.0) ( 0.0, 1.0) │ ( 1.0, 2.0) ( 0.0, 1.0) │
     │      .          .       │      .      (-1.0, 0.0) │( 1.0, 2.0) ( 1.0, 2.0) │ ( 0.0, 1.0) ( 1.0, 2.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 2   │      .          .       │      .          .       │(-1.0, 0.0) ( 0.0, 1.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │      .          .       │      .          .       │     .      ( 1.0, 0.0) │ ( 0.0, 1.0) ( 0.0, 1.0) │
     │ ------------------------|-------------------------|------------------------|------------------------ │
 3   │      .          .       │      .          .       │     .          .       │ ( 0.0, 0.0) ( 0.0, 1.0) │
     │      .          .       │      .          .       │     .          .       │      .      ( 0.0, 0.0) │
     └                                                                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                         0                         |                        1
-----|---------------------------------------------------|----------------------------------------------
     | ( 0.0,  . ) (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
     |      .      ( 1.0,  . ) ( 0.0, 1.0) ( 1.0, 2.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
 0   |      .          .       (-1.0,  . ) ( 0.0, 1.0) |      .           .      ( 0.0, 1.0) ( 0.0, 1.0)
     |      .          .            .      ( 1.0,  . ) |      .           .      ( 0.0, 1.0) ( 0.0, 1.0)
-----|---------------------------------------------------|----------------------------------------------
     |      .          .       ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0,  . ) (-1.0, 0.0) ( 1.0, 2.0) ( 0.0, 1.0)
     |      .          .       ( 1.0, 2.0) ( 1.0, 2.0) |      .      (-1.0,  . ) ( 0.0, 1.0) ( 1.0, 2.0)
 1   |      .          .            .           .      |      .           .      ( 0.0,  . ) ( 0.0, 1.0)
     |      .          .            .           .      |      .           .           .      ( 0.0,  . )
```

Global general 16 × 8 matrix ***B*** with block size 4 × 2:

```
 B,D            0                       1                       2                       3
       ┌                                                                                                   ┐
       | (-1.0,-3.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 1.0,-1.0) | (-1.0,-3.0) (-1.0,-3.0) |
       | (-1.0,-3.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) | (-1.0,-3.0) ( 1.0,-1.0) |
  0    | ( 1.0,-1.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) |
       | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 0.0,-2.0) |
       | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
  1    | ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) (-1.0,-3.0) |
       | ( 0.0,-2.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) |
       | (-1.0,-3.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) |
  2    | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
       | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
       | ------------------------|-------------------------|-------------------------|------------------------ |
       | ( 1.0,-1.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 1.0,-1.0) |
       | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) |
  3    | ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
       | (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 0.0,-2.0) |
       └                                                                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***B***:

```
 p,q  |                       0                       |                       1
 -----|-----------------------------------------------|-----------------------------------------------
      | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0) (-1.0,-3.0)
      | (-1.0,-3.0) (-1.0,-3.0) ( 1.0,-1.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) (-1.0,-3.0) ( 1.0,-1.0)
      | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
  0   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0)
      | (-1.0,-3.0) (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
 -----|-----------------------------------------------|-----------------------------------------------
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 1.0,-1.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0)
  1   | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0)
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) (-1.0,-3.0)
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
```

Output:

Global general 16 × 8 matrix ***C*** with block size 4 × 2:

```
B,D            0                          1                          2                          3
     ┌                                                                                                     ┐
     │ (-12.0,4.0) (-19.0,-5.0) │ (-9.0, 5.0) (-5.0,-5.0) │ ( 3.0,-3.0) ( 9.0,-5.0) │ (10.0, 2.0) (18.0,-6.0) │
     │ (-10.0,4.0) (-17.0,-7.0) │ (-9.0, 3.0) (-5.0,-9.0) │ ( 3.0,-1.0) ( 9.0,-9.0) │ (14.0,-2.0) (20.0,-8.0) │
 0   │ (-10.0,4.0) (-19.0,-5.0) │ (-7.0, 5.0) (-11.0,1.0) │ ( 5.0, 1.0) (11.0,-5.0) │ (12.0,-4.0) (17.0,-1.0) │
     │ (-10.0,6.0) (-22.0,-6.0) │ (-8.0, 4.0) (-10.0,-6.0)│ ( 4.0, 0.0) (10.0,-8.0) │ (12.0,-2.0) (19.0,-7.0) │
     │ -------------------------│-------------------------│-------------------------│------------------------ │
     │ (-8.0, 0.0) (-10.0,-8.0) │ (-6.0, 2.0) (-6.0,-4.0) │ ( 4.0, 2.0) (10.0, 0.0) │ ( 9.0, 1.0) (14.0, 4.0) │
     │ (-13.0,5.0) (-21.0,-5.0) │ (-11.0,5.0) (-15.0,-3.0)│ ( 3.0, 3.0) ( 9.0,-7.0) │ (13.0,-1.0) (19.0,-5.0) │
 1   │ (-10.0,2.0) (-17.0,-7.0) │ (-9.0, 3.0) (-7.0,-1.0) │ ( 1.0, 1.0) ( 7.0, 1.0) │ ( 7.0, 3.0) (14.0, 2.0) │
     │ (-7.0, 3.0) (-13.0,-7.0) │ (-5.0, 3.0) (-1.0,-5.0) │ ( 7.0,-3.0) (13.0,-7.0) │ (13.0,-5.0) (18.0,-4.0) │
     │ -------------------------│-------------------------│-------------------------│------------------------ │
     │ (-8.0, 2.0) (-14.0,-8.0) │ (-4.0, 2.0) ( 2.0,-6.0) │ ( 6.0,-6.0) (12.0,-8.0) │ (12.0,-2.0) (17.0,-5.0) │
     │ (-10.0,4.0) (-17.0,-7.0) │ (-7.0, 3.0) (-7.0,-9.0) │ ( 5.0,-1.0) (11.0,-11.0)│ (15.0,-3.0) (20.0,-8.0) │
 2   │ (-8.0, 2.0) (-14.0,-8.0) │ (-8.0, 2.0) (-6.0,-6.0) │ ( 2.0, 2.0) ( 8.0,-2.0) │ (10.0, 0.0) (16.0, 0.0) │
     │ (-11.0,3.0) (-17.0,-7.0) │ (-11.0,3.0) (-13.0,-5.0)│ ( 1.0, 5.0) ( 7.0,-3.0) │ (11.0, 1.0) (17.0,-1.0) │
     │ -------------------------│-------------------------│-------------------------│------------------------ │
     │ (-10.0,4.0) (-19.0,-5.0) │ (-7.0, 5.0) (-11.0,1.0) │ ( 5.0, 1.0) (11.0,-7.0) │ (14.0,-6.0) (18.0,-2.0) │
     │ (-10.0,4.0) (-20.0,-6.0) │ (-8.0, 4.0) (-8.0,-4.0) │ ( 2.0, 0.0) ( 8.0,-4.0) │ (10.0, 0.0) (17.0,-3.0) │
 3   │ (-11.0,3.0) (-17.0,-5.0) │ (-9.0, 5.0) (-9.0,-1.0) │ ( 3.0, 1.0) ( 9.0,-3.0) │ (11.0,-1.0) (17.0,-1.0) │
     │ (-7.0, 3.0) (-14.0,-6.0) │ (-2.0, 4.0) (-2.0,-6.0) │ ( 8.0,-4.0) (14.0,-8.0) │ (13.0,-5.0) (18.0,-4.0) │
     └                                                                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |                          0                             |                          1
-----|--------------------------------------------------------|-------------------------------------------------------
     | (-12.0,  4.0) (-19.0, -5.0) (  3.0, -3.0) (  9.0, -5.0) | ( -9.0,  5.0) ( -5.0, -5.0) ( 10.0,  2.0) ( 18.0, -6.0)
     | (-10.0,  4.0) (-17.0, -7.0) (  3.0, -1.0) (  9.0, -9.0) | ( -9.0,  3.0) ( -5.0, -9.0) ( 14.0, -2.0) ( 20.0, -8.0)
     | (-10.0,  4.0) (-19.0, -5.0) (  5.0,  1.0) ( 11.0, -5.0) | ( -7.0,  5.0) (-11.0,  1.0) ( 12.0, -4.0) ( 17.0, -1.0)
     | (-10.0,  6.0) (-22.0, -6.0) (  4.0,  0.0) ( 10.0, -8.0) | ( -8.0,  4.0) (-10.0, -6.0) ( 12.0, -2.0) ( 19.0, -7.0)
 0   | ( -8.0,  2.0) (-14.0, -8.0) (  6.0, -6.0) ( 12.0, -8.0) | ( -4.0,  2.0) (  2.0, -6.0) ( 12.0, -2.0) ( 17.0, -5.0)
     | (-10.0,  4.0) (-17.0, -7.0) (  5.0, -1.0) ( 11.0,-11.0) | ( -7.0,  3.0) ( -7.0, -9.0) ( 15.0, -3.0) ( 20.0, -8.0)
     | ( -8.0,  2.0) (-14.0, -8.0) (  2.0,  2.0) (  8.0, -2.0) | ( -8.0,  2.0) ( -6.0, -6.0) ( 10.0,  0.0) ( 16.0,  0.0)
     | (-11.0,  3.0) (-17.0, -7.0) (  1.0,  5.0) (  7.0, -3.0) | (-11.0,  3.0) (-13.0, -5.0) ( 11.0,  1.0) ( 17.0, -1.0)
-----|--------------------------------------------------------|-------------------------------------------------------
     | ( -8.0,  0.0) (-10.0, -8.0) (  4.0,  2.0) ( 10.0,  0.0) | ( -6.0,  2.0) ( -6.0, -4.0) (  9.0,  1.0) ( 14.0,  4.0)
     | (-13.0,  5.0) (-21.0, -5.0) (  3.0,  3.0) (  9.0, -7.0) | (-11.0,  5.0) (-15.0, -3.0) ( 13.0, -1.0) ( 19.0, -5.0)
     | (-10.0,  2.0) (-17.0, -7.0) (  1.0,  1.0) (  7.0,  1.0) | ( -9.0,  3.0) ( -7.0, -1.0) (  7.0,  3.0) ( 14.0,  2.0)
     | ( -7.0,  3.0) (-13.0, -7.0) (  7.0, -3.0) ( 13.0, -7.0) | ( -5.0,  3.0) ( -1.0, -5.0) ( 13.0, -5.0) ( 18.0, -4.0)
 1   | (-10.0,  4.0) (-19.0, -5.0) (  5.0,  1.0) ( 11.0, -7.0) | ( -7.0,  5.0) (-11.0,  1.0) ( 14.0, -6.0) ( 18.0, -2.0)
     | (-10.0,  4.0) (-20.0, -6.0) (  2.0,  0.0) (  8.0, -4.0) | ( -8.0,  4.0) ( -8.0, -4.0) ( 10.0,  0.0) ( 17.0, -3.0)
     | (-11.0,  3.0) (-17.0, -5.0) (  3.0,  1.0) (  9.0, -3.0) | ( -9.0,  5.0) ( -9.0, -1.0) ( 11.0, -1.0) ( 17.0, -1.0)
     | ( -7.0,  3.0) (-14.0, -6.0) (  8.0, -4.0) ( 14.0, -8.0) | ( -2.0,  4.0) ( -2.0, -6.0) ( 13.0, -5.0) ( 18.0, -4.0)
```

PDTRMM and PZTRMM—Triangular Matrix-Matrix Product

PDTRMM computes one of the following matrix-matrix products:

1. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$
2. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{T}$

| PZTRMM computes one of the following matrix-matrix products:

| 1. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$
| 2. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{T}$
5. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{H}\boldsymbol{B}$
6. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{H}$

where, in the formulas above:

A represents the global triangular submatrix:
- For *side* = `'L'`, it is $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+m-1}$.
- For *side* = `'R'`, it is $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.

B represents the global general submatrix $\boldsymbol{B}_{ib:ib+m-1,\ jb:jb+n-1}$.
α is a scalar.

Note: No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix ***A*** should always be stored in its untransposed form.

If $m = 0$ or $n = 0$, no computation is performed, and the subroutine returns after doing some parameter checking.

See references [14] and [15].

Table 51. Data Types

α, ***A***, ***B***	**Subprogram**
Long-precision real	PDTRMM
Long-precision complex	PZTRMM

Syntax

Fortran	CALL PDTRMM | PZTRMM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*)
C and C++	pdtrmm | pztrmm (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*);

On Entry

side
indicates whether ***A*** is located to the left or right of ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B***.

If *side* = `'R'`, ***A*** is to the right of ***B***.

Scope: **global**

Specified as: a single character; *side* = `'L'` or `'R'`.

uplo
indicates whether the upper or lower triangular part of the global triangular submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

Scope: **global**

Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Scope: **global**

Specified as: a single character; *diag* = `'U'` or `'N'`.

m

is the number of rows in submatrix ***B***, and:

If *side* = `'L'`, it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrix ***B***, and:

If *side* = `'R'`, it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 51 on page 321.

a

is the local part of the global triangular matrix ***A***. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, assuming the following:

If *side* = `'L'`, *numa* = *m*

If *side* = `'R'`, *numa* = *n*

the leading LOCp(*ia*+*numa*–1) by LOCq(*ja*+*numa*–1) part of the local array `A` must contain the local pieces of the leading *ia*+*numa*–1 by *ja*+ *numa*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading *numa* × *numa* upper triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading *numa* × *numa* lower triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Note: No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 51 on page 321. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+numa-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+numa-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If *m* = 0 and *side* = 'L' or *n* = 0 and *side* = 'R': M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If *m* = 0 and *side* = 'L' or *n* = 0 and *side* = 'R': N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the *p* × *q* grid over which the first row of the global matrix is distributed	0 ≤ RSRC_A < *p*	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ib*+*m*–1) by LOCq(*jb*+*n*–1) part of the local array B must contain the local pieces of the leading *ib*+*m*–1 by *jb*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 51 on page 321. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B and $ib+m-1 \le$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B and $jb+n-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global

desc_b	**Name**	**Description**	**Limits**	**Scope**
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_B $< p$	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_B $< q$	Global
9	LLD_B	The leading dimension of the local array	LLD_B $\ge$ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

b

is the updated local part of the global matrix ***B***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 51 on page 321.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *side*, *uplo*, *transa*, and *diag* arguments.
2. | For PDTRMM, if you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
3. The matrices must have no common elements; otherwise, results are unpredictable.
4. | PDTRMM and PZTRMM assume certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the lower and upper triangular part, respectively, are assumed to be zero.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
6. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
7. The following values must be equal: CTXT_A = CTXT_B.
8. If ***A*** is **not** contained within a single block, that is:

   numa+mod(*ia*–1, MB_A) $>$ MB_A
   numa+mod(*ja*–1, NB_A) $>$ NB_A
   where:
   If *side* = `'L'`, *numa* = *m*
   If *side* = `'R'`, *numa* = *n*

then:

- The global triangular matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
- The global triangular matrix ***A*** must be aligned on a block boundary, that is:

  ia–1 must be a multiple of MB_A.
  ja–1 must be a multiple of NB_A.

9. If *side* = `'L'`:
   - If ***A*** is **not** contained within a single block, then:
     - The following block sizes must be equal: MB_B = NB_A.
     - The global matrix ***B*** must be aligned on a block row boundary; that is, *ib*–1 must be a multiple of MB_B.
   - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

     iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
     ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
   - If ***A*** is contained within a single block, then ***B*** must be a block row matrix; that is, if $p > 1$:

     m+mod(*ib*–1, MB_B) $\le$ MB_B
10. If *side* = `'R'`:
    - If ***A*** is **not** contained within a single block, then:
      - The following block sizes must be equal: NB_B = MB_A
      - The global matrix ***B*** must be aligned on a block column boundary; that is, *jb*–1 must be a multiple of NB_B.
    - In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***B***, that is, *iacol* = *ibcol*, where:

      iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
      ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
    - If ***A*** is contained within a single block, then ***B*** must be a block column matrix; that is, if $q > 1$:

      n+mod(*jb*–1, NB_B) $\le$ NB_B

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *side* $\neq$ 'L' or 'R'
2. *uplo* $\neq$ 'U' or 'L'
3. *transa* $\neq$ 'N', 'T', or 'C'
4. *diag* $\neq$ 'N' or 'U'
5. $m < 0$
6. $n < 0$
7. M_A < 0 and $m = 0$ and *side* = 'L'; M_A < 0 and $n = 0$ and *side* = 'R'; M_A < 1 otherwise
8. N_A < 0 and $m = 0$ and *side* = 'L'; N_A < 0 and $n = 0$ and *side* = 'R'; N_A < 1 otherwise
9. MB_A < 1
10. NB_A < 1
11. M_B < 0 and ($m = 0$ or $n = 0$); M_B < 1 otherwise
12. N_B < 0 and ($m = 0$ or $n = 0$); N_B < 1 otherwise
13. MB_B < 1
14. NB_B < 1
15. RSRC_A < 0 or RSRC_A $\geq p$
16. CSRC_A < 0 or CSRC_A $\geq q$
17. RSRC_B < 0 or RSRC_B $\geq p$
18. CSRC_B < 0 or CSRC_B $\geq q$
19. $ia < 1$
20. $ja < 1$
21. $ib < 1$
22. $jb < 1$
23. CTXT_A $\neq$ CTXT_B

Stage 5

1. MB_A $\neq$ NB_A

If ***A*** is **not** contained within a single block, that is:

numa+mod(*ia*–1, MB_A) > MB_A
numa+mod(*ja*–1, NB_A) > NB_A
where:
If *side* = 'L', *numa* = *m*
If *side* = 'R', *numa* = *n*

and:

2. *side* = 'L' and MB_B $\neq$ NB_A
3. *side* = 'R' and NB_B $\neq$ MB_A

If ($m \neq 0$ or *side* $\neq$ 'L') and ($n \neq 0$ or *side* $\neq$ 'R'):

1. *ia* > M_A
2. *ja* > N_A
3. *ia*+*numa*–1 > M_A

4. *ja*+*numa*–1 > N_A

   where *numa* = *m* if *side* = `'L'` and *numa* = *n* if *side* = `'R'`.

If *m* ≠ 0 and *n* ≠ 0:

1. *ib* > M_B
2. *jb* > N_B
3. *ib*+*m*–1 > M_B
4. *jb*+*n*–1 > N_B

If ***A*** is not contained in a single block:

1. mod(*ia*–1, MB_A) ≠ 0
2. mod(*ja*–1, NB_A) ≠ 0
3. *side* = `'L'` and mod(*ib*–1, MB_B) ≠ 0
4. *side* = `'R'` and mod(*jb*–1, NB_B) ≠ 0

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_B < max(1, LOCp(M_B))

If *side* = `'L'`:

3. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* ≠ *ibrow*, where:
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
4. If ***A*** is contained in a single block:
   p > 1 and *m*+mod(*ib*–1, MB_B) > MB_B

If *side* = `'R'`:

5. In the process grid, the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***B***; that is, *iacol* ≠ *ibcol*, where:
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
   ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
6. If ***A*** is contained in a single block:
   q > 1 and *n*+mod(*jb*–1, NB_B) > NB_B

Example 1: This example computes ***B*** = α***AB*** using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                SIDE  UPLO  TRANSA  DIAG  M   N    ALPHA    A  IA  JA   DESC_A
                 |     |      |      |    |   |      |      |   |   |      |
CALL PDTRMM( 'L' , 'U'  , 'N'  , 'N' , 5 , 3 ,  1.0D0  , A , 1 , 1 , DESC_A ,

             B  IB  JB   DESC_B
             |   |   |     |
             B , 1 , 1 , DESC_B )
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	5	5
N_	5	3
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 2 on P_{10} and P_{11}.

Global triangular matrix ***A*** of order 5 is upper triangular with block size 2 × 2:

```
B,D       0            1          2
     ┌                                  ┐
 0   |  3.0 -1.0  |   2.0  2.0  |   1.0 |
     |    .  -2.0 |   4.0 -1.0  |   3.0 |
     | -----------|-------------|------ |
 1   |    .    .  |  -3.0  0.0  |   2.0 |
     |    .    .  |     .  4.0  |  -2.0 |
     | -----------|-------------|------ |
 2   |    .    .  |     .    .  |   1.0 |
     └                                  ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |        0         |      1
-----|------------------|------------
     |  3.0 -1.0  1.0   |   2.0  2.0
 0   |   .  -2.0  3.0   |   4.0 -1.0
     |   .    .   1.0   |    .    .
-----|------------------|------------
 1   |   .    .   2.0   |  -3.0  0.0
     |   .    .  -2.0   |    .   4.0
```

Global rectangular 5 × 3 matrix ***B*** with block size 2 × 2:

```
B,D       0         1
     ┌                 ┐
 0   |  2.0  3.0  |   1.0 |
     |  5.0  5.0  |   4.0 |
     | -----------|------ |
 1   |  0.0  1.0  |   2.0 |
     |  3.0  1.0  |  -3.0 |
     | -----------|------ |
 2   | -1.0  2.0  |   1.0 |
     └                 ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |      0     |   1
-----|------------|-------
     |  2.0  3.0  |   1.0
 0   |  5.0  5.0  |   4.0
     | -1.0  2.0  |   1.0
-----|------------|-------
 1   |  0.0  1.0  |   2.0
     |  3.0  1.0  |  -3.0
```

Output:

Global rectangular 5 × 3 matrix ***B*** with block size 2 × 2:

```
B,D         0           1
     ┌                      ┐
 0   │   6.0  10.0  │   -2.0 │
     │ -16.0  -1.0  │    6.0 │
     │ -------------|------- │
 1   │  -2.0   1.0  │   -4.0 │
     │  14.0   0.0  │  -14.0 │
     │ -------------|------- │
 2   │  -1.0   2.0  │    1.0 │
     └                      ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  │      0       │    1
-----|--------------|--------
     │   6.0  10.0  │   -2.0
 0   │ -16.0  -1.0  │    6.0
     │  -1.0   2.0  │    1.0
-----|--------------|--------
 1   │  -2.0   1.0  │   -4.0
     │  14.0   0.0  │  -14.0
```

Example 2: This example computes ***B*** = α***AB*** using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             SIDE  UPLO  TRANSA  DIAG  M   N    ALPHA    A  IA  JA   DESC_A
               |     |      |      |   |   |      |      |   |   |     |
CALL PZTRMM( 'L' , 'U'  , 'C'  , 'N' , 5 , 1 ,  ALPHA  , A , 1 , 1 , DESC_A ,

             B  IB  JB   DESC_B
             |   |   |     |
             B , 1 , 1 , DESC_B )

             ALPHA = (1.0, 0.0)
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	5	5

	Desc_A	Desc_B
N_	5	1
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 2 on P_{10} and P_{11}.

Global triangular matrix ***A*** of order 5 is upper triangular with block size 2 × 2:

```
B,D             0                        1                  2
     ┌                                                            ┐
 0   | (-4.0, 1.0) ( 4.0,-3.0) | (-1.0, 3.0) ( 0.0, 0.0) | (-1.0, 0.0) |
     |      .      (-2.0, 0.0) | (-3.0,-1.0) (-2.0,-1.0) | ( 4.0, 3.0) |
     | ------------------------|-------------------------|------------ |
 1   |      .           .      | (-5.0, 3.0) (-3.0,-3.0) | (-5.0,-5.0) |
     |      .           .      |      .      ( 4.0,-4.0) | ( 2.0, 0.0) |
     | ------------------------|-------------------------|------------ |
 2   |      .           .      |      .           .      | ( 2.0,-1.0) |
     └                                                            ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                 0                   |            1
-----|-------------------------------------|-------------------------
     | (-4.0, 1.0) ( 4.0,-3.0) (-1.0, 0.0) | (-1.0, 3.0) ( 0.0, 0.0)
 0   |      .      (-2.0, 0.0) ( 4.0, 3.0) | (-3.0,-1.0) (-2.0,-1.0)
     |      .           .      ( 2.0,-1.0) |      .           .
-----|-------------------------------------|-------------------------
 1   |      .           .      (-5.0,-5.0) | (-5.0, 3.0) (-3.0,-3.0)
     |      .           .      ( 2.0, 0.0) |      .      ( 4.0,-4.0)
```

Global rectangular 5 × 1 matrix ***B*** with block size 2 × 2:

```
B,D        0
     ┌             ┐
 0   | ( 3.0, 4.0) |
     | (-4.0, 2.0) |
     | ----------- |
 1   | (-5.0, 0.0) |
     | ( 1.0, 3.0) |
     | ----------- |
 2   | ( 3.0, 1.0) |
     └             ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |      0      |      1
-----|-------------|------------
     | ( 3.0, 4.0) |      .
 0   | (-4.0, 2.0) |      .
     | ( 3.0, 1.0) |      .
-----|-------------|------------
 1   | (-5.0, 0.0) |      .
     | ( 1.0, 3.0) |      .
```

Output:

Global rectangular 5 × 1 matrix ***B*** with block size 2 × 2:

```
B,D        0
     ┌               ┐
 0   | (-8.0,-19.0)  |
     | ( 8.0, 21.0)  |
     | ------------- |
 1   | (44.0, -8.0)  |
     | (13.0, -7.0)  |
     | ------------- |
 2   | (19.0,  2.0)  |
     └               ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |       0       |      1
-----|---------------|------------
     | (-8.0,-19.0)  |      .
 0   | ( 8.0, 21.0)  |      .
     | (19.0,  2.0)  |      .
-----|---------------|------------
 1   | (44.0, -8.0)  |      .
     | (13.0, -7.0)  |      .
```

PDTRSM and PZTRSM—Solution of Triangular System of Equations with Multiple Right-Hand Sides

PDTRSM perform one of the following solves for a triangular system of equations with multiple right-hand sides, using scalar α, rectangular matrix $\boldsymbol{B}$, and triangular matrix $\boldsymbol{A}$ or its transpose:

Solution	Equation
1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$	$\boldsymbol{AX} = \alpha\boldsymbol{B}$
2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$	$\boldsymbol{A}^{T}\boldsymbol{X} = \alpha\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$	$\boldsymbol{XA} = \alpha\boldsymbol{B}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$	$\boldsymbol{XA}^{T} = \alpha\boldsymbol{B}$

PZTRSM performs one of the following solves for a triangular system of equations with multiple right-hand sides, using scalar α, rectangular matrix $\boldsymbol{B}$, and triangular matrix $\boldsymbol{A}$, its transpose, or its conjugate transpose:

Solution	Equation
1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$	$\boldsymbol{AX} = \alpha\boldsymbol{B}$
2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$	$\boldsymbol{A}^{T}\boldsymbol{X} = \alpha\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$	$\boldsymbol{XA} = \alpha\boldsymbol{B}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$	$\boldsymbol{XA}^{T} = \alpha\boldsymbol{B}$
5. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-H})\boldsymbol{B}$	$\boldsymbol{A}^{H}\boldsymbol{X} = \alpha\boldsymbol{B}$
6. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-H})$	$\boldsymbol{XA}^{H} = \alpha\boldsymbol{B}$

where, in the formulas above:

$\boldsymbol{A}$ represents the global triangular submatrix:
- For *side* = 'L', it is $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+m-1}$.
- For *side* = 'R', it is $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.

$\boldsymbol{B}$ represents the global general submatrix $\boldsymbol{B}_{ib:ib+m-1,\ jb:jb+n-1}$.
α is a scalar.

Notes:

1. The term $\boldsymbol{X}$ used in the systems of equations listed above represents the output solution matrix. It is important to note that, in this subroutine, the solution matrix is actually returned in the input-output argument *b*.
2. No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix $\boldsymbol{A}$ should always be stored in its untransposed form.

If $m = 0$ or $n = 0$, no computation is performed, and the subroutine returns after doing some parameter checking.

See references [14] and [15].

Table 52. Data Types

α, $\boldsymbol{A}$, $\boldsymbol{B}$	**Subprogram**
Long-precision real	PDTRSM
Long-precision complex	PZTRSM

Syntax

Fortran	CALL PDTRSM | PZTRSM (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*)
C and C++	pdtrsm | pztrsm (*side*, *uplo*, *transa*, *diag*, *m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*);

On Entry

side

indicates whether ***A*** is located to the left or right of ***B*** in the system of equations, where:

If *side* = 'L', ***A*** is to the left of ***B***.

If *side* = 'R', ***A*** is to the right of ***B***.

Scope: **global**

Specified as: a single character; *side* = 'L' or 'R'.

uplo

indicates whether the upper or lower triangular part of the global triangular submatrix ***A*** is referenced, where:

If *uplo* = 'U', the upper triangular part is referenced.

If *uplo* = 'L', the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = 'U' or 'L'.

transa

indicates the form of matrix ***A*** used in the system of equations, where:

If *transa* = 'N', ***A*** is used.

If *transa* = 'T', $\boldsymbol{A}^T$ is used.

If *transa* = 'C', $\boldsymbol{A}^H$ is used.

Scope: **global**

Specified as: a single character; *transa* = 'N', 'T', or 'C'.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = 'U', ***A*** is a unit triangular matrix.

If *diag* = 'N', ***A*** is not a unit triangular matrix.

Scope: **global**

Specified as: a single character; *diag* = 'U' or 'N'.

m

is the number of rows in submatrix ***B***, and:

If *side* = 'L', it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \ge 0$.

n

is the number of columns in submatrix ***B***, and:

If *side* = 'R', it is the number of rows and columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 52 on page 335.

a

is the local part of the global triangular matrix ***A***, used in the system of equations. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, assuming the following:

If *side* = 'L', *numa* = *m*

If *side* = 'R', *numa* = *n*

the leading LOCp(*ia*+*numa*–1) by LOCq(*ja*+*numa*–1) part of the local array A must contain the local pieces of the leading *ia*+*numa*–1 by *ja*+*numa*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading *numa* × *numa* upper triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading *numa* × *numa* lower triangular part of the global triangular submatrix $\boldsymbol{A}_{ia:ia+numa-1,\ ja:ja+numa-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

| **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 52 on page 335. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+numa-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+numa-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If *side* = 'L' and *m* = 0: M_A ≥ 0 If *side* = 'R' and *n* = 0: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If *side* = 'L' and *m* = 0: N_A ≥ 0 If *side* = 'R' and *n* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the *p* × *q* grid over which the first row of the global matrix is distributed	0 ≤ RSRC_A < *p*	Global
8	CSRC_A	The process column of the *p* × *q* grid over which the first column of the global matrix is distributed	0 ≤ CSRC_A < *q*	Global
9	LLD_A	The leading dimension of the local array	LLD_A ≥ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the right-hand sides of the triangular system to be solved. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ib*+*m*–1) by LOCq(*jb*+*n*–1) part of the local array B must contain the local pieces of the leading *ib*+*m*–1 by *jb*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 52 on page 335. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ib* ≤ M_B and *ib*+*m*–1 ≤ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B and $jb+n-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If *side* = 'L' and *m* = 0: M_B ≥ 0 If *side* = 'R' and *n* = 0: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	If *side* = 'L' and *m* = 0: N_B ≥ 0 If *side* = 'R' and *n* = 0: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_B < *q*	Global
9	LLD_B	The leading dimension of the local array	LLD_B ≥ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

b

is the updated local part of the global matrix ***B***, containing the *n* solution vectors of length *m*.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 52 on page 335.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *side*, *uplo*, *transa*, and *diag* arguments.
2. For PDTRSM, if you specify 'C' for *transa*, it is interpreted as though you specified 'T'.

3. The matrices must have no common elements; otherwise, results are unpredictable.
4. PDTRSM and PZTRSM assume certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the lower and upper triangular part, respectively, are assumed to be zero.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
6. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
7. The following values must be equal: CTXT_A = CTXT_B.
8. If looping is required—that is, **either** of the following is true:

   side = `'L'` and *m*+mod(*ia*–1, MB_A) > MB_A
   side = `'R'` and *n*+mod(*ja*–1, NB_A) > NB_A

   then the global triangular matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
9. If ***A*** is **not** contained within a single block, that is:

   numa+mod(*ia*–1, MB_A) > MB_A
   numa+mod(*ja*–1, NB_A) > NB_A
   where:
   If *side* = `'L'`, *numa* = *m*
   If *side* = `'R'`, *numa* = *n*

   then the global triangular matrix ***A*** must be aligned on a block boundary, that is:

   ia–1 must be a multiple of MB_A.
   ja–1 must be a multiple of NB_A.
10. If *side* = `'L'`:
    - If ***A*** is **not** contained within a single block, then:
      – The following block sizes must be equal: MB_B = NB_A
      – The global matrix ***B*** must be aligned on a block row boundary; that is, *ib*–1 must be a multiple of MB_B.
    - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

      iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
      ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
11. If *side* = `'R'`:
    - If ***A*** is **not** contained within a single block, then:
      – The following block sizes must be equal: NB_B = MB_A

  - The global matrix ***B*** must be aligned on a block column boundary; that is, $jb-1$ must be a multiple of NB_B.
- In the process grid, the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***B***, that is, $iacol = ibcol$, where:

  $iacol$ = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
  $ibcol$ = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)

12. If ***A*** is contained within a single block, then:
    - If *side* = `'L'`, then ***B*** must be a block row matrix; that is, if $p > 1$:

      m+mod(*ib*–1, MB_B) ≤ MB_B
    - If *side* = `'R'`, then ***B*** must be a block column matrix; that is, if $q > 1$:

      n+mod(*jb*–1, NB_B) ≤ NB_B

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. *side* ≠ `'L'` or `'R'`
2. *uplo* ≠ `'U'` or `'L'`
3. *transa* ≠ `'N'`, `'T'`, or `'C'`
4. *diag* ≠ `'N'` or `'U'`
5. $m < 0$
6. $n < 0$
7. M_A < 0 and *m* = 0 and *side* = `'L'`; M_A < 0 and *n* = 0 and *side* = `'R'`; M_A < 1 otherwise
8. N_A < 0 and *m* = 0 and *side* = `'L'`; N_A < 0 and *n* = 0 and *side* = `'R'`; N_A < 1 otherwise
9. MB_A < 1
10. NB_A < 1
11. M_B < 0 and (*m* = 0 or *n* = 0); M_B < 1 otherwise
12. N_B < 0 and (*m* = 0 or *n* = 0); N_B < 1 otherwise
13. MB_B < 1
14. NB_B < 1
15. RSRC_A < 0 or RSRC_A ≥ *p*

16. CSRC_A $<$ 0 or CSRC_A $\geq$ *q*
17. RSRC_B $<$ 0 or RSRC_B $\geq$ *p*
18. CSRC_B $<$ 0 or CSRC_B $\geq$ *q*
19. *ia* $<$ 1
20. *ja* $<$ 1
21. *ib* $<$ 1
22. *jb* $<$ 1
23. CTXT_A $\neq$ CTXT_B

Stage 5: If ***A*** is **not** contained within a single block, that is:

numa+mod(*ia*–1, MB_A) $>$ MB_A
numa+mod(*ja*–1, NB_A) $>$ NB_A
where:
If *side* = `'L'`, *numa* = *m*
If *side* = `'R'`, *numa* = *n*

then:

1. MB_A $\neq$ NB_A
2. *side* = `'L'` and MB_B $\neq$ NB_A
3. *side* = `'R'` and NB_B $\neq$ MB_A

If (*m* $\neq$ 0 or *side* $\neq$ `'L'`) and (*n* $\neq$ 0 or *side* $\neq$ `'R'`):

1. *ia* $>$ M_A
2. *ja* $>$ N_A
3. *ia*+*numa*–1 $>$ M_A
4. *ja*+*numa*–1 $>$ N_A

   where *numa* = *m* if *side* = `'L'` and *numa* = *n* if *side* = `'R'`.

If *m* $\neq$ 0 and *n* $\neq$ 0:

1. *ib* $>$ M_B
2. *jb* $>$ N_B
3. *ib*+*m*–1 $>$ M_B
4. *jb*+*n*–1 $>$ N_B

If ***A*** is not contained in a single block:

1. mod(*ia*–1, MB_A) $\neq$ 0
2. mod(*ja*–1, NB_A) $\neq$ 0
3. *side* = `'L'` and mod(*ib*–1, MB_B) $\neq$ 0
4. *side* = `'R'` and mod(*jb*–1, NB_B) $\neq$ 0

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_B $<$ max(1, LOCp(M_B))

If *side* = `'L'`:

1. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* $\neq$ *ibrow*, where:

   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
   ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

2. If ***A*** is contained in a single block:

$p > 1$ and m+mod(ib–1, MB_B) > MB_B

If *side* = 'R':

1. In the process grid, the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***B***; that is, *iacol* ≠ *ibcol*, where:

   iacol = mod((((ja–1)/NB_A)+CSRC_A), q)
   ibcol = mod((((jb–1)/NB_B)+CSRC_B), q)

2. If ***A*** is contained in a single block:

   $q > 1$ and n+mod(jb–1, NB_B) > NB_B

Example 1: This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$ using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             SIDE  UPLO  TRANSA   DIAG   M   N    ALPHA    A  IA  JA    DESC_A
              |     |      |       |     |   |      |      |   |   |      |
CALL PDTRSM( 'L'  , 'U' ,  'N'  ,  'N' , 5 , 3 ,  1.0D0  , A , 1 , 1 ,  DESC_A ,

             B  IB  JB   DESC_B
             |   |   |     |
             B , 1 , 1 , DESC_B )
```

	Desc_A	**Desc_B**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	5	5
N_	5	3
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 2 on P_{10} and P_{11}.

Global triangular matrix ***A*** of order 5 is upper triangular with block size 2 × 2:

```
B,D       0           1         2
     ┌                                ┐
 0   │  3.0 -1.0  │   2.0  2.0  │   1.0 │
     │   .  -2.0  │   4.0 -1.0  │   3.0 │
     │ -----------|-------------|------ │
 1   │   .    .   │  -3.0  0.0  │   2.0 │
     │   .    .   │    .   4.0  │  -2.0 │
     │ -----------|-------------|------ │
 2   │   .    .   │    .    .   │   1.0 │
     └                                ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │       0          │      1
-----|------------------|------------
     │  3.0 -1.0  1.0   │   2.0  2.0
 0   │   .  -2.0  3.0   │   4.0 -1.0
     │   .    .   1.0   │    .    .
-----|------------------|------------
 1   │   .    .   2.0   │  -3.0  0.0
     │   .    .  -2.0   │    .   4.0
```

Global general 5 × 3 matrix ***B*** with block size 2 × 2:

```
B,D        0           1
     ┌                      ┐
 0   │   6.0  10.0  │  -2.0  │
     │ -16.0  -1.0  │   6.0  │
     │ -------------|------- │
 1   │  -2.0   1.0  │   -4.0 │
     │  14.0   0.0  │  -14.0 │
     │ -------------|------- │
 2   │  -1.0   2.0  │    1.0 │
     └                      ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |      0       |    1
-----|--------------|--------
     |   6.0  10.0  |   -2.0
 0   | -16.0  -1.0  |    6.0
     |  -1.0   2.0  |    1.0
-----|--------------|--------
 1   |  -2.0   1.0  |   -4.0
     |  14.0   0.0  |  -14.0
```

Output:

Global general 5 × 3 matrix ***B*** with block size 2 × 2:

```
B,D        0          1
     ┌                    ┐
 0   |  2.0  3.0  |   1.0 |
     |  5.0  5.0  |   4.0 |
     | -----------|------ |
 1   |  0.0  1.0  |   2.0 |
     |  3.0  1.0  |  -3.0 |
     | -----------|------ |
 2   | -1.0  2.0  |   1.0 |
     └                    ┘
```

The following is the 2 × 2 process grid:

B,D	0	1
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |      0     |   1
-----|------------|-------
     |  2.0  3.0  |   1.0
 0   |  5.0  5.0  |   4.0
     | -1.0  2.0  |   1.0
-----|------------|-------
 1   |  0.0  1.0  |   2.0
     |  3.0  1.0  |  -3.0
```

| ***Example 2:*** This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-H})\boldsymbol{B}$ using a 2 × 2 process grid.

| *Call Statements and Input*

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET (0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|              SIDE  UPLO  TRANSA   DIAG   M   N    ALPHA    A  IA  JA    DESC_A
|               |     |      |       |     |   |      |      |   |   |       |
| CALL PZTRSM( 'L'  , 'U' ,  'C'  ,  'N' , 5 , 2 ,  ALPHA  , A , 1 , 1 ,  DESC_A ,

|              B  IB  JB   DESC_B
|              |   |   |     |
|              B , 1 , 1 , DESC_B )

|              ALPHA = (1.0D0, -1.0D0)
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	5	5
N_	5	2
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 2 on P_{10} and P_{11}.

Global triangular matrix ***A*** of order 5 is upper triangular with block size 2 × 2:

```
B,D           0                        1                  2
     ┌                                                                ┐
 0   | (-4.0, 1.0) ( 4.0,-3.0) | (-1.0, 3.0) ( 0.0, 0.0) | (-1.0, 0.0) |
     |      .      (-2.0, 0.0) | (-3.0,-1.0) (-2.0,-1.0) | ( 4.0, 3.0) |
     | ------------------------|-------------------------|------------ |
 1   |      .           .      | (-5.0, 3.0) (-3.0,-3.0) | (-5.0,-5.0) |
     |      .           .      |      .      ( 4.0,-4.0) | ( 2.0, 0.0) |
     | ------------------------|-------------------------|------------ |
 2   |      .           .      |      .           .      | ( 2.0,-1.0) |
     └                                                                ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                 0                  |      1
-----|------------------------------------|------------------------------
     | (-4.0, 1.0) ( 4.0,-3.0) (-1.0, 0.0) | (-1.0, 3.0) ( 0.0, 0.0)     .
 0   |      .      (-2.0, 0.0) ( 4.0, 3.0) | (-3.0,-1.0) (-2.0,-1.0)     .
     |      .           .      ( 2.0,-1.0) |      .           .          .
-----|------------------------------------|------------------------------
 1   |      .           .      (-5.0,-5.0) | (-5.0, 3.0) (-3.0,-3.0)     .
     |      .           .      ( 2.0, 0.0) |      .      ( 4.0,-4.0)     .
```

Global general 5 × 2 matrix ***B*** with block size 2 × 2:

```
B,D             0
     ┌                          ┐
 0   | ( 5.5,-13.5) (-3.0, -5.0) |
     | (-6.5, 14.5) (-3.0,  5.0) |
     | ------------------------- |
 1   | (26.0, 18.0) ( 4.0, -3.0) |
     | (10.0,  3.0) ( 6.0, -6.0) |
     | ------------------------- |
 2   | ( 8.5, 10.5) (13.0,-12.0) |
     └                          ┘
```

The following is the 2 × 2 process grid:

B,D	0	—
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |             0             |            1
-----|---------------------------|------------------------
     | ( 5.5,-13.5) (-3.0, -5.0) |            .
 0   | (-6.5, 14.5) (-3.0,  5.0) |            .
     | ( 8.5, 10.5) (13.0,-12.0) |            .
-----|---------------------------|------------------------
     | (26.0, 18.0) ( 4.0, -3.0) |            .
 1   | (10.0,  3.0) ( 6.0, -6.0) |            .
```

Output:

Global general 5 × 2 matrix ***B*** with block size 2 × 2:

```
B,D             0
     ┌                        ┐
 0   | ( 3.0,4.0) ( 2.0, 0.0) |
     | (-4.0,2.0) ( 3.0,-1.0) |
     | ---------------------- |
 1   | (-5.0,0.0) (-1.0, 2.0) |
     | ( 1.0,3.0) ( 0.0,-2.0) |
     | ---------------------- |
 2   | ( 3.0,1.0) ( 1.0, 3.0) |
     └                        ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**—**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |            0             |          1
-----|--------------------------|-----------------------
     | ( 3.0, 4.0) ( 2.0, 0.0)  |          .
 0   | (-4.0, 2.0) ( 3.0,-1.0)  |          .
     | ( 3.0, 1.0) ( 1.0, 3.0)  |          .
-----|--------------------------|-----------------------
 1   | (-5.0, 0.0) (-1.0, 2.0)  |          .
     | ( 1.0, 3.0) ( 0.0,-2.0)  |          .
```

PDSYRK, PZSYRK, and PZHERK—Rank-K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix

PDSYRK and PZSYRK compute one of the following rank-k updates:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{T}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{A}+\beta \boldsymbol{C}$

PZHERK computes one of the following rank-k updates:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^{H}+\beta \boldsymbol{C}$
4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{H}\boldsymbol{A}+\beta \boldsymbol{C}$

where, in the formulas above:

A represents the global general submatrix:
- For *trans* = `'N'`, it is $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+k-1}$.
- For *trans* = `'T'` or `'C'`, it is $\boldsymbol{A}_{ia:ia+k-1,\ ja:ja+n-1}$.

C represents the global submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$.

and:

- For PDSYRK, submatrix ***C*** is real symmetric.
- For PZSYRK, submatrix ***C*** is complex symmetric.
- For PZHERK, submatrix ***C*** is complex Hermitian.

Note: No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix ***A*** should always be stored in its untransposed form.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $n = 0$
- β is one, and α is zero or $k = 0$.

See references [14] and [15].

Table 53. Data Types

A, ***C***	α, β	**Subprogram**
Long-precision real	Long-precision real	PDSYRK
Long-precision complex	Long-precision complex	PZSYRK
Long-precision complex	Long-precision real	PZHERK

Syntax

Fortran	CALL PDSYRK | PZSYRK | PZHERK (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *beta*, *c*, *ic*, *jc*, *desc_c*)
C and C++	pdsyrk | pzsyrk | pzherk (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *beta*, *c*, *ic*, *jc*, *desc_c*);

On Entry

uplo

indicates whether the upper or lower triangular part of the global submatrix ***C*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

| *trans*
| indicates which computation is performed, where:

| If *trans* = `'N'`, ***A*** is used.

| If *trans* = `'T'`, $\boldsymbol{A}^T$ is used.

| If *trans* = `'C'`, $\boldsymbol{A}^H$ is used.

| Scope: **global**

| Specified as: a single character, where:

| For PDSYRK, it must be `'N'`, `'T'`, or `'C'`.

| For PZSYRK, it must be `'N'` or `'T'`.

| For PZHERK, it must be `'N'` or `'C'`.

n
is the order of the global submatrix ***C*** used in the computation, and:

If *trans* = `'N'`, it is the number of rows in submatrix ***A*** used in the computation.

| If *trans* = `'T'` or `'C'`, it is the number of columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

k
has the following meaning:

If *trans* = `'N'`, it is the number of columns in submatrix ***A*** used in the computation.

| If *trans* = `'T'` or `'C'`, it is the number of rows in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $k \geq 0$.

alpha
is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 53 on page 349.

a
is the local part of the global general matrix ***A***. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *trans* = `'N'`, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*k*–1) part of the local array `A` must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*k*–1 part of the global matrix.
| - If *trans* = `'T'` or `'C'`, the leading LOCp(*ia*+*k*–1) by LOCq(*ja*+*n*–1) part of the local array `A` must contain the local pieces of the leading *ia*+*k*–1 by *ja*+*n*–1 part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 53 on page 349. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia
: is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A, and:

If *trans* = 'N', then $ia+n-1 \le$ M_A.

| If *trans* = 'T' or 'C', then $ia+k-1 \le$ M_A.

ja
: is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A, and:

If *trans* = 'N', then $ja+k-1 \le$ N_A.

| If *trans* = 'T' or 'C', then $ja+n-1 \le$ N_A.

desc_a
: is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$ or $k = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$ or $k = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 53 on page 349.

| *c*

| is the local part of the global real or complex symmetric or complex Hermitian
| matrix ***C***. This identifies the **first element** of the local array `C`. This subroutine
| computes the location of the first element of the local subarray used, based on
| *ic*, *jc*, *desc_c*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ic+n-1$) by
| LOCq($jc+n-1$) part of the local array `C` must contain the local pieces of the
| leading $ic+n-1$ by $jc+n-1$ part of the global matrix, and:

| • If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global
| submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$ must contain the upper triangular part of the
| submatrix, and the strictly lower triangular part is not referenced.

| • If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global
| submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$ must contain the lower triangular part of the
| submatrix, and the strictly upper triangular part is not referenced.

| When β is zero, ***C*** need not be set on input.

| Scope: **local**

| Specified as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of
| the data type indicated in Table 53 on page 349. Details about the block-cyclic
| data distribution of global matrix ***C*** are stored in *desc_c*.

ic

is the row index of the global matrix ***C***, identifying the first row of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le ic \le$ M_C and $ic+n-1 \le$ M_C.

jc

is the column index of the global matrix ***C***, identifying the first column of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le jc \le$ N_C and $jc+n-1 \le$ N_C.

desc_c

is the array descriptor for global matrix ***C***, described in the following table:

desc_c	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_C	Descriptor type	DTYPE_C=1	Global
2	CTXT_C	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_C	Number of rows in the global matrix	If $n = 0$: M_C ≥ 0 Otherwise: M_C ≥ 1	Global

desc_c	**Name**	**Description**	**Limits**	**Scope**
4	N_C	Number of columns in the global matrix	If $n = 0$: N_C $\geq$ 0 Otherwise: N_C $\geq$ 1	Global
5	MB_C	Row block size	MB_C $\geq$ 1	Global
6	NB_C	Column block size	NB_C $\geq$ 1	Global
7	RSRC_C	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_C $<$ p	Global
8	CSRC_C	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_C $<$ q	Global
9	LLD_C	The leading dimension of the local array	LLD_C $\geq$ max(1,LOCp(M_C))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

| *c*

| is the updated local part of the global real or complex symmetric or complex
| Hermitian matrix ***C***, containing the results of the computation.

| Scope: **local**

| Returned as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of
| the data type indicated in Table 53 on page 349.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo* and *trans* arguments.
2. | For PDSYRK, if you specify `'C'` for the *trans* argument, it is interpreted as | though you specified `'T'`.
3. | The imaginary parts of the diagonal elements of a complex Hermitian matrix ***C*** | are assumed to be zero, so you do not have to set these values. On output, | they are set to zero, except when β is one and α or *k* is zero, in which case no | computation is performed.
4. The matrices must have no common elements; otherwise, results are unpredictable.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
6. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
7. The following values must be equal: CTXT_A = CTXT_C.
8. If ***C*** is **not** contained within a single block, that is:

   n+mod(ic–1, MB_C) $>$ MB_C

n+mod(jc–1, NB_C) > NB_C

then:

- The global matrix ***C*** must be distributed using a square block-cyclic distribution; that is, MB_C = NB_C.
- The global matrix ***C*** must be aligned on a block boundary, that is:

  ic–1 must be a multiple of MB_C.
  jc–1 must be a multiple of NB_C.

9. If *trans* = `'N'`:
   - If ***C*** is **not** contained within a single block, then:
     – The following block sizes must be equal: MB_A = NB_C.
     – The global matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
   - In the process grid, the process row containing the first row of the submatrix ***C*** must also contain the first row of the submatrix ***A***; that is, *icrow* = *iarow*, where:

     icrow = mod((((ic–1)/MB_C)+RSRC_C), p)
     iarow = mod((((ia–1)/MB_A)+RSRC_A), p)

| 10. If *trans* = `'T'` or `'C'`:
   - If ***C*** is **not** contained within a single block, then:
     – The following block sizes must be equal: NB_A = MB_C.
     – The global matrix ***A*** must be aligned on a block column boundary; that is, *ja*–1 must be a multiple of NB_A.
   - In the process grid, the process column containing the first column of the submatrix ***C*** must also contain the first column of the submatrix ***A***; that is, *iccol* = *iacol*, where:

     iccol = mod((((jc–1)/NB_C)+CSRC_C), q)
     iacol = mod((((ja–1)/NB_A)+CSRC_A), q)

11. If ***C*** is contained within a single block:
   - If *trans* = `'N'`, ***A*** must be a block row matrix; that is, if $p > 1$:

     n+mod(ia–1, MB_A) ≤ MB_A

| - If *trans* = `'T'` or `'C'`, ***A*** must be a block column matrix; that is, if $q > 1$:

| n+mod(ja–1, NB_A) ≤ NB_A

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_C is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* $\neq$ `'U'` or `'L'`
2. *trans* $\neq$
   - `'N'`, `'T'`, or `'C'` for PDSYRK
   - `'N'` or `'T'` for PZSYRK
   - `'N'` or `'C'` for PZHERK
3. $n < 0$ and *trans* = `'N'`
4. $n < 0$ and *trans* = `'T'` or `'C'`
5. $n < 0$ and *trans* is invalid.
6. $k < 0$ and *trans* = `'N'`
7. $k < 0$ and *trans* = `'T'` or `'C'`
8. $k < 0$ and *trans* is invalid.
9. M_A < 0 and ($n = 0$ or $k = 0$); M_A < 1 otherwise
10. N_A < 0 and ($n = 0$ or $k = 0$); N_A < 1 otherwise
11. MB_A < 1
12. NB_A < 1
13. RSRC_A < 0 or RSRC_A $\geq p$
14. CSRC_A < 0 or CSRC_A $\geq q$
15. $ia < 1$
16. $ja < 1$
17. M_C < 0 and $n = 0$; M_C < 1 otherwise
18. N_C < 0 and $n = 0$; N_C < 1 otherwise
19. MB_C < 1
20. NB_C < 1
21. RSRC_C < 0 or RSRC_C $\geq p$
22. CSRC_C < 0 or CSRC_C $\geq q$
23. $ic < 1$
24. $jc < 1$
25. CTXT_A $\neq$ CTXT_C

If $n \neq 0$ and $k \neq 0$:

1. $ia >$ M_A
2. $ja >$ N_A
3. *trans* = `'N'` and $ia+n-1 >$ M_A
4. *trans* = `'N'` and $ja+k-1 >$ N_A
5. *trans* = `'T'` or `'C'` and $ia+k-1 >$ M_A
6. *trans* = `'T'` or `'C'` and $ja+n-1 >$ N_A

If $n \neq 0$:

1. $ic >$ M_C
2. $jc >$ N_C

3. $ic+n-1 > \text{M\_C}$
4. $jc+n-1 > \text{N\_C}$

Stage 5

1. If ***C*** is **not** contained within a single block, that is:
   $n+\text{mod}(ic-1, \text{MB\_C}) > \text{MB\_C}$
   $n+\text{mod}(jc-1, \text{NB\_C}) > \text{NB\_C}$

   and $\text{NB\_C} \neq \text{MB\_C}$.
2. *trans* = 'N' and $\text{NB\_C} \neq \text{MB\_A}$.
3. | *trans* = 'T' or 'C' and $\text{MB\_C} \neq \text{NB\_A}$.

If ***C*** is **not** contained within a single block:

1. $\text{mod}(ic-1, \text{MB\_C}) \neq 0$
2. $\text{mod}(jc-1, \text{NB\_C}) \neq 0$
3. *trans* = 'N' and $\text{mod}(ia-1, \text{MB\_A}) \neq 0$
4. | *trans* = 'T' or 'C' and $\text{mod}(ja-1, \text{NB\_A}) \neq 0$

Stage 6

1. $\text{LLD\_A} < \max(1, \text{LOCp(M\_A)})$
2. $\text{LLD\_C} < \max(1, \text{LOCp(M\_C)})$
3. If *trans* = 'N', then (in the process grid) the process row containing the first row of the submatrix ***C*** does not contain the first row of the submatrix ***A***; that is, $icrow \neq iarow$, where:
   $icrow = \text{mod}((((ic-1)/\text{MB\_C})+\text{RSRC\_C}), p)$
   $iarow = \text{mod}((((ia-1)/\text{MB\_A})+\text{RSRC\_A}), p)$
4. | If *trans* = 'T' or 'C', then (in the process grid) the process column containing the first column of the submatrix ***C*** does not contain the first column of the submatrix ***A***; that is, $iccol \neq iacol$, where:
   $iccol = \text{mod}((((jc-1)/\text{NB\_C})+\text{CSRC\_C}), q)$
   $iacol = \text{mod}((((ja-1)/\text{NB\_A})+\text{CSRC\_A}), q)$

If ***C*** is contained within a single block:

1. If *trans* = 'N':
   $p > 1$ and $n+\text{mod}(ia-1, \text{MB\_A}) > \text{MB\_A}$
2. | If *trans* = 'T' or 'C':
   $q > 1$ and $n+\text{mod}(ja-1, \text{NB\_A}) > \text{NB\_A}$

Example 1: This example computes $\boldsymbol{C} = \alpha \boldsymbol{A}\boldsymbol{A}^T+\beta \boldsymbol{C}$ using a 2×3 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A    BETA
                  |      |     |    |       |      |   |   |      |         |
CALL PDSYRK( 'L' ,  'N' ,  8  , 5  ,  1.0D0  , A , 1 , 1 ,  DESC_A ,  1.0D0 ,

                  C  IC  JC   DESC_C
                  |   |   |     |
                  C , 1 , 1 , DESC_C )
```

	Desc_A	**Desc_C**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	8	8
N_	5	8
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = LLD_C = 4 on all processes.

Global general 8 × 5 matrix ***A*** with block size 2 × 2:

```
B,D        0               1             2
     ┌                                          ┐
 0   |   0.0    8.0  |   16.0   24.0  |   32.0 |
     |   1.0    9.0  |   17.0   25.0  |   33.0 |
     | --------------|----------------|------- |
 1   |   2.0   10.0  |   18.0   26.0  |   34.0 |
     |   3.0   11.0  |   19.0   27.0  |   35.0 |
     | --------------|----------------|------- |
 2   |   4.0   12.0  |   20.0   28.0  |   36.0 |
     |   5.0   13.0  |   21.0   29.0  |   37.0 |
     | --------------|----------------|------- |
 3   |   6.0   14.0  |   22.0   30.0  |   38.0 |
     |   7.0   15.0  |   23.0   31.0  |   39.0 |
     └                                          ┘
```

The following is the 2 × 3 process grid:

B,D	0	1	2
0 2	P_{00}	P_{01}	P_{02}
1 3	P_{10}	P_{11}	P_{12}

Local arrays for ***A***:

```
p,q  |      0       |       1        |   2
-----|--------------|----------------|--------
     |  0.0   8.0   |   16.0   24.0  |  32.0
     |  1.0   9.0   |   17.0   25.0  |  33.0
 0   |  4.0  12.0   |   20.0   28.0  |  36.0
     |  5.0  13.0   |   21.0   29.0  |  37.0
-----|--------------|----------------|--------
     |  2.0  10.0   |   18.0   26.0  |  34.0
     |  3.0  11.0   |   19.0   27.0  |  35.0
 1   |  6.0  14.0   |   22.0   30.0  |  38.0
     |  7.0  15.0   |   23.0   31.0  |  39.0
```

Global real symmetric matrix ***C*** of order 8 block size 2 × 2:

```
B,D        0              1              2              3
     ┌                                                      ┐
 0   |   0.0    .   |    .     .   |    .     .   |    .     .  |
     |   1.0   8.0  |    .     .   |    .     .   |    .     .  |
     |  ------------|--------------|--------------|------------ |
 1   |   2.0   9.0  |   15.0    .  |    .     .   |    .     .  |
     |   3.0  10.0  |   16.0  21.0 |    .     .   |    .     .  |
     |  ------------|--------------|--------------|------------ |
 2   |   4.0  11.0  |   17.0  22.0 |   26.0    .  |    .     .  |
     |   5.0  12.0  |   18.0  23.0 |   27.0  30.0 |    .     .  |
     |  ------------|--------------|--------------|------------ |
 3   |   6.0  13.0  |   19.0  24.0 |   28.0  31.0 |   33.0    . |
     |   7.0  14.0  |   20.0  25.0 |   29.0  32.0 |   34.0  35.0|
     └                                                      ┘
```

The following is the 2 × 3 process grid:

B,D	0 3	1	2
0 2	P_{00}	P_{01}	P_{02}
1 3	P_{10}	P_{11}	P_{12}

Local arrays for ***C***:

```
p,q  |            0             |       1       |      2
-----|--------------------------|---------------|--------------
     |  0.0    .     .     .    |    .     .    |    .     .
     |  1.0   8.0    .     .    |    .     .    |    .     .
 0   |  4.0  11.0    .     .    |  17.0  22.0   |  26.0    .
     |  5.0  12.0    .     .    |  18.0  23.0   |  27.0  30.0
-----|--------------------------|---------------|--------------
     |  2.0   9.0    .     .    |  15.0    .    |    .     .
     |  3.0  10.0    .     .    |  16.0  21.0   |    .     .
 1   |  6.0  13.0  33.0    .    |  19.0  24.0   |  28.0  31.0
     |  7.0  14.0  34.0  35.0   |  20.0  25.0   |  29.0  32.0
```

Output:

Global real symmetric matrix ***C*** of order 8 with block size 2 × 2:

```
B,D         0                   1                   2                   3
      ┌                                                                            ┐
 0    |  1920.0      .    |       .       .    |       .       .    |      .       .   |
      |  2001.0  2093.0   |       .       .    |       .       .    |      .       .   |
      | ------------------|--------------------|--------------------|------------------ |
 1    |  2082.0  2179.0   |   2275.0      .    |       .       .    |      .       .   |
      |  2163.0  2265.0   |   2366.0  2466.0   |       .       .    |      .       .   |
      | ------------------|--------------------|--------------------|------------------ |
 2    |  2244.0  2351.0   |   2457.0  2562.0   |   2666.0      .    |      .       .   |
      |  2325.0  2437.0   |   2548.0  2658.0   |   2767.0  2875.0   |      .       .   |
      | ------------------|--------------------|--------------------|------------------ |
 3    |  2406.0  2523.0   |   2639.0  2754.0   |   2868.0  2981.0   |   3093.0      .  |
      |  2487.0  2609.0   |   2730.0  2850.0   |   2969.0  3087.0   |   3204.0  3320.0 |
      └                                                                            ┘
```

The following is the 2 × 3 process grid:

B,D	**0 3**	**1**	**2**
0 **2**	P_{00}	P_{01}	P_{02}
1 **3**	P_{10}	P_{11}	P_{12}

Local arrays for ***C***:

```
p,q  |                 0                  |         1          |         2
-----|------------------------------------|--------------------|-------------------
     |  1920.0      .       .       .     |      .       .     |      .       .
     |  2001.0  2093.0      .       .     |      .       .     |      .       .
 0   |  2244.0  2351.0      .       .     |   2457.0  2562.0   |   2666.0      .
     |  2325.0  2437.0      .       .     |   2548.0  2658.0   |   2767.0  2875.0
-----|------------------------------------|--------------------|-------------------
     |  2082.0  2179.0      .       .     |   2275.0      .    |      .       .
     |  2163.0  2265.0      .       .     |   2366.0  2466.0   |      .       .
 1   |  2406.0  2523.0  3093.0      .     |   2639.0  2754.0   |   2868.0  2981.0
     |  2487.0  2609.0  3204.0  3320.0    |   2730.0  2850.0   |   2969.0  3087.0
```

Example 2: This example computes $\boldsymbol{C} = \alpha \boldsymbol{A}\boldsymbol{A}^T + \beta \boldsymbol{C}$ using a 2 × 3 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

               UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A    BETA
                |      |      |    |       |      |   |   |      |         |
CALL PZSYRK( 'U' ,  'N' ,  3  , 5  ,  ALPHA  , A , 1 , 1 ,  DESC_A ,  BETA ,

                 C  IC  JC   DESC_C
                 |   |   |     |
                 C , 1 , 1 , DESC_C )

                 ALPHA = (1.0, 1.0)

                 BETA  = (1.0, 1.0)
```

	Desc_A	Desc_C
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	3	3
N_	5	3
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example:

LLD_A = LLD_C = 2 on P_{00}, P_{01}, and P_{02}
LLD_A = LLD_C = 1 on P_{10}, P_{11}, and P_{12}

Global general 3 × 5 matrix ***A*** with block size 2 × 2:

```
B,D           0                    1                2
      ┌                                                    ┐
 0    | (2.0,0.0) (3.0,2.0) | (4.0,1.0) (1.0,7.0) | (0.0,0.0) |
      | (30.,3.0) (8.0,0.0) | (2.0,5.0) (2.0,4.0) | (1.0,2.0) |
      | --------------------|---------------------|---------- |
 1    | (1.0,3.0) (2.0,1.0) | (6.0,0.0) (3.0,2.0) | (2.0,2.0) |
      └                                                    ┘
```

The following is the 2 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}
1	P_{10}	P_{11}	P_{12}

Local arrays for ***A***:

```
p,q  |          0          |          1          |     2
-----|---------------------|---------------------|-----------
     | (2.0,0.0) (3.0,2.0) | (4.0,1.0) (1.0,7.0) | (0.0,0.0)
 0   | (3.0,3.0) (8.0,0.0) | (2.0,5.0) (2.0,4.0) | (1.0,2.0)
-----|---------------------|---------------------|-----------
 1   | (1.0,3.0) (2.0,1.0) | (6.0,0.0) (3.0,2.0) | (2.0,2.0)
```

Global complex symmetric matrix ***C*** of order 3 with block size 2 × 2:

```
B,D          0                1
     ┌                               ┐
 0   | (2.0,1.0) (1.0,9.0) | (4.0,5.0) |
     |     .     (3.0,1.0) | (6.0,7.0) |
     | --------------------|---------- |
 1   |     .         .     | (8.0,1.0) |
     └                               ┘
```

The following is the 2 × 3 process grid:

B,D	0	1	—
0	P_{00}	P_{01}	P_{02}
1	P_{10}	P_{11}	P_{12}

Local arrays for ***C***:

```
p,q  |          0          |     1     |     2
-----|---------------------|-----------|-----------
     | (2.0,1.0) (1.0,9.0) | (4.0,5.0) |     .
 0   |     .     (3.0,1.0) | (6.0,7.0) |     .
-----|---------------------|-----------|-----------
 1   |     .         .     | (8.0,1.0) |     .
```

Output:

Global complex symmetric matrix ***C*** of order 3 with block size 2 × 2:

```
B,D              0                        1
     ┌                                               ┐
 0   | (-57.0, 13.0) (-63.0, 79.0) | (-24.0, 70.0) |
     |       .       (-28.0, 90.0) | (-55.0,103.0) |
     | ----------------------------|---------------|
 1   |       .             .       | ( 13.0, 75.0) |
     └                                               ┘
```

The following is the 2 × 3 process grid:

B,D	0	1	—
0	P_{00}	P_{01}	P_{02}
1	P_{10}	P_{11}	P_{12}

Local arrays for ***C***:

```
p,q  |             0              |       1       |       2
-----|----------------------------|---------------|---------------
     | (-57.0, 13.0) (-63.0, 79.0) | (-24.0, 70.0) |       .
 0   |       .       (-28.0, 90.0) | (-55.0,103.0) |       .
-----|----------------------------|---------------|---------------
 1   |       .             .       | ( 13.0, 75.0) |       .
```

Example 3: This example computes ***C*** = $\alpha A^H A + \beta C$ using a 3 × 2 process grid.

Note: On output, the imaginary parts of the diagonal elements of a complex Hermitian matrix are set to zero, except when β is one and α or *k* is zero.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A    BETA
              |      |     |    |       |      |   |   |      |         |
CALL PZHERK( 'L' ,  'C' ,  3  , 5  ,  ALPHA  , A , 1 , 1 ,  DESC_A ,  BETA ,

              C  IC  JC   DESC_C
              |   |   |     |
              C , 1 , 1 , DESC_C )

              ALPHA = 1.0

              BETA  = 1.0
```

	Desc_A	Desc_C
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	5	3
N_	3	3
MB_	2	2
NB_	2	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

	Desc_A	Desc_C

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example:

LLD_A = 2 on P_{00}, P_{01}, P_{10}, and P_{11}
LLD_A = 1 on P_{20} and P_{21}
LLD_C = 2 on P_{00} and P_{01}
LLD_C = 1 on P_{10} and P_{11}

Global general 5 × 3 matrix ***A*** with block size 2 × 2:

```
B,D           0              1
      ┌                                ┐
 0    │ (2.0,0.0) (3.0,2.0) │ (4.0,1.0) │
      │ (3.0,3.0) (8.0,0.0) │ (2.0,5.0) │
      │ --------------------│-----------│
 1    │ (1.0,3.0) (2.0,1.0) │ (6.0,0.0) │
      │ (3.0,3.0) (8.0,0.0) │ (2.0,5.0) │
      │ --------------------│-----------│
 2    │ (1.0,9.0) (3.0,0.0) │ (6.0,7.0) │
      └                                ┘
```

The following is the 3 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}
2	P_{20}	P_{21}

Local arrays for ***A***:

```
p,q  │          0          │     1     │
-----│---------------------│-----------│
     │ (2.0,0.0) (3.0,2.0) │ (4.0,1.0) │
 0   │ (3.0,3.0) (8.0,0.0) │ (2.0,5.0) │
-----│---------------------│-----------│
     │ (1.0,3.0) (2.0,1.0) │ (6.0,0.0) │
 1   │ (3.0,3.0) (8.0,0.0) │ (2.0,5.0) │
-----│---------------------│-----------│
 2   │ (1.0,9.0) (3.0,0.0) │ (6.0,7.0) │
-----│---------------------│-----------│
```

Global complex Hermitian matrix ***C*** of order 3 with block size 2 × 2:

```
B,D           0             1
      ┌                                  ┐
 0    │ (6.0,0.0)      .     │     .     │
      │ (3.0,4.0) (10.0,0.0) │     .     │
      │ ---------------------│-----------│
 1    │ (9.0,1.0) (12.0,2.0) │ (3.0,0.0) │
      └                                  ┘
```

The following is the 3 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}
—	P_{20}	P_{21}

Local arrays for ***C***:

```
p,q  |          0           |     1     |
-----|----------------------|-----------|
     | (6.0, . )      .     |     .     |
 0   | (3.0,4.0) (10.0, . ) |     .     |
-----|----------------------|-----------|
 1   | (9.0,1.0) (12.0,2.0) | (3.0, . ) |
-----|----------------------|-----------|
 2   |     .          .     |     .     |
```

Output:

Global complex Hermitian matrix ***C*** of order 3 with block size 2 × 2:

```
B,D                0                     1
     ┌                                         ┐
 0   | (138.0,  0.0)       .        |      .        |
     | ( 65.0, 80.0) (165.0,  0.0)  |      .        |
     | ---------------------------- |---------------|
 1   | (134.0, 46.0) ( 88.0,-88.0)  | (199.0,  0.0) |
     └                                         ┘
```

The following is the 3 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}
1	P_{10}	P_{11}
—	P_{20}	P_{21}

Local arrays for ***C***:

```
p,q  |              0              |       1       |
-----|-----------------------------|---------------|
     | (138.0,  0.0)       .       |       .       |
 0   | ( 65.0, 80.0) (165.0,  0.0) |       .       |
-----|-----------------------------|---------------|
 1   | (134.0, 46.0) ( 88.0,-88.0) | (199.0,  0.0) |
-----|-----------------------------|---------------|
 2   |       .             .       |       .       |
```

PDSYR2K, PZSYR2K, and PZHER2K—Rank-2K Update of a Real or Complex Symmetric or a Complex Hermitian Matrix

| PDSYR2K and PZSYR2K compute one of the following rank-2k updates:

| 1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{T}+\alpha \boldsymbol{B}\boldsymbol{A}^{T}+\beta \boldsymbol{C}$
| 2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{T}\boldsymbol{B}+\alpha \boldsymbol{B}^{T}\boldsymbol{A}+\beta \boldsymbol{C}$

| PZHER2K computes one of the following rank-2k updates:

3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^{H} + \overline{\alpha}\boldsymbol{B}\boldsymbol{A}^{H} + \beta \boldsymbol{C}$

4. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^{H}\boldsymbol{B} + \overline{\alpha}\boldsymbol{B}^{H}\boldsymbol{A} + \beta \boldsymbol{C}$

where, in the formulas above:

A represents the global general submatrix:
- For *trans* = 'N', it is $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+k-1}$.
- | For *trans* = 'T' or 'C', it is $\boldsymbol{A}_{ia:ia+k-1,\ ja:ja+n-1}$.

B represents the global general submatrix:
- For *trans* = 'N', it is $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+k-1}$.
- | For *trans* = 'T' or 'C', it is $\boldsymbol{B}_{ib:ib+k-1,\ jb:jb+n-1}$.

C represents the global submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$.
α and β are scalars.

| **Note:** No data should be moved to form $\boldsymbol{A}^{T}$, $\boldsymbol{A}^{H}$, $\boldsymbol{B}^{T}$, or $\boldsymbol{B}^{H}$; that is, the ***A*** and ***B***
| matrices should always be stored in their untransposed forms.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $n = 0$
- β is one, and α is zero or $k = 0$.

See references [14] and [15].

Table 54. Data Types

A, ***B***, ***C***, α	β	**Subprogram**
Long-precision real	Long-precision real	PDSYR2K
Long-precision complex	Long-precision complex	PZSYR2K
Long-precision complex	Long-precision real	PZHER2K

Syntax

Fortran	CALL PDSYR2K | PZSYR2K | PZHER2K (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*)
C and C++	pdsyr2k | pzsyr2k | pzher2k (*uplo*, *trans*, *n*, *k*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *beta*, *c*, *ic*, *jc*, *desc_c*);

On Entry

uplo

indicates whether the upper or lower triangular part of the global submatrix ***C*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

| *trans*

| indicates which computation is performed, where:

| If *trans* = `'N'`, ***A*** and ***B*** are used.

| If *trans* = `'T'`, $\boldsymbol{A}^{T}$ and $\boldsymbol{B}^{T}$ are used.

| If *trans* = `'C'`, $\boldsymbol{A}^{H}$ and $\boldsymbol{B}^{H}$ are used.

| Scope: **global**

| Specified as: a single character, where:

| For PDSYR2K, it must be `'N'`, `'T'`, or `'C'`.

| For PZSYR2K, it must be `'N'` or `'T'`.

| For PZHER2K, it must be `'N'` or `'C'`.

n

is the order of the global submatrix ***C*** used in the computation, and:

If *trans* = `'N'`, it is the number of rows in submatrices ***A*** and ***B*** used in the computation.

| If *trans* = `'T'` or `'C'`, it is the number of columns in submatrices ***A*** and ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

k

has the following meaning:

If *trans* = `'N'`, it is the number of columns in submatrices ***A*** and ***B*** used in the computation.

| If *trans* = `'T'` or `'C'`, it is the number of rows in submatrices ***A*** and ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $k \geq 0$.

alpha

is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 54 on page 365.

a

is the local part of the global general matrix ***A***. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *trans* = 'N', the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*k*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*k*–1 part of the global matrix.
- | If *trans* = 'T' or 'C', the leading LOCp(*ia*+*k*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*k*–1 by *ja*+*n*–1 part of the global matrix.

| | **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 54 on page 365. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ia* ≤ M_A, and:

If *trans* = 'N', then *ia*+*n*–1 ≤ M_A.

| If *trans* = 'T' or 'C', then *ia*+*k*–1 ≤ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ja* ≤ N_A, and:

If *trans* = 'N', then *ja*+*k*–1 ≤ N_A.

| If *trans* = 'T' or 'C', then *ja*+*n*–1 ≤ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If *n* = 0 or *k* = 0: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If *n* = 0 or *k* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore:

- If *trans* = 'N', the leading LOCp(*ib*+*n*–1) by LOCq(*jb*+*k*–1) part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *jb*+*k*–1 part of the global matrix.
- | If *trans* = 'T' or 'C', the leading LOCp(*ib*+*k*–1) by LOCq(*jb*+*n*–1) part of the local array B must contain the local pieces of the leading *ib*+*k*–1 by *jb*+*n*–1 part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix ***B*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 54 on page 365. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B, and:

If *trans* = 'N', then $ib+n-1 \le$ M_B.

| If *trans* = 'T' or 'C', then $ib+k-1 \le$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B, and:

If *trans* = 'N', then $jb+k-1 \le$ N_B.

| If *trans* = 'T' or 'C', then $jb+n-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$ or $k = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $n = 0$ or $k = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_B < q	Global
9	LLD_B	The leading dimension of the local array	LLD_B ≥ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 54 on page 365.

c

| is the local part of the global real symmetric, complex symmetric, or complex
| Hermitian matrix ***C***. This identifies the **first element** of the local array C. This subroutine computes the location of the first element of the local subarray used, based on *ic*, *jc*, *desc_c*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ic*+*n*–1) by LOCq(*jc*+*n*–1) part of the local array C must contain the local pieces of the leading *ic*+*n*–1 by *jc*+*n*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{C}_{ic:ic+n-1,\ jc:jc+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

When β is zero, ***C*** need not be set on input.

Scope: **local**

Specified as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 54 on page 365. Details about the block-cyclic data distribution of global matrix ***C*** are stored in *desc_c*.

ic

is the row index of the global matrix ***C***, identifying the first row of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le ic \le$ M_C and $ic+n-1 \le$ M_C.

jc

is the column index of the global matrix ***C***, identifying the first column of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \le jc \le$ N_C and $jc+n-1 \le$ N_C.

desc_c

is the array descriptor for global matrix ***C***, described in the following table:

desc_c	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_C	Descriptor type	DTYPE_C=1	Global
2	CTXT_C	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_C	Number of rows in the global matrix	If $n = 0$: M_C ≥ 0 Otherwise: M_C ≥ 1	Global
4	N_C	Number of columns in the global matrix	If $n = 0$: N_C ≥ 0 Otherwise: N_C ≥ 1	Global
5	MB_C	Row block size	MB_C ≥ 1	Global
6	NB_C	Column block size	NB_C ≥ 1	Global
7	RSRC_C	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_C $< p$	Global
8	CSRC_C	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_C $< q$	Global
9	LLD_C	The leading dimension of the local array	LLD_C $\ge$ max(1,LOCp(M_C))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

c

| is the updated local part of the global real symmetric, complex symmetric, or
| complex Hermitian matrix ***C***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 54 on page 365.

Notes and Coding Rules

1. These subroutines accept lowercase letters for the *uplo* and *trans* arguments.
2. For PDSYR2K, if you specify `'C'` for the *trans* argument, it is interpreted as though you specified `'T'`.
3. The imaginary parts of the diagonal elements of a complex Hermitian matrix ***C*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero, except when β is one and α or *k* is zero, in which case no computation is performed.
4. The matrices must have no common elements; otherwise, results are unpredictable.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
6. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
7. The following values must be equal: CTXT_A = CTXT_B = CTXT_C.
8. If *trans* = `'N'`:
   - In the process grid, the process row containing the first row of the submatrix ***C*** must also contain the first row of the submatrices ***A*** and ***B***; that is:

     icrow = *iarow*
     icrow = *ibrow*
     where:
     iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
     ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
     icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)
   - If looping is required—that is, **either** of the following is true:

     k+mod(*ja*–1, NB_A) > NB_A
     k+mod(*jb*–1, NB_B) > NB_B

     then the block column offset of ***A*** must be equal to the block column offset of ***B***; that is, mod(*ja*–1, NB_A) = mod(*jb*–1, NB_B).
9. If *trans* = `'T'` or `'C'`:
   - In the process grid, the process column containing the first column of the submatrix ***C*** must also contain the first column of the submatrices ***A*** and ***B***; that is:

     iccol = *iacol*
     iccol = *ibcol*
     where:
     iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
     ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)
     iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)
   - If looping is required—that is, **either** of the following is true:

     k+mod(*ia*–1, MB_A) > MB_A

k+mod(ib–1, MB_B) > MB_B

then the block row offset of ***A*** must be equal to the block row offset of ***B***; that is, mod(ia–1, MB_A) = mod(ib–1, MB_B)

10. If all the following are true:
    - ***C*** is contained within a single block, that is:

      n+mod(ic–1, MB_C) ≤ MB_C
      n+mod(jc–1, NB_C) ≤ NB_C

    - If *trans* = 'N', then (in the process grid) the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***B***; that is, *iacol* = *ibcol*, where:

      iacol = mod((((ja–1)/NB_A)+CSRC_A), q)
      ibcol = mod((((jb–1)/NB_B)+CSRC_B), q)

    - | If *trans* = 'T' or 'C', then (in the process grid) the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

      iarow = mod((((ia–1)/MB_A)+RSRC_A), p)
      ibrow = mod((((ib–1)/MB_B)+RSRC_B), p)

    then you must follow these rules:

    - If *trans* = 'N':
      – ***A*** and ***B*** must be block row matrices; that is, if $p > 1$:

        n+mod(ia–1, MB_A) ≤ MB_A
        n+mod(ib–1, MB_B) ≤ MB_B

      – If looping is required, the following block sizes must be equal: NB_A = NB_B.
    - | If *trans* = 'T' or 'C':
      – ***A*** and ***B*** must be block column matrices; that is, if $q > 1$:

        n+mod(ja–1, NB_A) ≤ NB_A
        n+mod(jb–1, NB_B) ≤ NB_B

      – If looping is required, the following block sizes must be equal: MB_A = MB_B.

11. If the following is true:
    - ***C*** is **not** contained within a single block.

    or if all the following are true:

    - ***C*** is contained within a single block.
    - If *trans* = 'N', then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***B***; that is, *iacol* ≠ *ibcol*, where:

      iacol = mod((((ja–1)/NB_A)+CSRC_A), q)
      ibcol = mod((((jb–1)/NB_B)+CSRC_B), q)

    - | If *trans* = 'T' or 'C', then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* ≠ *ibrow*, where:

iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

then you must follow these rules:

- The global symmetric matrix ***C*** must be distributed using a square block-cyclic distribution; that is, MB_C = NB_C.
- The global symmetric matrix ***C*** must be aligned on a block boundary, that is:

  ic–1 must be a multiple of MB_C.
  jc–1 must be a multiple of NB_C.
- If *trans* = `'N'`:
  - The following block sizes must be equal:

    NB_A = NB_B
    MB_A = MB_B = NB_C.
  - The global matrices ***A*** and ***B*** must be aligned on a block row boundary, that is:

    ia–1 must be a multiple of MB_A.
    ib–1 must be a multiple of MB_B.
- If *trans* = `'T'` or `'C'`:
  - The following block sizes must be equal:

    MB_A = MB_B
    NB_A = NB_B = MB_C.
  - The global matrices ***A*** and ***B*** must be aligned on a block column boundary, that is:

    ja–1 must be a multiple of NB_A.
    jb–1 must be a multiple of NB_B.

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.
3. DTYPE_C is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* ≠ 'U' or 'L'
2. *trans* ≠
   - 'N', 'T', or 'C' for PDSYR2K
   - 'N' or 'T' for PZSYR2K
   - 'N' or 'C' for PZHER2K
3. $n < 0$ and *trans* = 'N'; $n < 0$ and *trans* = 'T' or 'C'; $n < 0$ and *trans* is invalid.
4. $k < 0$ and *trans* = 'N'; $k < 0$ and *trans* = 'T' or 'C'; $k < 0$ and *trans* is invalid.
5. M_A < 0 and ($n = 0$ or $k = 0$); M_A < 1 otherwise
6. N_A < 0 and ($n = 0$ or $k = 0$); N_A < 1 otherwise
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A ≥ p
10. CSRC_A < 0 or CSRC_A ≥ q
11. $ia < 1$
12. $ja < 1$
13. M_B < 0 and ($n = 0$ or $k = 0$); M_B < 1 otherwise
14. N_B < 0 and ($n = 0$ or $k = 0$); N_B < 1 otherwise
15. MB_B < 1
16. NB_B < 1
17. RSRC_B < 0 or RSRC_B ≥ p
18. CSRC_B < 0 or CSRC_B ≥ q
19. $ib < 1$
20. $jb < 1$
21. M_C < 0 and $n = 0$; M_C < 1 otherwise
22. N_C < 0 and $n = 0$; N_C < 1 otherwise
23. MB_C < 1
24. NB_C < 1
25. RSRC_C < 0 or RSRC_C ≥ p
26. CSRC_C < 0 or CSRC_C ≥ q
27. $ic < 1$
28. $jc < 1$
29. CTXT_A ≠ CTXT_B
30. CTXT_A ≠ CTXT_C

Stage 5: If $n \neq 0$ and $k \neq 0$:

1. $ia >$ M_A
2. $ja >$ N_A
3. *trans* = 'N' and $ia+n-1 >$ M_A
4. *trans* = 'N' and $ja+k-1 >$ N_A
5. *trans* = 'T' or 'C' and $ia+k-1 >$ M_A
6. *trans* = 'T' or 'C' and $ja+n-1 >$ N_A
7. $ib >$ M_B
8. $jb >$ N_B
9. *trans* = 'N' and $ib+n-1 >$ M_B
10. *trans* = 'N' and $jb+k-1 >$ N_B
11. *trans* = 'T' or 'C' and $ib+k-1 >$ M_B
12. *trans* = 'T' or 'C' and $jb+n-1 >$ N_B

If $n \neq 0$:

13. $ic >$ M_C

14. $jc > N\_C$
15. $ic+n-1 > M\_C$
16. $jc+n-1 > N\_C$

Stage 6: If ***C*** is contained within a single block, that is:

$n+\text{mod}(ic-1, MB\_C) \le MB\_C$
$n+\text{mod}(jc-1, NB\_C) \le NB\_C$

and:

- If *trans* = `'N'`, then (in the process grid) the process column containing the first column of the submatrix ***A*** must also contain the first column of the submatrix ***B***; that is, *iacol* = *ibcol*, where:
  $iacol = \text{mod}((((ja-1)/NB\_A)+CSRC\_A), q)$
  $ibcol = \text{mod}((((jb-1)/NB\_B)+CSRC\_B), q)$
- | If *trans* = `'T'` or `'C'`, then (in the process grid) the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:
  $iarow = \text{mod}((((ia-1)/MB\_A)+RSRC\_A), p)$
  $ibrow = \text{mod}((((ib-1)/MB\_B)+RSRC\_B), p)$

then:

- If *trans* = `'N'`:
  1. $p > 1$ and $n+\text{mod}(ia-1, MB\_A) > MB\_A$
  2. $p > 1$ and $n+\text{mod}(ib-1, MB\_B) > MB\_B$
  3. Looping is required—that is, **either** of the following is true:
     $k+\text{mod}(ja-1, NB\_A) > NB\_A$
     $k+\text{mod}(jb-1, NB\_B) > NB\_B$

     and $NB\_A \ne NB\_B$.
- | If *trans* = `'T'` or `'C'`:
  1. $q > 1$ and $n+\text{mod}(ja-1, NB\_A) > NB\_A$
  2. $q > 1$ and $n+\text{mod}(jb-1, NB\_B) > NB\_B$
  3. Looping is required—that is, **either** of the following is true:
     $k+\text{mod}(ia-1, MB\_A) > MB\_A$
     $k+\text{mod}(ib-1, MB\_B) > MB\_B$

     and $MB\_A \ne MB\_B$.

If ***C*** is **not** contained within a single block, or if ***C*** is contained within a single block and:

- If *trans* = `'N'`, then (in the process grid) the process column containing the first column of the submatrix ***A*** does not contain the first column of the submatrix ***B***; that is, *iacol* ≠ *ibcol*, where:
  $iacol = \text{mod}((((ja-1)/NB\_A)+CSRC\_A), q)$
  $ibcol = \text{mod}((((jb-1)/NB\_B)+CSRC\_B), q)$
- | If *trans* = `'T'` or `'C'`, then (in the process grid) the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* ≠ *ibrow*, where:
  $iarow = \text{mod}((((ia-1)/MB\_A)+RSRC\_A), p)$
  $ibrow = \text{mod}((((ib-1)/MB\_B)+RSRC\_B), p)$

then:

1. MB_C $\neq$ NB_C
2. mod(*ic*–1, MB_C) $\neq$ 0
3. mod(*jc*–1, NB_C) $\neq$ 0

If *trans* = 'N':

4. NB_C $\neq$ MB_A
5. NB_C $\neq$ MB_B
6. NB_A $\neq$ NB_B
7. mod(*ia*–1, MB_A) $\neq$ 0
8. mod(*ib*–1, MB_B) $\neq$ 0

| If *trans* = 'T' or 'C':

9. MB_C $\neq$ NB_A
10. MB_C $\neq$ NB_B
11. MB_A $\neq$ MB_B
12. mod(*ja*–1, NB_A) $\neq$ 0
13. mod(*jb*–1, NB_B) $\neq$ 0

In all cases:

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_B $<$ max(1, LOCp(M_B))
3. LLD_C $<$ max(1, LOCp(M_C))

If *trans* = 'N':

4. Looping is required and mod(*ja*–1, NB_A) $\neq$ mod(*jb*–1, NB_B).
5. In the process grid, the process row containing the first row of the submatrix ***C*** does not contain the first row of the submatrix ***A***; that is, *icrow* $\neq$ *iarow*, where:
   icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)
   iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
6. In the process grid, the process row containing the first row of the submatrix ***C*** does not contain the first row of the submatrix ***B***; that is, *icrow* $\neq$ *ibrow*, where:
   icrow = mod((((*ic*–1)/MB_C)+RSRC_C), *p*)
   ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

| If *trans* = 'T' or 'C':

7. Looping is required and mod(*ia*–1, MB_A) $\neq$ mod(*ib*–1, MB_B).
8. In the process grid, the process column containing the first column of the submatrix ***C*** does not contain the first column of the submatrix ***A***; that is, *iccol* $\neq$ *iacol*, where:
   iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)
   iacol = mod((((*ja*–1)/NB_A)+CSRC_A), *q*)
9. In the process grid, the process column containing the first column of the submatrix ***C*** does not contain the first column of the submatrix ***B***; that is, *iccol* $\neq$ *ibcol*, where:
   iccol = mod((((*jc*–1)/NB_C)+CSRC_C), *q*)
   ibcol = mod((((*jb*–1)/NB_B)+CSRC_B), *q*)

Example 1: This example computes ***C*** = $\alpha \boldsymbol{A}^T \boldsymbol{B} + \alpha \boldsymbol{B}^T \boldsymbol{A} + \beta \boldsymbol{C}$ using a 2 $\times$ 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A   B  IB  JB
                 |       |     |    |       |      |   |   |      |      |   |   |
CALL PDSYR2K( 'U' ,  'T' ,  9  , 8  ,  1.0D0  , A , 1 , 1 ,  DESC_A , B , 1 , 1 ,

              DESC_B    BETA    C  IC  JC   DESC_C
                |         |     |   |   |     |
              DESC_B ,  0.0D0 , C , 1 , 1 , DESC_C )
```

	Desc_A	**Desc_B**	**Desc_C**
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	8	9
N_	9	9	9
MB_	2	2	4
NB_	4	4	4
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = LLD_B = 4 on all processes, LLD_C = 5 on P_{00} and P_{01}, and LLD_C = 4 on P_{10} and P_{11}.

Global general 8 × 9 matrix ***A*** with block size 2 × 4:

```
B,D          0                        1              2
       ┌                                                 ┐
 0     |  0.0 -1.0 -1.0  0.0  |   0.0  0.0  0.0  0.0  |   1.0 |
       |  0.0  1.0  0.0  1.0  |   0.0  1.0  0.0  1.0  |   1.0 |
       | ---------------------|-----------------------|------ |
 1     |  0.0  0.0 -1.0 -1.0  |   0.0  0.0  1.0  0.0  |   1.0 |
       |  0.0  1.0  0.0 -1.0  |   1.0  1.0  0.0  1.0  |   1.0 |
       | ---------------------|-----------------------|------ |
 2     |  1.0  0.0  0.0  0.0  |  -1.0  0.0  0.0  0.0  |   1.0 |
       |  1.0  0.0  0.0  0.0  |   1.0  1.0  0.0  0.0  |   1.0 |
       | ---------------------|-----------------------|------ |
 3     |  0.0  0.0 -1.0  0.0  |  -1.0  0.0  0.0  0.0  |   1.0 |
       | -1.0  0.0  0.0  0.0  |   0.0  0.0 -1.0  0.0  |   1.0 |
       └                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |             0              |          1
-----|----------------------------|----------------------
     |  0.0 -1.0 -1.0  0.0  1.0   |   0.0  0.0  0.0  0.0
     |  0.0  1.0  0.0  1.0  1.0   |   0.0  1.0  0.0  1.0
 0   |  1.0  0.0  0.0  0.0  1.0   |  -1.0  0.0  0.0  0.0
     |  1.0  0.0  0.0  0.0  1.0   |   1.0  1.0  0.0  0.0
-----|----------------------------|----------------------
     |  0.0  0.0 -1.0 -1.0  1.0   |   0.0  0.0  1.0  0.0
     |  0.0  1.0  0.0 -1.0  1.0   |   1.0  1.0  0.0  1.0
 1   |  0.0  0.0 -1.0  0.0  1.0   |  -1.0  0.0  0.0  0.0
     | -1.0  0.0  0.0  0.0  1.0   |   0.0  0.0 -1.0  0.0
```

Global general 8 × 9 matrix ***B*** with block size 2 × 4:

```
B,D           0                       1              2
     ┌                                                      ┐
 0   |  0.0  1.0  1.0  0.0  |   0.0  0.0  0.0  0.0  |  -1.0 |
     |  0.0 -1.0  0.0 -1.0  |   0.0 -1.0  0.0 -1.0  |  -1.0 |
     | ---------------------|-----------------------|------ |
 1   |  0.0  0.0  1.0  1.0  |   0.0  0.0 -1.0  0.0  |  -1.0 |
     |  0.0 -1.0  0.0  1.0  |  -1.0 -1.0  0.0 -1.0  |  -1.0 |
     | ---------------------|-----------------------|------ |
 2   | -1.0  0.0  0.0  0.0  |   1.0  0.0  0.0  0.0  |  -1.0 |
     | -1.0  0.0  0.0  0.0  |  -1.0 -1.0  0.0  0.0  |  -1.0 |
     | ---------------------|-----------------------|------ |
 3   |  0.0  0.0  1.0  0.0  |   1.0  0.0  0.0  0.0  |  -1.0 |
     |  1.0  0.0  0.0  0.0  |   0.0  0.0  1.0  0.0  |  -1.0 |
     └                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***B***:

```
p,q  |             0            |          1
-----|--------------------------|---------------------
     |  0.0  1.0  1.0  0.0 -1.0 |   0.0  0.0  0.0  0.0
     |  0.0 -1.0  0.0 -1.0 -1.0 |   0.0 -1.0  0.0 -1.0
 0   | -1.0  0.0  0.0  0.0 -1.0 |   1.0  0.0  0.0  0.0
     | -1.0  0.0  0.0  0.0 -1.0 |  -1.0 -1.0  0.0  0.0
-----|--------------------------|---------------------
     |  0.0  0.0  1.0  1.0 -1.0 |   0.0  0.0 -1.0  0.0
     |  0.0 -1.0  0.0  1.0 -1.0 |  -1.0 -1.0  0.0 -1.0
 1   |  0.0  0.0  1.0  0.0 -1.0 |   1.0  0.0  0.0  0.0
     |  1.0  0.0  0.0  0.0 -1.0 |   0.0  0.0  1.0  0.0
```

Output:

Global real symmetric matrix ***C*** of order 9 with block size 4 × 4:

```
B,D          0                     1              2
     ┌                                                 ┐
     | -6.0  0.0  0.0  0.0 |  0.0 -2.0 -2.0  0.0 |  -2.0 |
     |    . -6.0 -2.0  0.0 | -2.0 -4.0  0.0 -4.0 |  -2.0 |
 0   |    .    . -6.0 -2.0 | -2.0  0.0  2.0  0.0 |   6.0 |
     |    .    .    . -6.0 |  2.0  0.0  2.0  0.0 |   2.0 |
     | --------------------|---------------------|------- |
     |    .    .    .    . | -8.0 -4.0  0.0 -2.0 |   0.0 |
     |    .    .    .    . |    . -6.0  0.0 -4.0 |  -6.0 |
 1   |    .    .    .    . |    .    . -4.0  0.0 |   0.0 |
     |    .    .    .    . |    .    .    . -4.0 |  -4.0 |
     | --------------------|---------------------|------- |
 2   |    .    .    .    . |    .    .    .    . | -16.0 |
     └                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |             0             |          1
-----|---------------------------|---------------------
     | -6.0  0.0  0.0  0.0  -2.0 |   0.0 -2.0 -2.0  0.0
     |    . -6.0 -2.0  0.0  -2.0 |  -2.0 -4.0  0.0 -4.0
 0   |    .    . -6.0 -2.0   6.0 |  -2.0  0.0  2.0  0.0
     |    .    .    . -6.0   2.0 |   2.0  0.0  2.0  0.0
     |    .    .    .    . -16.0 |     .    .    .    .
-----|---------------------------|---------------------
     |    .    .    .    .   0.0 |  -8.0 -4.0  0.0 -2.0
     |    .    .    .    .  -6.0 |     . -6.0  0.0 -4.0
 1   |    .    .    .    .   0.0 |     .    . -4.0  0.0
     |    .    .    .    .  -4.0 |     .    .    . -4.0
```

| ***Example 2:*** This example computes $\boldsymbol{C} = \alpha \boldsymbol{A}^T\boldsymbol{B}+\alpha \boldsymbol{B}^T\boldsymbol{A}+\beta \boldsymbol{C}$ using a 2 × 2 process
| grid.

| *Call Statements and Input*

```
|   ORDER = 'R'
|   NPROW = 2
|   NPCOL = 2
|   CALL BLACS_GET (0, 0, ICONTXT)
|   CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
|   CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|              UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A   B  IB  JB
|                |      |     |    |       |      |   |   |       |     |   |   |
|   CALL PZSYR2K( 'U' ,  'T' ,  7  , 8  ,  ALPHA  , A , 1 , 1 ,  DESC_A , B , 1 , 1 ,

|               DESC_B    BETA    C  IC  JC   DESC_C
|                 |         |     |   |   |     |
|               DESC_B ,  BETA  , C , 1 , 1 , DESC_C )

|               ALPHA = (1.0,0.0)

|               BETA  = (0.0,0.0)
```

	Desc_A	Desc_B	Desc_C
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	8	7
N_	7	7	7
MB_	2	2	3
NB_	3	3	3
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = LLD_B = 4 on all processes, LLD_C = 4 on P_{00} and P_{01}, and LLD_C = 3 on P_{10} and P_{11}.

Global general 8 × 7 matrix ***A*** with block size 2 × 3:

```
 B,D                   0                                       1                             2
      ┌                                                                                           ┐
  0   | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
      | ------------------------------------|--------------------------------------|------------ |
  1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)  | ( 1.0, 2.0) |
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
      | ------------------------------------|--------------------------------------|------------ |
  2   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
      | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
      | ------------------------------------|--------------------------------------|------------ |
  3   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
      | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)  | (-1.0, 0.0) |
      └                                                                                           ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
 p,q  |                       0                         |                 1
 -----|-------------------------------------------------|------------------------------------
      | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
  0   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
      | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)
 -----|-------------------------------------------------|------------------------------------
      | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 1.0, 2.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)
  1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
      | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
```

Global general 8 × 7 matrix ***B*** with block size 2 × 3:

```
 B,D                   0                                       1                             2
      ┌                                                                                           ┐
  0   | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
      | ------------------------------------|--------------------------------------|------------ |
  1   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)  | (-1.0,-3.0) |
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
      | ------------------------------------|--------------------------------------|------------ |
  2   | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
      | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
      | ------------------------------------|--------------------------------------|------------ |
  3   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
      | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)  | ( 1.0,-1.0) |
      └                                                                                           ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***B***:

```
 p,q  |                       0                         |                 1
 -----|-------------------------------------------------|-----------------------------------
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0)
  0   | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0)
 -----|-------------------------------------------------|-----------------------------------
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0)
  1   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
```

Output:

Global complex symmetric matrix ***C*** of order 7 with block size 3 × 3:

```
 B,D                  0                                          1                            2
     ┌                                                                                                   ┐
     | ( 38.0,-18.0) ( 38.0, -6.0) ( 26.0,  6.0) | ( 32.0,  0.0) ( 35.0, -3.0) ( 44.0,-16.0) | ( 35.0, -7.0) |
  0  |       .       ( 38.0,-18.0) ( 26.0,  2.0) | ( 32.0,  0.0) ( 35.0, -7.0) ( 44.0,-20.0) | ( 35.0, -3.0) |
     |       .             .       ( 14.0,  6.0) | ( 20.0,  8.0) ( 23.0,  5.0) ( 32.0,  0.0) | ( 23.0, 13.0) |
     | ------------------------------------------|-------------------------------------------|-------------- |
     |       .             .             .       | ( 26.0, -6.0) ( 29.0,  7.0) ( 38.0, -6.0) | ( 29.0,  7.0) |
  1  |       .             .             .       |       .       ( 32.0,-16.0) ( 41.0,-17.0) | ( 32.0,  0.0) |
     |       .             .             .       |       .             .       ( 50.0,-30.0) | ( 41.0, -9.0) |
     | ------------------------------------------|-------------------------------------------|-------------- |
  2  |       .             .             .       |       .             .             ,       | ( 32.0, -8.0) |
     └                                                                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
 p,q  |                          0                           |                  1
 -----|------------------------------------------------------|-----------------------------------------
      | ( 38.0, -18.0) ( 38.0, -6.0) ( 26.0,  6.0) ( 35.0, -7.0) | ( 32.0,  0.0) ( 35.0, -3.0) ( 44.0,-16.0)
      |       .        ( 38.0,-18.0) ( 26.0,  2.0) ( 35.0, -3.0) | ( 32.0,  0.0) ( 35.0, -7.0) ( 44.0,-20.0)
  0   |       .              .       ( 14.0,  6.0) ( 23.0, 13.0) | ( 20.0,  8.0) ( 23.0,  5.0) ( 32.0,  0.0)
      |       .              .             .       ( 32.0, -8.0) |       .             .             .
 -----|------------------------------------------------------|-----------------------------------------
      |       .              .             .       ( 29.0,  7.0) | ( 26.0, -6.0) ( 29.0,  7.0) ( 38.0, -6.0)
  1   |       .              .             .       ( 32.0,  0.0) |       .       ( 32.0,-16.0) ( 41.0,-17.0)
      |       .              .             .       ( 41.0, -9.0) |       .             .       ( 50.0,-30.0)
```

| ***Example 3:*** This example computes:

$$C \leftarrow \alpha A^{H} B + \bar{\alpha} B^{H} A + \beta C$$

| using a 2 × 2 process grid.

| **Note:** The imaginary parts of the diagonal elements of a complex Hermitian matrix
| are assumed to be zero, so you do not have to set these values. On output,
| they are set to zero except when β is one and α or *k* is zero.

| *Call Statements and Input*

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET (0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|              UPLO   TRANS   N    K     ALPHA    A  IA  JA    DESC_A   B  IB  JB
|                |      |     |    |       |      |   |   |      |      |   |   |
| CALL PZHER2K( 'U' ,  'C' ,  7  , 8  ,  ALPHA  , A , 1 , 1 ,  DESC_A , B , 1 , 1 ,

|               DESC_B    BETA    C  IC  JC   DESC_C
|                 |         |     |   |   |     |
|               DESC_B ,  BETA  , C , 1 , 1 , DESC_C )

|               ALPHA = (1.0,0.0)

|               BETA  = 0.0
```

	Desc_A	Desc_B	Desc_C
DTYPE_	1	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]	*icontxt*[1]
M_	8	8	7
N_	7	7	7
MB_	2	2	3
NB_	3	3	3
RSRC_	0	0	0
CSRC_	0	0	0
LLD_	See below[2]	See below[2]	See below[2]

| [1] *icontxt* is the output of the BLACS_GRIDINIT call.

| [2] Each process should set the LLD_ as follows:

```
| LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
| LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
| LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

| In this example, LLD_A = LLD_B = 4 on all processes, LLD_C = 4 on P_{00} and
| P_{01}, and LLD_C = 3 on P_{10} and P_{11}.

| Global general 8 × 7 matrix ***A*** with block size 2 × 3:

```
B,D                   0                                 1                         2
     ┌                                                                                      ┐
 0   | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
     | ------------------------------------|--------------------------------------|------------ |
 1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)  | ( 1.0, 2.0) |
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
     | ------------------------------------|--------------------------------------|------------ |
 2   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
     | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)  | ( 0.0, 1.0) |
     | ------------------------------------|--------------------------------------|------------ |
 3   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)  | ( 0.0, 1.0) |
     | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)  | (-1.0, 0.0) |
     └                                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                       0                         |                  1
-----|-------------------------------------------------|------------------------------------
     | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
 0   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
     | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)
-----|-------------------------------------------------|------------------------------------
     | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 1.0, 2.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)
 1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
     | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
```

Global general 8 × 7 matrix ***B*** with block size 2 × 3:

```
B,D                   0                                 1                         2
     ┌                                                                                      ┐
 0   | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
     | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
     | ------------------------------------|--------------------------------------|------------ |
 1   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)  | (-1.0,-3.0) |
     | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
     | ------------------------------------|--------------------------------------|------------ |
 2   | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
     | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0)  | ( 0.0,-2.0) |
     | ------------------------------------|--------------------------------------|------------ |
 3   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)  | ( 0.0,-2.0) |
     | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)  | ( 1.0,-1.0) |
     └                                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
2		
1	P_{10}	P_{11}
3		

Local arrays for ***B***:

```
 p,q  |                      0                       |                 1
 -----|----------------------------------------------|-----------------------------------
      | ( 0.0,-2.0) ( 1.0,-1.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0)
  0   | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 0.0,-2.0) (-1.0,-3.0) (-1.0,-3.0)
 -----|----------------------------------------------|-----------------------------------
      | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) (-1.0,-3.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0)
      | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) (-1.0,-3.0) (-1.0,-3.0)
  1   | ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 0.0,-2.0)
      | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
```

Output:

Global complex Hermitian matrix ***C*** of order 7 with block size 3 × 3:

```
 B,D                   0                                        1                            2
      ┌                                                                                                    ┐
      | (-50.0,  0.0) (-38.0,  0.0) (-26.0,-12.0) | (-32.0, -6.0) (-35.0, -3.0) (-48.0,  6.0) | (-39.0, -3.0) |
  0   |       .       (-50.0,  0.0) (-30.0,-12.0) | (-32.0, -6.0) (-39.0, -3.0) (-52.0,  6.0) | (-35.0, -3.0) |
      |       .             .       (-26.0,  0.0) | (-24.0,  6.0) (-27.0,  9.0) (-32.0, 18.0) | (-19.0,  9.0) |
      | ----------------------------------------- |------------------------------------------|-------------- |
      |       .             .             .       | (-38.0,  0.0) (-25.0,  3.0) (-38.0, 12.0) | (-25.0,  3.0) |
  1   |       .             .             .       |       .       (-48.0,  0.0) (-49.0,  9.0) | (-32.0,  0.0) |
      |       .             .             .       |       .             .       (-62.0,  0.0) | (-41.0, -9.0) |
      | ----------------------------------------- |------------------------------------------|-------------- |
  2   |       .             .             .       |       .             .             ,       | (-40.0,  0.0) |
      └                                                                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0	P_{00}	P_{01}
2		
1	P_{10}	P_{11}

Local arrays for ***C***:

```
 p,q  |                          0                            |                   1
 -----|-------------------------------------------------------|----------------------------------------
      | (-50.0,  0.0) (-38.0,  0.0) (-26.0,-12.0) (-39.0, -3.0) | (-32.0, -6.0) (-35.0, -3.0) (-48.0,  6.0)
      |       .       (-50.0,  0.0) (-30.0,-12.0) (-35.0, -3.0) | (-32.0, -6.0) (-39.0, -3.0) (-52.0,  6.0)
  0   |       .             .       (-26.0,  0.0) (-19.0,  9.0) | (-24.0,  6.0) (-27.0,  9.0) (-32.0, 18.0)
      |       .             .             .       (-40.0,  0.0) |       .             .             .
 -----|-------------------------------------------------------|----------------------------------------
      |       .             .             .       (-25.0,  3.0) | (-38.0,  0.0) (-25.0,  3.0) (-38.0, 12.0)
  1   |       .             .             .       (-32.0,  0.0) |       .       (-48.0,  0.0) (-49.0,  9.0)
      |       .             .             .       (-41.0, -9.0) |       .             .       (-62.0,  0.0)
```

PDTRAN, PZTRANC, and PZTRANU—Matrix Transpose for a General Matrix

| PDTRAN and PZTRANU perform the following matrix computation:

| $\boldsymbol{C} \leftarrow \beta\boldsymbol{C}+\alpha\boldsymbol{A}^{T}$

| PZTRANC performs the following matrix computation:

| $\boldsymbol{C} \leftarrow \beta\boldsymbol{C}+\alpha\boldsymbol{A}^{H}$

where, in the formula above:

$\boldsymbol{A}$ represents the global general submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+m-1}$.
$\boldsymbol{C}$ represents the global general submatrix $\boldsymbol{C}_{ic:ic+m-1,\ jc:jc+n-1}$.
α and β are scalars.

| **Note:** No data should be moved to form $\boldsymbol{A}^{T}$ or $\boldsymbol{A}^{H}$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

In the following three cases, no computation is performed and the subroutine returns after doing some parameter checking:

- $m = 0$
- $n = 0$
- α is zero and β is one.

See references [14] and [15].

Table 55. Data Types

α, β, ***A***, ***C***	**Subprogram**
Long-precision real	PDTRAN
Long-precision complex	PZTRANC and PZTRANU

Syntax

Fortran	CALL PDTRAN | PZTRANC | PZTRANU (*m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *beta*, *c*, *ic*, *jc*, *desc_c*)
C and C++	pdtran | pztranc | pztranu (*m*, *n*, *alpha*, *a*, *ia*, *ja*, *desc_a*, *beta*, *c*, *ic*, *jc*, *desc_c*);

On Entry

m

is the number of rows in submatrix ***C*** and the number of columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n

is the number of columns in submatrix ***C*** and the number of rows in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

alpha
is the scalar α.

Scope: **global**

Specified as: a number of the data type indicated in Table 55 on page 386.

a
is the local part of the global general matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*m*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*m*–1 part of the global matrix.

| **Note:** No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix ***A*** should
| always be stored in its untransposed form.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 55 on page 386. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia
is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja
is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+m-1 \le$ N_A.

desc_a
is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

beta

is the scalar β.

Scope: **global**

Specified as: a number of the data type indicated in Table 55 on page 386.

c

is the local part of the global general matrix ***C***. This identifies the **first element** of the local array C. This subroutine computes the location of the first element of the local subarray used, based on *ic*, *jc*, *desc_c*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ic*+*m*–1) by LOCq(*jc*+*n*–1) part of the local array C must contain the local pieces of the leading *ic*+*m*–1 by *jc*+*n*–1 part of the global matrix.

When β is zero, ***C*** need not be set on input.

Scope: **local**

Specified as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 55 on page 386. Details about the block-cyclic data distribution of global matrix ***C*** are stored in *desc_c*.

ic

is the row index of the global matrix ***C***, identifying the first row of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ic \leq$ M_C and $ic+m-1 \leq$ M_C.

jc

is the column index of the global matrix ***C***, identifying the first column of the submatrix ***C***.

Scope: **global**

Specified as: a fullword integer; $1 \leq jc \leq$ N_C and $jc+n-1 \leq$ N_C.

desc_c

is the array descriptor for global matrix ***C***, described in the following table:

desc_c	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_C	Descriptor type	DTYPE_C=1	Global
2	CTXT_C	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_C	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_C ≥ 0 Otherwise: M_C ≥ 1	Global

desc_c	**Name**	**Description**	**Limits**	**Scope**
4	N_C	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_C $\geq$ 0 Otherwise: N_C $\geq$ 1	Global
5	MB_C	Row block size	MB_C $\geq$ 1	Global
6	NB_C	Column block size	NB_C $\geq$ 1	Global
7	RSRC_C	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_C $< p$	Global
8	CSRC_C	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_C $< q$	Global
9	LLD_C	The leading dimension of the local array	LLD_C $\geq$ max(1,LOCp(M_C))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

On Return

c

is the updated local part of the global general matrix ***C***, containing the results of the computation.

Scope: **local**

Returned as: an LLD_C by (at least) LOCq(N_C) array, containing numbers of the data type indicated in Table 55 on page 386.

Notes and Coding Rules

1. The matrices must have no common elements; otherwise, results are unpredictable.
2. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
3. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
4. The following values must be equal: CTXT_A = CTXT_C.
5. The coding rules (given in this section) and the error conditions (given in the next section) are written in terms of *adist*. To determine a value for *adist*, check the following conditions, in order, and chose the first value having a true condition:
   a. If ***A*** is a block column matrix, that is:

      m+mod(ja–1, NB_A) $\leq$ NB_A

      then *adist* = `'C'`
   b. If ***A*** is a block row matrix, that is:

n+mod(ia–1, MB_A) ≤ MB_A

then *adist* = `'R'`

c. If ***A*** is neither a block column or a block row matrix, then:

   - If $m \le n$, then *adist* = `'C'`.
   - Otherwise, *adist* = `'R'`.

6. If *adist* = `'C'`, then you must follow these coding rules:

   - ***A*** must be aligned on a block row boundary, that is:

     ia–1 must be a multiple of MB_A.

   - ***C*** must be aligned on a block column boundary, that is:

     jc–1 must be a multiple of NB_C.

   - MB_A = NB_C
   - If looping is required—that is, **either** of the following is true:

     m+mod(ja–1, NB_A) > NB_A
     m+mod(ic–1, MB_C) > MB_C

     then:

     – The block column offset of ***A*** must be equal to the block row offset of ***C***; that is, mod(*ja*–1, NB_A) = mod(*ic*–1, MB_C).

     – NB_A = MB_C

7. If *adist* = `'R'`, then you must follow these coding rules:

   - ***A*** must be aligned on a block column boundary, that is:

     ja–1 must be a multiple of NB_A.

   - ***C*** must be aligned on a block row boundary, that is:

     ic–1 must be a multiple of MB_C.

   - NB_A = MB_C
   - If looping is required—that is, **either** of the following is true:

     n+mod(ia–1, MB_A) > MB_A
     n+mod(jc–1, NB_C) > NB_C

     then:

     – The block row offset of ***A*** must be equal to the block column offset of ***C***; that is, mod(*ia*–1, MB_A) = mod(*jc*–1, NB_C).

     – MB_A = NB_C

Error Conditions

Computational Errors: None

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_C is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

| 1. This subroutine was called from outside the process grid.

Stage 4

1. $m < 0$
2. $n < 0$
3. M_A < 0 and ($m = 0$ or $n = 0$); M_A < 1 otherwise
4. N_A < 0 and ($m = 0$ or $n = 0$); N_A < 1 otherwise
5. MB_A < 1
6. NB_A < 1
7. RSRC_A < 0 or RSRC_A $\geq p$
8. CSRC_A < 0 or CSRC_A $\geq q$
9. $ia < 1$
10. $ja < 1$
11. M_C < 0 and ($m = 0$ or $n = 0$); M_C < 1 otherwise
12. N_C < 0 and ($m = 0$ or $n = 0$); N_C < 1 otherwise
13. MB_C < 1
14. NB_C < 1
15. RSRC_C < 0 or RSRC_C $\geq p$
16. CSRC_C < 0 or CSRC_C $\geq q$
17. $ic < 1$
18. $jc < 1$
19. CTXT_A $\neq$ CTXT_C

Stage 5

Note: Some of the following error conditions depend on the value of *adist*—that is, *adist* = `'C'` or or *adist* = `'R'`. For details on determining the value, see "Notes and Coding Rules" on page 389.

If $m \neq 0$ and $n \neq 0$:

1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+n-1 >$ M_A
4. $ja+m-1 >$ N_A
5. $ic >$ M_C
6. $jc >$ N_C
7. $ic+m-1 >$ M_C
8. $jc+n-1 >$ N_C

If *adist* = `'C'`:

1. mod($ia-1$, MB_A) $\neq 0$
2. mod($jc-1$, NB_C) $\neq 0$
3. MB_A $\neq$ NB_C
4. If looping is required—that is, **either** of the following is true:

m+mod(ja–1, NB_A) $>$ NB_A
m+mod(ic–1, MB_C) $>$ MB_C

then:
a. mod(ja–1, NB_A) $\neq$ mod(ic–1, MB_C)
b. NB_A $\neq$ MB_C.

If *adist* = 'R':

1. mod(ja–1, NB_A) $\neq$ 0
2. mod(ic–1, MB_C) $\neq$ 0
3. NB_A $\neq$ MB_C
4. If looping is required—that is, **either** of the following is true:
   n+mod(ia–1, MB_A) $>$ MB_A
   n+mod(jc–1, NB_C) $>$ NB_C

   then:
   a. mod(ia–1, MB_A) $\neq$ mod(jc–1, NB_C)
   b. MB_A $\neq$ NB_C.

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_C $<$ max(1, LOCp(M_C))

Example 1: This example computes $\boldsymbol{C} = \beta\boldsymbol{C}+\alpha\boldsymbol{A}^T$ using a 2 $\times$ 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              M    N     ALPHA    A  IA  JA    DESC_A    BETA    C  IC  JC   DESC_C
              |    |       |      |   |   |      |         |     |   |   |     |
CALL PDTRAN(  9  , 8  ,  1.0D0  , A , 1 , 1 ,  DESC_A ,  1.0D0 , C , 1 , 1 , DESC_C )
```

	Desc_A	**Desc_C**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	8	9
N_	9	8
MB_	2	4
NB_	4	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

	Desc_A	Desc_C

[1] *icontxt* is the output of the BLACS_GRIDINIT call.
[2] Each process should set the LLD_ as follows:
`LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))`
`LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))`
In this example, LLD_A = 4 on all processes, LLD_C = 5 on P_{00} and P_{01}, and LLD_C = 4 on P_{10} and P_{11}.		

Global general 8 × 9 matrix ***A*** with block size 2 × 4:

```
B,D          0                    1             2
     ┌                                                ┐
 0   │  0.0 -1.0 -1.0  0.0  │   0.0  0.0  0.0  0.0  │   1.0 │
     │  0.0  1.0  0.0  1.0  │   0.0  1.0  0.0  1.0  │   1.0 │
     │ ---------------------|-----------------------|------ │
 1   │  0.0  0.0 -1.0 -1.0  │   0.0  0.0  1.0  0.0  │   1.0 │
     │  0.0  1.0  0.0 -1.0  │   1.0  1.0  0.0  1.0  │   1.0 │
     │ ---------------------|-----------------------|------ │
 2   │  1.0  0.0  0.0  0.0  │  -1.0  0.0  0.0  0.0  │   1.0 │
     │  1.0  0.0  0.0  0.0  │   1.0  1.0  0.0  0.0  │   1.0 │
     │ ---------------------|-----------------------|------ │
 3   │  0.0  0.0 -1.0  0.0  │  -1.0  0.0  0.0  0.0  │   1.0 │
     │ -1.0  0.0  0.0  0.0  │   0.0  0.0 -1.0  0.0  │   1.0 │
     └                                                ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │            0              │            1
-----|---------------------------|----------------------
     │  0.0 -1.0 -1.0  0.0  1.0  │   0.0  0.0  0.0  0.0
     │  0.0  1.0  0.0  1.0  1.0  │   0.0  1.0  0.0  1.0
 0   │  1.0  0.0  0.0  0.0  1.0  │  -1.0  0.0  0.0  0.0
     │  1.0  0.0  0.0  0.0  1.0  │   1.0  1.0  0.0  0.0
-----|---------------------------|----------------------
     │  0.0  0.0 -1.0 -1.0  1.0  │   0.0  0.0  1.0  0.0
     │  0.0  1.0  0.0 -1.0  1.0  │   1.0  1.0  0.0  1.0
 1   │  0.0  0.0 -1.0  0.0  1.0  │  -1.0  0.0  0.0  0.0
     │ -1.0  0.0  0.0  0.0  1.0  │   0.0  0.0 -1.0  0.0
```

Global general 9 × 8 matrix ***C*** with block size 4 × 2:

```
B,D       0            1            2            3
     ┌                                                   ┐
     |  0.0  1.0  |   1.0  5.0  |   6.0  7.0  |   8.0  9.0 |
     |  0.0 -1.0  |   0.0 -1.0  |   0.0 -1.0  |   0.0  1.0 |
 0   |  0.0  0.0  |   1.0  1.0  |   0.0  0.0  |  -1.0  0.0 |
     |  0.0 -1.0  |   0.0  1.0  |  -1.0 -1.0  |   0.0  1.0 |
     | -----------|-------------|-------------|----------- |
     | -1.0  2.0  |   0.0  0.0  |   1.0  0.0  |   0.0  0.0 |
     | -1.0  3.0  |   0.0  0.0  |  -1.0 -1.0  |   0.0  0.0 |
 1   |  0.0  4.0  |   1.0  0.0  |   1.0  0.0  |   0.0  0.0 |
     |  1.0  5.0  |   0.0  0.0  |   0.0  0.0  |   1.0  0.0 |
     | -----------|-------------|-------------|----------- |
 2   |  1.0  2.0  |   3.0  4.0  |   1.0  1.0  |   1.0  1.0 |
     └                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |          0           |          1
-----|----------------------|---------------------
     |  0.0  1.0  6.0  7.0  |  1.0  5.0  8.0  9.0
     |  0.0 -1.0  0.0 -1.0  |  0.0 -1.0  0.0  1.0
 0   |  0.0  0.0  0.0  0.0  |  1.0  1.0 -1.0  0.0
     |  0.0 -1.0 -1.0 -1.0  |  0.0  1.0  0.0  1.0
     |  1.0  2.0  1.0  1.0  |  3.0  4.0  1.0  1.0
-----|----------------------|---------------------
     | -1.0  2.0  1.0  0.0  |  0.0  0.0  0.0  0.0
     | -1.0  3.0 -1.0 -1.0  |  0.0  0.0  0.0  0.0
 1   |  0.0  4.0  1.0  0.0  |  1.0  0.0  0.0  0.0
     |  1.0  5.0  0.0  0.0  |  0.0  0.0  1.0  0.0
```

Output:

Global general 9 × 8 matrix ***C*** with block size 4 × 2:

```
B,D       0            1            2            3
     ┌                                                   ┐
     |  0.0  1.0  |   1.0  5.0  |   7.0  8.0  |   8.0  8.0 |
     | -1.0  0.0  |   0.0  0.0  |   0.0 -1.0  |   0.0  1.0 |
 0   | -1.0  0.0  |   0.0  1.0  |   0.0  0.0  |  -2.0  0.0 |
     |  0.0  0.0  |  -1.0  0.0  |  -1.0 -1.0  |   0.0  1.0 |
     | -----------|-------------|-------------|----------- |
     | -1.0  2.0  |   0.0  1.0  |   0.0  1.0  |  -1.0  0.0 |
     | -1.0  4.0  |   0.0  1.0  |  -1.0  0.0  |   0.0  0.0 |
 1   |  0.0  4.0  |   2.0  0.0  |   1.0  0.0  |   0.0 -1.0 |
     |  1.0  6.0  |   0.0  1.0  |   0.0  0.0  |   1.0  0.0 |
     | -----------|-------------|-------------|----------- |
 2   |  2.0  3.0  |   4.0  5.0  |   2.0  2.0  |   2.0  2.0 |
     └                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |          0           |          1
-----|----------------------|----------------------
     |  0.0  1.0  7.0  8.0  |   1.0  5.0  8.0  8.0
     | -1.0  0.0  0.0 -1.0  |   0.0  0.0  0.0  1.0
 0   | -1.0  0.0  0.0  0.0  |   0.0  1.0 -2.0  0.0
     |  0.0  0.0 -1.0 -1.0  |  -1.0  0.0  0.0  1.0
     |  2.0  3.0  2.0  2.0  |   4.0  5.0  2.0  2.0
-----|----------------------|----------------------
     | -1.0  2.0  0.0  1.0  |   0.0  1.0 -1.0  0.0
     | -1.0  4.0 -1.0  0.0  |   0.0  1.0  0.0  0.0
 1   |  0.0  4.0  1.0  0.0  |   2.0  0.0  0.0 -1.0
     |  1.0  6.0  0.0  0.0  |   0.0  1.0  1.0  0.0
```

Example 2: This example computes $\boldsymbol{C} = \beta \boldsymbol{C} + \alpha \boldsymbol{A}^H$ using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

               M    N     ALPHA    A  IA  JA    DESC_A    BETA    C  IC  JC   DESC_C
               |    |       |      |   |   |      |         |     |   |   |     |
CALL PZTRANC(  7  , 8  ,  ALPHA  , A , 1 , 1 ,  DESC_A ,  BETA  , C , 1 , 1 , DESC_C )

               ALPHA = (1.0,0.0)

               BETA  = (1.0,0.0)
```

	Desc_A	**Desc_C**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	8	7
N_	7	8
MB_	2	3
NB_	3	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

	Desc_A	Desc_C

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on all processes, LLD_C = 4 on P_{00} and P_{01}, and LLD_C = 3 on P_{10} and P_{11}.

Global general 8 × 7 matrix ***A*** with block size 2 × 3:

```
B,D                 0                                     1                           2
     ┌                                                                                      ┐
 0   | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
     | ------------------------------------|-------------------------------------|------------ |
 1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) |
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
     | ------------------------------------|-------------------------------------|------------ |
 2   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
     | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
     | ------------------------------------|-------------------------------------|------------ |
 3   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
     | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) |
     └                                                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                       0                         |                  1
-----|-------------------------------------------------|-----------------------------------
     | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
 0   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
     | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)
-----|-------------------------------------------------|-----------------------------------
     | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 1.0, 2.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)
     | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)
 1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
     | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
```

Global general 7 × 8 matrix ***C*** with block size 3 × 2:

```
B,D             0                       1                       2                       3
        ┌                                                                                               ┐
        | ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 5.0, 3.0) | ( 6.0, 4.0) ( 7.0, 5.0) | ( 8.0, 6.0) ( 9.0, 7.0) |
  0     | ( 1.0,-1.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) |
        | ( 2.0, 0.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 2.0, 0.0) ( 3.0, 1.0) | (-1.0,-3.0) ( 0.0,-2.0) |
        | ------------------------|-------------------------|-------------------------|------------------------ |
        | ( 3.0, 1.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 3.0, 1.0) ( 1.0,-1.0) |
  1     | (-1.0,-3.0) ( 2.0, 0.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 2.0, 0.0) ( 0.0,-2.0) |
        | (-1.0,-3.0) ( 3.0, 1.0) | ( 0.0,-2.0) ( 0.0,-2.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
        | ------------------------|-------------------------|-------------------------|------------------------ |
  2     | ( 5.0, 3.0) ( 4.0, 2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
        └                                                                                               ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |                       0                         |                      1
-----|-------------------------------------------------|---------------------------------------------
     | ( 0.0,-2.0) ( 1.0,-1.0) ( 6.0, 4.0) ( 7.0, 5.0) | ( 1.0,-1.0) ( 5.0, 3.0) ( 8.0, 6.0) ( 9.0, 7.0)
     | ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0)
  0  | ( 2.0, 0.0) ( 0.0,-2.0) ( 2.0, 0.0) ( 3.0, 1.0) | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0)
     | ( 5.0, 3.0) ( 4.0, 2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
-----|-------------------------------------------------|---------------------------------------------
     | ( 3.0, 1.0) (-1.0,-3.0) (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 3.0, 1.0) ( 1.0,-1.0)
  1  | (-1.0,-3.0) ( 2.0, 0.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 2.0, 0.0) ( 0.0,-2.0)
     | (-1.0,-3.0) ( 3.0, 1.0) (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
```

Output:

Global general 7 × 8 matrix ***C*** with block size 3 × 2:

```
B,D             0                       1                       2                       3
        ┌                                                                                               ┐
        | ( 0.0,-3.0) ( 1.0,-2.0) | ( 1.0,-2.0) ( 5.0, 2.0) | ( 7.0, 2.0) ( 8.0, 3.0) | ( 8.0, 5.0) ( 8.0, 7.0) |
  0     | ( 0.0,-1.0) ( 0.0,-5.0) | ( 0.0,-3.0) ( 0.0,-5.0) | ( 0.0,-3.0) (-1.0,-4.0) | ( 0.0,-3.0) ( 1.0,-2.0) |
        | ( 1.0, 0.0) ( 0.0,-3.0) | ( 0.0,-1.0) ( 1.0,-2.0) | ( 2.0,-1.0) ( 3.0, 0.0) | (-2.0,-3.0) ( 0.0,-3.0) |
        | ------------------------|-------------------------|-------------------------|------------------------ |
        | ( 3.0, 0.0) ( 0.0,-5.0) | (-1.0,-2.0) ( 0.0,-1.0) | (-1.0,-4.0) (-1.0,-4.0) | ( 3.0, 0.0) ( 1.0,-2.0) |
  1     | (-1.0,-4.0) ( 2.0,-1.0) | ( 0.0,-3.0) ( 1.0,-4.0) | ( 0.0,-1.0) ( 2.0,-3.0) | ( 1.0, 0.0) ( 0.0,-3.0) |
        | (-1.0,-4.0) ( 4.0,-1.0) | ( 0.0,-3.0) ( 1.0,-4.0) | (-1.0,-4.0) ( 0.0,-5.0) | ( 0.0,-3.0) ( 0.0,-3.0) |
        | ------------------------|-------------------------|-------------------------|------------------------ |
  2     | ( 5.0, 2.0) ( 4.0, 1.0) | ( 2.0,-3.0) ( 0.0,-3.0) | ( 1.0,-2.0) ( 0.0,-3.0) | ( 0.0,-3.0) (-1.0,-2.0) |
        └                                                                                               ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
| p,q |                       0                        |                     1
| -----|------------------------------------------------|----------------------------------------------
|      | ( 0.0,-3.0) ( 1.0,-2.0) ( 7.0, 2.0) ( 8.0, 3.0) | ( 1.0,-2.0) ( 5.0, 2.0) ( 8.0, 5.0) ( 8.0, 7.0)
|      | ( 0.0,-1.0) ( 0.0,-5.0) ( 0.0,-3.0) (-1.0,-4.0) | ( 0.0,-3.0) ( 0.0,-5.0) ( 0.0,-3.0) ( 1.0,-2.0)
|  0   | ( 1.0, 0.0) ( 0.0,-3.0) ( 2.0,-1.0) ( 3.0, 0.0) | ( 0.0,-1.0) ( 1.0,-2.0) (-2.0,-3.0) ( 0.0,-3.0)
|      | ( 5.0, 2.0) ( 4.0, 1.0) ( 1.0,-2.0) ( 0.0,-3.0) | ( 2.0,-3.0) ( 0.0,-3.0) ( 0.0,-3.0) (-1.0,-2.0)
| -----|------------------------------------------------|----------------------------------------------
|      | ( 3.0, 0.0) ( 0.0,-5.0) (-1.0,-4.0) (-1.0,-4.0) | (-1.0,-2.0) ( 0.0,-1.0) ( 3.0, 0.0) ( 1.0,-2.0)
|  1   | (-1.0,-4.0) ( 2.0,-1.0) ( 0.0,-1.0) ( 2.0,-3.0) | ( 0.0,-3.0) ( 1.0,-4.0) ( 1.0, 0.0) ( 0.0,-3.0)
|      | (-1.0,-4.0) ( 4.0,-1.0) (-1.0,-4.0) ( 0.0,-5.0) | ( 0.0,-3.0) ( 1.0,-4.0) ( 0.0,-3.0) ( 0.0,-3.0)
```

| ***Example 3:*** This example computes $\boldsymbol{C} = \beta \boldsymbol{C} + \alpha \boldsymbol{A}^T$ using a 2 × 2 process grid.

| *Call Statements and Input*

```
| ORDER = 'R'
| NPROW = 2
| NPCOL = 2
| CALL BLACS_GET (0, 0, ICONTXT)
| CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
| CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

|               M    N     ALPHA    A  IA  JA    DESC_A    BETA    C  IC  JC   DESC_C
|               |    |       |      |   |   |      |        |      |   |   |     |
| CALL PZTRANU( 7  , 8  ,  ALPHA  , A , 1 , 1 ,  DESC_A ,  BETA  , C , 1 , 1 , DESC_C )

|               ALPHA = (1.0,0.0)

|               BETA  = (1.0,0.0)
```

	Desc_A	Desc_C
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	8	7
N_	7	8
MB_	2	3
NB_	3	2
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_C = MAX(1,NUMROC(M_C, MB_C, MYROW, RSRC_C, NPROW))
```

In this example, LLD_A = 4 on all processes, LLD_C = 4 on P_{00} and P_{01}, and LLD_C = 3 on P_{10} and P_{11}.

| Global general 8 × 7 matrix $\boldsymbol{A}$ with block size 2 × 3:

```
 B,D                 0                                 1                           2
      ┌                                                                                  ┐
  0   | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
      | ------------------------------------|-------------------------------------|------------ |
  1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) |
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
      | ------------------------------------|-------------------------------------|------------ |
  2   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
      | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0) | ( 0.0, 1.0) |
      | ------------------------------------|-------------------------------------|------------ |
  3   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) |
      | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) |
      └                                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
 p,q  |                       0                       |                 1
 -----|-----------------------------------------------|-------------------------------------
      | ( 0.0, 1.0) (-1.0, 0.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 1.0, 2.0) ( 0.0, 1.0) ( 1.0, 2.0)
  0   | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
      | ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0) | ( 0.0, 1.0) ( 1.0, 2.0) ( 1.0, 2.0)
 -----|-----------------------------------------------|-------------------------------------
      | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 1.0, 2.0) | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0)
      | ( 0.0, 1.0) ( 1.0, 2.0) ( 0.0, 1.0) ( 0.0, 1.0) | (-1.0, 0.0) ( 1.0, 2.0) ( 1.0, 2.0)
  1   | ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0) | ( 0.0, 1.0) (-1.0, 0.0) ( 0.0, 1.0)
      | (-1.0, 0.0) ( 0.0, 1.0) ( 0.0, 1.0) (-1.0, 0.0) | ( 0.0, 1.0) ( 0.0, 1.0) ( 0.0, 1.0)
```

Global general 7 × 8 matrix ***C*** with block size 3 × 2:

```
 B,D            0                         1                         2                         3
      ┌                                                                                                    ┐
      | ( 0.0,-2.0) ( 1.0,-1.0) | ( 1.0,-1.0) ( 5.0, 3.0) | ( 6.0, 4.0) ( 7.0, 5.0) | ( 8.0, 6.0) ( 9.0, 7.0) |
  0   | ( 1.0,-1.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) |
      | ( 2.0, 0.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 2.0, 0.0) ( 3.0, 1.0) | (-1.0,-3.0) ( 0.0,-2.0) |
      | ------------------------|-------------------------|-------------------------|------------------------ |
      | ( 3.0, 1.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 3.0, 1.0) ( 1.0,-1.0) |
  1   | (-1.0,-3.0) ( 2.0, 0.0) | ( 0.0,-2.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 1.0,-1.0) | ( 2.0, 0.0) ( 0.0,-2.0) |
      | (-1.0,-3.0) ( 3.0, 1.0) | ( 0.0,-2.0) ( 0.0,-2.0) | (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
      | ------------------------|-------------------------|-------------------------|------------------------ |
  2   | ( 5.0, 3.0) ( 4.0, 2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) | ( 0.0,-2.0) ( 0.0,-2.0) |
      └                                                                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |                      0                        |                    1
-----|-----------------------------------------------|---------------------------------------------
     | ( 0.0,-2.0) ( 1.0,-1.0) ( 6.0, 4.0) ( 7.0, 5.0) | ( 1.0,-1.0) ( 5.0, 3.0) ( 8.0, 6.0) ( 9.0, 7.0)
     | ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0) (-1.0,-3.0) | ( 0.0,-2.0) (-1.0,-3.0) ( 0.0,-2.0) ( 1.0,-1.0)
  0  | ( 2.0, 0.0) ( 0.0,-2.0) ( 2.0, 0.0) ( 3.0, 1.0) | ( 1.0,-1.0) ( 1.0,-1.0) (-1.0,-3.0) ( 0.0,-2.0)
     | ( 5.0, 3.0) ( 4.0, 2.0) ( 1.0,-1.0) ( 0.0,-2.0) | ( 1.0,-1.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
-----|-----------------------------------------------|---------------------------------------------
     | ( 3.0, 1.0) (-1.0,-3.0) (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 1.0,-1.0) ( 3.0, 1.0) ( 1.0,-1.0)
  1  | (-1.0,-3.0) ( 2.0, 0.0) ( 1.0,-1.0) ( 1.0,-1.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 2.0, 0.0) ( 0.0,-2.0)
     | (-1.0,-3.0) ( 3.0, 1.0) (-1.0,-3.0) (-1.0,-3.0) | ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0) ( 0.0,-2.0)
```

Output:

Global general 7 × 8 matrix ***C*** with block size 3 × 2:

```
B,D           0                       1                       2                        3
     ┌                                                                                                  ┐
     | ( 0.0,-1.0) ( 1.0, 0.0) | ( 1.0, 0.0) ( 5.0, 4.0) | ( 7.0, 6.0) ( 8.0, 7.0) | ( 8.0, 7.0) ( 8.0, 7.0) |
  0  | ( 0.0,-1.0) ( 0.0,-1.0) | ( 0.0,-1.0) ( 0.0,-1.0) | ( 0.0,-1.0) (-1.0,-2.0) | ( 0.0,-1.0) ( 1.0, 0.0) |
     | ( 1.0, 0.0) ( 0.0,-1.0) | ( 0.0,-1.0) ( 1.0, 0.0) | ( 2.0, 1.0) ( 3.0, 2.0) | (-2.0,-3.0) ( 0.0,-1.0) |
     | ------------------------|-------------------------|-------------------------|------------------------ |
     | ( 3.0, 2.0) ( 0.0,-1.0) | (-1.0,-2.0) ( 0.0,-1.0) | (-1.0,-2.0) (-1.0,-2.0) | ( 3.0, 2.0) ( 1.0, 0.0) |
  1  | (-1.0,-2.0) ( 2.0, 1.0) | ( 0.0,-1.0) ( 1.0, 0.0) | ( 0.0,-1.0) ( 2.0, 1.0) | ( 1.0, 0.0) ( 0.0,-1.0) |
     | (-1.0,-2.0) ( 4.0, 3.0) | ( 0.0,-1.0) ( 1.0, 0.0) | (-1.0,-2.0) ( 0.0,-1.0) | ( 0.0,-1.0) ( 0.0,-1.0) |
     | ------------------------|-------------------------|-------------------------|------------------------ |
  2  | ( 5.0, 4.0) ( 4.0, 3.0) | ( 2.0, 1.0) ( 0.0,-1.0) | ( 1.0, 0.0) ( 0.0,-1.0) | ( 0.0,-1.0) (-1.0,-2.0) |
     └                                                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  |                      0                        |                    1
-----|-----------------------------------------------|---------------------------------------------
     | ( 0.0,-1.0) ( 1.0, 0.0) ( 7.0, 6.0) ( 8.0, 7.0) | ( 1.0, 0.0) ( 5.0, 4.0) ( 8.0, 7.0) ( 8.0, 7.0)
     | ( 0.0,-1.0) ( 0.0,-1.0) ( 0.0,-1.0) (-1.0,-2.0) | ( 0.0,-1.0) ( 0.0,-1.0) ( 0.0,-1.0) ( 1.0, 0.0)
  0  | ( 1.0, 0.0) ( 0.0,-1.0) ( 2.0, 1.0) ( 3.0, 2.0) | ( 0.0,-1.0) ( 1.0, 0.0) (-2.0,-3.0) ( 0.0,-1.0)
     | ( 5.0, 4.0) ( 4.0, 3.0) ( 1.0, 0.0) ( 0.0,-1.0) | ( 2.0, 1.0) ( 0.0,-1.0) ( 0.0,-1.0) (-1.0,-2.0)
-----|-----------------------------------------------|---------------------------------------------
     | ( 3.0, 2.0) ( 0.0,-1.0) (-1.0,-2.0) (-1.0,-2.0) | (-1.0,-2.0) ( 0.0,-1.0) ( 3.0, 2.0) ( 1.0, 0.0)
  1  | (-1.0,-2.0) ( 2.0, 1.0) ( 0.0,-1.0) ( 2.0, 1.0) | ( 0.0,-1.0) ( 1.0, 0.0) ( 1.0, 0.0) ( 0.0,-1.0)
     | (-1.0,-2.0) ( 4.0, 3.0) (-1.0,-2.0) ( 0.0,-1.0) | ( 0.0,-1.0) ( 1.0, 0.0) ( 0.0,-1.0) ( 0.0,-1.0)
```

Chapter 8. Linear Algebraic Equations (Message Passing)

The linear algebraic equation subroutines are described in this chapter. These subroutines include a subset of the ScaLAPACK subroutines.

Note: The dense and banded linear algebraic equation subroutines are designed in accordance with the proposed ScaLAPACK standard. See references [16], [18], [27], and [28]. If these subroutines do not comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Overview of the Dense Linear Algebraic Equation Subroutines

The dense linear algebraic equation subroutines provide solutions to linear systems of equations for real and complex general matrices and their transposes, and for positive definite real symmetric and complex Hermitian matrices.

Table 56. List of Dense Linear Algebraic Equation Subroutines

Descriptive Name	Long-Precision Subroutine	Page
General Matrix Factorization and Solve	PDGESV PZGESV	405
General Matrix Factorization	PDGETRF PZGETRF	422
General Matrix Solve	PDGETRS PZGETRS	434
Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization and Solve	PDPOSV PZPOSV	446
Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization	PDPOTRF PZPOTRF	461
Positive Definite Real Symmetric or Complex Hermitian Matrix Solve	PDPOTRS PZPOTRS	471

Overview of the Banded Linear Algebraic Equation Subroutines

The banded linear algebraic equation subroutines provide solutions to linear systems of equations for real positive definite symmetric band matrices, real general tridiagonal matrices, diagonally-dominant real general tridiagonal matrices, and real positive definite symmetric tridiagonal matrices.

Table 57 (Page 1 of 2). List of Banded Linear Algebraic Equation Subroutines

Descriptive Name	Long-Precision Subroutine	Page
Positive Definite Symmetric Band Matrix Factorization and Solve	PDPBSV	484
Positive Definite Symmetric Band Matrix Factorization	PDPBTRF	497
Positive Definite Symmetric Band Matrix Solve	PDPBTRS	507

Table 57 (Page 2 of 2). *List of Banded Linear Algebraic Equation Subroutines*

Descriptive Name	**Long-Precision Subroutine**	**Page**
General Tridiagonal Matrix Factorization and Solve	PDGTSV	519
General Tridiagonal Matrix Factorization	PDGTTRF	535
General Tridiagonal Matrix Solve	PDGTTRS	553
Diagonally-Dominant General Tridiagonal Matrix Factorization and Solve	PDDTSV	519
Diagonally-Dominant General Tridiagonal Matrix Factorization	PDDTTRF	535
Diagonally-Dominant General Tridiagonal Matrix Solve	PDDTTRS	553
Positive Definite Symmetric Tridiagonal Matrix Factorization and Solve	PDPTSV	573
Positive Definite Symmetric Tridiagonal Matrix Factorization	PDPTTRF	588
Positive Definite Symmetric Tridiagonal Matrix Solve	PDPTTRS	602

Overview of the Fortran 90 Sparse Linear Algebraic Equation Subroutines

The Fortran 90 sparse linear algebraic equation subroutines provide solutions to linear systems of equations for a real general sparse matrix. The sparse utility subroutines provided in Parallel ESSL must be used in conjunction with the sparse linear algebraic equation subroutines.

Table 58. *List of Fortran 90 Sparse Linear Algebraic Equation Subroutines*

Descriptive Name	**Long-Precision Subroutine**	**Page**
Allocates Space for an Array Descriptor for a General Sparse Matrix	PADALL	619
Allocates Space for a General Sparse Matrix	PSPALL	621
Allocates Space for a Dense Vector	PGEALL	623
Inserts Local Data into a General Sparse Matrix	PSPINS	625
Inserts Local Data into a Dense Vector	PGEINS	630
Assembles a General Sparse Matrix	PSPASB	632
Assembles a Dense Vector	PGEASB	635
Preconditioner for a General Sparse Matrix	PSPGPR	637
Iterative Linear System Solver for a General Sparse Matrix	PSPGIS	640
Deallocates Space for a Dense Vector	PGEFREE	645
Deallocates Space for a General Sparse Matrix	PSPFREE	646
Deallocates Space for an Array Descriptor for a General Sparse Matrix	PADFREE	648

Overview of the Fortran 77 Sparse Linear Algebraic Equation Subroutines

The Fortran 77 sparse linear algebraic equation subroutines provide solutions to linear systems of equations for a real general sparse matrix. The sparse utility subroutines provided in Parallel ESSL must be used in conjunction with the sparse linear algebraic equation subroutines.

Table 59. List of The Fortran 77 Sparse Linear Algebraic Equation Subroutines

Descriptive Name	**Long-Precision Subroutine**	**Page**
Initializes an Array Descriptor for a General Sparse Matrix	PADINIT	658
Initializes a General Sparse Matrix	PDSPINIT	660
Inserts Local Data into a General Sparse Matrix	PDSPINS	662
Inserts Local Data into a Dense Vector	PDGEINS	667
Assembles a General Sparse Matrix	PDSPASB	670
Assembles a Dense Vector	PDGEASB	674
Preconditioner for a General Sparse Matrix	PDSPGPR	676
Iterative Linear System Solver for a General Sparse Matrix	PDSPGIS	679

Dense Linear Algebraic Equation Subroutines

This section contains the dense linear algebraic equation subroutine descriptions.

PDGESV and PZGESV—General Matrix Factorization and Solve

These subroutines solve the following systems of equations for multiple right-hand sides:

$\boldsymbol{AX} = \boldsymbol{B}$

In the formula above:

A represents the global general submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
B represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the right-hand sides in its columns.
X represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the solution vectors in its columns.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 60. Data Types

A, ***B***	***ipvt***	**Subroutine**
Long-precision real	Integer	PDGESV
Long-precision complex	Integer	PZGESV

Syntax

Fortran	CALL PDGESV | PZGESV (*n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *b*, *ib*, *jb*, *desc_b*, *info*)
C and C++	pdgesv | pzgesv (*n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *b*, *ib*, *jb*, *desc_b*, *info*);

On Entry

n

is the order of the submatrix ***A*** and the number of rows in submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

nrhs

is the number of right-hand sides— that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \geq 0$.

a

is the local part of the global general matrix ***A***, used in the system of equations. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 60. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

ipvt

See On Return.

b

is the local part of the global general matrix ***B***, containing the right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ib+n-1$) by LOCq($jb+nrhs-1$) part of the local array B must contain the local pieces of the leading $ib+n-1$ by $jb+nrhs-1$ part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 60 on page 405. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ib \leq$ M_B and $ib+n-1 \leq$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \leq jb \leq$ N_B and $jb+nrhs-1 \leq$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$ or $nrhs = 0$: M_B $\geq$ 0 Otherwise: M_B $\geq$ 1	Global
4	N_B	Number of columns in the global matrix	If $n = 0$ or $nrhs = 0$: N_B $\geq$ 0 Otherwise: N_B $\geq$ 1	Global
5	MB_B	Row block size	MB_B $\geq$ 1	Global
6	NB_B	Column block size	NB_B $\geq$ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_B $< p$	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_B $< q$	Global
9	LLD_B	The leading dimension of the local array	LLD_B $\geq$ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

info

See On Return.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the factorization.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 60 on page 405.

ipvt

is the local part of the global vector ***ipvt***, containing the pivot indices. This identifies the **first element** of the local array `IPVT`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, and *myrow*; therefore, the leading LOCp(ia+m−1) part of the local array `IPVT` must contain the local pieces of the leading ia+m−1 part of the global vector.

A copy of the vector ***ipvt***, with a block size of MB_A and global index *ia*, is returned to each column of the process grid. The process row over which the first row of ***ipvt*** is distributed is RSRC_A.

Scope: **local**

Returned as: an array of (at least) length LOCp(ia+m−1), containing fullword integers, where $ia \le$ (pivoting indices) $\le ia+m-1$. Details about the block-cyclic data distribution of global vector ***ipvt*** are stored in *desc_a*.

b

is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 60 on page 405.

info

has the following meaning:

If $info = 0$, global submatrix ***A*** is not singular, and the factorization and solve completed normally.

If $info > 0$, global submatrix ***A*** is singular; that is, one or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros. All columns of ***L*** are checked. *info* is set equal to *i*, the first column of ***L*** with a corresponding ***U*** = 0 diagonal element, encountered at $\boldsymbol{A}_{ia+i-1,\ ja+i-1}$. The factorization is completed; however, the solution submatrix ***B*** is not computed.

Scope: **global**

Returned as: a fullword integer; $info \ge 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. If $n > 0$ and $nrhs = 0$, only the factorization is computed.
3. The matrices and vector must have no common elements; otherwise, results are unpredictable.
4. The way these subroutines handle singularity differs from ScaLAPACK. These subroutines use the *info* argument to provide information about the singularity of ***A***, like ScaLAPACK, but also provide an error message.
5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.

6. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
7. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
8. The following values must be equal: CTXT_A = CTXT_B.
9. The global general matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
10. The following block sizes must be equal: MB_A = MB_B.
11. The global general matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
12. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
13. The block row offset of ***A*** must be equal to the block row offset of ***B***; that is, mod(*ia*–1, MB_A) = mod(*ib*–1, MB_B).
14. In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
15. There is no array descriptor for ***ipvt***. It is a column-distributed vector with block size MB_A, local arrays of dimension LOCp(*ia*+*m*–1) by 1, and global index *ia*. A copy of this vector exists on each column of the process grid, and the process row over which the first column of ***ipvt*** is distributed is RSRC_A.

Error Conditions

Computational Errors: Matrix ***A*** is a singular matrix. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. $n < 0$
2. $nrhs < 0$
3. M_A < 0 and $n = 0$; M_A < 1 otherwise
4. N_A < 0 and $n = 0$; N_A < 1 otherwise
5. $ia < 1$

6. $ja < 1$
7. MB_A $<$ 1
8. NB_A $<$ 1
9. RSRC_A $<$ 0 or RSRC_A $\geq p$
10. CSRC_A $<$ 0 or CSRC_A $\geq q$
11. M_B $<$ 0 and ($n = 0$ or $nrhs = 0$); M_B $<$ 1 otherwise
12. N_B $<$ 0 and ($n = 0$ or $nrhs = 0$); N_B $<$ 1 otherwise
13. $ib < 1$
14. $jb < 1$
15. MB_B $<$ 1
16. NB_B $<$ 1
17. RSRC_B $<$ 0 or RSRC_B $\geq p$
18. CSRC_B $<$ 0 or CSRC_B $\geq q$
19. CTXT_A $\neq$ CTXT_B

Stage 5

If $n \neq 0$:
1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+n-1 >$ M_A
4. $ja+n-1 >$ N_A

If $n \neq 0$ and $nrhs \neq 0$:
5. $ib >$ M_B
6. $jb >$ N_B
7. $ib+n-1 >$ M_B
8. $jb+nrhs-1 >$ N_B

In all cases:
9. MB_A $\neq$ NB_A
10. mod(ia–1, MB_A) $\neq$ mod(ja–1, NB_A)
11. MB_B $\neq$ MB_A
12. mod(ia–1, MB_A) $\neq$ mod(ib–1, MB_B).
13. mod(ia–1, MB_A) $\neq$ 0
14. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, $iarow \neq ibrow$, where:
    $iarow$ = mod((((ia–1)/MB_A)+RSRC_A), p)
    $ibrow$ = mod((((ib–1)/MB_B)+RSRC_B), p)

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_B $<$ max(1, LOCp(M_B))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

3. n differs.
4. $nrhs$ differs.
5. ia differs.
6. ja differs.
7. DTYPE_A differs.
8. M_A differs.
9. N_A differs.
10. MB_A differs.
11. NB_A differs.

12. RSRC_A differs.
13. CSRC_A differs.
14. *ib* differs.
15. *jb* differs.
16. DTYPE_B differs.
17. M_B differs.
18. N_B differs.
19. MB_B differs.
20. NB_B differs.
21. RSRC_B differs.
22. CSRC_B differs.

Example 1: This example solves the real system ***AX*** = ***B*** where ***A*** is a 9 × 9 real general matrix and ***B*** contains 5 right-hand sides using a 2 × 2 process grid. By specifying RSRC_A = 1, the rows of global matrix ***A*** and the elements of global vector ***ipvt*** are distributed over the process grid starting in the second row of the process grid.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying RSRC_B = 1, the rows of global matrix ***B*** are distributed over the process grid starting in the second row of the process grid. In addition, by specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N  NRHS  A  IA  JA   DESC_A   IPVT   B  IB   JB  DESC_B   INFO
              |  |     |  |   |    |        |      |  |    |   |        |
CALL PDGESV (9 , 5   , A , 1 , 1 , DESC_A , IPVT , B , 1 , 2 , DESC_B , INFO)
```

	Desc_A	**Desc_B**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	1	1
CSRC_	0	1
LLD_	See below[2]	See below[2]

	Desc_A	Desc_B

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 6 on P_{10} and P_{11}.

Global general 9 × 9 matrix ***A*** with block size 3 × 3:

```
B,D        0                   1                   2
      ┌                                                      ┐
      │  1.0  1.2  1.4  │   1.6  1.8  2.0  │   2.2  2.4  2.6 │
 0    │  1.2  1.0  1.2  │   1.4  1.6  1.8  │   2.0  2.2  2.4 │
      │  1.4  1.2  1.0  │   1.2  1.4  1.6  │   1.8  2.0  2.2 │
      │ ----------------|------------------|---------------- │
      │  1.6  1.4  1.2  │   1.0  1.2  1.4  │   1.6  1.8  2.0 │
 1    │  1.8  1.6  1.4  │   1.2  1.0  1.2  │   1.4  1.6  1.8 │
      │  2.0  1.8  1.6  │   1.4  1.2  1.0  │   1.2  1.4  1.6 │
      │ ----------------|------------------|---------------- │
      │  2.2  2.0  1.8  │   1.6  1.4  1.2  │   1.0  1.2  1.4 │
 2    │  2.4  2.2  2.0  │   1.8  1.6  1.4  │   1.2  1.0  1.2 │
      │  2.6  2.4  2.2  │   2.0  1.8  1.6  │   1.4  1.2  1.0 │
      └                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
1	P_{00}	P_{01}
0 **2**	P_{10}	P_{11}

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  │               0                │        1
-----|--------------------------------|-----------------
     │  1.6  1.4  1.2  1.6  1.8  2.0  │   1.0  1.2  1.4
 0   │  1.8  1.6  1.4  1.4  1.6  1.8  │   1.2  1.0  1.2
     │  2.0  1.8  1.6  1.2  1.4  1.6  │   1.5  1.3  1.0
-----|--------------------------------|-----------------
     │  1.0  1.2  1.4  2.2  2.4  2.6  │   1.6  1.8  2.0
     │  1.2  1.0  1.2  2.0  2.2  2.4  │   1.4  1.6  1.8
     │  1.4  1.2  1.0  1.8  2.0  2.2  │   1.2  1.4  1.6
 1   │  2.2  2.0  1.8  1.0  1.2  1.4  │   1.6  1.4  1.2
     │  2.4  2.2  2.0  1.2  1.0  1.2  │   1.8  1.6  1.4
     │  2.6  2.4  2.2  1.4  1.2  1.0  │   2.0  1.8  1.6
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D        0              1              2
      ┌                                            ┐
      |   .  93.0 |  186.0  279.0 |  372.0  465.0 |
 0    |   .  84.4 |  168.8  253.2 |  337.6  422.0 |
      |   .  76.6 |  153.2  229.8 |  306.4  383.0 |
      | ----------|---------------|-------------- |
      |   .  70.0 |  140.0  210.0 |  280.0  350.0 |
 1    |   .  65.0 |  130.0  195.0 |  260.0  325.0 |
      |   .  62.0 |  124.0  186.0 |  248.0  310.0 |
      | ----------|---------------|-------------- |
      |   .  61.4 |  122.8  184.2 |  245.6  307.0 |
 2    |   .  63.6 |  127.2  190.8 |  254.4  318.0 |
      |   .  69.0 |  138.0  207.0 |  276.0  345.0 |
      └                                            ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |       0        |            1
-----|----------------|---------------------------
     | 140.0  210.0   |    .  70.0  280.0  350.0
 0   | 130.0  195.0   |    .  65.0  260.0  325.0
     | 124.0  186.0   |    .  62.0  248.0  310.0
-----|----------------|---------------------------
     | 186.0  279.0   |    .  93.0  372.0  465.0
     | 168.8  253.2   |    .  84.4  337.6  422.0
     | 153.2  229.8   |    .  76.6  306.4  383.0
 1   | 122.8  184.2   |    .  61.4  245.6  307.0
     | 127.2  190.8   |    .  63.6  254.4  318.0
     | 138.0  207.0   |    .  69.0  276.0  345.0
```

Output:

Global general 9 × 9 transformed matrix ***A*** with block size 3 × 3:

```
B,D        0                 1                 2
     ┌                                                   ┐
     │ 2.6  2.4  2.2  │  2.0  1.8  1.6  │  1.4  1.2  1.0 │
 0   │ 0.4  0.3  0.6  │  0.8  1.1  1.4  │  1.7  1.9  2.2 │
     │ 0.5 -0.4  0.4  │  0.8  1.2  1.6  │  2.0  2.4  2.8 │
     | ---------------|-----------------|--------------- |
     │ 0.5 -0.3  0.0  │  0.4  0.8  1.2  │  1.6  2.0  2.4 │
 1   │ 0.6 -0.3  0.0  │  0.0  0.4  0.8  │  1.2  1.6  2.0 │
     │ 0.7 -0.2  0.0  │  0.0  0.0  0.4  │  0.8  1.2  1.6 │
     | ---------------|-----------------|--------------- |
     │ 0.8 -0.2  0.0  │  0.0  0.0  0.0  │  0.4  0.8  1.2 │
 2   │ 0.8 -0.1  0.0  │  0.0  0.0  0.0  │  0.0  0.4  0.8 │
     │ 0.9 -0.1  0.0  │  0.0  0.0  0.0  │  0.0  0.0  0.4 │
     └                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  |               0               |        1
-----|-------------------------------|-----------------
     │ 0.5 -0.3  0.0  1.6  2.0  2.4  │  0.4  0.8  1.2
 0   │ 0.6 -0.3  0.0  1.2  1.6  2.0  │  0.0  0.4  0.8
     │ 0.7 -0.2  0.0  0.8  1.2  1.6  │  0.0  0.0  0.4
-----|-------------------------------|-----------------
     │ 2.6  2.4  2.2  1.4  1.2  1.0  │  2.0  1.8  1.6
     │ 0.4  0.3  0.6  1.7  1.9  2.2  │  0.8  1.1  1.4
     │ 0.5 -0.4  0.4  2.0  2.4  2.8  │  0.8  1.2  1.6
 1   │ 0.8 -0.2  0.0  0.4  0.8  1.2  │  0.0  0.0  0.0
     │ 0.8 -0.1  0.0  0.0  0.4  0.8  │  0.0  0.0  0.0
     │ 0.9 -0.1  0.0  0.0  0.0  0.4  │  0.0  0.0  0.0
```

Global vector ***ipvt*** of length 9 with block size 3:

```
B,D   0
     ┌    ┐
     │  9 │
 0   │  9 │
     │  9 │
     | -- |
     │  9 │
 1   │  9 │
     │  9 │
     | -- |
     │  9 │
 2   │  9 │
     │  9 │
     └    ┘
```

Note: A copy of ***ipvt*** is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D		
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***ipvt*** begins in the second row of the process grid.

Local arrays for ***ipvt***:

```
p,q  |  0  |  1
-----|-----|-----
     |  9  |  9
 0   |  9  |  9
     |  9  |  9
-----|-----|-----
     |  9  |  9
     |  9  |  9
     |  9  |  9
 1   |  9  |  9
     |  9  |  9
     |  9  |  9
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D       0            1             2
     ┌                                        ┐
     |  .   1.0 |   2.0   3.0 |   4.0   5.0 |
 0   |  .   2.0 |   4.0   6.0 |   8.0  10.0 |
     |  .   3.0 |   6.0   9.0 |  12.0  15.0 |
     | ---------|-------------|------------ |
     |  .   4.0 |   8.0  12.0 |  16.0  20.0 |
 1   |  .   5.0 |  10.0  15.0 |  20.0  25.0 |
     |  .   6.0 |  12.0  18.0 |  24.0  30.0 |
     | ---------|-------------|------------ |
     |  .   7.0 |  14.0  21.0 |  28.0  35.0 |
 2   |  .   8.0 |  16.0  24.0 |  32.0  40.0 |
     |  .   9.0 |  18.0  27.0 |  36.0  45.0 |
     └                                        ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |      0       |           1
-----|--------------|------------------------
     |  8.0  12.0   |    .   4.0  16.0  20.0
 0   | 10.0  15.0   |    .   5.0  20.0  25.0
     | 12.0  18.0   |    .   6.0  24.0  30.0
-----|--------------|------------------------
     |  2.0   3.0   |    .   1.0   4.0   5.0
     |  4.0   6.0   |    .   2.0   8.0  10.0
     |  6.0   9.0   |    .   3.0  12.0  15.0
 1   | 14.0  21.0   |    .   7.0  28.0  35.0
     | 16.0  24.0   |    .   8.0  32.0  40.0
     | 18.0  27.0   |    .   9.0  36.0  45.0
```

The value of *info* is 0 on all processes.

Example 2: This example solves the complex system ***AX*** = ***B*** where ***A*** is a 9 × 9 complex general matrix and ***B*** contains 5 right-hand sides using a 2 × 2 process grid. By specifying RSRC_A = 1, the rows of global matrix ***A*** and the elements of global vector ***ipvt*** are distributed over the process grid starting in the second row of the process grid.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying RSRC_B = 1, the rows of global matrix ***B*** are distributed over the process grid starting in the second row of the process grid. In addition, by specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             N  NRHS  A  IA  JA   DESC_A   IPVT   B  IB  JB   DESC_B   INFO
             |   |    |   |   |     |       |     |   |   |     |       |
CALL PZGESV (9 , 5  , A , 1 , 1 , DESC_A , IPVT , B , 1 , 2 , DESC_B , INFO)
```

	Desc_A	**Desc_B**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	1	1
CSRC_	0	1
LLD_	See below[2]	See below[2]

	Desc_A	Desc_B

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 6 on P_{10} and P_{11}.

Global general 9 × 9 matrix ***A*** with block size 3 × 3:

```
B,D               0                                     1                                     2
     ┌                                                                                                       ┐
     │ (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  │  (4.4,-1.0)  (4.8,-1.0)  (5.2,-1.0) │
 0   │ (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  │  (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0) │
     │ (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  │  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0) │
     │ ------------------------------------|--------------------------------------|------------------------------------ │
     │ (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  │  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0) │
 1   │ (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  │  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0) │
     │ (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  │  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0) │
     │ ------------------------------------|--------------------------------------|------------------------------------ │
     │ (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  │  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  │  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0) │
 2   │ (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  │  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  │  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0) │
     │ (5.2, 1.0)  (4.8, 1.0)  (4.4, 1.0)  │  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  │  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) │
     └                                                                                                       ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0	P_{10}	P_{11}
2		

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  │                                 0                                         │                 1
-----│---------------------------------------------------------------------------│------------------------------------
     │ (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  │  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)
 0   │ (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  │  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)
     │ (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  │  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)
-----│---------------------------------------------------------------------------│------------------------------------
     │ (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (4.4,-1.0)  (4.8,-1.0)  (5.2,-1.0)  │  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)
     │ (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0)  │  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)
     │ (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0)  │  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)
 1   │ (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)
     │ (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)
     │ (5.2, 1.0)  (4.8, 1.0)  (4.4, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
| B,D          0                             1                                   2
|       ┌                                                                                    ┐
|       |   .  (193.0,-10.6) |  (200.0, 21.8)  (207.0, 54.2)  |  (214.0, 86.6)  (221.0,119.0) |
|  0    |   .  (173.8, -9.4) |  (178.8, 20.2)  (183.8, 49.8)  |  (188.8, 79.4)  (193.8,109.0) |
|       |   .  (156.2, -5.4) |  (159.2, 22.2)  (162.2, 49.8)  |  (165.2, 77.4)  (168.2,105.0) |
|       | -------------------|--------------------------------|------------------------------ |
|       |   .  (141.0,  1.4) |  (142.0, 27.8)  (143.0, 54.2)  |  (144.0, 80.6)  (145.0,107.0) |
|  1    |   .  (129.0, 11.0) |  (128.0, 37.0)  (127.0, 63.0)  |  (126.0, 89.0)  (125.0,115.0) |
|       |   .  (121.0, 23.4) |  (118.0, 49.8)  (115.0, 76.2)  |  (112.0,102.6)  (109.0,129.0) |
|       | -------------------|--------------------------------|------------------------------ |
|       |   .  (117.8, 38.6) |  (112.8, 66.2)  (107.8, 93.8)  |  (102.8,121.4)   (97.8,149.0) |
|  2    |   .  (120.2, 56.6) |  (113.2, 86.2)  (106.2,115.8)  |   (99.2,145.4)   (92.2,175.0) |
|       |   .  (129.0, 77.4) |  (120.0,109.8)  (111.0,142.2)  |  (102.0,174.6)   (93.0,207.0) |
|       └                                                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
| p,q |              0                |                      1
| -----|-------------------------------|---------------------------------------------------
|      | (142.0, 27.8)  (143.0, 54.2)  |     .   (141.0,  1.4)  (144.0, 80.6)  (145.0,107.0)
|  0   | (128.0, 37.0)  (127.0, 63.0)  |     .   (129.0, 11.0)  (126.0, 89.0)  (125.0,115.0)
|      | (118.0, 49.8)  (115.0, 76.2)  |     .   (121.0, 23.4)  (112.0,102.6)  (109.0,129.0)
| -----|-------------------------------|---------------------------------------------------
|      | (200.0, 21.8)  (207.0, 54.2)  |     .   (193.0,-10.6)  (214.0, 86.6)  (221.0,119.0)
|      | (178.8, 20.2)  (183.8, 49.8)  |     .   (173.8, -9.4)  (188.8, 79.4)  (193.8,109.0)
|      | (159.2, 22.2)  (162.2, 49.8)  |     .   (156.2, -5.4)  (165.2, 77.4)  (168.2,105.0)
|  1   | (112.8, 66.2)  (107.8, 93.8)  |     .   (117.8, 38.6)  (102.8,121.4)   (97.8,149.0)
|      | (113.2, 86.2)  (106.2,115.8)  |     .   (120.2, 56.6)   (99.2,145.4)   (92.2,175.0)
|      | (120.0,109.8)  (111.0,142.2)  |     .   (129.0, 77.4)  (102.0,174.6)   (93.0,207.0)
```

Output:

Global general 9 × 9 transformed matrix ***A*** with block size 3 × 3:

```
B,D                 0                                    1                                    2
     ┌                                                                                                       ┐
     | (5.2, 1.0)  (4.8, 1.0)   (4.4, 1.0) |   (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0) |  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) |
 0   | (0.4, 0.1)  (0.6,-2.0)   (1.1,-1.9) |   (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8) |  (3.4,-1.7)  (3.9,-1.7)  (4.5,-1.6) |
     | (0.5, 0.1)  (0.0,-0.1)   (0.6,-1.9) |   (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6) |  (3.1,-1.5)  (3.7,-1.4)  (4.3,-1.3) |
     | ------------------------------------|---------------------------------------|------------------------------------ |
     | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1) |   (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6) |  (2.7,-1.5)  (3.4,-1.4)  (4.0,-1.2) |
 1   | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1) |  (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7) |  (2.2,-1.6)  (2.9,-1.5)  (3.7,-1.3) |
     | (0.7, 0.1)  (0.0,-0.1)   (0.0, 0.0) |  (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9) |  (1.6,-1.8)  (2.4,-1.6)  (3.2,-1.5) |
     | ------------------------------------|---------------------------------------|------------------------------------ |
     | (0.8, 0.0)  (0.0, 0.0)   (0.0, 0.0) |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) |  (0.8,-1.9)  (1.7,-1.8)  (2.5,-1.8) |
 2   | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0) |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) |  (0.0, 0.0)  (0.8,-2.0)  (1.7,-1.9) |
     | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0) |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0) |  (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0) |
     └                                                                                                       ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  |                                   0                                      |                  1
-----|--------------------------------------------------------------------------|-------------------------------------
     | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  (2.7,-1.5)  (3.4,-1.4)  (4.0,-1.2)  |   (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6)
 0   | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  (2.2,-1.6)  (2.9,-1.5)  (3.7,-1.3)  |  (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7)
     | (0.7, 0.1)  (0.0,-0.1)   (0.0, 0.0)  (1.6,-1.8)  (2.4,-1.6)  (3.2,-1.5)  |  (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9)
-----|--------------------------------------------------------------------------|-------------------------------------
     | (5.2, 1.0)  (4.8, 1.0)   (4.4, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  |   (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0)
     | (0.4, 0.1)  (0.6,-2.0)   (1.1,-1.9)  (3.4,-1.7)  (3.9,-1.7)  (4.5,-1.6)  |   (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8)
     | (0.5, 0.1)  (0.0,-0.1)   (0.6,-1.9)  (3.1,-1.5)  (3.7,-1.4)  (4.3,-1.3)  |   (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6)
 1   | (0.8, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.8,-1.9)  (1.7,-1.8)  (2.5,-1.8)  |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
     | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0)  (1.7,-1.9)  |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
     | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0)  |   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
```

Global vector ***ipvt*** of length 9 with block size 3:

```
B,D     0

      ┌    ┐
      | 9  |
 0    | 9  |
      | 9  |
      | -- |
      | 9  |
 1    | 9  |
      | 9  |
      | -- |
      | 9  |
 2    | 9  |
      | 9  |
      └    ┘
```

Note: A copy of ***ipvt*** is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0	P_{10}	P_{11}
2		

Note: The first row of ***ipvt*** begins in the second row of the process grid.

Local arrays for ***ipvt***:

```
p,q  |  0  |  1
-----|-----|-----
     |  9  |  9
 0   |  9  |  9
     |  9  |  9
-----|-----|-----
     |  9  |  9
     |  9  |  9
     |  9  |  9
 1   |  9  |  9
     |  9  |  9
     |  9  |  9
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D         0                    1                          2
       ┌                                                                  ┐
       |   .   (1.0, 1.0)  |  (1.0, 2.0)  (1.0, 3.0)  |  (1.0, 4.0)  (1.0, 5.0) |
 0     |   .   (2.0, 1.0)  |  (2.0, 2.0)  (2.0, 3.0)  |  (2.0, 4.0)  (2.0, 5.0) |
       |   .   (3.0, 1.0)  |  (3.0, 2.0)  (3.0, 3.0)  |  (3.0, 4.0)  (3.0, 5.0) |
       | ------------------|--------------------------|------------------------- |
       |   .   (4.0, 1.0)  |  (4.0, 2.0)  (4.0, 3.0)  |  (4.0, 4.0)  (4.0, 5.0) |
 1     |   .   (5.0, 1.0)  |  (5.0, 2.0)  (5.0, 3.0)  |  (5.0, 4.0)  (5.0, 5.0) |
       |   .   (6.0, 1.0)  |  (6.0, 2.0)  (6.0, 3.0)  |  (6.0, 4.0)  (6.0, 5.0) |
       | ------------------|--------------------------|------------------------- |
       |   .   (7.0, 1.0)  |  (7.0, 2.0)  (7.0, 3.0)  |  (7.0, 4.0)  (7.0, 5.0) |
 2     |   .   (8.0, 1.0)  |  (8.0, 2.0)  (8.0, 3.0)  |  (8.0, 4.0)  (8.0, 5.0) |
       |   .   (9.0, 1.0)  |  (9.0, 2.0)  (9.0, 3.0)  |  (9.0, 4.0)  (9.0, 5.0) |
       └                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0	P_{10}	P_{11}
2		

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
 p,q |            0             |                    1
-----|--------------------------|-----------------------------------------
     |  (3.0, 2.0)  (3.0, 3.0)  |    .   (3.0, 1.0)  (3.0, 4.0)  (3.0, 5.0)
  0  |  (4.0, 2.0)  (4.0, 3.0)  |    .   (4.0, 1.0)  (4.0, 4.0)  (4.0, 5.0)
     |  (5.0, 2.0)  (5.0, 3.0)  |    .   (5.0, 1.0)  (5.0, 4.0)  (5.0, 5.0)
-----|--------------------------|-----------------------------------------
     |  (1.0, 2.0)  (1.0, 3.0)  |    .   (1.0, 1.0)  (1.0, 4.0)  (1.0, 5.0)
     |  (2.0, 2.0)  (2.0, 3.0)  |    .   (2.0, 1.0)  (2.0, 4.0)  (2.0, 5.0)
     |  (3.0, 2.0)  (3.0, 3.0)  |    .   (3.0, 1.0)  (3.0, 4.0)  (3.0, 5.0)
  1  |  (7.0, 2.0)  (7.0, 3.0)  |    .   (7.0, 1.0)  (7.0, 4.0)  (7.0, 5.0)
     |  (8.0, 2.0)  (8.0, 3.0)  |    .   (8.0, 1.0)  (8.0, 4.0)  (8.0, 5.0)
     |  (9.0, 2.0)  (9.0, 3.0)  |    .   (9.0, 1.0)  (9.0, 4.0)  (9.0, 5.0)
```

The value of *info* is 0 on all processes.

PDGETRF and PZGETRF—General Matrix Factorization

These subroutines factor general matrix ***A*** using Gaussian elimination with partial pivoting, ***ipvt***, to compute the ***LU*** factorization of ***A***, where, in this description:

> ***A*** represents the global general submatrix $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$ to be factored.
> ***ipvt*** represents the global vector $\boldsymbol{ipvt}_{ia:ia+m-1}$ containing the pivoting indices.
> ***L*** is a lower triangular matrix.
> ***U*** is an upper triangular matrix.

On output, the transformed matrix ***A*** contains ***U*** in the upper triangle (if $m \geq n$) or upper trapezoid (if $m < n$) and ***L*** in the strict lower triangle (if $m \leq n$) or lower trapezoid (if $m > n$). ***ipvt*** contains the pivots representing permutation ***P***, such that $\boldsymbol{A} = \boldsymbol{PLU}$.

To solve the system of equations with any number of right-hand sides, follow the call to these subroutines with one or more calls to PDGETRS or PZGETRS, respectively.

If $m = 0$ or $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 61. Data Types

A	***ipvt***	**Subroutine**
Long-precision real	Integer	PDGETRF
Long-precision complex	Integer	PZGETRF

Syntax

Fortran	CALL PDGETRF | PZGETRF (*m*, *n*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *info*)
C and C++	pdgetrf | pzgetrf (*m*, *n*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *info*);

On Entry

m
is the number of rows in submatrix ***A*** and the number of elements in vector ***ipvt*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$.

n
is the number of columns in submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

a
is the local part of the global general matrix ***A***, used in the system of equations. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*m*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*m*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 61 on page 422. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+m-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

ipvt

See On Return.

info

See On Return.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the factorization.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 61 on page 422.

ipvt

is the local part of the global vector ***ipvt***, containing the pivot indices. This identifies the **first element** of the local array `IPVT`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, and *myrow*; therefore, the leading LOCp($ia+m-1$) part of the local array `IPVT` must contain the local pieces of the leading $ia+m-1$ part of the global vector.

A copy of the vector ***ipvt***, with a block size of MB_A and global index *ia*, is returned to each column of the process grid. The process row over which the first row of ***ipvt*** is distributed is RSRC_A.

Scope: **local**

Returned as: an array of (at least) length LOCp($ia+m-1$), containing fullword integers, where $ia \le$ (pivoting indices) $\le ia+m-1$. Details about the block-cyclic data distribution of global vector ***ipvt*** are stored in *desc_a*.

info

has the following meaning:

If $info = 0$, global submatrix ***A*** is not singular, and the factorization completed normally.

If $info > 0$, global submatrix ***A*** is singular; that is, one or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros. All columns of ***L*** are checked. *info* is set equal to *i*, the first column of ***L*** with a corresponding $\boldsymbol{U} = 0$ diagonal element, encountered at $\boldsymbol{A}_{ia+i-1,\ ja+i-1}$. The factorization is completed; however, if you call PDGETRS/PZGETRS with these factors, results are unpredictable.

Scope: **global**

Returned as: a fullword integer; $info \ge 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The matrix and vector must have no common elements; otherwise, results are unpredictable.
3. The scalar data specified for input argument *n* must be the same for both PDGETRF/PZGETRF and PDGETRS/PZGETRS. In addition, the scalar data specified for input argument *m* in PDGETRF/PZGETRF **must be the same** as input argument *n* in both PDGETRF/PZGETRF and PDGETRS/PZGETRS.

   If, however, you do **not** plan to call PDGETRS/PZGETRS after calling PDGETRF/PZGETRF, then input arguments *m* and *n* in PDGETRF/PZGETRF do not need to be equal.
4. The global submatrices for ***A*** and ***ipvt*** input to PDGETRS/PZGETRS must be the same as for the corresponding output arguments for PDGETRF/PZGETRF;

and thus, the scalar data specified for *ia*, *ja*, and the contents of *desc_a* must also be the same.

5. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
6. The way these subroutines handle singularity differs from ScaLAPACK. These subroutines use the *info* argument to provide information about the singularity of ***A***, like ScaLAPACK, but also provide an error message.
7. On both input and output, matrix ***A*** conforms to ScaLAPACK format.
8. The global general matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
9. The global general matrix ***A*** must be aligned on a block row boundary; that is, *ia*−1 must be a multiple of MB_A.
10. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*−1, MB_A) = mod(*ja*−1, NB_A).
11. There is no array descriptor for ***ipvt***. It is a column-distributed vector with block size MB_A, local arrays of dimension LOCp(*ia*+*m*−1) by 1, and global index *ia*. A copy of this vector exists on each column of the process grid, and the process row over which the first column of ***ipvt*** is distributed is RSRC_A.

Performance Considerations

1. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
2. Pivoting imposes additional communication requirements over the process grid columns; therefore, you achieve optimal performance by using a process grid with $p < q$. On the other hand, a $p \times 1$ grid provides the worse possible configuration.
3. For optimal performance, take the following items into consideration when choosing the NB_A (= MB_A) value:
    - The cache size of the computational nodes. NB_A determines the granularity of the most expensive part of the computation, which tends to increase the optimal value of NB_A.
    - The communication and synchronization overhead. This has two aspects, the cost of internal synchronization points and the cost of broadcasts. These tend to slightly decrease the optimal value of NB_A.
    - The model of communication adapter you are using. The High Performance Switch Adapter-2 allows a larger NB.
    - Load balancing. For the best processor utilization, it is necessary for the processor nodes to be active for as long as possible; therefore, each one should have as many blocks as possible. For a given problem size, this tends to decrease the optimal value of NB_A (best load balancing: 1) and is most relevant at very small problem sizes.
    - If NB_A is equal to a power of 2, performance may be degraded.
    - Use the following rules of thumb for reasonably-sized problems:

- For the POWER processors, choose NB_A in the following range:
  - For PDGETRF, use [30, 50], avoiding 32.
  - For PZGETRF, use [10, 25], avoiding 16.
- For the POWER2 processors, choose NB_A in the following range:
  - For PDGETRF, use [60, 80], avoiding 64.
  - For PZGETRF, use [20, 40], avoiding 32.
- For the SMP processors, choose NB_A in the following range:
  - For PDGETRF, use [70, 100].
  - For PZGETRF, use [30, 50], avoiding 32.

Error Conditions

Computational Errors: Matrix ***A*** is a singular matrix. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. $m < 0$
2. $n < 0$
3. M_A < 0 and ($m = 0$ or $n = 0$); M_A < 1 otherwise
4. N_A < 0 and ($m = 0$ or $n = 0$); N_A < 1 otherwise
5. $ia < 1$
6. $ja < 1$
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A $\geq p$
10. CSRC_A < 0 or CSRC_A $\geq q$

Stage 5

If $m \neq 0$ and $n \neq 0$:

1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+m-1 >$ M_A
4. $ja+n-1 >$ N_A

In all cases:

5. MB_A $\neq$ NB_A
6. mod(ia–1, MB_A) $\neq$ mod(ja–1, NB_A)
7. mod(ia–1, MB_A) $\neq 0$

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

2. *m* differs.
3. *n* differs.
4. *ia* differs.
5. *ja* differs.
6. DTYPE_A differs.
7. M_A differs.
8. N_A differs.
9. MB_A differs.
10. NB_A differs.
11. RSRC_A differs.
12. CSRC_A differs.

Example 1: This example factors a 9 $\times$ 9 real general matrix using a 2 $\times$ 2 process grid. By specifying RSRC_A = 1, the rows of global matrix ***A*** and the elements of global vector ***ipvt*** are distributed over the process grid starting in the second row of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              M    N   A  IA  JA    DESC_A   IPVT   INFO
              |    |   |   |   |      |       |      |
CALL PDGETRF( 9  , 9 , A , 1 , 1 ,  DESC_A , IPVT , INFO )
```

	Desc_A
DTYPE_	1
CTXT_	*icontxt*[1]
M_	9
N_	9
MB_	3
NB_	3
RSRC_	1
CSRC_	0
LLD_	See below[2]

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
```

In this example, LLD_A = 3 on P_{00} and P_{01}, and LLD_A = 6 on P_{10} and P_{11}.

Global general 9 $\times$ 9 matrix ***A*** with block size 3 $\times$ 3:

```
B,D         0                   1                   2
     ┌                                                      ┐
     │  1.0  1.2  1.4  │  1.6  1.8  2.0  │  2.2  2.4  2.6 │
 0   │  1.2  1.0  1.2  │  1.4  1.6  1.8  │  2.0  2.2  2.4 │
     │  1.4  1.2  1.0  │  1.2  1.4  1.6  │  1.8  2.0  2.2 │
     │ ----------------|-----------------|----------------│
     │  1.6  1.4  1.2  │  1.0  1.2  1.4  │  1.6  1.8  2.0 │
 1   │  1.8  1.6  1.4  │  1.2  1.0  1.2  │  1.4  1.6  1.8 │
     │  2.0  1.8  1.6  │  1.4  1.2  1.0  │  1.2  1.4  1.6 │
     │ ----------------|-----------------|----------------│
     │  2.2  2.0  1.8  │  1.6  1.4  1.2  │  1.0  1.2  1.4 │
 2   │  2.4  2.2  2.0  │  1.8  1.6  1.4  │  1.2  1.0  1.2 │
     │  2.6  2.4  2.2  │  2.0  1.8  1.6  │  1.4  1.2  1.0 │
     └                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
1	P_{00}	P_{01}
0	P_{10}	P_{11}
2		

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  │              0                │        1
-----|-------------------------------|-----------------
     │  1.6  1.4  1.2  1.6  1.8  2.0 │  1.0  1.2  1.4
 0   │  1.8  1.6  1.4  1.4  1.6  1.8 │  1.2  1.0  1.2
     │  2.0  1.8  1.6  1.2  1.4  1.6 │  1.5  1.3  1.0
-----|-------------------------------|-----------------
     │  1.0  1.2  1.4  2.2  2.4  2.6 │  1.6  1.8  2.0
     │  1.2  1.0  1.2  2.0  2.2  2.4 │  1.4  1.6  1.8
     │  1.4  1.2  1.0  1.8  2.0  2.2 │  1.2  1.4  1.6
 1   │  2.2  2.0  1.8  1.0  1.2  1.4 │  1.6  1.4  1.2
     │  2.4  2.2  2.0  1.2  1.0  1.2 │  1.8  1.6  1.4
     │  2.6  2.4  2.2  1.4  1.2  1.0 │  2.0  1.8  1.6
```

Output:

Global general 9 × 9 transformed matrix ***A*** with block size 3 × 3:

```
B,D         0                   1                   2
     ┌                                                      ┐
     │  2.6  2.4  2.2  │  2.0  1.8  1.6  │  1.4  1.2  1.0 │
 0   │  0.4  0.3  0.6  │  0.8  1.1  1.4  │  1.7  1.9  2.2 │
     │  0.5 -0.4  0.4  │  0.8  1.2  1.6  │  2.0  2.4  2.8 │
     │ ----------------|-----------------|----------------│
     │  0.5 -0.3  0.0  │  0.4  0.8  1.2  │  1.6  2.0  2.4 │
 1   │  0.6 -0.3  0.0  │  0.0  0.4  0.8  │  1.2  1.6  2.0 │
     │  0.7 -0.2  0.0  │  0.0  0.0  0.4  │  0.8  1.2  1.6 │
     │ ----------------|-----------------|----------------│
     │  0.8 -0.2  0.0  │  0.0  0.0  0.0  │  0.4  0.8  1.2 │
 2   │  0.8 -0.1  0.0  │  0.0  0.0  0.0  │  0.0  0.4  0.8 │
     │  0.9 -0.1  0.0  │  0.0  0.0  0.0  │  0.0  0.0  0.4 │
     └                                                      ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  |                0               |        1
-----|--------------------------------|-----------------
     |  0.5 -0.3  0.0  1.6  2.0  2.4  |   0.4  0.8  1.2
 0   |  0.6 -0.3  0.0  1.2  1.6  2.0  |   0.0  0.4  0.8
     |  0.7 -0.2  0.0  0.8  1.2  1.6  |   0.0  0.0  0.4
-----|--------------------------------|-----------------
     |  2.6  2.4  2.2  1.4  1.2  1.0  |   2.0  1.8  1.6
     |  0.4  0.3  0.6  1.7  1.9  2.2  |   0.8  1.1  1.4
     |  0.5 -0.4  0.4  2.0  2.4  2.8  |   0.8  1.2  1.6
 1   |  0.8 -0.2  0.0  0.4  0.8  1.2  |   0.0  0.0  0.0
     |  0.8 -0.1  0.0  0.0  0.4  0.8  |   0.0  0.0  0.0
     |  0.9 -0.1  0.0  0.0  0.0  0.4  |   0.0  0.0  0.0
```

Global vector ***ipvt*** of length 9 with block size 3:

```
B,D    0
     ┌    ┐
     |  9 |
 0   |  9 |
     |  9 |
     | -- |
     |  9 |
 1   |  9 |
     |  9 |
     | -- |
     |  9 |
 2   |  9 |
     |  9 |
     └    ┘
```

Note: A copy of ***ipvt*** is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D		
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***ipvt*** begins in the second row of the process grid.

Local arrays for ***ipvt***:

```
p,q  |  0  |  1
-----|-----|-----
     |  9  |  9
 0   |  9  |  9
     |  9  |  9
-----|-----|-----
     |  9  |  9
     |  9  |  9
     |  9  |  9
 1   |  9  |  9
     |  9  |  9
     |  9  |  9
```

The value of *info* is 0 on all processes.

Example 2: This example factors a 9 × 9 complex matrix using a 2 × 2 process grid. By specifying RSRC_A = 1, the rows of global matrix ***A*** and the elements of global vector ***ipvt*** are distributed over the process grid starting in the second row of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              M    N   A  IA  JA    DESC_A   IPVT   INFO
              |    |   |   |   |      |       |      |
CALL PZGETRF( 9  , 9 , A , 1 , 1 ,  DESC_A , IPVT , INFO )
```

	Desc_A
DTYPE_	1
CTXT_	*icontxt*1
M_	9
N_	9
MB_	3
NB_	3
RSRC_	1
CSRC_	0
LLD_	See below2

1 *icontxt* is the output of the BLACS_GRIDINIT call.

2 Each process should set the LLD_ as follows:

`LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))`

In this example, LLD_A = 3 on P_{00} and P_{01}, and LLD_A = 6 on P_{10} and P_{11}.

Global general 9 × 9 matrix ***A*** with block size 3 × 3:

```
B,D                 0                                      1                                      2
      ┌                                                                                                                 ┐
      │ (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │   (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  │   (4.4,-1.0)  (4.8,-1.0)  (5.2,-1.0) │
 0    │ (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │   (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  │   (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0) │
      │ (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │   (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  │   (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0) │
      │ ------------------------------------|-------------------------------------|------------------------------------ │
      │ (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  │   (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │   (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0) │
 1    │ (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  │   (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │   (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0) │
      │ (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  │   (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │   (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0) │
      │ ------------------------------------|-------------------------------------|------------------------------------ │
      │ (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  │   (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  │   (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0) │
 2    │ (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  │   (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  │   (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0) │
      │ (5.2, 1.0)  (4.8, 1.0)  (4.4, 1.0)  │   (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  │   (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) │
      └                                                                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0	P_{10}	P_{11}
2		

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  │                             0                                        │                 1
-----│----------------------------------------------------------------------│-----------------------------------
     │ (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  │  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)
 0   │ (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  │  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)
     │ (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  │  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)
-----│----------------------------------------------------------------------│-----------------------------------
     │ (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (4.4,-1.0)  (4.8,-1.0)  (5.2,-1.0)  │  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)
     │ (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0)  │  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)
     │ (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0)  │  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)
 1   │ (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  │  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)
     │ (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  │  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)
     │ (5.2, 1.0)  (4.8, 1.0)  (4.4, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  │  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)
```

Output:

Global general 9 × 9 transformed matrix ***A*** with block size 3 × 3:

```
B,D                 0                                       1                                        2
      ┌                                                                                                                     ┐
      │ (5.2, 1.0)  (4.8, 1.0)   (4.4, 1.0)  │   (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0)  │   (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) │
 0    │ (0.4, 0.1)  (0.6,-2.0)   (1.1,-1.9)  │   (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8)  │   (3.4,-1.7)  (3.9,-1.7)  (4.5,-1.6) │
      │ (0.5, 0.1)  (0.0,-0.1)   (0.6,-1.9)  │   (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6)  │   (3.1,-1.5)  (3.7,-1.4)  (4.3,-1.3) │
      │ -------------------------------------|---------------------------------------|------------------------------------ │
      │ (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  │   (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6)  │   (2.7,-1.5)  (3.4,-1.4)  (4.0,-1.2) │
 1    │ (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  │  (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7)  │   (2.2,-1.6)  (2.9,-1.5)  (3.7,-1.3) │
      │ (0.7, 0.1)  (0.0,-0.1)   (0.0, 0.0)  │  (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9)  │   (1.6,-1.8)  (2.4,-1.6)  (3.2,-1.5) │
      │ -------------------------------------|---------------------------------------|------------------------------------ │
      │ (0.8, 0.0)  (0.0, 0.0)   (0.0, 0.0)  │   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  │   (0.8,-1.9)  (1.7,-1.8)  (2.5,-1.8) │
 2    │ (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  │   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  │   (0.0, 0.0)  (0.8,-2.0)  (1.7,-1.9) │
      │ (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  │   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  │   (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0) │
      └                                                                                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***A*** begins in the second row of the process grid.

Local arrays for ***A***:

```
p,q  |                                   0                                    |                 1
-----|------------------------------------------------------------------------|---------------------------------------
     | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  (2.7,-1.5)  (3.4,-1.4)  (4.0,-1.2) |  (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6)
 0   | (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  (2.2,-1.6)  (2.9,-1.5)  (3.7,-1.3) | (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7)
     | (0.7, 0.1)  (0.0,-0.1)   (0.0, 0.0)  (1.6,-1.8)  (2.4,-1.6)  (3.2,-1.5) | (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9)
-----|------------------------------------------------------------------------|---------------------------------------
     | (5.2, 1.0)  (4.8, 1.0)   (4.4, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) |  (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0)
     | (0.4, 0.1)  (0.6,-2.0)   (1.1,-1.9)  (3.4,-1.7)  (3.9,-1.7)  (4.5,-1.6) |  (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8)
     | (0.5, 0.1)  (0.0,-0.1)   (0.6,-1.9)  (3.1,-1.5)  (3.7,-1.4)  (4.3,-1.3) |  (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6)
 1   | (0.8, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.8,-1.9)  (1.7,-1.8)  (2.5,-1.8) |  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
     | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0)  (1.7,-1.9) |  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
     | (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0) |  (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)
```

Global vector ***ipvt*** of length 9 with block size 3:

```
B,D     0
      ┌    ┐
      |  9 |
 0    |  9 |
      |  9 |
      | -- |
      |  9 |
 1    |  9 |
      |  9 |
      | -- |
      |  9 |
 2    |  9 |
      |  9 |
      └    ┘
```

Note: A copy of ***ipvt*** is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D	0 2	1
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***ipvt*** begins in the second row of the process grid.

Local arrays for ***ipvt***:

```
p,q  |  0  |  1
-----|-----|-----
     |  9  |  9
 0   |  9  |  9
     |  9  |  9
-----|-----|-----
     |  9  |  9
     |  9  |  9
     |  9  |  9
 1   |  9  |  9
     |  9  |  9
     |  9  |  9
```

The value of *info* is 0 on all processes.

PDGETRS and PZGETRS—General Matrix Solve

PDGETRS solves one of the following systems of equations for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{B}$

PZGETRS solves one of the following systems of equations for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{X} = \boldsymbol{B}$
3. $\boldsymbol{A}^{\mathrm{H}}\boldsymbol{X} = \boldsymbol{B}$

In the formulas above:

$\boldsymbol{A}$ represents the global general submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ containing the ***LU*** factorization.
$\boldsymbol{B}$ represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the right-hand sides in its columns.
$\boldsymbol{X}$ represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the solution vectors in its columns.

These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to PDGETRF or PZGETRF, respectively. For details on the factorization, see "PDGETRF and PZGETRF—General Matrix Factorization" on page 422.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 62. Data Types

A, ***B***	***ipvt***	**Subroutine**
Long-precision real	Integer	PDGETRS
Long-precision complex	Integer	PZGETRS

Syntax

Fortran	CALL PDGETRS | PZGETRS (*transa*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *b*, *ib*, *jb*, *desc_b*, *info*)
C and C++	pdgetrs | pzgetrs (*transa*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *ipvt*, *b*, *ib*, *jb*, *desc_b*, *info*);

On Entry

transa
indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in solution 1.

If *transa* = `'T'`, $\boldsymbol{A}^{\mathrm{T}}$ is used in the computation, resulting in solution 2.

If *transa* = `'C'`, $\boldsymbol{A}^{\mathrm{H}}$ is used in the computation, resulting in solution 3.

Scope: **global**

Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

n

is the order of the factored submatrix ***A*** and the number of rows in submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

nrhs

is the number of right-hand sides— that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \geq 0$.

a

is the local part of the global general matrix ***A***, containing the factorization of matrix ***A*** produced by a preceding call to PDGETRF or PZGETRF, respectively. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 62 on page 434. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global

desc_a	Name	Description	Limits	Scope
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ p	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ q	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

ipvt

is the local part of the global vector ***ipvt***, containing the pivoting indices produced on a preceding call to PDGETRF or PZGETRF, respectively. This identifies the **first element** of the local array IPVT. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, and *myrow*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array IPVT must contain the local pieces of the leading *ia*+*n*–1 part of the global vector.

A copy of the vector ***ipvt***, with a block size of MB_A and global index *ia*, is contained in each column of the process grid. The process row over which the first row of ***ipvt*** is distributed is RSRC_A.

Scope: **local**

Specified as: an array of (at least) length LOCp(*ia*+*n*–1), containing fullword integers, where *ia* $\leq$ (pivoting index values) $\leq$ *ia*+*m*–1, and *m* is an argument in PDGETRF and PZGETRF. Details about the block-cyclic data distribution of global vector ***ipvt*** are stored in *desc_a*.

b

is the local part of the global general matrix ***B***, containing the right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ib*+*n*–1) by LOCq(*jb*+*nrhs*–1) part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *jb*+*nrhs*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 62 on page 434. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 $\leq$ *ib* $\leq$ M_B and *ib*+*n*–1 $\leq$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le jb \le$ N_B and $jb+nrhs-1 \le$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$ or $nrhs = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $n = 0$ or $nrhs = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_B $< p$	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_B $< q$	Global
9	LLD_B	The leading dimension of the local array	LLD_B $\ge$ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

info

See On Return.

On Return

b

is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 62 on page 434.

info

indicates that a successful computation occurred.

Scope: **global**

Returned as: a fullword integer; $info = 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. This subroutine accepts lowercase letters for the *transa* argument.

3. For PDGETRS, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
4. The matrices and vector must have no common elements; otherwise, results are unpredictable.
5. The scalar data specified for input argument *n* must be the same for both PDGETRF/PZGETRF and PDGETRS/PZGETRS. In addition, the scalar data specified for input argument *m* in PDGETRF/PZGETRF **must be the same** as input argument *n* in both PDGETRF/PZGETRF and PDGETRS/PZGETRS.

   If, however, you do **not** plan to call PDGETRS/PZGETRS after calling PDGETRF/PZGETRF, then input arguments *m* and *n* in PDGETRF/PZGETRF do not need to be equal.
6. The global submatrices for ***A*** and ***ipvt*** input to PDGETRS/PZGETRS must be the same as for the corresponding output arguments for PDGETRF/PZGETRF; and thus, the scalar data specified for *ia*, *ja*, and the contents of *desc_a* must also be the same.
7. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
8. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
9. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
10. The following values must be equal: CTXT_A = CTXT_B.
11. The global general matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
12. The following block sizes must be equal: MB_A = MB_B.
13. The global general matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
14. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
15. The block row offset of ***A*** must be equal to the block row offset of ***B***; that is, mod(*ia*–1, MB_A) = mod(*ib*–1, MB_B).
16. In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)
17. There is no array descriptor for ***ipvt***. It is a column-distributed vector with block size MB_A, local arrays of dimension LOCp(*ia*+*m*–1) by 1, and global index *ia*. A copy of this vector exists on each column of the process grid, and the process row over which the first column of ***ipvt*** is distributed is RSRC_A.

Error Conditions

Computational Errors: None

Note: If the factorization performed by PDGETRF/PZGETRF failed because of a singular matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDGETRF/PZGETRF.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *transa* ≠ 'N', 'T', or 'C'
2. $n < 0$
3. $nrhs < 0$
4. M_A < 0 and $n = 0$; M_A < 1 otherwise
5. N_A < 0 and $n = 0$; N_A < 1 otherwise
6. $ia < 1$
7. $ja < 1$
8. MB_A < 1
9. NB_A < 1
10. RSRC_A < 0 or RSRC_A ≥ p
11. CSRC_A < 0 or CSRC_A ≥ q
12. M_B < 0 and ($n = 0$ or $nrhs = 0$); M_B < 1 otherwise
13. N_B < 0 and ($n = 0$ or $nrhs = 0$); N_B < 1 otherwise
14. $ib < 1$
15. $jb < 1$
16. MB_B < 1
17. NB_B < 1
18. RSRC_B < 0 or RSRC_B ≥ p
19. CSRC_B < 0 or CSRC_B ≥ q
20. CTXT_A ≠ CTXT_B

Stage 5

If $n \neq 0$:
1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+n-1 >$ M_A
4. $ja+n-1 >$ N_A

If $n \neq 0$ and $nrhs \neq 0$:
5. $ib >$ M_B
6. $jb >$ N_B
7. $ib+n-1 >$ M_B

8. $jb+nrhs-1 > N\_B$

In all cases:

9. MB_A ≠ NB_A
10. mod(*ia*–1, MB_A) ≠ mod(*ja*–1, NB_A)
11. MB_B ≠ MB_A
12. mod(*ia*–1, MB_A) ≠ mod(*ib*–1, MB_B).
13. mod(*ia*–1, MB_A) ≠ 0
14. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* ≠ *ibrow*, where:
    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_B < max(1, LOCp(M_B))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

3. *transa* differs.
4. *n* differs.
5. *nrhs* differs.
6. *ia* differs.
7. *ja* differs.
8. DTYPE_A differs.
9. M_A differs.
10. N_A differs.
11. MB_A differs.
12. NB_A differs.
13. RSRC_A differs.
14. CSRC_A differs.
15. *ib* differs.
16. *jb* differs.
17. DTYPE_B differs.
18. M_B differs.
19. N_B differs.
20. MB_B differs.
21. NB_B differs.
22. RSRC_B differs.
23. CSRC_B differs.

Example 1: This example solves the real system ***AX*** = ***B*** with 5 right-hand sides using a 2 × 2 process grid. The input ***ipvt*** vector and transformed matrix ***A*** are the output from "Example 1" on page 427.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying RSRC_B = 1, the rows of global matrix ***B*** are distributed over the process grid starting in the second row of the process grid. In addition, by specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

           TRANSA N  NRHS  A  IA  JA   DESC_A   IPVT   B  IB   JB   DESC_B   INFO
             |    |   |    |   |   |     |       |     |   |   |     |       |
CALL PDGETRS( 'N' , 9 , 5  , A , 1 , 1 , DESC_A , IPVT , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	1	1
CSRC_	0	1
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 6 on P_{10} and P_{11}.

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D        0              1               2
      ┌                                           ┐
      │   .   93.0 │  186.0  279.0 │   372.0  465.0 │
 0    │   .   84.4 │  168.8  253.2 │   337.6  422.0 │
      │   .   76.6 │  153.2  229.8 │   306.4  383.0 │
      │ -----------│---------------│-------------- │
      │   .   70.0 │  140.0  210.0 │   280.0  350.0 │
 1    │   .   65.0 │  130.0  195.0 │   260.0  325.0 │
      │   .   62.0 │  124.0  186.0 │   248.0  310.0 │
      │ -----------│---------------│-------------- │
      │   .   61.4 │  122.8  184.2 │   245.6  307.0 │
 2    │   .   63.6 │  127.2  190.8 │   254.4  318.0 │
      │   .   69.0 │  138.0  207.0 │   276.0  345.0 │
      └                                           ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |       0        |            1
-----|----------------|---------------------------
     |  140.0  210.0  |     .  70.0  280.0  350.0
 0   |  130.0  195.0  |     .  65.0  260.0  325.0
     |  124.0  186.0  |     .  62.0  248.0  310.0
-----|----------------|---------------------------
     |  186.0  279.0  |     .  93.0  372.0  465.0
     |  168.8  253.2  |     .  84.4  337.6  422.0
     |  153.2  229.8  |     .  76.6  306.4  383.0
 1   |  122.8  184.2  |     .  61.4  245.6  307.0
     |  127.2  190.8  |     .  63.6  254.4  318.0
     |  138.0  207.0  |     .  69.0  276.0  345.0
```

Output:

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D       0            1            2
     ┌                                      ┐
     |   .   1.0 |   2.0   3.0 |   4.0   5.0 |
 0   |   .   2.0 |   4.0   6.0 |   8.0  10.0 |
     |   .   3.0 |   6.0   9.0 |  12.0  15.0 |
     |  ---------|-------------|------------ |
     |   .   4.0 |   8.0  12.0 |  16.0  20.0 |
 1   |   .   5.0 |  10.0  15.0 |  20.0  25.0 |
     |   .   6.0 |  12.0  18.0 |  24.0  30.0 |
     |  ---------|-------------|------------ |
     |   .   7.0 |  14.0  21.0 |  28.0  35.0 |
 2   |   .   8.0 |  16.0  24.0 |  32.0  40.0 |
     |   .   9.0 |  18.0  27.0 |  36.0  45.0 |
     └                                      ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |      0       |           1
-----|--------------|-----------------------
     |  8.0  12.0   |    .   4.0  16.0  20.0
 0   | 10.0  15.0   |    .   5.0  20.0  25.0
     | 12.0  18.0   |    .   6.0  24.0  30.0
-----|--------------|-----------------------
     |  2.0   3.0   |    .   1.0   4.0   5.0
     |  4.0   6.0   |    .   2.0   8.0  10.0
     |  6.0   9.0   |    .   3.0  12.0  15.0
 1   | 14.0  21.0   |    .   7.0  28.0  35.0
     | 16.0  24.0   |    .   8.0  32.0  40.0
     | 18.0  27.0   |    .   9.0  36.0  45.0
```

The value of *info* is 0 on all processes.

Example 2: This example solves the complex system ***AX*** = ***B*** with 5 right-hand sides using a 2 × 2 process grid. The input ***ipvt*** vector and transformed matrix ***A*** are the output from "Example 2" on page 430.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying RSRC_B = 1, the rows of global matrix ***B*** are distributed over the process grid starting in the second row of the process grid. In addition, by specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             TRANSA N  NRHS  A  IA  JA   DESC_A   IPVT   B  IB  JB   DESC_B   INFO
               |    |   |    |   |   |     |       |     |   |   |     |       |
CALL PZGETRS( 'N' , 9 , 5  , A , 1 , 1 , DESC_A , IPVT , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	1	1
CSRC_	0	1
LLD_	See below[2]	See below[2]

	Desc_A	Desc_B
[1] *icontxt* is the output of the BLACS_GRIDINIT call. [2] Each process should set the LLD_ as follows: `LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))` `LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))` In this example, LLD_A = LLD_B = 3 on P_{00} and P_{01}, and LLD_A = LLD_B = 6 on P_{10} and P_{11}.		

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D           0                           1                                2
       ┌                                                                               ┐
       │   .  (193.0,-10.6)  │  (200.0, 21.8)  (207.0, 54.2)  │  (214.0, 86.6)  (221.0,119.0) │
 0     │   .  (173.8, -9.4)  │  (178.8, 20.2)  (183.8, 49.8)  │  (188.8, 79.4)  (193.8,109.0) │
       │   .  (156.2, -5.4)  │  (159.2, 22.2)  (162.2, 49.8)  │  (165.2, 77.4)  (168.2,105.0) │
       │ --------------------|--------------------------------|------------------------------ │
       │   .  (141.0,  1.4)  │  (142.0, 27.8)  (143.0, 54.2)  │  (144.0, 80.6)  (145.0,107.0) │
 1     │   .  (129.0, 11.0)  │  (128.0, 37.0)  (127.0, 63.0)  │  (126.0, 89.0)  (125.0,115.0) │
       │   .  (121.0, 23.4)  │  (118.0, 49.8)  (115.0, 76.2)  │  (112.0,102.6)  (109.0,129.0) │
       │ --------------------|--------------------------------|------------------------------ │
       │   .  (117.8, 38.6)  │  (112.8, 66.2)  (107.8, 93.8)  │  (102.8,121.4)   (97.8,149.0) │
 2     │   .  (120.2, 56.6)  │  (113.2, 86.2)  (106.2,115.8)  │   (99.2,145.4)   (92.2,175.0) │
       │   .  (129.0, 77.4)  │  (120.0,109.8)  (111.0,142.2)  │  (102.0,174.6)   (93.0,207.0) │
       └                                                                               ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |              0                |                      1
-----|-------------------------------|-----------------------------------------------
     | (142.0, 27.8)  (143.0, 54.2)  |    .  (141.0,  1.4)  (144.0, 80.6)  (145.0,107.0)
 0   | (128.0, 37.0)  (127.0, 63.0)  |    .  (129.0, 11.0)  (126.0, 89.0)  (125.0,115.0)
     | (118.0, 49.8)  (115.0, 76.2)  |    .  (121.0, 23.4)  (112.0,102.6)  (109.0,129.0)
-----|-------------------------------|-----------------------------------------------
     | (200.0, 21.8)  (207.0, 54.2)  |    .  (193.0,-10.6)  (214.0, 86.6)  (221.0,119.0)
     | (178.8, 20.2)  (183.8, 49.8)  |    .  (173.8, -9.4)  (188.8, 79.4)  (193.8,109.0)
     | (159.2, 22.2)  (162.2, 49.8)  |    .  (156.2, -5.4)  (165.2, 77.4)  (168.2,105.0)
 1   | (112.8, 66.2)  (107.8, 93.8)  |    .  (117.8, 38.6)  (102.8,121.4)   (97.8,149.0)
     | (113.2, 86.2)  (106.2,115.8)  |    .  (120.2, 56.6)   (99.2,145.4)   (92.2,175.0)
     | (120.0,109.8)  (111.0,142.2)  |    .  (129.0, 77.4)  (102.0,174.6)   (93.0,207.0)
```

Output:

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D         0                     1                      2
      ┌                                                                  ┐
      |  .  (1.0, 1.0) | (1.0, 2.0)  (1.0, 3.0) |  (1.0, 4.0)  (1.0, 5.0) |
 0    |  .  (2.0, 1.0) | (2.0, 2.0)  (2.0, 3.0) |  (2.0, 4.0)  (2.0, 5.0) |
      |  .  (3.0, 1.0) | (3.0, 2.0)  (3.0, 3.0) |  (3.0, 4.0)  (3.0, 5.0) |
      | ---------------|------------------------|------------------------ |
      |  .  (4.0, 1.0) | (4.0, 2.0)  (4.0, 3.0) |  (4.0, 4.0)  (4.0, 5.0) |
 1    |  .  (5.0, 1.0) | (5.0, 2.0)  (5.0, 3.0) |  (5.0, 4.0)  (5.0, 5.0) |
      |  .  (6.0, 1.0) | (6.0, 2.0)  (6.0, 3.0) |  (6.0, 4.0)  (6.0, 5.0) |
      | ---------------|------------------------|------------------------ |
      |  .  (7.0, 1.0) | (7.0, 2.0)  (7.0, 3.0) |  (7.0, 4.0)  (7.0, 5.0) |
 2    |  .  (8.0, 1.0) | (8.0, 2.0)  (8.0, 3.0) |  (8.0, 4.0)  (8.0, 5.0) |
      |  .  (9.0, 1.0) | (9.0, 2.0)  (9.0, 3.0) |  (9.0, 4.0)  (9.0, 5.0) |
      └                                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
1	P_{00}	P_{01}
0 2	P_{10}	P_{11}

Note: The first row of ***B*** begins in the second row of the process grid, and the first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |            0            |                    1
-----|-------------------------|----------------------------------------
     | (3.0, 2.0)  (3.0, 3.0)  |   .   (3.0, 1.0)  (3.0, 4.0)  (3.0, 5.0)
 0   | (4.0, 2.0)  (4.0, 3.0)  |   .   (4.0, 1.0)  (4.0, 4.0)  (4.0, 5.0)
     | (5.0, 2.0)  (5.0, 3.0)  |   .   (5.0, 1.0)  (5.0, 4.0)  (5.0, 5.0)
-----|-------------------------|----------------------------------------
     | (1.0, 2.0)  (1.0, 3.0)  |   .   (1.0, 1.0)  (1.0, 4.0)  (1.0, 5.0)
     | (2.0, 2.0)  (2.0, 3.0)  |   .   (2.0, 1.0)  (2.0, 4.0)  (2.0, 5.0)
     | (3.0, 2.0)  (3.0, 3.0)  |   .   (3.0, 1.0)  (3.0, 4.0)  (3.0, 5.0)
 1   | (7.0, 2.0)  (7.0, 3.0)  |   .   (7.0, 1.0)  (7.0, 4.0)  (7.0, 5.0)
     | (8.0, 2.0)  (8.0, 3.0)  |   .   (8.0, 1.0)  (8.0, 4.0)  (8.0, 5.0)
     | (9.0, 2.0)  (9.0, 3.0)  |   .   (9.0, 1.0)  (9.0, 4.0)  (9.0, 5.0)
```

The value of *info* is 0 on all processes.

|
|
PDPOSV and PZPOSV—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization and Solve

| These subroutines solve the following systems of equations for multiple right-hand sides:

| $\boldsymbol{AX} = \boldsymbol{B}$

| where, in the formula above:

| $\boldsymbol{A}$ represents the global positive definite real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
| $\boldsymbol{B}$ represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the right-hand sides in its columns.
| $\boldsymbol{X}$ represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the solution vectors in its columns.

| If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

| *Table 63. Data Types*

A, ***B***	**Subroutine**
Long-precision real	PDPOSV
Long-precision complex	PZPOSV

| ***Syntax***

Fortran	CALL PDPOSV | PZPOSV (*uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *info*)
C and C++	pdposv | pzposv (*uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *info*);

| *On Entry*

| *uplo*
| indicates whether the upper or lower triangular part of the global real symmetric or complex Hermitian submatrix ***A*** is referenced, where:

| If *uplo* = `'U'`, the upper triangular part is referenced.

| If *uplo* = `'L'`, the lower triangular part is referenced.

| Scope: **global**

| Specified as: a single character; *uplo* = `'U'` or `'L'`.
| *n*
| is the order of the submatrix ***A***.

| Scope: **global**

| Specified as: a fullword integer; $n \geq 0$.
| *nrhs*
| is the number of right-hand sides— that is, the number of columns in submatrix ***B*** used in the computation.

| Scope: **global**

| Specified as: a fullword integer; $nrhs \geq 0$.

a

is the local part of the global real symmetric or complex Hermitian matrix ***A***, used in the system of equations. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ia+n-1$) by LOCq($ja+n-1$) part of the local array A must contain the local pieces of the leading $ia+n-1$ by $ja+n-1$ part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 64 on page 461. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global

desc_a	Name	Description	Limits	Scope
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ib*+*n*–1) by LOCq(*jb*+*nrhs*–1) part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *jb*+*nrhs*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 65 on page 471. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ib \leq$ M_B and $ib+n-1 \leq$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \leq jb \leq$ N_B and $jb+nrhs-1 \leq$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	Name	Description	Limits	Scope
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$ or $nrhs = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $n = 0$ or $nrhs = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global

desc_b	Name	Description	Limits	Scope
5	MB_B	Row block size	MB_B $\geq$ 1	Global
6	NB_B	Column block size	NB_B $\geq$ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_B $<$ p	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_B $<$ q	Global
9	LLD_B	The leading dimension of the local array	LLD_B $\geq$ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

info

See On Return.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the factorization.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 64 on page 461.

b

is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 65 on page 471.

info

has the following meaning:

If *info* = 0, the global real symmetric or complex Hermitian submatrix ***A*** is positive definite, and the factorization and solve completed normally.

If *info* > 0, the leading minor of order *k* of the global real symmetric or complex Hermitian submatrix ***A*** is not positive definite. *info* is set equal to *k*, where the leading minor was encountered at $\boldsymbol{A}_{ia+k-1,\ ja+k-1}$. The factorization is not completed. ***A*** is overwritten with the partial factors. The solution submatrix ***B*** is not computed.

Scope: **global**

Returned as: a fullword integer; *info* $\geq$ 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. If $n > 0$ and $nrhs = 0$, only the factorization is computed.
3. This subroutine accepts lowercase letters for the *uplo* argument.

4. On input to PZPOSV, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.
5. The matrices must have no common elements; otherwise, results are unpredictable.
6. The way these subroutines handle nonpositive definiteness differs from ScaLAPACK. These subroutines use the *info* argument to provide information about the nonpositive definiteness of ***A***, like ScaLAPACK, but also provides an error message.
7. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
8. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
9. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
10. The following values must be equal: CTXT_A = CTXT_B.
11. The global real symmetric or complex Hermitian matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
12. The following block sizes must be equal: MB_A = MB_B.
13. The global real symmetric or complex Hermitian matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
14. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
15. The block row offset of ***A*** must be equal to the block row offset of ***B***; that is, mod(*ia*–1, MB_A) = mod(*ib*–1, MB_B).
16. In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

| *Stage 2*

| 1. CTXT_A is invalid.

| *Stage 3*

| 1. This subroutine was called from outside the process grid.

| *Stage 4*

| 1. *uplo* $\neq$ `'U'` or `'L'`
| 2. $n < 0$
| 3. $nrhs < 0$
| 4. M_A < 0 and $n = 0$; M_A < 1 otherwise
| 5. N_A < 0 and $n = 0$; N_A < 1 otherwise
| 6. $ia < 1$
| 7. $ja < 1$
| 8. MB_A < 1
| 9. NB_A < 1
| 10. RSRC_A < 0 or RSRC_A $\geq p$
| 11. CSRC_A < 0 or CSRC_A $\geq q$
| 12. M_B < 0 and ($n = 0$ or $nrhs = 0$); M_B < 1 otherwise
| 13. N_B < 0 and ($n = 0$ or $nrhs = 0$); N_B < 1 otherwise
| 14. $ib < 1$
| 15. $jb < 1$
| 16. MB_B < 1
| 17. NB_B < 1
| 18. RSRC_B < 0 or RSRC_B $\geq p$
| 19. CSRC_B < 0 or CSRC_B $\geq q$
| 20. CTXT_A $\neq$ CTXT_B

| *Stage 5*

| If $n \neq 0$:
| 1. $ia >$ M_A
| 2. $ja >$ N_A
| 3. $ia+n-1 >$ M_A
| 4. $ja+n-1 >$ N_A

| If $n \neq 0$ and $nrhs \neq 0$:
| 5. $ib >$ M_B
| 6. $jb >$ N_B
| 7. $ib+n-1 >$ M_B
| 8. $jb+nrhs-1 >$ N_B

| In all cases:
| 9. MB_A $\neq$ NB_A
| 10. mod($ia-1$, MB_A) $\neq$ mod($ja-1$, NB_A)
| 11. MB_B $\neq$ MB_A
| 12. mod($ia-1$, MB_A) $\neq$ mod($ib-1$, MB_B).
| 13. mod($ia-1$, MB_A) $\neq 0$
| 14. In the process grid, the process row containing the first row of the submatrix ***A***
| does not contain the first row of the submatrix ***B***; that is, *iarow* $\neq$ *ibrow*, where:
| *iarow* = mod(((($ia-1$)/MB_A)+RSRC_A), p)
| *ibrow* = mod(((($ib-1$)/MB_B)+RSRC_B), p)

Stage 6

1. LLD_A < max(1, LOCp(M_A))
2. LLD_B < max(1, LOCp(M_B))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

3. *uplo* differs.
4. *n* differs.
5. *nrhs* differs.
6. *ia* differs.
7. *ja* differs.
8. DTYPE_A differs.
9. M_A differs.
10. N_A differs.
11. MB_A differs.
12. NB_A differs.
13. RSRC_A differs.
14. CSRC_A differs.
15. *ib* differs.
16. *jb* differs.
17. DTYPE_B differs.
18. M_B differs.
19. N_B differs.
20. MB_B differs.
21. NB_B differs.
22. RSRC_B differs.
23. CSRC_B differs.

Example 1: This example solves the positive definite real symmetric system ***AX*** = ***B*** where ***A*** is a 9 × 9 positive definite real symmetric matrix and ***B*** contains 5 right-hand sides using a 2 × 2 process grid.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
```

```
             UPLO  N  NRHS A  IA  JA   DESC_A   B  IB  JB   DESC_B   INFO
              |    |   |   |   |   |     |      |   |   |     |       |
CALL PDPOSV( 'L' , 9 , 5 , A , 1 , 1 , DESC_A , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]

	Desc_A	Desc_B
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	0	0
CSRC_	0	1
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 6 on P_{00} and P_{01}, and LLD_A = LLD_B = 3 on P_{10} and P_{11}.

Global real symmetric matrix ***A*** of order 9 with block size 3 × 3:

```
B,D        0                  1                  2
     ┌                                                   ┐
     │ 1.0   .    .  │   .    .    .  │   .    .    .  │
 0   │ 1.0  2.0   .  │   .    .    .  │   .    .    .  │
     │ 1.0  2.0  3.0 │   .    .    .  │   .    .    .  │
     │ --------------|----------------|---------------- │
     │ 1.0  2.0  3.0 │  4.0   .    .  │   .    .    .  │
 1   │ 1.0  2.0  3.0 │  4.0  5.0   .  │   .    .    .  │
     │ 1.0  2.0  3.0 │  4.0  5.0  6.0 │   .    .    .  │
     │ --------------|----------------|---------------- │
     │ 1.0  2.0  3.0 │  4.0  5.0  6.0 │  7.0   .    .  │
 2   │ 1.0  2.0  3.0 │  4.0  5.0  6.0 │  7.0  8.0   .  │
     │ 1.0  2.0  3.0 │  4.0  5.0  6.0 │  7.0  8.0  9.0 │
     └                                                   ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                 0               |        1
-----|---------------------------------|-----------------
     |  1.0   .    .    .    .    .    |    .    .    .
     |  1.0  2.0   .    .    .    .    |    .    .    .
     |  1.0  2.0  3.0   .    .    .    |    .    .    .
 0   |  1.0  2.0  3.0  7.0   .    .    |   4.0  5.0  6.0
     |  1.0  2.0  3.0  7.0  8.0   .    |   4.0  5.0  6.0
     |  1.0  2.0  3.0  7.0  8.0  9.0   |   4.0  5.0  6.0
-----|---------------------------------|-----------------
     |  1.0  2.0  3.0   .    .    .    |   4.0   .    .
 1   |  1.0  2.0  3.0   .    .    .    |   4.0  5.0   .
     |  1.0  2.0  3.0   .    .    .    |   4.0  5.0  6.0
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D          0                1                2
     ┌                                                  ┐
     |      .    18.0  |   27.0   36.0  |    45.0    9.0 |
 0   |      .    34.0  |   51.0   68.0  |    85.0   17.0 |
     |      .    48.0  |   72.0   96.0  |   120.0   24.0 |
     |  ---------------|----------------|--------------- |
     |      .    60.0  |   90.0  120.0  |   150.0   30.0 |
 1   |      .    70.0  |  105.0  140.0  |   175.0   35.0 |
     |      .    78.0  |  117.0  156.0  |   195.0   39.0 |
     |  ---------------|----------------|--------------- |
     |      .    84.0  |  126.0  168.0  |   210.0   42.0 |
 2   |      .    88.0  |  132.0  176.0  |   220.0   44.0 |
     |      .    90.0  |  135.0  180.0  |   225.0   45.0 |
     └                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	**1**	**0 2**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |       0         |              1
-----|-----------------|------------------------------
     |   27.0   36.0   |      .    18.0   45.0    9.0
     |   51.0   68.0   |      .    34.0   85.0   17.0
     |   72.0   96.0   |      .    48.0  120.0   24.0
 0   |  126.0  168.0   |      .    84.0  210.0   42.0
     |  132.0  176.0   |      .    88.0  220.0   44.0
     |  135.0  180.0   |      .    90.0  225.0   45.0
-----|-----------------|------------------------------
     |   90.0  120.0   |      .    60.0  150.0   30.0
 1   |  105.0  140.0   |      .    70.0  175.0   35.0
     |  117.0  156.0   |      .    78.0  195.0   39.0
```

Output:

Global real symmetric matrix ***A*** of order 9 with block size 3 × 3:

```
B,D        0                  1                  2
     ┌                                                    ┐
     | 1.0   .    .  |   .    .    .  |   .    .    .  |
 0   | 1.0  1.0   .  |   .    .    .  |   .    .    .  |
     | 1.0  1.0  1.0 |   .    .    .  |   .    .    .  |
     | ---------------|-----------------|---------------- |
     | 1.0  1.0  1.0 |  1.0   .    .  |   .    .    .  |
 1   | 1.0  1.0  1.0 |  1.0  1.0   .  |   .    .    .  |
     | 1.0  1.0  1.0 |  1.0  1.0  1.0 |   .    .    .  |
     | ---------------|-----------------|---------------- |
     | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0   .    .  |
 2   | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0   .  |
     | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0  1.0 |
     └                                                    ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                0                |        1
-----|---------------------------------|-----------------
     | 1.0   .    .    .    .    .    |   .    .    .
     | 1.0  1.0   .    .    .    .    |   .    .    .
     | 1.0  1.0  1.0   .    .    .    |   .    .    .
 0   | 1.0  1.0  1.0  1.0   .    .    |  1.0  1.0  1.0
     | 1.0  1.0  1.0  1.0  1.0   .    |  1.0  1.0  1.0
     | 1.0  1.0  1.0  1.0  1.0  1.0   |  1.0  1.0  1.0
-----|---------------------------------|-----------------
     | 1.0  1.0  1.0   .    .    .    |  1.0   .    .
 1   | 1.0  1.0  1.0   .    .    .    |  1.0  1.0   .
     | 1.0  1.0  1.0   .    .    .    |  1.0  1.0  1.0
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D       0           1          2
     ┌                                ┐
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
 0   │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     │ ---------|-----------|---------- │
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
 1   │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     │ ---------|-----------|---------- │
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
 2   │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     │  .   2.0 │  3.0  4.0 │  5.0  1.0 │
     └                                ┘
```

The following is the 2 × 2 process grid:

B,D	**1**	**0 2**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  │     0      │          1
-----|------------|---------------------
     │ 3.0  4.0   │    .   2.0  5.0  1.0
     │ 3.0  4.0   │    .   2.0  5.0  1.0
     │ 3.0  4.0   │    .   2.0  5.0  1.0
 0   │ 3.0  4.0   │    .   2.0  5.0  1.0
     │ 3.0  4.0   │    .   2.0  5.0  1.0
     │ 3.0  4.0   │    .   2.0  5.0  1.0
-----|------------|---------------------
     │ 3.0  4.0   │    .   2.0  5.0  1.0
 1   │ 3.0  4.0   │    .   2.0  5.0  1.0
     │ 3.0  4.0   │    .   2.0  5.0  1.0
```

The value of *info* is 0 on all processes.

Example 2: This example solves the positive definite complex Hermitian system ***AX*** = ***B*** where ***A*** is a 9 × 9 positive definite complex Hermitian matrix and ***B*** contains 5 right-hand sides using a 2 × 2 process grid.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
```

```
                UPLO  N  NRHS  A  IA  JA   DESC_A   B  IB  JB   DESC_B   INFO
                 |    |   |    |   |   |     |      |   |   |     |       |
CALL PZPOSV( 'L' , 9 , 5  , A , 1 , 1 , DESC_A , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	Desc_B
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	0	0
CSRC_	0	1
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 6 on P_{00} and P_{01}, and LLD_A = LLD_B = 3 on P_{10} and P_{11}.

Global complex Hermitian matrix ***A*** of order 9 with block size 3 × 3:

```
B,D                 0                                    1                                    2
     ┌                                                                                                           ┐
     │ (18.0, 0.0)      .           .      │      .           .           .      │      .           .           .      │
 0   │  (1.0, 1.0) (18.0, 0.0)      .      │      .           .           .      │      .           .           .      │
     │  (1.0, 1.0)  (3.0, 1.0) (18.0, 0.0) │      .           .           .      │      .           .           .      │
     │ ------------------------------------│-------------------------------------│------------------------------------ │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │  (18.0, 0.0)      .           .     │      .           .           .      │
 1   │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │   (7.0, 1.0) (18.0, 0.0)      .     │      .           .           .      │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │   (7.0, 1.0)  (9.0, 1.0) (18.0, 0.0)│      .           .           .      │
     │ ------------------------------------│-------------------------------------│------------------------------------ │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)│  (18.0, 0.0)      .           .     │
 2   │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)│  (13.0, 1.0) (18.0, 0.0)      .     │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0) │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)│  (13.0, 1.0) (15.0, 1.0) (18.0, 0.0)│
     └                                                                                                           ┘
```

Note: On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
 p,q |                                    0                                       |                  1
-----|----------------------------------------------------------------------------|---------------------------------------
     | (18.0,  . )      .          .          .          .          .           |     .          .          .
     |  (1.0, 1.0)  (18.0,  . )    .          .          .          .           |     .          .          .
     |  (1.0, 1.0)   (3.0, 1.0)  (18.0,  . )  .          .          .           |     .          .          .
  0  |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (18.0,  . )  .          .         |  (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
     |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (13.0, 1.0)  (18.0,  . )   .      |  (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
     |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (13.0, 1.0)  (15.0, 1.0)  (18.0,  . ) |  (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
-----|----------------------------------------------------------------------------|---------------------------------------
     |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .          .          .       | (18.0,  . )      .          .
  1  |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .          .          .       |  (7.0, 1.0)  (18.0,  . )     .
     |  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .          .          .       |  (7.0, 1.0)   (9.0, 1.0)  (18.0,  . )
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D           0                          1                               2
     ┌                                                                                 ┐
     |    .     (60.0, 10.0) |    (86.0, 2.0)   (112.0, -6.0) | (138.0, -14.0)   (34.0, 18.0) |
 0   |    .     (86.0, 28.0) |  (126.0, 22.0)   (166.0, 16.0) |  (206.0, 10.0)   (46.0, 34.0) |
     |    .    (108.0, 44.0) |  (160.0, 40.0)   (212.0, 36.0) |  (264.0, 32.0)   (56.0, 48.0) |
     | ----------------------|--------------------------------|------------------------------ |
     |    .    (126.0, 58.0) |  (188.0, 56.0)   (250.0, 54.0) |  (312.0, 52.0)   (64.0, 60.0) |
 1   |    .    (140.0, 70.0) |  (210.0, 70.0)   (280.0, 70.0) |  (350.0, 70.0)   (70.0, 70.0) |
     |    .    (150.0, 80.0) |  (226.0, 82.0)   (302.0, 84.0) |  (378.0, 86.0)   (74.0, 78.0) |
     | ----------------------|--------------------------------|------------------------------ |
     |    .    (156.0, 88.0) |  (236.0, 92.0)   (316.0, 96.0) | (396.0, 100.0)   (76.0, 84.0) |
 2   |    .    (158.0, 94.0) | (240.0, 100.0)  (322.0, 106.0) | (404.0, 112.0)   (76.0, 88.0) |
     |    .    (156.0, 98.0) | (238.0, 106.0)  (320.0, 114.0) | (402.0, 122.0)   (74.0, 90.0) |
     └                                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
 p,q |                0                 |                        1
-----|----------------------------------|----------------------------------------------------
     |    (86.0, 2.0)   (112.0, -6.0)   |       .      (60.0, 10.0)  (138.0, -14.0)   (34.0, 18.0)
     |   (126.0, 22.0)  (166.0, 16.0)   |       .      (86.0, 28.0)   (206.0, 10.0)   (46.0, 34.0)
     |   (160.0, 40.0)  (212.0, 36.0)   |       .     (108.0, 44.0)   (264.0, 32.0)   (56.0, 48.0)
  0  |   (236.0, 92.0)  (316.0, 96.0)   |       .     (156.0, 88.0)  (396.0, 100.0)   (76.0, 84.0)
     |  (240.0, 100.0) (322.0, 106.0)   |       .     (158.0, 94.0)  (404.0, 112.0)   (76.0, 88.0)
     |  (238.0, 106.0) (320.0, 114.0)   |       .     (156.0, 98.0)  (402.0, 122.0)   (74.0, 90.0)
-----|----------------------------------|----------------------------------------------------
     |   (188.0, 56.0)  (250.0, 54.0)   |       .     (126.0, 58.0)   (312.0, 52.0)   (64.0, 60.0)
  1  |   (210.0, 70.0)  (280.0, 70.0)   |       .     (140.0, 70.0)   (350.0, 70.0)   (70.0, 70.0)
     |   (226.0, 82.0)  (302.0, 84.0)   |       .     (150.0, 80.0)   (378.0, 86.0)   (74.0, 78.0)
```

Output:

Global complex Hermitian matrix ***A*** of order 9 with block size 3 × 3:

```
B,D                   0                                     1                                     2
      ┌                                                                                                        ┐
      |   (4.2, 0.0)        .            .      |       .            .           .      |       .          .          .     |
  0   | (0.24, 0.24)   (4.2, 0.0)        .      |       .            .           .      |       .          .          .     |
      | (0.24, 0.24) (0.68, 0.24)   (4.2, 0.0)  |       .            .           .      |       .          .          .     |
      | ----------------------------------------|---------------------------------------|-------------------------------------- |
      | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (4.0, 0.0)      .           .      |       .          .          .     |
  1   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |   (1.3, 0.25)  (3.8, 0.0)      .      |       .          .          .     |
      | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |   (1.3, 0.25)  (1.4, 0.26)  (3.5, 0.0) |       .          .          .     |
      | ----------------------------------------|---------------------------------------|-------------------------------------- |
      | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28) |   (3.2, 0.0)      .          .     |
  2   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28) |  (1.6, 0.32)  (2.7, 0.0)      .     |
      | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28) |  (1.6, 0.32)  (1.6, 0.37)  (2.2, 0.0) |
      └                                                                                                        ┘
```

Note: On output, the imaginary parts of the diagonal elements of the matrix are set to zero.

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
 p,q |                                     0                                       |                   1
-----|-----------------------------------------------------------------------------|----------------------------------------
     |   (4.2, 0.0)       .            .            .            .            .     |       .            .            .
     | (0.24, 0.24)   (4.2, 0.0)       .            .            .            .     |       .            .            .
     | (0.24, 0.24) (0.68, 0.24)   (4.2, 0.0)       .            .            .     |       .            .            .
  0  | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)   (3.2, 0,0)       .            .     |  (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  (1.6, 0.32)   (2.7, 0.0)       .     |  (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  (1.6, 0.32)  (1.6, 0.37)   (2.2, 0.0) |  (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
-----|-----------------------------------------------------------------------------|----------------------------------------
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)       .            .            .     |   (4.0, 0.0)       .            .
  1  | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)       .            .            .     |  (1.3, 0.25)   (3.8, 0.0        .
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)       .            .            .     |  (1.3, 0.25)  (1.4, 0.26)   (3.5, 0.0)
```

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
| B,D          0                    1                       2
|       ┌                                                                ┐
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|  0    │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       │ ---------------|-------------------------|----------------------- │
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|  1    │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       │ ---------------|-------------------------|----------------------- │
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|  2    │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       │  .  (2.0, 1.0) │  (3.0, 1.0)  (4.0, 1.0) │  (5.0, 1.0)  (1.0, 1.0) │
|       └                                                                ┘
```

| The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

| **Note:** The first column of ***B*** begins in the second column of the process grid.

| Local arrays for ***B***:

```
| p,q  │           0              │                  1
| -----|--------------------------|----------------------------------------
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|  0   │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
| -----|--------------------------|----------------------------------------
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|  1   │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
|      │ (3.0, 4.0)  (3.0, 4.0)   │   .  (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
```

| The value of *info* is 0 on all processes.

PDPOTRF and PZPOTRF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization

PDPOTRF uses Cholesky factorization to factor a positive definite real symmetric matrix $\boldsymbol{A}$ into one of the following forms:

$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^T$ if $\boldsymbol{A}$ is lower triangular.
$\boldsymbol{A} = \boldsymbol{U}^T\boldsymbol{U}$ if $\boldsymbol{A}$ is upper triangular.

PZPOTRF uses Cholesky factorization to factor a positive definite complex Hermitian matrix $\boldsymbol{A}$ into one of the following forms:

$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^H$ if $\boldsymbol{A}$ is lower triangular.
$\boldsymbol{A} = \boldsymbol{U}^H\boldsymbol{U}$ if $\boldsymbol{A}$ is upper triangular.

In the formulas above:

$\boldsymbol{A}$ represents the global positive definite real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ to be factored.
$\boldsymbol{L}$ is a lower triangular matrix.
$\boldsymbol{U}$ is an upper triangular matrix.

To solve the system of equations with any number of right-hand sides, follow the call to these subroutines with one or more calls to PDPOTRS or PZPOTRS, respectively.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 64. Data Types

A	**Subroutine**
Long-precision real	PDPOTRF
Long-precision complex	PZPOTRF

Syntax

Fortran	CALL PDPOTRF | PZPOTRF (*uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *info*)
C and C++	pdpotrf | pzpotrf (*uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *info*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}$ is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n
is the number of rows and columns in submatrix $\boldsymbol{A}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

a

is the local part of the global real symmetric or complex Hermitian matrix ***A***, used in the system of equations. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix, and:

- If *uplo* = 'U', the leading $n \times n$ upper triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = 'L', the leading $n \times n$ lower triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 64 on page 461. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

info

See On Return.

On Return

a

is the updated local part of the global matrix ***A***, containing the results of the factorization.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 64 on page 461.

info

has the following meaning:

If *info* = 0, global real symmetric or complex Hermitian submatrix ***A*** is positive definite, and the factorization completed normally.

If *info* > 0, the leading minor of order *k* of the global real symmetric or complex Hermitian submatrix ***A*** is not positive definite. *info* is set equal to *k*, where the leading minor was encountered at $\boldsymbol{A}_{ia+k-1,\ ja+k-1}$. The factorization is not completed. ***A*** is overwritten with the partial factors.

Scope: **global**

Returned as: a fullword integer; *info* $\ge$ 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. On input to PZPOTRF, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.
4. The scalar data specified for input argument *n* must be the same for both PDPOTRF/PZPOTRF and PDPOTRS/PZPOTRS.
5. The global submatrix ***A*** input to PDPOTRS/PZPOTRS must be the same as for the corresponding output argument for PDPOTRF/PZPOTRF; and thus, the scalar data specified for *ia*, *ja*, and the contents of *desc_a* must also be the same.
6. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details,

see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.

7. The way these subroutines handle nonpositive definiteness differs from ScaLAPACK. These subroutines use the *info* argument to provide information about the nonpositive definiteness of ***A***, like ScaLAPACK, but also provides an error message.
8. On both input and output, matrix ***A*** conforms to ScaLAPACK format.
9. The global real symmetric or complex Hermitian matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
10. The global real symmetric or complex Hermitian matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
11. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).

Performance Considerations

1. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
2. For optimal performance, you should use a square process grid to minimize the communication path length in both directions.
3. For optimal performance, take the following items into consideration when choosing the NB_A (= MB_A) value:
   - Whether you are using the upper or lower triangular part of ***A***, which may affect performance.
   - The cache size of the computational nodes. NB_A determines the granularity of the most expensive part of the computation, which tends to increase the optimal value of NB_A.
   - The communication and synchronization overhead. This has two aspects, the cost of internal synchronization points and the cost of broadcasts. These tend to slightly decrease the optimal value of NB_A.
   - The model of communication adapter you are using. The High Performance Switch Adapter-2 allows a larger NB.
   - Load balancing. For the best processor utilization, it is necessary for the processor nodes to be active for as long as possible; therefore, each one should have as many blocks as possible. For a given problem size, this tends to decrease the optimal value of NB_A (best load balancing: 1) and is most relevant at very small problem sizes.
   - If NB_A is equal to a power of 2, performance may be degraded.
   - Use the following rules of thumb for reasonably-sized problems:
     – For the POWER processors, choose NB_A in the following range:
       - For PDPOTRF, use [30, 50], avoiding 32.
       - For PZPOTRF, use [10, 25], avoiding 16.

     – For the POWER2 processors, choose NB_A in the following range:
       - For PDPOTRF, use [60, 80], avoiding 64.

| - For PZPOTRF, use [20, 40], avoiding 32.
| – For the SMP processors, choose NB_A in the following range:
| - For PDPOTRF, use [70, 100].
| - For PZPOTRF, use [30, 50], avoiding 32.

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* $\neq$ `'U'` or `'L'`
2. *n* $<$ 0
3. M_A $<$ 0 and *n* $=$ 0; M_A $<$ 1 otherwise
4. N_A $<$ 0 and *n* $=$ 0; N_A $<$ 1 otherwise
5. *ia* $<$ 1
6. *ja* $<$ 1
7. MB_A $<$ 1
8. NB_A $<$ 1
9. RSRC_A $<$ 0 or RSRC_A $\geq$ *p*
10. CSRC_A $<$ 0 or CSRC_A $\geq$ *q*

Stage 5

If *n* $\neq$ 0:
1. *ia* $>$ M_A
2. *ja* $>$ N_A
3. *ia*+*n*–1 $>$ M_A
4. *ja*+*n*–1 $>$ N_A

In all cases:
5. MB_A $\neq$ NB_A
6. mod(*ia*–1, MB_A) $\neq$ mod(*ja*–1, NB_A)
7. mod(*ia*–1, MB_A) $\neq$ 0

Stage 6

1. LLD_A < max(1, LOCp(M_A))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

2. *uplo* differs.
3. *n* differs.
4. *ia* differs.
5. *ja* differs.
6. DTYPE_A differs.
7. M_A differs.
8. N_A differs.
9. MB_A differs.
10. NB_A differs.
11. RSRC_A differs.
12. CSRC_A differs.

Example 1: This example factors a 9 × 9 positive definite real symmetric matrix using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

               UPLO   N    A  IA  JA    DESC_A   INFO
                |     |    |   |   |      |       |
CALL PDPOTRF(  'L'  , 9  , A , 1 , 1 ,  DESC_A , INFO )
```

	Desc_A
DTYPE_	1
CTXT_	*icontxt*[1]
M_	9
N_	9
MB_	3
NB_	3
RSRC_	0
CSRC_	0
LLD_	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
```

In this example, LLD_A = 6 on P_{00} and P_{01}, and LLD_A = 3 on P_{10} and P_{11}.

Global real symmetric matrix ***A*** of order 9 with block size 3 × 3:

```
B,D         0               1               2
        ┌                                               ┐
        | 1.0   .    .  |   .    .    .  |   .    .    . |
 0      | 1.0  2.0   .  |   .    .    .  |   .    .    . |
        | 1.0  2.0  3.0 |   .    .    .  |   .    .    . |
        | --------------|----------------|---------------|
        | 1.0  2.0  3.0 |  4.0   .    .  |   .    .    . |
 1      | 1.0  2.0  3.0 |  4.0  5.0   .  |   .    .    . |
        | 1.0  2.0  3.0 |  4.0  5.0  6.0 |   .    .    . |
        | --------------|----------------|---------------|
        | 1.0  2.0  3.0 |  4.0  5.0  6.0 |  7.0   .    . |
 2      | 1.0  2.0  3.0 |  4.0  5.0  6.0 |  7.0  8.0   . |
        | 1.0  2.0  3.0 |  4.0  5.0  6.0 |  7.0  8.0  9.0|
        └                                               ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |              0                |        1
-----|-------------------------------|-----------------
     | 1.0   .    .    .    .    .   |   .    .    .
     | 1.0  2.0   .    .    .    .   |   .    .    .
     | 1.0  2.0  3.0   .    .    .   |   .    .    .
 0   | 1.0  2.0  3.0  7.0   .    .   |  4.0  5.0  6.0
     | 1.0  2.0  3.0  7.0  8.0   .   |  4.0  5.0  6.0
     | 1.0  2.0  3.0  7.0  8.0  9.0  |  4.0  5.0  6.0
-----|-------------------------------|-----------------
     | 1.0  2.0  3.0   .    .    .   |  4.0   .    .
 1   | 1.0  2.0  3.0   .    .    .   |  4.0  5.0   .
     | 1.0  2.0  3.0   .    .    .   |  4.0  5.0  6.0
```

Output:

Global real symmetric matrix ***A*** of order 9 with block size 3 × 3:

```
B,D         0               1               2
        ┌                                               ┐
        | 1.0   .    .  |   .    .    .  |   .    .    . |
 0      | 1.0  1.0   .  |   .    .    .  |   .    .    . |
        | 1.0  1.0  1.0 |   .    .    .  |   .    .    . |
        | --------------|----------------|---------------|
        | 1.0  1.0  1.0 |  1.0   .    .  |   .    .    . |
 1      | 1.0  1.0  1.0 |  1.0  1.0   .  |   .    .    . |
        | 1.0  1.0  1.0 |  1.0  1.0  1.0 |   .    .    . |
        | --------------|----------------|---------------|
        | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0   .    . |
 2      | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0   . |
        | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0  1.0|
        └                                               ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |              0               |       1
-----|------------------------------|----------------
     |  1.0   .    .    .    .    . |   .    .    .
     |  1.0  1.0   .    .    .    . |   .    .    .
     |  1.0  1.0  1.0   .    .    . |   .    .    .
 0   |  1.0  1.0  1.0  1.0   .    . |  1.0  1.0  1.0
     |  1.0  1.0  1.0  1.0  1.0   . |  1.0  1.0  1.0
     |  1.0  1.0  1.0  1.0  1.0  1.0|  1.0  1.0  1.0
-----|------------------------------|----------------
     |  1.0  1.0  1.0   .    .    . |  1.0   .    .
 1   |  1.0  1.0  1.0   .    .    . |  1.0  1.0   .
     |  1.0  1.0  1.0   .    .    . |  1.0  1.0  1.0
```

The value of *info* is 0 on all processes.

Example 2: This example factors a 9 × 9 positive definite complex Hermitian matrix using a 2 × 2 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

               UPLO   N    A  IA  JA    DESC_A   INFO
                |     |    |   |   |      |       |
CALL PZPOTRF(  'L'  , 9  , A , 1 , 1 ,  DESC_A , INFO )
```

	Desc_A
DTYPE_	1
CTXT_	*icontxt*[1]
M_	9
N_	9
MB_	3
NB_	3
RSRC_	0
CSRC_	0
LLD_	See below[2]

	Desc_A
1 *icontxt* is the output of the BLACS_GRIDINIT call. 2 Each process should set the LLD_ as follows: `LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))` In this example, LLD_A = 6 on P_{00} and P_{01}, and LLD_A = 3 on P_{10} and P_{11}.	

Global complex Hermitian matrix ***A*** of order 9 with block size 3 × 3:

```
B,D                 0                                    1                                    2
     ┌                                                                                                        ┐
     │ (18.0, 0.0)      .           .       │       .          .          .       │       .          .          .      │
 0   │  (1.0, 1.0) (18.0, 0.0)      .       │       .          .          .       │       .          .          .      │
     │  (1.0, 1.0)  (3.0, 1.0) (18.0, 0.0)  │       .          .          .       │       .          .          .      │
     │ ------------------------------------|------------------------------------|------------------------------------ │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │  (18.0, 0.0)      .          .       │       .          .          .      │
 1   │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │   (7.0, 1.0) (18.0, 0.0)      .       │       .          .          .      │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │   (7.0, 1.0)  (9.0, 1.0) (18.0, 0.0)  │       .          .          .      │
     │ ------------------------------------|------------------------------------|------------------------------------ │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)  │  (18.0, 0.0)      .          .      │
 2   │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)  │  (13.0, 1.0) (18.0, 0.0)      .      │
     │  (1.0, 1.0)  (3.0, 1.0)  (5.0, 1.0)  │   (7.0, 1.0)  (9.0, 1.0) (11.0, 1.0)  │  (13.0, 1.0) (15.0, 1.0) (18.0, 0.0) │
     └                                                                                                        ┘
```

Note: On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                                        0                                          |                   1
-----|-----------------------------------------------------------------------------------|-----------------------------------------
     |  (18.0,  . )      .            .            .            .            .           |       .            .            .
     |   (1.0, 1.0)  (18.0,  . )      .            .            .            .           |       .            .            .
     |   (1.0, 1.0)   (3.0, 1.0)  (18.0,  . )      .            .            .           |       .            .            .
  0  |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (18.0,  . )      .            .           |   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
     |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (13.0, 1.0)  (18.0,  . )      .           |   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
     |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (13.0, 1.0)  (15.0, 1.0)  (18.0,  . )     |   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)
-----|-----------------------------------------------------------------------------------|-----------------------------------------
     |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .            .            .           |  (18.0,  . )      .            .
  1  |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .            .            .           |   (7.0, 1.0)  (18.0,  . )      .
     |   (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)      .            .            .           |   (7.0, 1.0)   (9.0, 1.0)  (18.0,  . )
```

Output:

Global complex Hermitian matrix ***A*** of order 9 with block size 3 × 3:

```
B,D                   0                                        1                                        2
     ┌                                                                                                                   ┐
     |   (4.2, 0.0)      .           .        |        .            .            .       |        .            .            .        |
 0   | (0.24, 0.24)   (4.2, 0.0)      .        |        .            .            .       |        .            .            .        |
     | (0.24, 0.24) (0.68, 0.24)   (4.2, 0.0)  |        .            .            .       |        .            .            .        |
     | ----------------------------------------|-----------------------------------------|----------------------------------------- |
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |     (4.0, 0.0)       .            .       |        .            .            .        |
 1   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (1.3, 0.25)   (3.8, 0.0)        .       |        .            .            .        |
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (1.3, 0.25)  (1.4, 0.26)   (3.5, 0.0)  |        .            .            .        |
     | ----------------------------------------|-----------------------------------------|----------------------------------------- |
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)  |     (3.2, 0.0)        .            .        |
 2   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)  |    (1.6, 0.32)   (2.7, 0.0)        .        |
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  |    (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)  |    (1.6, 0.32)  (1.6, 0.37)   (2.2, 0.0)  |
     └                                                                                                                   ┘
```

Note: On output, the imaginary parts of the diagonal elements of the matrix are set to zero.

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |                                      0                                             |                    1
-----|------------------------------------------------------------------------------------|------------------------------------------
     |   (4.2, 0.0)      .            .            .            .            .         |        .            .            .
     | (0.24, 0.24)   (4.2, 0.0)      .            .            .            .         |        .            .            .
     | (0.24, 0.24) (0.68, 0.24)   (4.2, 0.0)      .            .            .         |        .            .            .
 0   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)   (3.2, 0,0)      .            .         |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  (1.6, 0.32)   (2.7, 0.0)      .         |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)  (1.6, 0.32)  (1.6, 0.37)   (2.2, 0.0)   |   (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)
-----|------------------------------------------------------------------------------------|------------------------------------------
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)      .            .            .         |    (4.0, 0.0)       .            .
 1   | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)      .            .            .         |   (1.3, 0.25)   (3.8, 0.0        .
     | (0.24, 0.24) (0.68, 0.24)  (1.1, 0.24)      .            .            .         |   (1.3, 0.25)  (1.4, 0.26)   (3.5, 0.0)
```

The value of *info* is 0 on all processes.

PDPOTRS and PZPOTRS—Positive Definite Real Symmetric or Complex Hermitian Matrix Solve

These subroutines solve the following systems of equations for multiple right-hand sides:

$\boldsymbol{AX} = \boldsymbol{B}$

where, in the formula above:

A represents the global positive definite real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ factored by Cholesky factorization.
B represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the right-hand sides in its columns.
X represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ jb:jb+nrhs-1}$ containing the solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to PDPOTRF or PZPOTRF, respectively. For details on the factorization, see "PDPOTRF and PZPOTRF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization" on page 461.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 65. Data Types

A, ***B***	**Subroutine**
Long-precision real	PDPOTRS
Long-precision complex	PZPOTRS

Syntax

Fortran	CALL PDPOTRS | PZPOTRS (*uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *info*)
C and C++	pdpotrs | pzpotrs (*uplo*, *n*, *nrhs*, *a*, *ia*, *ja*, *desc_a*, *b*, *ib*, *jb*, *desc_b*, *info*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global real symmetric or complex Hermitian submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n
is the order of the factored submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

nrhs
is the number of right-hand sides— that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \geq 0$.

a

is the local part of the global real symmetric or complex Hermitian matrix ***A***, containing the factorization of matrix ***A*** produced by a preceding call to PDPOTRF or PZPOTRF, respectively. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ia+n-1$) by LOCq($ja+n-1$) part of the local array `A` must contain the local pieces of the leading $ia+n-1$ by $ja+n-1$ part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global real symmetric or complex Hermitian submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 65 on page 471. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *jb*, *desc_b*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ib*+*n*–1) by LOCq(*jb*+*nrhs*–1) part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *jb*+*nrhs*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 65 on page 471. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 $\leq$ *ib* $\leq$ M_B and *ib*+*n*–1 $\leq$ M_B.

jb

is the column index of the global matrix ***B***, identifying the first column of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 $\leq$ *jb* $\leq$ N_B and *jb*+*nrhs*–1 $\leq$ N_B.

desc_b

is the array descriptor for global matrix ***B***, described in the following table:

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B=1	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global

desc_b	**Name**	**Description**	**Limits**	**Scope**
3	M_B	Number of rows in the global matrix	If $n = 0$ or $nrhs = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	If $n = 0$ or $nrhs = 0$: N_B ≥ 0 Otherwise: N_B ≥ 1	Global
5	MB_B	Row block size	MB_B ≥ 1	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
8	CSRC_B	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	0 ≤ CSRC_B < q	Global
9	LLD_B	The leading dimension of the local array	LLD_B ≥ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

info

See On Return.

On Return

b

is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) LOCq(N_B) array, containing numbers of the data type indicated in Table 65 on page 471.

info

indicates that a successful computation occurred.

Scope: **global**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. The matrices must have no common elements; otherwise, results are unpredictable.
4. The scalar data specified for input argument *n* must be the same for both PDPOTRF/PZPOTRF and PDPOTRS/PZPOTRS.
5. The global submatrix ***A*** input to PDPOTRS/PZPOTRS must be the same as for the corresponding output argument for PDPOTRF/PZPOTRS; and thus, the scalar data specified for *ia*, *ja*, and the contents of *desc_a* must also be the same.

6. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
7. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.
8. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
9. The following values must be equal: CTXT_A = CTXT_B.
10. The global real symmetric or complex Hermitian matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
11. The following block sizes must be equal: MB_A = MB_B.
12. The global real symmetric or complex Hermitian matrix ***A*** must be aligned on a block row boundary; that is, *ia*–1 must be a multiple of MB_A.
13. The block row offset of ***A*** must be equal to the block column offset of ***A***; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A).
14. The block row offset of ***A*** must be equal to the block row offset of ***B***; that is, mod(*ia*–1, MB_A) = mod(*ib*–1, MB_B).
15. In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***B***; that is, *iarow* = *ibrow*, where:

    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

Error Conditions

Computational Errors: None

Note: If the factorization performed by PDPOTRF/PZPOTRF failed because of a nonpositive definite matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDPOTRF/PZPOTRF.

Resource Errors: Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *uplo* $\neq$ `'U'` or `'L'`
2. $n < 0$
3. *nrhs* < 0
4. M_A < 0 and $n = 0$; M_A < 1 otherwise
5. N_A < 0 and $n = 0$; N_A < 1 otherwise
6. *ia* < 1
7. *ja* < 1
8. MB_A < 1
9. NB_A < 1
10. RSRC_A < 0 or RSRC_A $\geq p$
11. CSRC_A < 0 or CSRC_A $\geq q$
12. M_B < 0 and ($n = 0$ or *nrhs* $= 0$); M_B < 1 otherwise
13. N_B < 0 and ($n = 0$ or *nrhs* $= 0$); N_B < 1 otherwise
14. *ib* < 1
15. *jb* < 1
16. MB_B < 1
17. NB_B < 1
18. RSRC_B < 0 or RSRC_B $\geq p$
19. CSRC_B < 0 or CSRC_B $\geq q$
20. CTXT_A $\neq$ CTXT_B

Stage 5

If $n \neq 0$:
1. *ia* $>$ M_A
2. *ja* $>$ N_A
3. *ia*+n–1 $>$ M_A
4. *ja*+n–1 $>$ N_A

If $n \neq 0$ and *nrhs* $\neq 0$:

5. *ib* $>$ M_B
6. *jb* $>$ N_B
7. *ib*+n–1 $>$ M_B
8. *jb*+*nrhs*–1 $>$ N_B

In all cases:

9. MB_A $\neq$ NB_A
10. mod(*ia*–1, MB_A) $\neq$ mod(*ja*–1, NB_A)
11. MB_B $\neq$ MB_A
12. mod(*ia*–1, MB_A) $\neq$ mod(*ib*–1, MB_B).
13. mod(*ia*–1, MB_A) $\neq 0$
14. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***B***; that is, *iarow* $\neq$ *ibrow*, where:
    iarow = mod((((*ia*–1)/MB_A)+RSRC_A), *p*)
    ibrow = mod((((*ib*–1)/MB_B)+RSRC_B), *p*)

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. LLD_B $<$ max(1, LOCp(M_B))

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

3. *uplo* differs.
4. *n* differs.

5. *nrhs* differs.
6. *ia* differs.
7. *ja* differs.
8. DTYPE_A differs.
9. M_A differs.
10. N_A differs.
11. MB_A differs.
12. NB_A differs.
13. RSRC_A differs.
14. CSRC_A differs.
15. *ib* differs.
16. *jb* differs.
17. DTYPE_B differs.
18. M_B differs.
19. N_B differs.
20. MB_B differs.
21. NB_B differs.
22. RSRC_B differs.
23. CSRC_B differs.

Example 1: This example solves the positive definite real symmetric system ***AX*** = ***B*** with 5 right-hand sides using a 2 × 2 process grid. The transformed matrix ***A*** is the output from "Example 1" on page 466.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              UPLO  N  NRHS A  IA  JA   DESC_A   B  IB  JB   DESC_B   INFO
               |    |   |   |   |   |     |      |   |   |     |       |
CALL PDPOTRS( 'L' , 9 , 5 , A , 1 , 1 , DESC_A , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	**Desc_B**
DTYPE_	1	1
CTXT_	*icontxt*1	*icontxt*1
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	0	0
CSRC_	0	1

	Desc_A	Desc_B
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 6 on P_{00} and P_{01}, and LLD_A = LLD_B = 3 on P_{10} and P_{11}.

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D         0                1                2
      ┌                                                  ┐
      |    .    18.0  |   27.0   36.0  |    45.0    9.0  |
 0    |    .    34.0  |   51.0   68.0  |    85.0   17.0  |
      |    .    48.0  |   72.0   96.0  |   120.0   24.0  |
      | --------------|----------------|---------------  |
      |    .    60.0  |   90.0  120.0  |   150.0   30.0  |
 1    |    .    70.0  |  105.0  140.0  |   175.0   35.0  |
      |    .    78.0  |  117.0  156.0  |   195.0   39.0  |
      | --------------|----------------|---------------  |
      |    .    84.0  |  126.0  168.0  |   210.0   42.0  |
 2    |    .    88.0  |  132.0  176.0  |   220.0   44.0  |
      |    .    90.0  |  135.0  180.0  |   225.0   45.0  |
      └                                                  ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |       0        |              1
-----|----------------|-----------------------------
     |   27.0   36.0  |     .    18.0   45.0    9.0
     |   51.0   68.0  |     .    34.0   85.0   17.0
     |   72.0   96.0  |     .    48.0  120.0   24.0
 0   |  126.0  168.0  |     .    84.0  210.0   42.0
     |  132.0  176.0  |     .    88.0  220.0   44.0
     |  135.0  180.0  |     .    90.0  225.0   45.0
-----|----------------|-----------------------------
     |   90.0  120.0  |     .    60.0  150.0   30.0
 1   |  105.0  140.0  |     .    70.0  175.0   35.0
     |  117.0  156.0  |     .    78.0  195.0   39.0
```

Output:

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D       0            1            2
     ┌                                     ┐
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
 0   |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     | ----------|------------|---------- |
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
 1   |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     | ----------|------------|---------- |
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
 2   |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     |   .   2.0 |  3.0  4.0  |  5.0  1.0 |
     └                                     ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |     0      |         1
-----|------------|---------------------
     | 3.0  4.0   |    .   2.0  5.0  1.0
     | 3.0  4.0   |    .   2.0  5.0  1.0
     | 3.0  4.0   |    .   2.0  5.0  1.0
 0   | 3.0  4.0   |    .   2.0  5.0  1.0
     | 3.0  4.0   |    .   2.0  5.0  1.0
     | 3.0  4.0   |    .   2.0  5.0  1.0
-----|------------|---------------------
     | 3.0  4.0   |    .   2.0  5.0  1.0
 1   | 3.0  4.0   |    .   2.0  5.0  1.0
     | 3.0  4.0   |    .   2.0  5.0  1.0
```

The value of *info* is 0 on all processes.

Example 2: This example solves the positive definite complex Hermitian system ***AX*** = ***B*** with 5 right-hand sides using a 2 × 2 process grid. The transformed matrix ***A*** is the output from “Example 2” on page 468.

This example uses a global submatrix ***B*** within a global matrix ***B*** by specifying *ib* = 1 and *jb* = 2.

By specifying CSRC_B = 1, the columns of global matrix ***B*** are distributed over the process grid starting in the second column of the process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              UPLO  N  NRHS  A  IA  JA   DESC_A   B  IB  JB   DESC_B   INFO
               |    |   |    |   |   |     |      |   |   |     |       |
CALL PZPOTRS( 'L' , 9 , 5  , A , 1 , 1 , DESC_A , B , 1 , 2 , DESC_B , INFO )
```

	Desc_A	**Desc_B**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	9	9
N_	9	6
MB_	3	3
NB_	3	2
RSRC_	0	0
CSRC_	0	1
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_B = MAX(1,NUMROC(M_B, MB_B, MYROW, RSRC_B, NPROW))
```

In this example, LLD_A = LLD_B = 6 on P_{00} and P_{01}, and LLD_A = LLD_B = 3 on P_{10} and P_{11}.

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D             0                                1                                  2
     ┌                                                                                                 ┐
     |      .      (60.0, 10.0)  |    (86.0, 2.0)   (112.0, -6.0)  |  (138.0, -14.0)   (34.0, 18.0)  |
 0   |      .      (86.0, 28.0)  |  (126.0, 22.0)   (166.0, 16.0)  |   (206.0, 10.0)   (46.0, 34.0)  |
     |      .     (108.0, 44.0)  |  (160.0, 40.0)   (212.0, 36.0)  |   (264.0, 32.0)   (56.0, 48.0)  |
     | --------------------------|---------------------------------|-------------------------------- |
     |      .     (126.0, 58.0)  |  (188.0, 56.0)   (250.0, 54.0)  |   (312.0, 52.0)   (64.0, 60.0)  |
 1   |      .     (140.0, 70.0)  |  (210.0, 70.0)   (280.0, 70.0)  |   (350.0, 70.0)   (70.0, 70.0)  |
     |      .     (150.0, 80.0)  |  (226.0, 82.0)   (302.0, 84.0)  |   (378.0, 86.0)   (74.0, 78.0)  |
     | --------------------------|---------------------------------|-------------------------------- |
     |      .     (156.0, 88.0)  |  (236.0, 92.0)   (316.0, 96.0)  |  (396.0, 100.0)   (76.0, 84.0)  |
 2   |      .     (158.0, 94.0)  | (240.0, 100.0)  (322.0, 106.0)  |  (404.0, 112.0)   (76.0, 88.0)  |
     |      .     (156.0, 98.0)  | (238.0, 106.0)  (320.0, 114.0)  |  (402.0, 122.0)   (74.0, 90.0)  |
     └                                                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |               0                |                          1
-----|--------------------------------|-------------------------------------------------------
     |    (86.0, 2.0)   (112.0, -6.0) |      .       (60.0, 10.0)  (138.0, -14.0)   (34.0, 18.0)
     |  (126.0, 22.0)   (166.0, 16.0) |      .       (86.0, 28.0)   (206.0, 10.0)   (46.0, 34.0)
     |  (160.0, 40.0)   (212.0, 36.0) |      .      (108.0, 44.0)   (264.0, 32.0)   (56.0, 48.0)
 0   |  (236.0, 92.0)   (316.0, 96.0) |      .      (156.0, 88.0)  (396.0, 100.0)   (76.0, 84.0)
     | (240.0, 100.0)  (322.0, 106.0) |      .      (158.0, 94.0)  (404.0, 112.0)   (76.0, 88.0)
     | (238.0, 106.0)  (320.0, 114.0) |      .      (156.0, 98.0)  (402.0, 122.0)   (74.0, 90.0)
-----|--------------------------------|-------------------------------------------------------
     |  (188.0, 56.0)   (250.0, 54.0) |      .      (126.0, 58.0)   (312.0, 52.0)   (64.0, 60.0)
 1   |  (210.0, 70.0)   (280.0, 70.0) |      .      (140.0, 70.0)   (350.0, 70.0)   (70.0, 70.0)
     |  (226.0, 82.0)   (302.0, 84.0) |      .      (150.0, 80.0)   (378.0, 86.0)   (74.0, 78.0)
```

Output:

After the global matrix ***B*** is distributed over the process grid, only a portion of the global data structure is used—that is, global submatrix ***B***. Following is the global 9 × 5 submatrix ***B***, starting at row 1 and column 2 in global general 9 × 6 matrix ***B*** with block size 3 × 2:

```
B,D         0                     1                        2
     ┌                                                                 ┐
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
 0   |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     | ---------------|--------------------------|------------------------ |
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
 1   |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     | ---------------|--------------------------|------------------------ |
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
 2   |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     |  .  (2.0, 1.0) |  (3.0, 1.0)  (4.0, 1.0)  |  (5.0, 1.0)  (1.0, 1.0) |
     └                                                                 ┘
```

The following is the 2 × 2 process grid:

B,D	1	0 2
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Note: The first column of ***B*** begins in the second column of the process grid.

Local arrays for ***B***:

```
p,q  |            0             |                    1
-----|--------------------------|------------------------------------------
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
 0   |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
-----|--------------------------|------------------------------------------
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
 1   |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
     |  (3.0, 4.0)  (3.0, 4.0)  |    .   (2.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)
```

The value of *info* is 0 on all processes.

Banded Linear Algebraic Equation Subroutines

This section contains the banded linear algebraic equation subroutine descriptions.

PDPBSV—Positive Definite Symmetric Band Matrix Factorization and Solve

This subroutine solves the following system of equations for multiple right-hand sides:

$$\boldsymbol{AX} = \boldsymbol{B}$$

where, in the formula above:

A represents the global positive definite symmetric band submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ to be factored by Cholesky factorization.
B represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
X represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

| If $n = 0$, no computation is performed and the subroutine returns after doing some
| parameter checking. See references [2], [23], [39], and [40].

Table 66. Data Types

A, ***B***, *work*	**Subroutine**
Long-precision real	PDPBSV

Syntax

Fortran	CALL PDPBSV (*uplo*, *n*, *k*, *nrhs*, *a*, *ja*, *desc_a*, *b*, *ib*, *desc_b*, *work*, *lwork*, *info*)
C and C++	pdpbsv (*uplo*, *n*, *k*, *nrhs*, *a*, *ja*, *desc_a*, *b*, *ib*, *desc_b*, *work*, *lwork*, *info*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n
is the number of columns in the submatrix ***A***, stored in the upper- or lower-band-packed storage mode. It is also the number of rows in the general submatrix ***B*** containing the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer; $0 \le n \le$ (NB_A)p–mod(ja–1,NB_A).

k
is the half bandwidth of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If *uplo* = `'U'`, $0 \le k \le$ NB_A.

- If *uplo* = `'L'`, $0 \le k < n$.

These limits for *k* are extensions of the ScaLAPACK standard.

nrhs

is the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \ge 0$.

a

is the local part of the global positive definite symmetric band matrix ***A***, stored in upper- or lower-band-packed storage mode, to be factored. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *k*, *ja*, *desc_a*, and *p*; therefore, the leading *k*+1 by LOCp(*ja*+*n*–1) part of the local array `A` must contain the local pieces of the leading *k*+1 by *ja*+*n*–1 part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCp(*ja*+*n*–1) array, containing numbers of the data type indicated in Table 66 on page 484. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, array `A` is overwritten; that is, original input is not preserved.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, which may be type 501 or type 1, as described in the following tables. For rules on using array descriptors, see “Notes and Coding Rules” on page 489.

desc_a	Name	Description	Limits	Scope
1	DTYPE_A	Descriptor Type	DTYPE_A = 501 for $1 \times p$ or $p \times 1$ where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
4	NB_A	Column block size	NB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (NB_A)*p*–mod(*ja*–1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ *p*	Global
6	LLD_A	Leading dimension	LLD_A $\geq$ *k*+1	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for 1 $\times$ *p* where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A > *k*	Global
4	N_A	Number of columns in the global matrix	If *n* = 0: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (NB_A)*p*–mod(*ja*–1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A=0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ *p*	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ *k*+1	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*–1) by *nrhs* part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 66 on page 484. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ib \leq$ M_B and $ib+n-1 \leq$ M_B.

desc_b

is the array descriptor for global matrix ***B***, which may be type 502 or type 1, as described in the following tables. For rules on using array descriptors, see "Notes and Coding Rules" on page 489.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and $0 \leq n \leq$ (MB_B)p–mod(ib–1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
6	LLD_B	Leading dimension	LLD_B ≥ max(1,LOCp(M_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and $0 \leq n \leq$ (MB_B)p–mod(ib–1,MB_B)	Global

desc_b	**Name**	**Description**	**Limits**	**Scope**
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B=0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1,LOCp(M_B)	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ -1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = -1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 66 on page 484.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer, where:

- If *lwork* = 0, PDPBSV dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard. It is suggested that you specify *lwork*=0.
- If *lwork* = -1, PDPBSV performs a work area query and returns the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  lwork ≥ (NB_A+2*k*)(*k*)+max(*nrhs*, *k*)(*k*)

info

See On Return.

On Return

a

a is overwritten; that is, the original input is not preserved. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.

b

b is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 66 on page 484.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, the size of *work* is (at least) of length *lwork*.

If $lwork = -1$, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 66 on page 484, where:

- If $lwork \geq 1$, $work_1$ is set to the minimum *lwork* value needed.
- If $lwork = -1$, $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info

has the following meaning:

If $info = 0$, global submatrix ***A*** is positive definite, and the factorization completed normally or the work area query completed successfully.

If $info > 0$, the leading minor of order *i* of the global submatrix ***A*** is not positive definite. *info* is set equal to *i*, where the first leading minor was encountered at $\boldsymbol{A}_{ja+i-1,\ ja+i-1}$. The results contained in matrix ***A*** are not defined.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. If $n > 0$ and $nrhs = 0$, only the factorization is completed.
3. The subroutine accepts lowercase letters for the *uplo* argument.
4. This subroutine gives the best performance for wide band widths, for example:

$$k > 100\sqrt{p}$$

   where *p* is the number of processes. For details, see references [2], [39], and [40]. Also, it is suggested that you specify *uplo* = `'L'`.
5. ***A***, ***B***, and *work* must have no common elements; otherwise, results are unpredictable.
6. In all cases, follow these rules:

- *ib* = *ja*
- DTYPE_A=501 or 1
- DTYPE_B=502 or 1
- NB_A = MB_B
- If DTYPE_A=1, RSRC_A=0, M_A ≥ *k*+1, and MB_A ≥ 1.
- If DTYPE_B=1, CSRC_B=0, N_B ≥ *nrhs*, and NB_B ≥ 1.
- CTXT_A = CTXT_B
- Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	DTYPE_B	Process Grid
501	502	$p \times 1$ or $1 \times p$
501	1	$1 \times p$
1	502	$p \times 1$
1	1	1×1

7. To determine the values of LOCp(*n*) used in the argument descriptions, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 for descriptor type-1 or "Determining the Number of Rows or Columns in Your Local Arrays" on page 30 for descriptor type-501 and type-502.
8. The global band matrix ***A*** must be positive definite. If ***A*** is not positive definite, this subroutine uses the *info* argument to provide information about ***A*** and issues an error message. This differs from ScaLAPACK, which only uses the *info* argument to provide information about ***A***.
9. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. See the section on block-cyclically distributing a symmetric matrix in "Matrices" on page 40.

   Matrix ***A*** must be distributed over a one-dimensional process grid using block-cyclic data distribution. For more information on using block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26.
10. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information on block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26. Also, see the section on distributing the right-hand side matrix in "Matrices" on page 40.
11. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
12. Although global matrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ja*, *ib*, NB_A and MB_B, must be chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite (corresponding computational error messages are issued by both PDPBTRF and PDPBSV). For details, see the description of the *info* argument.

Resource Errors: *lwork* = 0 and unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDPBSV was called from outside the process grid.

Stage 4

1. The process grid is not $1 \times p$ or $p \times 1$.
2. $uplo \neq$ `'U'` or `'L'`
3. $n < 0$
4. $k < 0$
5. $k+1 > n$
6. $ja < 1$
7. DTYPE_A = 1 and:
   a. M_A < k+1
   b. MB_A < 1
   c. RSRC_A ≠ 0
   d. The process grid is not $1 \times p$.
8. N_A < 0 and (n = 0); N_A < 1 otherwise
9. NB_A < 1
10. n+mod(ja–1, NB_A) > (NB_A)p
11. CSRC_A < 0 or CSRC_A ≥ p
12. $uplo$ = `'U'` and k > NB_A
13. $nrhs < 0$
14. $ib \neq ja$
15. $ib < 1$
16. DTYPE_B = 1 and:
    a. N_B < $nrhs$
    b. NB_B < 1
    c. CSRC_B ≠ 0
    d. The process grid is not $p \times 1$.
17. M_B < 0 and (n = 0); M_B < 1 otherwise
18. MB_B < 1
19. n+mod(ib–1,MB_B) > (MB_B)p
20. MB_B ≠ NB_A
21. RSRC_B < 0 or RSRC_B ≥ p
22. CTXT_A ≠ CTXT_B

Stage 5: If $n \neq 0$:

1. $ja+n-1 > \text{N\_A}$
2. $ja > \text{N\_A}$
3. $ib > \text{M\_B}$
4. $ib+n-1 > \text{M\_B}$
5. $\text{LLD\_A} < k+1$

Stage 6

1. $\text{LLD\_B} < \max(1, \text{LOCp}(\text{M\_B}))$
2. $lwork \neq 0$, $lwork \neq -1$, and $lwork < (\text{NB\_A}+2k)(k)+\max(nrhs, k)(k)$

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *uplo* differs.
2. *n* differs.
3. *k* differs.
4. *nrhs* differs.
5. *ja* differs.
6. DTYPE_A differs.
7. DTYPE_A does not differ and:
    a. N_A differs.
    b. NB_A differs.
    c. CSRC_A differs.
    d. DTYPE_A = 1 and:
        1) M_A differs.
        2) MB_A differs.
        3) RSRC_A differs.
8. *ib* differs.
9. DTYPE_B differs.
10. DTYPE_B does not differ and:
    a. M_B differs.
    b. MB_B differs.
    c. RSRC_B differs.
    d. DTYPE_A = 1 and:
        1) N_B differs.
        2) NB_B differs.
        3) CSRC_B differs.

Also:
11. *lwork* = –1 on a subset of processes.

Example: This example shows a factorization of the positive definite symmetric band matrix ***A*** of order 9 with a half bandwidth of 7:

```
┌                                             ┐
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0 |
| 1.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  1.0 |
| 1.0  2.0  3.0  3.0  3.0  3.0  3.0  3.0  2.0 |
| 1.0  2.0  3.0  4.0  4.0  4.0  4.0  4.0  3.0 |
| 1.0  2.0  3.0  4.0  5.0  5.0  5.0  5.0  4.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  6.0  6.0  5.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0  6.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  7.0 |
| 0.0  1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0 |
└                                             ┘
```

Matrix ***A*** is stored in lower-band-packed storage mode:

```
┌                                             ┐
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  8.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0   .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  6.0   .    .  |
| 1.0  2.0  3.0  4.0  5.0  5.0   .    .    .  |
| 1.0  2.0  3.0  4.0  4.0   .    .    .    .  |
| 1.0  2.0  3.0  3.0   .    .    .    .    .  |
| 1.0  2.0  2.0   .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
└                                             ┘
```

where "." means you do not have to store a value in that position in the local array. However, these storage positions are required and are overwritten during the computation.

Notes:

1. On output, the submatrix ***A*** is overwritten; that is, the original input is not preserved.
2. Notice **only one process grid was created**, even though, DTYPE_A = 501 and DTYPE_B = 502.
3. Because *lwork* = 0, PDPBSV dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            UPLO   N   K  NRHS A  JA   DESC_A   B   IB  DESC_B   WORK LWORK INFO
             |     |   |   |   |   |      |     |   |     |       |     |     |
CALL PDPBSV( 'L' , 9 , 7 , 3 , A , 1 , DESC_A , B , 1 , DESC_B , WORK , 0 , INFO )
```

	Desc_A
DTYPE_	501
CTXT_	*icontxt*[1]

	Desc_A
N_	9
NB_	3
CSRC_	0
LLD_A	8
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	9
MB_	3
RSRC_	0
LLD_B	3
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global matrix ***A*** stored in lower-band-packed storage mode with block size of 3:

```
B,D         0                  1                  2
     ┌                                                     ┐
     |  1.0  2.0  3.0  |   4.0  5.0  6.0  |   7.0  8.0  8.0 |
     |  1.0  2.0  3.0  |   4.0  5.0  6.0  |   7.0  7.0   .  |
     |  1.0  2.0  3.0  |   4.0  5.0  6.0  |   6.0   .    .  |
     |  1.0  2.0  3.0  |   4.0  5.0  5.0  |    .    .    .  |
 0   |  1.0  2.0  3.0  |   4.0  4.0   .   |    .    .    .  |
     |  1.0  2.0  3.0  |   3.0   .    .   |    .    .    .  |
     |  1.0  2.0  2.0  |    .    .    .   |    .    .    .  |
     |  1.0  1.0   .   |    .    .    .   |    .    .    .  |
     └                                                     ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local array A with block size of 3:

```
p,q  |        0         |        1         |        2
-----|------------------|------------------|-----------------
     |  1.0  2.0  3.0   |   4.0  5.0  6.0  |   7.0  8.0  8.0
     |  1.0  2.0  3.0   |   4.0  5.0  6.0  |   7.0  7.0   .
     |  1.0  2.0  3.0   |   4.0  5.0  6.0  |   6.0   .    .
     |  1.0  2.0  3.0   |   4.0  5.0  5.0  |    .    .    .
 0   |  1.0  2.0  3.0   |   4.0  4.0   .   |    .    .    .
     |  1.0  2.0  3.0   |   3.0   .    .   |    .    .    .
     |  1.0  2.0  2.0   |    .    .    .   |    .    .    .
     |  1.0  1.0   .    |    .    .    .   |    .    .    .
```

Global matrix ***B*** with block size of 3:

```
B,D          0
     ┌                 ┐
     |   8.0  36.0  44.0 |
 0   |  16.0  80.0  80.0 |
     |  23.0 122.0 108.0 |
     | ----------------- |
     |  29.0 161.0 129.0 |
 1   |  34.0 196.0 144.0 |
     |  38.0 226.0 154.0 |
     | ----------------- |
     |  41.0 250.0 160.0 |
 2   |  43.0 267.0 163.0 |
     |  36.0 240.0 120.0 |
     └                 ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array `B` with block size of 3:

```
p,q  |         0          |          1          |          2
-----|--------------------|---------------------|--------------------
     |   8.0  36.0  44.0  |   29.0 161.0 129.0  |   41.0 250.0 160.0
 0   |  16.0  80.0  80.0  |   34.0 196.0 144.0  |   43.0 267.0 163.0
     |  23.0 122.0 108.0  |   38.0 226.0 154.0  |   36.0 240.0 120.0
```

Output:

Global matrix ***B*** with block size of 3:

```
B,D        0
     ┌              ┐
     |  1.0  1.0  9.0 |
 0   |  1.0  2.0  8.0 |
     |  1.0  3.0  7.0 |
     | -------------- |
     |  1.0  4.0  6.0 |
 1   |  1.0  5.0  5.0 |
     |  1.0  6.0  4.0 |
     | -------------- |
     |  1.0  7.0  3.0 |
 2   |  1.0  8.0  2.0 |
     |  1.0  9.0  1.0 |
     └              ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array `B` with block size of 3:

```
p,q  |        0        |        1         |        2
-----|-----------------|------------------|----------------
     |  1.0  1.0  9.0  |   1.0  4.0  6.0  |   1.0  7.0  3.0
 0   |  1.0  2.0  8.0  |   1.0  5.0  5.0  |   1.0  8.0  2.0
     |  1.0  3.0  7.0  |   1.0  6.0  4.0  |   1.0  9.0  1.0
```

The value of *info* is 0 on all processes.

PDPBTRF—Positive Definite Symmetric Band Matrix Factorization

This subroutine uses Cholesky factorization to factor a positive definite symmetric band matrix ***A***, stored in upper- or lower-band-packed storage mode, into one of the following forms:

$\boldsymbol{A} = \boldsymbol{U}^T\boldsymbol{U}$ if ***A*** is upper triangular.
$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^T$ if ***A*** is lower triangular.

where, in the formulas above:

A represents the global positive definite symmetric band submatrix $\boldsymbol{A}_{ja:ja+n-1, ja:ja+n-1}$ to be factored.
U is an upper triangular matrix.
L is a lower triangular matrix.

To solve the system of equations with multiple right-hand sides, follow the call to this subroutine with one of more calls to PDPBTRS. The output from this factorization subroutine should be used only as input to PDPBTRS.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [2], [23], [39], and [40].

Table 67. Data Types

A, *af*, *work*	**Subroutine**
Long-precision real	PDPBTRF

Syntax

Fortran	CALL PDPBTRF (*uplo*, *n*, *k*, *a*, *ja*, *desc_a*, *af*, *laf*, *work*, *lwork*, *info*)
C and C++	pdpbtrf (*uplo*, *n*, *k*, *a*, *ja*, *desc_a*, *af*, *laf*, *work*, *lwork*, *info*);

On Entry

uplo
indicates whether the upper or lower triangular part of the global submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n
is the number of columns in the submatrix ***A***, stored in upper- or lower-band-packed storage mode, to be factored.

Scope: **global**

Specified as: a fullword integer; $0 \leq n \leq (\text{NB\_A})p - \text{mod}(ja-1, \text{NB\_A})$.

k
is the half bandwidth of the submatrix ***A*** to be factored.

Scope: **global**

Specified as: a fullword integer, where:

- If *uplo* = `'U'`, $0 \le k \le$ NB_A.
- If *uplo* = `'L'`, $0 \le k < n$.

These limits for *k* are extensions of the ScaLAPACK standard.

a

is the local part of the global positive definite symmetric band matrix ***A***, stored in upper- or lower-band-packed storage mode, to be factored. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *k*, *ja*, *desc_a*, and *p*; therefore, the leading k+1 by LOCp(ja+n–1) part of the local array `A` must contain the local pieces of the leading k+1 by ja+n–1 part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the submatrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the submatrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCp(ja+n–1) array, containing numbers of the data type indicated in Table 67 on page 497. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, array `A` is overwritten; that is, original input is not preserved.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, which may be type 501 or type 1, as described in the following tables.

desc_a	Name	Description	Limits	Scope
1	DTYPE_A	Descriptor Type	DTYPE_A = 501 for $1 \times p$ or $p \times 1$ where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
4	NB_A	Column block size	NB_A ≥ 1 and $0 \le n \le$ (NB_A)p–mod(ja–1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < *p*	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
6	LLD_A	Leading dimension	LLD_A ≥ k+1	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for 1 × p where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A > k	Global
4	N_A	Number of columns in the global matrix	If n = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1 and 0 ≤ n ≤ (NB_A)p–mod(ja–1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A=0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < p	Global
9	LLD_A	The leading dimension of the local array	LLD_A ≥ k+1	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

af

is a reserved output area and its size is specified by LAF.

Scope: **local**

Specified as: for migration purposes, you should specify a one-dimensional, long-precision array of (at least) length LAF.

laf

is the number of elements in array AF.

The *laf* argument must be specified; however, this subroutine currently ignores its value. For migration purposes, you should specify *laf* using the formula below.

Scope: **local**

Specified as: a fullword integer, *laf* ≥ (NB_A+2k)(k).

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ -1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = -1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 67 on page 497.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDPBTRF dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard. It is suggested that you specify *lwork*=0.
- If *lwork* = -1, PDPBTRF performs a work area query and returns the optimum required size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  $lwork \geq k^2$

info

See On Return.

On Return

a

a is the updated local part of the global matrix ***A***, containing the results of the factorization, where:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ contains the results of the factorization. The remaining elements stored in submatrix ***A*** were overwritten by this subroutine.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ contains the results of the factorization. The remaining elements stored in submatrix ***A*** were overwritten by this subroutine.

Scope: **local**

Returned as: an LLD_A by (at least) LOCp(*ja*+*n*–1) array, containing numbers of the data type indicated in Table 67 on page 497.

On output, array `A` is overwritten; that is, original input is not preserved.

af

is a reserved area.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ or $lwork \neq -1$, the size of *work* is (at least) of length *lwork*.

If $lwork = -1$, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 67 on page 497, where:

- If $lwork \geq 1$, $work_1$ is set to the minimum *lwork* value needed.
- If $lwork = -1$, $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info

has the following meaning:

If $info = 0$, global submatrix ***A*** is positive definite and the factorization completed normally, or the work area query completed successfully.

If $info > 0$, the leading minor of order *i* of the global submatrix ***A*** is not positive definite. *info* is set equal to *i*, where the first leading minor was encountered at $A_{ja+i-1, ja+i-1}$. The results contained in matrix ***A*** are not defined.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. This subroutine gives the best performance for wide band widths, for example:

$$k > 100\sqrt{p}$$

   where *p* is the number of processes. For details, see references [2], [39], and [40]. Also, it is suggested that you specify *uplo* = `'L'`.
4. The *k*+1 by *n* array specified for submatrix ***A*** must remain unchanged between calls to PDPBTRF and PDPBTRS. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.
5. The output from this factorization subroutine should be used only as input to the solve subroutine PDPBTRS.

   The data specified for input arguments *uplo*, *n*, and *k* must be the same for both PDPBTRF and PDPBTRS.

   The matrix ***A*** and *af* input to PDPBTRS must be the same as the corresponding output arguments for PDPBTRF; and thus, the scalar data specified for *ja*, *desc_a*, and *laf* must also be the same.
6. In all cases, follow these rules:

- DTYPE_A=501 or 1
- If DTYPE_A=1, RSRC_A=0, M_A ≥ k+1, and MB_A ≥ 1.
- Following are the allowable array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	**Process Grid**
501	$p \times 1$ or $1 \times p$
1	$1 \times p$

7. To determine the values of LOCp(*n*) used in the argument descriptions, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 for descriptor type-1 or "Determining the Number of Rows or Columns in Your Local Arrays" on page 30 for descriptor type-501 and type-502.
8. Matrix ***A***, *af*, and *work* must have no common elements; otherwise, results are unpredictable.
9. The global symmetric band matrix ***A*** must be positive definite. If ***A*** is not positive definite, this subroutine uses the *info* argument to provide information about ***A*** and issues an error message. This differs from ScaLAPACK, which only uses the *info* argument to provide information about ***A***.
10. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. See the section on block-cyclically distributing a symmetric matrix in "Matrices" on page 40.

    Matrix ***A*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information on using block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26.
11. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
12. Although global matrix ***A*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ja*, and NB_A must be chosen so that each process has at most one full or partial block of the global submatrix ***A***.

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite. For details, see the description of the *info* argument.

Resource Errors: *lwork*= 0 and unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDPBTRF was called from outside the process grid.

Stage 4

1. The process grid is not $1 \times p$ or $p \times 1$.
2. *uplo* ≠ `'U'` or `'L'`
3. $n < 0$
4. $ja < 1$
5. $k < 0$
6. $k+1 > n$
7. DTYPE_A = 1 and:
   a. M_A < k+1
   b. MB_A < 1
   c. RSRC_A ≠ 0
   d. The process grid is not $1 \times p$.
8. N_A < 0 and (n = 0); N_A < 1 otherwise
9. NB_A < 1
10. n > (NB_A)p–mod(ja–1,NB_A)
11. CSRC_A < 0 or CSRC_A ≥ p
12. *uplo* = `'U'` and k > NB_A.

Stage 5

1. ja > N_A and ($n > 0$)
2. $ja+n-1$ > N_A and ($n > 0$)
3. LLD_A < k+1

Stage 6

1. *lwork* ≠ 0, *lwork* ≠ -1, and *lwork* < k^2

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *uplo* differs.
2. *n* differs.
3. *k* differs.
4. *ja* differs.
5. DTYPE_A differs.
6. DTYPE_A does not differ and:
   a. N_A differs.
   b. NB_A differs.
   c. CSRC_A differs.
   d. DTYPE_A = 1 and:
      1) M_A differs.
      2) MB_A differs.
      3) RSRC_A differs.

Also:

7. *lwork* = –1 on a subset of processes.

Example: This example shows a factorization of the positive definite symmetric band matrix ***A*** of order 9 with a half bandwidth of 7:

$$\begin{bmatrix}
1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.0 \\
1.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 1.0 \\
1.0 & 2.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 2.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 4.0 & 4.0 & 4.0 & 4.0 & 3.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 5.0 & 5.0 & 5.0 & 4.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.0 & 6.0 & 5.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 7.0 & 6.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 7.0 \\
0.0 & 1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0
\end{bmatrix}$$

Matrix ***A*** is stored in lower-band-packed storage mode:

$$\begin{bmatrix}
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 8.0 & 8.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 7.0 & 7.0 & . \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 6.0 & 6.0 & . & . \\
1.0 & 2.0 & 3.0 & 4.0 & 5.0 & 5.0 & . & . & . \\
1.0 & 2.0 & 3.0 & 4.0 & 4.0 & . & . & . & . \\
1.0 & 2.0 & 3.0 & 3.0 & . & . & . & . & . \\
1.0 & 2.0 & 2.0 & . & . & . & . & . & . \\
1.0 & 1.0 & . & . & . & . & . & . & .
\end{bmatrix}$$

where "." means you do not have to store a value in that position in the local array. However, these storage positions are required and are overwritten during the computation.

Matrix ***A*** is distributed over a 1 $\times$ 3 process grid using block-cyclic distribution.

Notes:

1. Matrix ***A***, output from PDPBTRF, must be passed, unchanged, to the solve subroutine PDPBTRS.
2. The *laf* argument must be specified; however, this subroutine currently ignores its value. For migration purposes, in this example, *laf* is specified as 119.
3. The *af* argument is reserved and not shown in this example.
4. Because *lwork* = 0, PDPBTRF dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             UPLO   N   K   A   JA  DESC_A   AF   LAF   WORK LWORK INFO
               |    |   |   |   |      |    |      |    |     |     |
CALL PDPBTRF( 'L' , 9 , 7 , A , 1 , DESC_A , AF , 119 , WORK , 0 , INFO )
```

	Desc_A
DTYPE_	501

	Desc_A
CTXT_	*icontxt*[1]
N_	9
NB_	3
CSRC_	0
LLD_A	8
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global matrix ***A*** stored in lower-band-packed storage mode with block size of 3:

```
B,D         0                  1                  2
      ┌                                                    ┐
      | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  7.0  8.0  8.0 |
      | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  7.0  7.0   .  |
      | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  6.0   .    .  |
      | 1.0  2.0  3.0  |  4.0  5.0  5.0  |   .    .    .  |
 0    | 1.0  2.0  3.0  |  4.0  4.0   .   |   .    .    .  |
      | 1.0  2.0  3.0  |  3.0   .    .   |   .    .    .  |
      | 1.0  2.0  2.0  |   .    .    .   |   .    .    .  |
      | 1.0  1.0   .   |   .    .    .   |   .    .    .  |
      └                                                    ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array A with block size of 3:

```
p,q  |       0        |        1        |       2
-----|----------------|-----------------|----------------
     | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  7.0  8.0  8.0
     | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  7.0  7.0   .
     | 1.0  2.0  3.0  |  4.0  5.0  6.0  |  6.0   .    .
     | 1.0  2.0  3.0  |  4.0  5.0  5.0  |   .    .    .
 0   | 1.0  2.0  3.0  |  4.0  4.0   .   |   .    .    .
     | 1.0  2.0  3.0  |  3.0   .    .   |   .    .    .
     | 1.0  2.0  2.0  |   .    .    .   |   .    .    .
     | 1.0  1.0   .   |   .    .    .   |   .    .    .
```

Output:

Global matrix ***A*** is returned in lower-band-packed storage mode with block size of 3:

```
B,D         0                 1                 2
     ┌                                                     ┐
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0  1.0  1.0 │
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0  1.0   .  │
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0   .    .  │
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │    .    .    .  │
 0   │  1.0  1.0  1.0  │   1.0  1.0   .   │    .    .    .  │
     │  1.0  1.0  1.0  │   1.0   .    .   │    .    .    .  │
     │  1.0  1.0  1.0  │    .    .    .   │    .    .    .  │
     │  1.0  1.0   .   │    .    .    .   │    .    .    .  │
     └                                                     ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array A with block size of 3:

```
p,q  │        0        │         1        │        2
-----│-----------------│------------------│-----------------
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0  1.0  1.0
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0  1.0   .
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │   1.0   .    .
     │  1.0  1.0  1.0  │   1.0  1.0  1.0  │    .    .    .
 0   │  1.0  1.0  1.0  │   1.0  1.0   .   │    .    .    .
     │  1.0  1.0  1.0  │   1.0   .    .   │    .    .    .
     │  1.0  1.0  1.0  │    .    .    .   │    .    .    .
     │  1.0  1.0   .   │    .    .    .   │    .    .    .
```

The value of *info* is 0 on all processes.

PDPBTRS—Positive Definite Symmetric Band Matrix Solve

This subroutine solves the following system of equations for multiple right-hand sides:

$\boldsymbol{AX} = \boldsymbol{B}$

where, in the formula above:

> ***A*** represents the global positive definite symmetric band submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ factored by Cholesky factorization.
> ***B*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
> ***X*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to PDPBTRF. The output from PDPBTRF should be used only as input to this solve subroutine.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See references [2], [23], [39], and [40].

Table 68. Data Types

A, ***B***, *af, work*	**Subroutine**
Long-precision real	PDPBTRS

Syntax

Fortran	CALL PDPBTRS (*uplo*, *n*, *k*, *nrhs*, *a*, *ja*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info*)
C and C++	pdpbtrs (*uplo*, *n*, *k*, *nrhs*, *a*, *ja*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info*);

On Entry

uplo

indicates whether the upper or lower triangular part of the global submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the number of columns in the submatrix ***A***, stored in the upper- or lower-band-packed storage mode. It is also the number of rows in the general submatrix ***B*** containing the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer; $0 \leq n \leq (\text{NB\_A})p - \text{mod}(ja-1,\text{NB\_A})$.

k

is the half bandwidth of the factored submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If *uplo* = `'U'`, $0 \le k \le$ NB_A.
- If *uplo* = `'L'`, $0 \le k < n$.

These limits for *k* are extensions of the ScaLAPACK standard.

nrhs
is the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \ge 0$.

a
is the local part of the global positive definite symmetric band matrix ***A***, stored in upper- or lower-band-packed storage mode, containing the factorization of matrix ***A*** produced from a preceding call to PDPBTRF. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *k*, *ja*, *desc_a*, and *p*; therefore, the leading *k*+1 by LOCp(*ja*+*n*–1) part of the local array `A` must contain the local pieces of the leading *k*+1 by *ja*+*n*–1 part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ contains the factorization.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global submatrix $\boldsymbol{A}_{ja:ja+n-1,\ ja:ja+n-1}$ contains the factorization.

Scope: **local**

Specified as: an LLD_A by (at least) LOCp(*ja*+*n*–1) array, containing numbers of the data type indicated in Table 68 on page 507. Details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, array `A` is overwritten; that is, original input is not preserved.

ja
is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a
is the array descriptor for global matrix ***A***, which may be type 501 or type 1, as described in the following tables. For rules on using array descriptors, see "Notes and Coding Rules" on page 512.

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 501 for $1 \times p$ or $p \times 1$ where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
4	NB_A	Column block size	NB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (NB_A)*p*–mod(*ja*–1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ *p*	Global
6	LLD_A	Leading dimension	LLD_A $\geq$ *k*+1	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for 1 $\times$ *p* where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A > *k*	Global
4	N_A	Number of columns in the global matrix	If *n* = 0: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (NB_A)*p*–mod(*ja*–1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A=0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 $\leq$ CSRC_A $<$ *p*	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ *k*+1	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array `B`. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*–1) by *nrhs* part of the local array `B` must contain the local pieces of the leading *ib*+*n*–1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 68 on page 507. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B.

desc_b

is the array descriptor for global matrix ***B***, which may be type 502 or type 1, as described in the following tables. For rules on using array descriptors, see "Notes and Coding Rules" on page 512.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and $0 \le n \le$ (MB_B)p–mod(ib–1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
6	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and $0 \le n \le$ (MB_B)p–mod(ib–1,MB_B)	Global

desc_b	**Name**	**Description**	**Limits**	**Scope**
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B=0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

af

is a reserved area. Its size is specified by LAF.

Scope: **local**

Specified as: for migration purposes, you should specify a one-dimensional, long-precision array of (at least) length *laf*.

laf

is the number of elements in array AF.

The *laf* argument must be specified; however, this subroutine currently ignores its value. For migration purposes, you should specify *laf* using the formula below.

Scope: **local**

Specified as: a fullword integer, *laf* ≥ (NB_A+2*k*)(*k*).

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ -1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = -1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 68 on page 507.

lwork

is the number of elements in array WORK.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDPBTRS dynamically allocates the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard. It is suggested that you specify *lwork*=0.

- If *lwork* = -1, PDPBTRS performs a work area query and returns the optimum required size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise,

  $lwork \geq (nrhs)(k)$

info
See On Return.

On Return

b
b is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 68 on page 507.

work
is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ or $lwork \neq -1$, the size of *work* is (at least) of length *lwork*.

If *lwork* = -1, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 68 on page 507, where:

- If *lwork* = -1, $work_1$ is set to the optimum *lwork* value needed.
- If $lwork \geq 1$, $work_1$ is set to the minimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info
indicates a successful computation or work area query occurred.

Scope: **global**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The subroutine accepts lowercase letters for the *uplo* argument.
3. This subroutine gives the best performance for wide band widths, for example:

$$k > 100\sqrt{p}$$

   where *p* is the number of processes). For details, see references [2], [39], and [40]. Also, it is suggested that you specify *uplo* = `'L'`.
4. The *k*+1 by *n* array specified for submatrix ***A*** must remain unchanged between calls to PDPBTRF and PDPBTRS. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.

5. The output from the PDPBTRF subroutine should be used only as input to the solve subroutine PDPBTRS.

   The input arguments *uplo*, *n*, and *k* must be the same for both PDPBTRF and PDPBTRS.

   The global matrix ***A*** and *af* input to PDPBTRS must be the same as the corresponding output arguments for PDPBTRF; and thus, the scalar data specified for *ja*, *desc_a*, and *laf* must also be the same.

6. In all cases, follow these rules:

   - *ib* = *ja*
   - DTYPE_A=501 or 1
   - DTYPE_B=502 or 1
   - NB_A = MB_B
   - If DTYPE_A=1, RSRC_A=0, M_A ≥ *k*+1, and MB_A ≥ 1.
   - If DTYPE_B=1, CSRC_B=0, N_B ≥ *nrhs*, and NB_B ≥ 1.
   - CTXT_A = CTXT_B
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	DTYPE_B	Process Grid
501	502	$p \times 1$ or $1 \times p$
501	1	$1 \times p$
1	502	$p \times 1$
1	1	1×1

7. To determine the values of LOCp(*n*) used in the argument descriptions, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 for descriptor type-1 or "Determining the Number of Rows or Columns in Your Local Arrays" on page 30 for descriptor type-501 and type-502.

8. ***A***, ***B***, *af* and *work* must have no common elements; otherwise, results are unpredictable.

9. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. See the section on block distributing a symmetric matrix in "Matrices" on page 40.

   Matrix ***A*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information on using block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26.

10. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information on using block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26. Also, see the section on distributing the right-hand side matrix in "Matrices" on page 40.

11. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1, they must all specify −1.

12. Although global submatrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ja*, *ib*, NB_A, and MB_B must be

chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.

Error Conditions

Computational Errors: None

Note: If the factorization performed by PDPBTRF failed because of a nonpositive definite matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDPBTRF.

Resource Errors: *lwork* = 0 and unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDPBTRS was called from outside the process grid.

Stage 4

1. The process grid is not 1 × p or p × 1.
2. $uplo \neq$ `'U'` or `'L'`
3. $n < 0$
4. $k < 0$
5. $k+1 > n$
6. $ja < 1$
7. DTYPE_A = 1 and:
   a. M_A < k+1
   b. MB_A < 1
   c. RSRC_A ≠ 0
   d. The process grid is not 1 × p.
8. N_A < 0 and (n = 0); N_A < 1 otherwise
9. NB_A < 1
10. n > (NB_A)p–mod(ja–1,NB_A)
11. $uplo$ = `'U'` and k > NB_A
12. CSRC_A < 0 or CSRC_A ≥ p
13. $nrhs < 0$
14. $ib \neq ja$
15. $ib < 1$
16. DTYPE_B = 1 and:
    a. N_B < $nrhs$
    b. NB_B < 1
    c. CSRC_B ≠ 0
    d. The process grid is not p × 1.
17. M_B < 0 and (n = 0); M_B < 1 otherwise
18. MB_B < 1
19. n > (MB_B)p–mod(ib–1,MB_B)

20. MB_B $\neq$ NB_A
21. RSRC_B $<$ 0 or RSRC_B $\geq$ *p*
22. CTXT_A $\neq$ CTXT_B

Stage 5: If *n* $>$ 0:

1. *ja*+*n*–1 $>$ N_A
2. *ja* $>$ N_A
3. *ib* $>$ M_B
4. *ib*+*n*–1 $>$ M_B
5. LLD_A $<$ *k*+1

Stage 6

1. LLD_B $<$ max(1, LOCp(M_B))
2. *lwork* $\neq$ 0,
3. *lwork* $\neq$ –1, and *lwork* $<$ (*nrhs*)(*k*)

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *uplo* differs.
2. *n* differs.
3. *k* differs.
4. *nrhs* differs.
5. *ja* differs.
6. DTYPE_A differs.
7. DTYPE_A does not differ and:
   a. N_A differs.
   b. NB_A differs.
   c. CSRC_A differs.
   d. DTYPE_A = 1 and:
      1) M_A differs.
      2) MB_A differs.
      3) RSRC_A differs.
8. *ib* differs.
9. DTYPE_B differs.
10. DTYPE_B does not differ and:
    a. M_B differs.
    b. MB_B differs.
    c. RSRC_B differs.
    d. DTYPE_A = 1 and:
       1) N_B differs.
       2) NB_B differs.
       3) CSRC_B differs.

Also:
11. *lwork* = –1 on a subset of processes.

Example: This example solves the ***AX***=***B*** system, where matrix ***A*** is the same positive definite symmetric band matrix factored in “Example” on page 503 for PDPBTRF.

Notes:

1. Matrix ***A***, output from PDPBTRF, must be passed, unchanged, to the solve subroutine PDPBTRS.

   The input values for *desc_a* are the same values shown in “Example” on page 503.

2. Notice **only one process grid was created**, even though, DTYPE_A = 501 and DTYPE_B = 502.
3. The *laf* argument must be specified; however, this subroutine currently ignores its value. For migration purposes, in this example, *laf* is specified as 119.
4. The *af* argument, output from PDPBTRF, must be passed, unchanged, to the solve subroutine PDPBTRS.
5. Because *lwork* = 0, PDPBTRS dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             UPLO   N   K  NRHS A  JA   DESC_A   B   IB  DESC_B   AF   LAF
               |    |   |   |   |   |     |      |   |     |      |     |
CALL PDPBTRS( 'L' , 9 , 7 , 3 , A , 1 , DESC_A , B , 1 , DESC_B , AF , 119 ,

             WORK LWORK INFO
               |     |    |
             WORK , 0 , INFO )
```

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	9
MB_	3
RSRC_	0
LLD_B	3
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global matrix ***A*** stored in lower-band-packed storage mode with block size of 3:

```
B,D        0              1              2
       ┌                                          ┐
       | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0  1.0 |
       | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0  1.0   .  |
       | 1.0  1.0  1.0 |  1.0  1.0  1.0 |  1.0   .    .  |
       | 1.0  1.0  1.0 |  1.0  1.0  1.0 |   .    .    .  |
 0     | 1.0  1.0  1.0 |  1.0  1.0   .  |   .    .    .  |
       | 1.0  1.0  1.0 |  1.0   .    .  |   .    .    .  |
       | 1.0  1.0  1.0 |   .    .    .  |   .    .    .  |
       | 1.0  1.0   .  |   .    .    .  |   .    .    .  |
       └                                          ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array A with block size of 3:

```
p,q  |       0         |        1         |       2
-----|-----------------|------------------|-----------------
     | 1.0  1.0  1.0   |  1.0  1.0  1.0   |  1.0  1.0  1.0
     | 1.0  1.0  1.0   |  1.0  1.0  1.0   |  1.0  1.0   .
     | 1.0  1.0  1.0   |  1.0  1.0  1.0   |  1.0   .    .
     | 1.0  1.0  1.0   |  1.0  1.0  1.0   |   .    .    .
 0   | 1.0  1.0  1.0   |  1.0  1.0   .    |   .    .    .
     | 1.0  1.0  1.0   |  1.0   .    .    |   .    .    .
     | 1.0  1.0  1.0   |   .    .    .    |   .    .    .
     | 1.0  1.0   .    |   .    .    .    |   .    .    .
```

Global matrix ***B*** with block size of 3:

```
B,D          0
       ┌                   ┐
       |   8.0  36.0  44.0 |
 0     |  16.0  80.0  80.0 |
       |  23.0 122.0 108.0 |
       | ----------------- |
       |  29.0 161.0 129.0 |
 1     |  34.0 196.0 144.0 |
       |  38.0 226.0 154.0 |
       | ----------------- |
       |  41.0 250.0 160.0 |
 2     |  43.0 267.0 163.0 |
       |  36.0 240.0 120.0 |
       └                   ┘
```

The following is the 1 × 3 process grid:

B,D	0	1	2
0	P_{00}	P_{01}	P_{02}

Local array B with block size of 3:

```
p,q  |         0          |          1          |         2
-----|--------------------|---------------------|--------------------
     |   8.0  36.0  44.0  |   29.0 161.0 129.0  |   41.0 250.0 160.0
 0   |  16.0  80.0  80.0  |   34.0 196.0 144.0  |   43.0 267.0 163.0
     |  23.0 122.0 108.0  |   38.0 226.0 154.0  |   36.0 240.0 120.0
```

Output:

Global matrix ***B*** with block size of 3:

```
B,D         0
     ┌                ┐
     |  1.0  1.0  9.0 |
 0   |  1.0  2.0  8.0 |
     |  1.0  3.0  7.0 |
     | -------------- |
     |  1.0  4.0  6.0 |
 1   |  1.0  5.0  5.0 |
     |  1.0  6.0  4.0 |
     | -------------- |
     |  1.0  7.0  3.0 |
 2   |  1.0  8.0  2.0 |
     |  1.0  9.0  1.0 |
     └                ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local array B with block size of 3:

```
p,q  |       0          |        1          |       2
-----|------------------|-------------------|-----------------
     |  1.0  1.0  9.0   |   1.0  4.0  6.0   |   1.0  7.0  3.0
 0   |  1.0  2.0  8.0   |   1.0  5.0  5.0   |   1.0  8.0  2.0
     |  1.0  3.0  7.0   |   1.0  6.0  4.0   |   1.0  9.0  1.0
```

The value of *info* is 0 on all processes.

PDGTSV and PDDTSV—General Tridiagonal Matrix Factorization and Solve

PDGTSV solves the tridiagonal systems of linear equations, ***AX*** = ***B***, using Gaussian elimination with partial pivoting for the general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

PDDTSV solves the tridiagonal systems of linear equations, ***AX*** = ***B***, using Gaussian elimination for the diagonally dominant general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

- ***A*** represents the global square general tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.
- ***B*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
- ***X*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

| If $n = 0$, no computation is performed and the subroutine returns after doing some
| parameter checking. See reference [51].

Table 69. Data Types

dl, ***d***, ***du***, ***B***, *work*	**Subroutine**
Long-precision real	PDGTSV and PDDTSV

Syntax

Fortran	CALL PDGTSV | PDDTSV *(n*, *nrhs*, *dl*, *d*, *du*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *work*, *lwork*, *info)*
C and C++	pdgtsv | pddtsv *(n*, *nrhs*, *dl*, *d*, *du*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *work*, *lwork*, *info);*

On Entry

n

is the order of the general tridiagonal matrix ***A*** and the number of rows in the general submatrix ***B***, which contains the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \le n \le (\text{MB\_A})(p)-\text{mod}(ia-1,\text{MB\_A})$.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \le n \le (\text{NB\_A})(p)-\text{mod}(ia-1,\text{NB\_A})$.

where *p* is the number of processes in a process grid.

nrhs

is the number of right-hand sides; that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \ge 0$.

dl

is the local part of the global vector ***dl***. This identifies the **first element** of the local array DL. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array `DL` contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***dl*** contains the subdiagonal of the global general tridiagonal submatrix ***A*** in elements *ia*+1 through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 69 on page 519. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, `DL` is overwritten; that is, the original input is not preserved.

d

is the local part of the global vector ***d***. This identifies the **first element** of the local array `D`. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array `D` contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***d*** contains the main diagonal of the global general tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1) containing numbers of the data type indicated in Table 69 on page 519. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, `D` is overwritten; that is, the original input is not preserved.

du

is the local part of the global vector ***du***. This identifies the **first element** of the local array `DU`. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array `DU` contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***du*** contains the superdiagonal of the global general tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–2.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 69 on page 519. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, `DU` is overwritten; that is, the original input is not preserved.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $1 \le ia \le$ N_A and $ia+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid.

The following tables describe the three types of array descriptors. For rules on using array descriptors, see "Notes and Coding Rules" on page 526.

Table 70. Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(p)−mod(ia−1,MB_A)	Global
5	RSRC_A	The process row over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 71 (Page 1 of 2). Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global

Table 71 (Page 2 of 2). Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	
5	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(p)−mod(ia−1,MB_A)	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 72. Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for $1 \times p$ or $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
4	NB_A	Column block size	NB_A ≥ 1 and $0 \le n \le$ (NB_A)(p)−mod(ia−1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < p	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 73. Type-1 Array Descriptor (1 × p Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for 1 × *p* where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A = 1	Global
4	N_A	Number of columns in the global matrix	If *n* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1 and 0 ≤ *n* ≤ (NB_A)(*p*)−mod(*ia*−1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A = 0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < *p*	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*−1) by *nrhs* part of the local array B must contain the local pieces of the leading *ib*+*n*−1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 69 on page 519. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ib* ≤ M_B and *ib*+*n*−1 ≤ M_B

desc_b

is the array descriptor for global matrix ***B***, which may be type-502 or type-1, as described in the following tables. For type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For rules on using array descriptors, see "Notes and Coding Rules" on page 526.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for $p \times 1$ or $1 \times p$ where *p* is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and $0 \leq n \leq$ (MB_B)p–mod(ib–1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
6	LLD_B	Leading dimension	LLD_B ≥ max(1,LOCp(M_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where *p* is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and $0 \leq n \leq$ (MB_B)p–mod(ib–1,MB_B)	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B = 0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1,LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

work

has the following meaning:

If *lwork* $= 0$, *work* is ignored.

If *lwork* $\neq 0$, *work* is the work area used by this subroutine, where:

- If *lwork* $\neq -1$, the size of *work* is (at least) of length *lwork*.
- If *lwork* $= -1$, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 69 on page 519.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* $= -1$, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* $= 0$, PDGTSV and PDDTSV dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* $= -1$, PDGTSV and PDDTSV perform a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, if (the process grid is $p \times 1$ and DTYPE_A $= 1$) or DTYPE_A $= 502$:
  - If $nrhs \leq 1$, then:
    - For PDGTSV, *lwork* $\geq$ 18P+MB_A+12.
    - For PDDTSV, *lwork* $\geq$ 10P+10
  - If $nrhs > 1$, then:
    - For PDGTSV, *lwork* $\geq$ 24P+5(MB_A+*nrhs*)
    - For PDDTSV, *lwork* $\geq$ 20P+2(MB_A)+4(*nrhs*)

  where, in the above formulas, P is the **actual** number of processes containing data.

  If (the process grid is $1 \times p$ and DTYPE_A $= 1$) or DTYPE_A $= 501$, you would substitute NB_A in place of MB_A in the formulas above.

  Note: In ScaLAPACK 1.5, PDDTSV requires *lwork* = 22P+3MB_A+4(*nrhs*). This value is greater than or equal to the value required by Parallel ESSL.

info

See On Return.

On Return

dl

is overwritten; that is, the original input is not preserved.

d
is overwritten; that is, the original input is not preserved.

du
is overwritten; that is, the original input is not preserved.

b
b is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 69 on page 519.

work
is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, its size is (at least) of length *lwork*.

If $lwork = -1$, its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 69 on page 519, where:

- If $lwork \geq 1$, $work_1$ is set to the minimum *lwork* value needed.
- If $lwork = -1$, $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info
has the following meaning:

If $info = 0$, the factorization or the work area query completed successfully.

Note: For PDDTSV, if the input matrix ***A*** is not diagonally dominant, the subroutine may still complete the factorization; however, results are unpredictable.

If $1 \leq info \leq p$, the portion of the global submatrix ***A*** stored on process $info-1$ and factored locally, is singular or reducible (for PDGTSV), or not diagonally dominant (for PDDTSV). The magnitude of a pivot element was zero or too small.

If $info > p$, the portion of the global submatrix ***A*** stored on process $info-p-1$ representing interactions with other processes, is singular or reducible (for PDGTSV), or not diagonally dominant (for PDDTSV). The magnitude of a pivot element was zero or too small.

If $info > 0$, the results are unpredictable.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. If $n > 0$ and $nrhs = 0$, only the factorization is completed. |
3. ***dl***, ***d***, ***du***, ***B***, and *work* must have no common elements; otherwise, results are unpredictable.

4. In all cases, follow these rules:
   - *ia* = *ib*
   - CTXT_A = CTXT_B
   - If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, MB_A = MB_B.
   - If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, NB_A = MB_B.
   - If DTYPE_A = 1, then:
     - For a $p \times 1$ process grid (where $p > 1$), N_A = 1, NB_A≥1, and CSRC_A = 0.
     - For a $1 \times p$ process grid, M_A = 1, MB_A≥1, and RSRC_A = 0.
     - For a 1×1 process grid:
       - If N_A = 1, NB_A≥1, and CSRC_A = 0.
       - If M_A = 1, MB_A≥1, and RSRC_A = 0.
   - If DTYPE_B = 1, N_B≥*nrhs*, NB_B≥1, and CSRC_B = 0.
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	**DTYPE_B**	**Process Grid**
501	502	$p \times 1$ or $1 \times p$
502	502	$p \times 1$ or $1 \times p$
501	1	$p \times 1$
502	1	$p \times 1$
1	502	$1 \times p$
1	1	1×1

5. To determine the values of LOCp(*n*) used in the argument descriptions, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 for descriptor type-1 or “Determining the Number of Rows or Columns in Your Local Arrays” on page 30 for descriptor type-501 and type-502.

6. For PDGTSV, the global general tridiagonal matrix ***A*** must be non-singular and irreducible. For PDDTSV, the global general tridiagonal matrix ***A*** must be diagonally dominant to ensure numerical accuracy, because no pivoting is performed. These subroutines use the *info* argument to provide information about ***A***, like ScaLAPACK. However, these subroutines also issue an error message, which differs from ScaLAPACK.

7. The global general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in “Matrices” on page 40.

   For more information on using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.

8. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26. Also, see the section on distributing the right-hand side matrix in “Matrices” on page 40.

9. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
10. Although global matrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p\times1$ process grid, the values of *n*, *ia*, *ib*, MB_A (if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.
11. For global tridiagonal matrix ***A***, use of the type-1 array descriptor with a $p \times 1$ process grid is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: Matrix ***A*** is a singular or reducible matrix (for PDGTSV), or not diagonally dominant (for PDDTSV). For details, see the description of the *info* argument.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A = 1 and DTYPE_A = 1, a 1×1 process grid is treated as a $1 \times p$ process grid.
- If N_A = 1 and DTYPE_A = 1, a 1×1 process grid is treated as a $p \times 1$ process grid.

1. The process grid is not $1 \times p$ or $p \times 1$.
2. CTXT_A ≠ CTXT_B
3. $n < 0$
4. $ia < 1$
5. DTYPE_A = 1 and M_A ≠ 1 and N_A ≠ 1

If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501:

6. N_A < 0 and (n = 0); N_A < 1 otherwise
7. NB_A < 1
8. n > (NB_A)(p)−mod(ia−1,NB_A)
9. ia > N_A and (n > 0)

10. ia+n–1 $>$ N_A and ($n > 0$)
11. CSRC_A $<$ 0 or CSRC_A $\geq$ p
12. NB_A $\neq$ MB_B
13. CSRC_A $\neq$ RSRC_B

If the process grid is 1 $\times$ p and DTYPE_A = 1:
14. M_A $\neq$ 1
15. MB_A $<$ 1
16. RSRC_A $\neq$ 0

If (the process grid is p $\times$ 1 and DTYPE_A = 1) or DTYPE_A = 502:
17. M_A $<$ 0 and ($n = 0$); M_A $<$ 1 otherwise
18. MB_A $<$ 1
19. n $>$ (MB_A)(p)–mod(ia–1,MB_A)
20. ia $>$ M_A and ($n > 0$)
21. ia+n–1 $>$ M_A and ($n > 0$)
22. RSRC_A $<$ 0 or RSRC_A $\geq$ p
23. MB_A $\neq$ MB_B
24. RSRC_A $\neq$ RSRC_B

If the process grid is p $\times$ 1 and DTYPE_A = 1:
25. N_A $\neq$ 1
26. NB_A $<$ 1
27. CSRC_A $\neq$ 0

In all cases:
28. $ia \neq ib$
29. DTYPE_B = 1 and the process grid is 1 $\times$ p and $p > 1$
30. $nrhs < 0$
31. $ib < 1$
32. M_B $<$ 0 and ($n = 0$); M_B $<$ 1 otherwise
33. MB_B $<$ 1
34. ib $>$ M_B and ($n > 0$)
35. ib+n–1 $>$ M_B and ($n > 0$)
36. RSRC_B $<$ 0 or RSRC_B $\geq$ p
37. LLD_B $<$ max(1,LOCp(M_B))

If DTYPE_B = 1:
38. N_B $<$ 0 and ($nrhs = 0$); N_B $<$ 1 otherwise
39. N_B $<$ $nrhs$
40. NB_B $<$ 1
41. CSRC_B $\neq$ 0

In all cases:
42. $lwork \neq 0$, $lwork \neq -1$, and $lwork$ $<$ (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:
1. *n* differs.
2. *nrhs* differs.
3. *ia* differs.
4. *ib* differs.
5. DTYPE_A differs.

If DTYPE_A = 1 on all processes:

6. M_A differs.
7. N_A differs.
8. MB_A differs.
9. NB_A differs.
10. RSRC_A differs.
11. CSRC_A differs.

If DTYPE_A = 501 on all processes:
12. N_A differs.
13. NB_A differs.
14. CSRC_A differs.

If DTYPE_A = 502 on all processes:
15. M_A differs.
16. MB_A differs.
17. RSRC_A differs.

In all cases:
18. DTYPE_B differs.

If DTYPE_B = 1 on all processes:
19. M_B differs.
20. N_B differs.
21. MB_B differs.
22. NB_B differs.
23. RSRC_B differs.
24. CSRC_B differs.

If DTYPE_B = 502 on all processes:
25. M_B differs.
26. MB_B differs.
27. RSRC_B differs.

Also:
28. *lwork* = −1 on a subset of processes.

Example: This example shows a factorization of the general tridiagonal matrix ***A*** of order 12:

$$\begin{bmatrix}
2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0
\end{bmatrix}$$

Matrix ***A*** is distributed over a 1 × 3 process grid using block-column distribution.

Notes:

1. On output, the vectors ***dl***, ***d***, and ***du*** are overwritten by this subroutine.
2. Notice **only one process grid was created**, even though, DTYPE_A = 501 and DTYPE_B = 502.
3. Because *lwork* = 0, this subroutine dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N  NRHS DL   D   DU  IA   DESC_A   B  IB   DESC_B   WORK LWORK INFO
              |   |   |    |    |   |      |     |   |      |      |     |    |
CALL PDGTSV( 12 , 3 , DL , D , DU , 1 , DESC_A , B , 1 , DESC_B , WORK , 0 , INFO )

-or-
              N  NRHS DL   D   DU  IA   DESC_A   B  IB   DESC_B   WORK LWORK INFO
              |   |   |    |    |   |      |     |   |      |      |     |    |
CALL PDDTSV( 12 , 3 , DL , D , DU , 1 , DESC_A , B , 1 , DESC_B , WORK , 0 , INFO )
```

	Desc_A
DTYPE_	501
CTXT_	*icontxt*[1]
N_	12
NB_	4
CSRC_	0
Not used	—
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
LLD_B	4
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global vector ***dl*** with block size of 4:

```
B,D           0                       1                       2
     ┌                                                                   ┐
 0   │   .   1.0  1.0  1.0  │  1.0  1.0  1.0  1.0  │  1.0  1.0  1.0  1.0 │
     └                                                                   ┘
```

Global vector ***d*** with block size of 4:

```
B,D           0                       1                       2
     ┌                                                                   ┐
 0   │ 2.0  3.0  3.0  3.0  │  3.0  3.0  3.0  3.0  │  3.0  3.0  3.0  3.0 │
     └                                                                   ┘
```

Global vector ***du*** with block size of 4:

```
B,D           0                       1                      2
     ┌                                                                  ┐
 0   │ 2.0  2.0  2.0  2.0  │  2.0  2.0  2.0  2.0  │  2.0  2.0  2.0   .  │
     └                                                                  ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local array `DL` with block size of 4:

```
p,q  │          0           │          1            │          2
-----│----------------------│-----------------------│----------------------
 0   │   .   1.0  1.0  1.0  │  1.0  1.0  1.0  1.0   │  1.0  1.0  1.0  1.0
```

Local array `D` with block size of 4:

```
p,q  │          0           │          1            │          2
-----│----------------------│-----------------------│----------------------
 0   │ 2.0  3.0  3.0  3.0   │  3.0  3.0  3.0  3.0   │  3.0  3.0  3.0  3.0
```

Local array `DU` with block size of 4:

```
p,q  │          0           │          1            │         2
-----│----------------------│-----------------------│---------------------
 0   │ 2.0  2.0  2.0  2.0   │  2.0  2.0  2.0  2.0   │  2.0  2.0  2.0   .
```

Global matrix ***B*** with a block size of 4:

```
B,D         0
     ┌                ┐
     | 46.0  6.0  4.0 |
     | 65.0 13.0  6.0 |
 0   | 59.0 19.0  6.0 |
     | 53.0 25.0  6.0 |
     | -------------- |
     | 47.0 31.0  6.0 |
     | 41.0 37.0  6.0 |
 1   | 35.0 43.0  6.0 |
     | 29.0 49.0  6.0 |
     | -------------- |
     | 23.0 55.0  6.0 |
     | 17.0 61.0  6.0 |
 2   | 11.0 67.0  6.0 |
     |  5.0 47.0  4.0 |
     └                ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local matrix ***B*** with a block size of 4:

```
p,q  |        0         |        1          |        2
-----|------------------|-------------------|-----------------
     | 46.0  6.0  4.0   |  47.0 31.0  6.0   |  23.0 55.0  6.0
     | 65.0 13.0  6.0   |  41.0 37.0  6.0   |  17.0 61.0  6.0
 0   | 59.0 19.0  6.0   |  35.0 43.0  6.0   |  11.0 67.0  6.0
     | 53.0 25.0  6.0   |  29.0 49.0  6.0   |   5.0 47.0  4.0
```

Output:

Global matrix ***B*** with a block size of 4:

```
p,q  |        0
-----|----------------
     | 12.0  1.0  1.0
     | 11.0  2.0  1.0
 0   | 10.0  3.0  1.0
     |  9.0  4.0  1.0
-----|----------------
     |  8.0  5.0  1.0
     |  7.0  6.0  1.0
 1   |  6.0  7.0  1.0
     |  5.0  8.0  1.0
-----|----------------
     |  4.0  9.0  1.0
     |  3.0 10.0  1.0
 2   |  2.0 11.0  1.0
     |  1.0 12.0  1.0
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local matrix ***B*** with a block size of 4:

```
p,q  |       0         |        1         |        2
-----|-----------------|------------------|-----------------
     | 12.0  1.0  1.0  |   8.0  5.0  1.0  |   4.0  9.0  1.0
     | 11.0  2.0  1.0  |   7.0  6.0  1.0  |   3.0 10.0  1.0
 0   | 10.0  3.0  1.0  |   6.0  7.0  1.0  |   2.0 11.0  1.0
     |  9.0  4.0  1.0  |   5.0  8.0  1.0  |   1.0 12.0  1.0
```

The value of *info* is 0 on all processes.

PDGTTRF and PDDTTRF—General Tridiagonal Matrix Factorization

PDGTTRF factors the general tridiagonal matrix ***A***, stored in tridiagonal storage mode, using Gaussian elimination with partial pivoting.

PDDTTRF factors the diagonally dominant general tridiagonal matrix ***A***, stored in tridiagonal storage mode, using Gaussian elimination.

In these subroutine descriptions, ***A*** represents the global square general tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.

To solve a tridiagonal system of linear equations with multiple right-hand sides, follow the call to PDGTTRF or PDDTTRF with one or more calls to PDGTTRS or PDDTTRS, respectively. The output from these factorization subroutines should be used only as input to the solve subroutines PDGTTRS and PDDTTRS, respectively.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See reference [51].

Table 74. Data Types

dl, ***d***, ***du***, ***du2***, *af*, *work*	***ipiv***	**Subroutine**
Long-precision real	Integer	PDGTTRF and PDDTTRF

Syntax

Fortran	CALL PDGTTRF *(n, dl, d, du, du2, ia, desc_a, ipiv, af, laf, work, lwork, info)*
	CALL PDDTTRF *(n, dl, d, du, ia, desc_a, af, laf, work, lwork, info)*
C and C++	pdgttrf *(n, dl, d, du, du2, ia, desc_a, ipiv, af, laf, work, lwork, info);*
	pddttrf *(n, dl, d, du, ia, desc_a, af, laf, work, lwork, info);*

On Entry

n

is the order of the general tridiagonal matrix ***A*** and the number of elements in vector ***ipiv*** used in the computation.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \leq n \leq$ (MB_A)(p)−mod(ia−1,MB_A).
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \leq n \leq$ (NB_A)(p)−mod(ia−1,NB_A).

where *p* is the number of processes in a process grid.

dl

is the local part of the global vector ***dl***. This identifies the **first element** of the local array DL. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array DL contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

The global vector ***dl*** contains the subdiagonal of the global general tridiagonal submatrix ***A*** in elements *ia*+1 through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 74 on page 535. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, DL is overwritten; that is, the original input is not preserved.

d

is the local part of the global vector ***d***. This identifies the **first element** of the local array D. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array D contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***d*** contains the main diagonal of the global general tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1). containing numbers of the data type indicated in Table 74 on page 535. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, D is overwritten; that is, the original input is not preserved.

du

is the local part of the global vector ***du***. This identifies the **first element** of the local array DU. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array DU contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***du*** contains the superdiagonal of the global general tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–2.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 74 on page 535. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, DU is overwritten; that is, the original input is not preserved.

du2

See On Return.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501 $1 \le ia \le$ N_A and $ia+n-1 \le$ N_A

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid. The following tables describe three types of array descriptors. For rules on using array descriptors, see “Notes and Coding Rules” on page 542.

Table 75. Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	MB_A	Row block size	MB_A ≥ 1 and 0 ≤ n ≤ (MB_A)(p)−mod(ia−1,MB_A)	Global
5	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 76 (Page 1 of 2). Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	

Table 76 (Page 2 of 2). Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
5	MB_A	Row block size	MB_A $\geq$ 1 and $0 \leq n \leq$ (MB_A)(p)−mod(ia−1,MB_A)	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 77. Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for $1 \times p$ or $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
4	NB_A	Column block size	NB_A $\geq$ 1 and $0 \leq n \leq$ (NB_A)(p)−mod(ia−1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< p$	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 78 (Page 1 of 2). Type-1 Array Descriptor ($1 \times p$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for $1 \times p$ where p is the number of processes in a process grid.	Global

Table 78 (Page 2 of 2). Type-1 Array Descriptor (1 × p Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A = 1	Global
4	N_A	Number of columns in the global matrix	If n = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1 and 0 ≤ n ≤ (NB_A)(p)−mod(ia−1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A = 0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < p	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

ipiv
See On Return.

af
See On Return.

laf
is the number of elements in array AF.

Scope: **local**

Specified as: a fullword integer, where:

If (the process grid is p × 1 and DTYPE_A = 1) or DTYPE_A = 502:
- For PDGTTRF, *laf* ≥ 12P+3(MB_A)
- For PDDTTRF, *laf* ≥ 12P+2(MB_A).

where, in the above formulas, P is the **actual** number of processes containing data.

If (the process grid is 1 × p and DTYPE_A = 1) or DTYPE_A = 501, you would substitute NB_A in place of MB_A in the formulas above.

Note: In ScaLAPACK 1.5, PDDTTRF requires *laf* = 12P+3NB_A. This value is greater than or equal to the value required by Parallel ESSL.

work
has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ -1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = -1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 74 on page 535.

lwork
is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDGTTRF and PDDTTRF dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = -1, PDGTTRF and PDDTTRF perform a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, *lwork* must have the following value:

  For PDGTTRF, *lwork* ≥ 10P

  For PDDTTRF, *lwork* ≥ 8P

  where, in the above formulas, P is the **actual** number of processes containing data.

info
See On Return.

On Return

dl
dl is the updated local part of the global vector ***dl***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 74 on page 535.

On output, `DL` is overwritten; that is, the original input is not preserved.

d
d is the updated local part of the global vector ***d***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 74 on page 535.

On output, `D` is overwritten; that is, the original input is not preserved.

du

du is the updated local part of the global vector ***du***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 74 on page 535.

On output, `DU` is overwritten; that is, the original input is not preserved.

du2

is the local part of the global vector ***du2***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 74 on page 535.

ipiv

is the local part of the global vector ***ipiv***, containing the pivot information needed by PDGTTRS. This identifies the **first element** of the local array `IPIV`. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array `IPIV` contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

Scope: **local**

Returned as: an array of (at least) length LOCp(*ia*+*n*−1), containing fullword integers. There is no array descriptor for ***ipiv***. The details about the block data distribution of global vector ***ipiv*** are stored in *desc_a*.

af

is a work area used by these subroutines and contains part of the factorization. Its size is specified by *laf*.

Scope: **local**

Returned as: a one-dimensional array of (at least) length *laf*, containing numbers of the data type indicated in Table 74 on page 535.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, the size of *work* is (at least) of length *lwork*.

If $lwork = -1$, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of data type indicated in Table 74 on page 535, where:

- If $lwork \geq 1$, the $work_1$ is set to the minimum *lwork* value needed.
- If $lwork = -1$, the $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info

has the following meaning:

If $info = 0$, the factorization or work area query completed successfully.

Note: For PDDTTRF, if the input matrix ***A*** is not diagonally dominant, the subroutine may still complete the factorization; however, results are unpredictable.

If $1 \leq info \leq p$, the portion of the global submatrix ***A*** stored on process *info*–1 and factored locally, is singular or reducible (for PDGTTRF), or not diagonally dominant (for PDDTTRF). The magnitude of a pivot element was zero or too small.

If $info > p$, the portion of the global submatrix ***A*** stored on process *info*–*p*–1 representing interactions with other processes, is singular or reducible (for PDGTTRF), or not diagonally dominant (for PDDTTRF). The magnitude of a pivot element was zero or too small.

If $info > 0$, the factorization is completed; however, if you call PDGTTRS/PDDTTRS with these factors, results are unpredictable.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The output from these factorization subroutines should be used only as input to the solve subroutines PDGTTRS and PDDTTRS, respectively.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   | The format of the output from PDDTTRF has changed. Therefore, the
   | factorization and solve must be performed using Parallel ESSL Version 2
   | Release 1.2.

   The scalar data specified for input argument *n* must be the same for both PDGTTRF/PDDTTRF and PDGTTRS/PDDTTRS.

   The global vectors for ***dl***, ***d***, ***du***, ***du2***, and *af* input to PDGTTRS/PDDTTRS must be the same as the corresponding output arguments for PDGTTRF/PDDTTRF; and thus, the scalar data specified for *ia*, *desc_a*, and *laf* must also be the same.
3. In all cases, follow these rules:
   - $ia = ib$
   - If DTYPE_A=1, then:
     - For a $p \times 1$ process grid (where $p>1$), N_A=1, NB_A≥1, and CSRC_A=0.
     - For a $1 \times p$ process grid (where $p>1$), M_A=1, MB_A≥1, and RSRC_A=0.
     - For a 1×1 process grid:
       - If N_A=1, NB_A≥1 and CSRC_A=0.
       - If M_A=1, MB_A≥1 and RSRC_A=0.
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	Process Grid
501	$p \times 1$ or $1 \times p$
502	$p \times 1$ or $1 \times p$
1	$p \times 1$ or $1 \times p$

4. To determine the values of LOCp(*n*) used in the argument descriptions, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 for descriptor type-1 or “Determining the Number of Rows or Columns in Your Local Arrays” on page 30 for descriptor type-501 and type-502.
5. ***dl***, ***d***, ***du***, ***du2***, ***ipiv***, *af*, and *work* must have no common elements; otherwise, results are unpredictable.
6. For PDGTTRF, the global general tridiagonal matrix ***A*** must be non-singular and irreducible. For PDDTTRF, the global general tridiagonal matrix ***A*** must be diagonally dominant to ensure numerical accuracy, because no pivoting is performed. These subroutines use the *info* argument to provide information about ***A***, like ScaLAPACK. However, these subroutines also issue an error message, which differs from ScaLAPACK.
7. The global general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in “Matrices” on page 40.

   For more information on using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.
8. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
9. Although global matrix ***A*** may be block-cyclically distributed on a 1 × *p* or *p* × 1 process grid, the values of *n*, *ia*, MB_A (if (the process grid is *p* × 1 and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is 1 × *p* and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of global submatrix ***A***.
10. For global tridiagonal matrix ***A***, use of the type-1 array descriptor is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: Matrix ***A*** is a singular or reducible matrix (for PDGTTRF), or not diagonally dominant (for PDDTTRF). For details, see the description of the *info* argument.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A = 1 and DTYPE_A = 1, a 1 × 1 process grid is treated as a 1 × *p* process grid.
- If N_A = 1 and DTYPE_A = 1, a 1 × 1 process grid is treated as a *p* × 1 process grid.

1. The process grid is not 1 × *p* or *p* × 1.
2. $n < 0$
3. $ia < 1$
4. DTYPE_A = 1 and M_A ≠ 1 and N_A ≠ 1

If (the process grid is 1 × *p* and DTYPE_A = 1) or DTYPE_A = 501:

5. N_A < 0 and ($n = 0$); N_A < 1 otherwise
6. NB_A < 1
7. $n >$ (NB_A)(p)−mod(ia−1,NB_A)
8. $ia >$ N_A and ($n > 0$)
9. $ia+n-1 >$ N_A and ($n > 0$)
10. CSRC_A < 0 or CSRC_A ≥ p

If the process grid is 1 × *p* and DTYPE_A = 1:

11. M_A ≠ 1
12. MB_A < 1
13. RSRC_A ≠ 0

If (the process grid is *p* × 1 and DTYPE_A = 1) or DTYPE_A = 502:

14. M_A < 0 and ($n = 0$); M_A < 1 otherwise
15. MB_A < 1
16. $n >$ (MB_A)(p)−mod(ia−1,MB_A)
17. $ia >$ M_A and ($n > 0$)
18. $ia+n-1 >$ M_A and ($n > 0$)
19. RSRC_A < 0 or RSRC_A ≥ p

If the process grid is *p* × 1 and DTYPE_A = 1:

20. N_A ≠ 1
21. NB_A < 1
22. CSRC_A ≠ 0

In all cases:

23. *laf* < (minimum value) (For the minimum value, see the *laf* argument description.)
24. *lwork* ≠ 0, *lwork* ≠ -1, and *lwork* < (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:

1. *n* differs.
2. *ia* differs.
3. DTYPE_A differs.

If DTYPE_A = 1 on all processes:

4. M_A differs.
5. N_A differs.
6. MB_A differs.
7. NB_A differs.
8. RSRC_A differs.
9. CSRC_A differs.

If DTYPE_A = 501 on all processes:

10. N_A differs.
11. NB_A differs.
12. CSRC_A differs.

If DTYPE_A = 502 on all processes:

13. M_A differs.
14. MB_A differs.
15. RSRC_A differs.

Also:

16. *lwork* = −1 on a subset of processes.

Example 1: This example shows a factorization of the general tridiagonal matrix ***A*** of order 12.

$$\begin{bmatrix}
2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0
\end{bmatrix}$$

Matrix ***A*** is stored in tridiagonal storage mode and is distributed over a 3 × 1 process grid using block-cyclic distribution.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from PDGTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDGTTRS.
2. The contents of the *du2* and *af* vectors, output from PDGTTRF, are not shown. These vectors are passed, unchanged, to the solve subroutine PDGTTRS.
3. Because *lwork* = 0, PDGTTRF dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

                N    DL   D   DU   DU2  IA   DESC_A   IPIV   AF   LAF  WORK LWORK INFO
                |    |    |   |    |    |      |       |     |    |      |    |    |
CALL PDGTTRF( 12 , DL , D , DU , DU2 , 1 , DESC_A , IPIV , AF , 48 , WORK , 0 , INFO )
```

	Desc_A
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
Not used	—
Reserved	—

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

Global vector ***dl*** with block size of 4:

```
B,D     0
     ┌     ┐
     |  .  |
     | 1.0 |
 0   | 1.0 |
     | 1.0 |
     | --- |
     | 1.0 |
     | 1.0 |
 1   | 1.0 |
     | 1.0 |
     | --- |
     | 1.0 |
     | 1.0 |
 2   | 1.0 |
     | 1.0 |
     └     ┘
```

Global vector ***d*** with block size of 4:

```
B,D     0
      ┌     ┐
      | 2.0 |
      | 3.0 |
 0    | 3.0 |
      | 3.0 |
      | --- |
      | 3.0 |
      | 3.0 |
 1    | 3.0 |
      | 3.0 |
      | --- |
      | 3.0 |
      | 3.0 |
 2    | 3.0 |
      | 3.0 |
      └     ┘
```

Global vector ***du*** with block size of 4:

```
B,D     0
      ┌     ┐
      | 2.0 |
      | 2.0 |
 0    | 2.0 |
      | 2.0 |
      | --- |
      | 2.0 |
      | 2.0 |
 1    | 2.0 |
      | 2.0 |
      | --- |
      | 2.0 |
      | 2.0 |
 2    | 2.0 |
      |  .  |
      └     ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local array `DL` with block size of 4:

```
p,q  |  0
-----|-----
     |  .
     | 1.0
 0   | 1.0
     | 1.0
-----|-----
     | 1.0
     | 1.0
 1   | 1.0
     | 1.0
-----|-----
     | 1.0
     | 1.0
 2   | 1.0
     | 1.0
```

Local array D with block size of 4:

```
p,q  |  0
-----|-----
     | 2.0
     | 3.0
 0   | 3.0
     | 3.0
-----|-----
     | 3.0
     | 3.0
 1   | 3.0
     | 3.0
-----|-----
     | 3.0
     | 3.0
 2   | 3.0
     | 3.0
```

Local array DU with block size of 4:

```
p,q  |  0
-----|-----
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|-----
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|-----
     | 2.0
     | 2.0
 2   | 2.0
     |  .
```

Output:

Global vector ***dl*** with block size of 4:

```
B,D     0
      ┌      ┐
      |  .   |
      | 0.5  |
 0    | 0.5  |
      | 0.5  |
      | ---- |
      | 1.0  |
      | 0.33 |
 1    | 0.43 |
      | 0.47 |
      | ---- |
      | 1.0  |
      | 1.0  |
 2    | 1.0  |
      | 1.0  |
      └      ┘
```

Global vector ***d*** with block size of 4:

```
B,D     0
      ┌      ┐
      | 0.5  |
      | 0.5  |
 0    | 0.5  |
      | 2.0  |
      | ---- |
      | 0.33 |
      | 0.43 |
 1    | 0.47 |
      | 2.07 |
      | ---- |
      | 2.07 |
      | 0.47 |
 2    | 0.43 |
      | 0.33 |
      └      ┘
```

Global vector ***du*** with block size of 4:

```
B,D     0
      ┌      ┐
      | 2.0  |
      | 2.0  |
 0    | 2.0  |
      | 2.0  |
      | ---- |
      | 2.0  |
      | 2.0  |
 1    | 2.0  |
      | 2.0  |
      | ---- |
      | 0.93 |
      | 0.86 |
 2    | 0.67 |
      |  .   |
      └      ┘
```

Global vector ***ipiv*** with block size of 4:

```
B,D    0
     ┌   ┐
     | 0 |
     | 0 |
 0   | 0 |
     | 0 |
     | - |
     | 0 |
     | 0 |
 1   | 0 |
     | 0 |
     | - |
     | 0 |
     | 0 |
 2   | 0 |
     | 0 |
     └   ┘
```

The following is the 3 × 1 process grid:

B,D	0
0	P_{00}
1	P_{10}
2	P_{20}

Local array DL with block size of 4:

```
p,q  |  0
-----|------
     |  .
     | 0.5
 0   | 0.5
     | 0.5
-----|------
     | 1.0
     | 0.33
 1   | 0.43
     | 0.47
-----|------
     | 1.0
     | 1.0
 2   | 1.0
     | 1.0
```

Local array D with block size of 4:

```
p,q  |  0
-----|------
     | 0.5
     | 0.5
 0   | 0.5
     | 2.0
-----|------
     | 0.33
     | 0.43
 1   | 0.47
     | 2.07
-----|------
     | 2.07
     | 0.47
 2   | 0.43
     | 0.33
```

Local array DU with block size of 4:

```
p,q  |  0
-----|------
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|------
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|------
     | 0.93
     | 0.86
 2   | 0.67
     |  .
```

Local array IPIV with block size of 4:

```
p,q  | 0
-----|---
     | 0
     | 0
 0   | 0
     | 0
-----|---
     | 0
     | 0
 1   | 0
     | 0
-----|---
     | 0
     | 0
 2   | 0
     | 0
```

The value of *info* is 0 on all processes.

Example 2: This example shows a factorization of the diagonally dominant general tridiagonal matrix ***A*** of order 12. Matrix ***A*** is stored in tridiagonal storage mode and distributed over a 3 × 1 process grid using block-cyclic distribution.

Matrix ***A*** and the input and/or output values for *dl*, *d*, *du*, *desc_a*, and *info* in this example are the same as shown for “Example 1” on page 545.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from PDDTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDDTTRS.
2. The contents of vector *af*, output from PDDTTRF, are not shown. This vector is passed, unchanged, to the solve subroutine PDDTTRS.
3. Because *lwork* = 0, PDDTTRF dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N    DL   D   DU   IA  DESC_A   AF   LAF  WORK LWORK  INFO
              |    |    |    |   |      |     |    |     |    |     |
CALL PDDTTRF( 12 , DL , D , DU , 1 , DESC_A , AF , 44 , WORK , 0  , INFO )
```

PDGTTRS and PDDTTRS—General Tridiagonal Matrix Solve

PDGTTRS solves the tridiagonal systems of linear equations, using Gaussian elimination with partial pivoting for the general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

1. $\boldsymbol{AX} = \boldsymbol{B}$

| PDDTTRS solves one of the following tridiagonal systems of linear equations, using
| Gaussian elimination for the diagonally dominant general tridiagonal matrix ***A*** stored
| in tridiagonal storage mode.

| 1. $\boldsymbol{AX} = \boldsymbol{B}$
| 2. $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$

In these subroutines:

- ***A*** represents the global square general tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.
- ***B*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
- ***X*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to PDGTTRF or PDDTTRF, respectively. The output from the factorization subroutines, PDGTTRF and PDDTTRF, should be used only as input to these solve subroutines, respectively.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See reference [51].

Table 79. Data Types

dl, ***d***, ***du***, ***du2***, ***B***, *af*, *work*	***ipiv***	**Subroutine**
Long-precision real	Integer	PDGTTRS and PDDTTRS

Syntax

Fortran	CALL PDGTTRS *(transa*, *n*, *nrhs*, *dl*, *d*, *du*, *du2*, *ia*, *desc_a*, *ipiv*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info)*
	CALL PDDTTRS *(transa*, *n*, *nrhs*, *dl*, *d*, *du*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info)*
C and C++	pdgttrs *(transa*, *n*, *nrhs*, *dl*, *d*, *du*, *du2*, *ia*, *desc_a*, *ipiv*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info);*
	pddttrs *(transa*, *n*, *nrhs*, *dl*, *d*, *du*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info);*

On Entry

transa
indicates submatrix ***A*** is used in the computation, resulting in solution 1.

Scope: **global**

Specified as: a single character, where:

| - For PDGTTRS, it must be 'N'.
| - For PDDTTRS, it must be 'N', 'T', or 'C'.

n

is the order of the general tridiagonal submatrix ***A*** and the number of rows in the general submatrix ***B***, which contains the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \le n \le$ (MB_A)(p)−mod(ia−1,MB_A).
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \le n \le$ (NB_A)(p)−mod(ia−1,NB_A).

where *p* is the number of processes in a process grid.

nrhs

is the number of right-hand sides; that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \ge 0$.

dl

is the local part of the global vector ***dl***, containing part of the factorization produced from a preceding call to PDGTTRF or PDDTTRF. This identifies the **first element** of the local array DL. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array DL contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 79 on page 553. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

d

is the local part of the global vector ***d***, containing part of the factorization produced from a preceding call to PDGTTRF or PDDTTRF. This identifies the **first element** of the local array D. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array D contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 79 on page 553. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

du

is the local part of the global vector ***du***, containing part of the factorization produced from a preceding call to PDGTTRF or PDDTTRF. This identifies the **first element** of the local array DU. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array DU contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 79 on page 553. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

du2

is the local part of the global vector ***du2***, containing part of the factorization produced from a preceding call to PDGTTRF. This identifies the **first element** of the local array DU2. These subroutines compute the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array DU2 contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 79 on page 553. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $1 \le ia \le$ N_A and $ia+n-1 \le$ N_A

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid. The following tables describe three types of array descriptors. For rules on using array descriptors, see “Notes and Coding Rules” on page 561.

Table 80 (Page 1 of 2). Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global

Table 80 (Page 2 of 2). Type-502 Array Descriptor				
desc_a	**Name**	**Description**	**Limits**	**Scope**
4	MB_A	Row block size	MB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (MB_A)(*p*)−mod(*ia*−1,MB_A)	Global
5	RSRC_A	The process row over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ *p*	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 81. Type-1 Array Descriptor (p × 1 Process Grid)				
desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for *p* × 1 where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If *n* = 0: M_A $\geq$ 0 Otherwise: M_A $\geq$ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	
5	MB_A	Row block size	MB_A $\geq$ 1 and 0 $\leq$ *n* $\leq$ (MB_A)(*p*)−mod(*ia*−1,MB_A)	Global
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	0 $\leq$ RSRC_A $<$ *p*	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 82 (Page 1 of 2). Type-501 Array Descriptor				
desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for 1 × *p* or *p* × 1 where *p* is the number of processes in a process grid.	Global

Table 82 (Page 2 of 2). Type-501 Array Descriptor

desc_a	Name	Description	Limits	Scope
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
4	NB_A	Column block size	NB_A $\geq$ 1 and $0 \leq n \leq$ (NB_A)(p)−mod(ia−1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< p$	Global
6	—	Not used by these subroutines.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 83. Type-1 Array Descriptor (1 × *p* Process Grid)

desc_a	Name	Description	Limits	Scope
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A = 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A $\geq$ 0 Otherwise: N_A $\geq$ 1	Global
5	MB_A	Row block size	MB_A $\geq$ 1	Global
6	NB_A	Column block size	NB_A $\geq$ 1 and $0 \leq n \leq$ (NB_A)(p)−mod(ia−1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A = 0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< p$	Global
9	—	Not used by these subroutines.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

ipiv

is the local part of the global vector ***ipiv***, containing the pivot indices produced on a preceding call to PDGTTRF. This identifies the **first element** of the local array IPIV. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array IPIV must contain the local pieces of the leading *ia*+*n*–1 part of the global vector.

Scope: **local**

Specified as: an array of (at least) LOCp(*ia*+*n*–1), containing fullword integers. There is no array descriptor for ***ipiv***. The details about the block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*–1) by *nrhs* part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 79 on page 553. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; 1 ≤ *ib* ≤ M_B and *ib*+*n*–1 ≤ M_B.

desc_b

is the array descriptor for global matrix ***B***, which may be type 502 or type 1, as described in the following tables. For type-502 array descriptor, the process grid is used as if it is a *p* × 1 process grid. For rules on using array descriptors, see “Notes and Coding Rules” on page 561.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for *p* × 1 or 1 × *p* where *p* is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If *n* = 0: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and 0 ≤ *n* ≤ (MB_B)*p*–mod(*ib*–1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global

desc_b	**Name**	**Description**	**Limits**	**Scope**
6	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and $0 \le n \le$ (MB_B)p–mod(ib–1,MB_B)	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B = 0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

af

is a work area used by these subroutines and contains part of the factorization produced on a preceding call to PDGTTRF or PDDTTRF. Its size is specified by *laf*.

Scope: **local**

Specified as: a one-dimensional array of (at least) length *laf*, containing numbers of the data type indicated in Table 79 on page 553.

laf

is the number of elements in array AF.

Scope: **local**

Specified as: a fullword integer, where:

If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502:
- For PDGTTRS, *laf* ≥ 12P+3(MB_A)
- For PDDTTRS, *laf* ≥ 12P+2(MB_A).

where, in the above formulas, P is the **actual** number of processes containing data.

If (the process grid is 1 × *p* and DTYPE_A = 1) or DTYPE_A = 501, you would substitute NB_A in place of MB_A in the formulas above.

Note: In ScaLAPACK 1.5, PDDTTRS requires *laf* = 12P+3(NB_A). This value is greater than or equal to the value required by Parallel ESSL.

work
has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 79 on page 553.

lwork
is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDGTTRS and PDDTTRS dynamically allocate the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = −1, PDGTTRS and PDDTTRS perform a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, *lwork* must have the following value:
  - For PDGTTRS, *lwork* ≥ 12P+5(*nrhs*).
  - For PDDTTRS, *lwork* ≥ 10P+4(*nrhs*)

  where, in the above formulas, P is the **actual** number of processes containing data.

info
See On Return.

On Return

b
b is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 79 on page 553.

work

is the work area used by this subroutine if *lwork* ≠ 0, where:

If *lwork* ≠ 0 and *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.

If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of data type indicated in Table 79 on page 553, where:

- If *lwork* = −1, the $work_1$ is set to the optimum *lwork* value needed.
- If *lwork* ≥ 1, the $work_1$ is set to the minimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info

indicates that a successful computation or work area query occurred.

Scope: **global**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The subroutine accepts lowercase letters for the *transa* argument.
3. The output from the factorization subroutines should be used only as input to the solve subroutines PDGTTRS and PDDTTRS, respectively.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   | The format of the output from PDDTTRF has changed. Therefore, the factorization and solve must be performed using Parallel ESSL Version 2 Release 1.2.

   The scalar data specified for input argument *n* must be the same for both PDGTTRF/PDDTTRF and PDGTTRS/PDDTTRS.

   The global vectors for ***dl***, ***d***, ***du***, ***du2***, ***ipiv***, and *af* input to PDGTTRS/PDDTTRS must be the same as the corresponding output arguments for PDGTTRF/PDDTTRF; and thus, the scalar data specified for *ia*, *desc_a*, and *laf* must also be the same.
4. In all cases, follow these rules:
   - *ia* = *ib*
   - CTXT_A = CTXT_B
   - If (the process grid is *p* × 1 and DTYPE_A = 1) or DTYPE_A = 502, MB_A = MB_B.
   - If (the process grid is 1 × *p* and DTYPE_A = 1) or DTYPE_A = 501, NB_A = MB_B.
   - If DTYPE_A=1, then:
     – For a *p* × 1 process grid (where *p*>1), N_A=1, NB_A≥1, and CSRC_A=0.

     – For a 1 × *p* process grid (where *p*>1), M_A=1, MB_A≥1, and RSRC_A=0.

     – For a 1 × 1 process grid:

  - If N_A=1, NB_A≥1 and CSRC_A=0.
  - If M_A=1, MB_A≥1 and RSRC_A=0.
- If DTYPE_B=1, N_B≥*nrhs*, NB_B≥1, and CSRC_B=0.
- Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	DTYPE_B	Process Grid
501	502	$p \times 1$ or $1 \times p$
502	502	$p \times 1$ or $1 \times p$
501	1	$p \times 1$
502	1	$p \times 1$
1	502	$p \times 1$ or $1 \times p$
1	1	$p \times 1$

5. To determine the values of LOCp(*n*) used in the argument descriptions, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 for descriptor type-1 or “Determining the Number of Rows or Columns in Your Local Arrays” on page 30 for descriptor type-501 and type-502.
6. ***dl***, ***d***, ***du***, ***du2***, ***ipiv***, *af* and *work* must have no common elements; otherwise, results are unpredictable.
7. The global general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in “Matrices” on page 40.

   For more information on using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.
8. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26. Also, see the section on distributing the right-hand side matrix in “Matrices” on page 40.
9. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
10. Although global matrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ia*, *ib*, MB_A (if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.
11. For global tridiagonal matrix ***A***, use of the type-1 array descriptor is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: None

Note: If the factorization performed by PDGTTRF or PDDTTRF failed because matrix ***A*** is singular or reducible, or is not diagonally dominant, respectively, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDGTTRF or PDDTTRF.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A $=$ 1 and DTYPE_A $=$ 1, a 1×1 process grid is treated as a $1 \times p$ process grid.
- If N_A $=$ 1 and DTYPE_A $=$ 1, a 1×1 process grid is treated as a $p \times 1$ process grid.

1. The process grid is not $1 \times p$ or $p \times 1$.
2. CTXT_A $\neq$ CTXT_B
3. *transa* $\neq$
   - | `'N'` for PDGTTRS
   - | `'N'`, `'T'`, or `'C'` for PDDTTRS
4. $n < 0$
5. $ia < 1$
6. DTYPE_A $=$ 1 and M_A $\neq$ 1 and N_A $\neq$ 1

If (the process grid is $1 \times p$ and DTYPE_A $=$ 1) or DTYPE_A $=$ 501:

7. N_A $<$ 0 and ($n = 0$); N_A $<$ 1 otherwise
8. NB_A $<$ 1
9. $n >$ (NB_A)(p)–mod(ia–1,NB_A)
10. $ia >$ N_A and ($n > 0$)
11. $ia+n-1 >$ N_A and ($n > 0$)
12. CSRC_A $<$ 0 or CSRC_A $\geq p$
13. NB_A $\neq$ MB_B
14. CSRC_A $\neq$ RSRC_B

If the process grid is $1 \times p$ and DTYPE_A $=$ 1:

15. M_A $\neq$ 1
16. MB_A $<$ 1
17. RSRC_A $\neq$ 0

If (the process grid is $p \times 1$ and DTYPE_A $=$ 1) or DTYPE_A $=$ 502:

18. M_A $<$ 0 and ($n = 0$); M_A $<$ 1 otherwise

19. MB_A $<$ 1
20. n $>$ (MB_A)(p)−mod(ia−1,MB_A)
21. ia $>$ MB_A and (n $>$ 0)
22. ia+n−1 $>$ M_A and (n $>$ 0)
23. RSRC_A $<$ 0 or RSRC_A $\geq$ p
24. MB_A $\neq$ MB_B
25. RSRC_A $\neq$ RSRC_B

If the process grid is $p \times 1$ and DTYPE_A = 1:
26. N_A $\neq$ 1
27. NB_A $<$ 1
28. CSRC_A $\neq$ 0

In all cases:
29. $ia \neq ib$
30. DTYPE_B = 1 and the process grid is $1 \times p$ and $p > 1$
31. $nrhs$ $<$ 0
32. ib $<$ 1
33. M_B $<$ 0 and (n = 0); M_B $<$ 1 otherwise
34. MB_B $<$ 1
35. ib $>$ M_B and (n $>$ 0)
36. ib+n−1 $>$ M_B and (n $>$ 0)
37. RSRC_B $<$ 0 or RSRC_B $\geq$ p
38. LLD_B $<$ max(1,LOCp(M_B))

If DTYPE_B = 1:
39. N_B $<$ 0 and ($nrhs$ = 0); N_B $<$ 1 otherwise
40. N_B $<$ $nrhs$
41. NB_B $<$ 1
42. CSRC_B $\neq$ 0

In all cases:
43. *laf* $<$ (minimum value) (For the minimum value, see the *laf* argument description.)
44. *lwork* $\neq$ 0, *lwork* $\neq$ −1, and *lwork* $<$ (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:
1. *n* differs.
2. *nrhs* differs.
3. *transa* differs.
4. *ia* differs.
5. *ib* differs.
6. DTYPE_A differs.

If DTYPE_A = 1 on all processes:
7. M_A differs.
8. N_A differs.
9. MB_A differs.
10. NB_A differs.
11. RSRC_A differs.
12. CSRC_A differs.

If DTYPE_A = 501 on all processes:

13. N_A differs.
14. NB_A differs.
15. CSRC_A differs.

If DTYPE_A = 502 on all processes:
16. M_A differs.
17. MB_A differs.
18. RSRC_A differs.

In all cases:
19. DTYPE_B differs.

If DTYPE_B = 1 on all processes:
20. M_B differs.
21. N_B differs.
22. MB_B differs.
23. NB_B differs.
24. RSRC_B differs.
25. CSRC_B differs.

If DTYPE_B = 502 on all processes:
26. M_B differs.
27. MB_B differs.
28. RSRC_B differs.

Also:
29. *lwork* = −1 on a subset of processes.

Example 1: This example shows how to solve the system ***AX***=***B***, where matrix ***A*** is the same general tridiagonal matrix factored in “Example 1” on page 545 for PDGTTRF.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from PDGTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDGTTRS.
2. The contents of these *du2* and *af* vectors, output from PDGTTRF, are not shown. These vectors are passed, unchanged, to the solve subroutine PDGTTRS.
3. Because *lwork* = 0, PDGTTRS dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            TRANSA N  NRHS DL   D   DU   DU2  IA   DESC_A   IPIV   B  IB
              |    |   |   |    |    |    |    |      |       |    |   |
CALL PDGTTRS( N , 12 , 3 , DL , D , DU , DU2 , 1 , DESC_A , IPIV , B , 1 ,

              DESC_B   AF   LAF  WORK  LWORK INFO
                  |     |     |     |     |    |
              DESC_B , AF , 48 , WORK , 0  , INFO )
```

	Desc_A
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
Not used	—
Reserved	—

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
LLD_B	4
Reserved	—

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

Global vector ***dl*** with block size of 4:

```
B,D     0
      ┌      ┐
      │  .   │
      │ 0.5  │
 0    │ 0.5  │
      │ 0.5  │
      │ ---- │
      │ 1.0  │
      │ 0.33 │
 1    │ 0.43 │
      │ 0.47 │
      │ ---- │
      │ 1.0  │
      │ 1.0  │
 2    │ 1.0  │
      │ 1.0  │
      └      ┘
```

Global vector ***d*** with block size of 4:

```
B,D     0
      ┌      ┐
      │ 0.5  │
      │ 0.5  │
 0    │ 0.5  │
      │ 2.0  │
      │ ---- │
      │ 0.33 │
      │ 0.43 │
 1    │ 0.47 │
      │ 2.07 │
      │ ---- │
      │ 2.07 │
      │ 0.47 │
 2    │ 0.43 │
      │ 0.33 │
      └      ┘
```

Global vector ***du*** with block size of 4:

```
B,D     0
      ┌      ┐
      │ 2.0  │
      │ 2.0  │
 0    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │ 2.0  │
      │ 2.0  │
 1    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │ 0.93 │
      │ 0.86 │
 2    │ 0.67 │
      │  .   │
      └      ┘
```

Global vector ***ipiv*** with block size of 4:

```
B,D    0
     ┌   ┐
     | 0 |
     | 0 |
 0   | 0 |
     | 0 |
     | - |
     | 0 |
     | 0 |
 1   | 0 |
     | 0 |
     | - |
     | 0 |
     | 0 |
 2   | 0 |
     | 0 |
     └   ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local array DL with block size of 4:

```
p,q  |  0
-----|------
     |  .
     | 0.5
 0   | 0.5
     | 0.5
-----|------
     | 1.0
     | 0.33
 1   | 0.43
     | 0.47
-----|------
     | 1.0
     | 1.0
 2   | 1.0
     | 1.0
```

Local array D with block size of 4:

```
p,q  |  0
-----|------
     | 0.5
     | 0.5
 0   | 0.5
     | 2.0
-----|------
     | 0.33
     | 0.43
 1   | 0.47
     | 2.07
-----|------
     | 2.07
     | 0.47
 2   | 0.43
     | 0.33
```

Local array DU with block size of 4:

```
p,q  |  0
-----|------
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|------
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|------
     | 0.93
     | 0.86
 2   | 0.67
     |  .
```

Local array IPIV with block size of 4:

```
p,q  | 0
-----|---
     | 0
     | 0
 0   | 0
     | 0
-----|---
     | 0
     | 0
 1   | 0
     | 0
-----|---
     | 0
     | 0
 2   | 0
     | 0
```

Global matrix ***B*** with block size of 4:

```
B,D         0
     ┌                ┐
     | 46.0  6.0  4.0 |
     | 65.0 13.0  6.0 |
 0   | 59.0 19.0  6.0 |
     | 53.0 25.0  6.0 |
     | -------------- |
     | 47.0 31.0  6.0 |
     | 41.0 37.0  6.0 |
 1   | 35.0 43.0  6.0 |
     | 29.0 49.0  6.0 |
     | -------------- |
     | 23.0 55.0  6.0 |
     | 17.0 61.0  6.0 |
 2   | 11.0 67.0  6.0 |
     |  5.0 47.0  4.0 |
     └                ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local matrix ***B*** with block size of 4:

```
p,q  |       0
-----|----------------
     | 46.0  6.0  4.0
     | 65.0 13.0  6.0
 0   | 59.0 19.0  6.0
     | 53.0 25.0  6.0
-----|----------------
     | 47.0 31.0  6.0
     | 41.0 37.0  6.0
 1   | 35.0 43.0  6.0
     | 29.0 49.0  6.0
-----|----------------
     | 23.0 55.0  6.0
     | 17.0 61.0  6.0
 2   | 11.0 67.0  6.0
     |  5.0 47.0  4.0
```

Output:

Global matrix ***B*** with block size of 4:

```
B,D         0
     ┌                 ┐
     | 12.0  1.0  1.0  |
     | 11.0  2.0  1.0  |
 0   | 10.0  3.0  1.0  |
     |  9.0  4.0  1.0  |
     | --------------- |
     |  8.0  5.0  1.0  |
     |  7.0  6.0  1.0  |
 1   |  6.0  7.0  1.0  |
     |  5.0  8.0  1.0  |
     | --------------- |
     |  4.0   9.0  1.0 |
     |  3.0  10.0  1.0 |
 2   |  2.0  11.0  1.0 |
     |  1.0  12.0  1.0 |
     └                 ┘
```

The following is the 3 × 1 process grid:

B,D	0
0	P_{00}
1	P_{10}
2	P_{20}

Local matrix ***B*** with block size of 4:

```
p,q  |       0
-----|-----------------
     | 12.0  1.0  1.0
     | 11.0  2.0  1.0
 0   | 10.0  3.0  1.0
     |  9.0  4.0  1.0
-----|-----------------
     |  8.0  5.0  1.0
     |  7.0  6.0  1.0
 1   |  6.0  7.0  1.0
     |  5.0  8.0  1.0
-----|-----------------
     |  4.0   9.0  1.0
     |  3.0  10.0  1.0
 2   |  2.0  11.0  1.0
     |  1.0  12.0  1.0
```

The value of *info* is 0 on all processes.

Example 2: This example shows how to solve the system ***AX***=***B***, where matrix ***A*** is the same diagonally dominant general tridiagonal matrix factored in “Example 2” on page 552 for PDDTTRF. The input and/or output values for *dl*, *d*, *du*, *desc_a*, and *info* in this example are the same as shown for “Example 1” on page 545.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from PDDTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDDTTRS.
2. The contents of vector *af*, output from PDDTTRF, are not shown. This vector is passed, unchanged, to the solve subroutine PDDTTRS.
3. Because *lwork* = 0, PDDTTRS dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

            TRANSA N NRHS  DL   D   DU  IA   DESC_A   B  IB   DESC_B
              |    |   |   |    |    |   |     |      |   |     |
CALL PDDTTRS( N , 12 , 3 , DL , D , DU , 1 , DESC_A , B , 1 , DESC_B ,

              AF   LAF  WORK  LWORK INFO
               |    |     |     |     |
              AF , 44 , WORK ,  0 , INFO )
```

PDPTSV—Positive Definite Symmetric Tridiagonal Matrix Factorization and Solve

This subroutine solves the tridiagonal systems of linear equations, ***AX*** = ***B***, where the positive definite symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode. In this description:

- ***A*** represents the global positive definite symmetric tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.
- ***B*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
- ***X*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

| If $n = 0$, no computation is performed and the subroutine returns after doing some
| parameter checking. See reference [51].

Table 84. Data Types

d, ***e***, ***B***, *work*	**Subroutine**
Long-precision real	PDPTSV

Syntax

Fortran	CALL PDPTSV *(n, nrhs, d, e, ia, desc_a, b, ib, desc_b, work, lwork, info)*
C and C++	pdptsv *(n, nrhs, d, e, ia, desc_a, b, ib, desc_b, work, lwork, info);*

On Entry

n

is the order of the positive definite symmetric tridiagonal matrix ***A*** and the number of rows in the general submatrix ***B***, which contains the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \le n \le (\text{MB\_A})(p) - \text{mod}(ia-1, \text{MB\_A})$.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \le n \le (\text{NB\_A})(p) - \text{mod}(ia-1, \text{NB\_A})$.

where *p* is the number of processes in a process grid.

nrhs

is the number of right-hand sides; that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \ge 0$.

d

is the local part of the global vector ***d***. This identifies the **first element** of the local array D. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading

LOCp(*ia*+*n*–1) part of the local array D contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***d*** contains the main diagonal of the global positive definite symmetric tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1) containing numbers of the data type indicated in Table 84 on page 573. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, D is overwritten; that is, the original input is not preserved.

e

is the local part of the global vector ***e***. This identifies the **first element** of the local array E. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array E contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***e*** contains the off-diagonal of the global positive definite symmetric tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–2.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 84 on page 573. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, E is overwritten; that is, the original input is not preserved.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $1 \le ia \le$ N_A and $ia+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid. The following tables describe three types of array descriptors. For rules on using array descriptors, see “Notes and Coding Rules” on page 580.

Table 85. Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(p)–mod(ia–1,MB_A)	Global
5	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 86. Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	Global
5	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(p)–mod(ia–1,MB_A)	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 87. Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for $1 \times p$ or $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
4	NB_A	Column block size	NB_A ≥ 1 and $0 \leq n \leq$ (NB_A)(p)–mod(ia–1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< p$	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 88. Type-1 Array Descriptor (1 × p Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A = 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1 and $0 \leq n \leq$ (NB_A)(p)–mod(ia–1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A=0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< p$	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*–1) by *nrhs* part of the local array B must contain the local pieces of the leading *ib*+*n*–1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 84 on page 573. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B and $ib+n-1 \le$ M_B.

desc_b

is the array descriptor for global matrix ***B***, which may be type 502 or type 1, as described in the following tables. For type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For rules on using array descriptors, see “Notes and Coding Rules” on page 580.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for $p \times 1$ or $1 \times p$ where *p* is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and 0 ≤ *n* ≤ (MB_B)(*p*)–mod(*ia*–1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
6	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(MB_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	Name	Description	Limits	Scope
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and 0 ≤ n ≤ (MB_B)(p)−mod(ib−1,MB_B)	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B = 0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1,LOCp(MB_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 84 on page 573.

lwork

is the number of elements in array WORK.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDPTSV dynamically allocates the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.

- If *lwork* = −1, PDPTSV performs a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502:
  - If $nrhs \le 1$, $lwork \ge$ 10P+MB_A+10.
  - If $nrhs > 1$, ($lwork \ge$(20+2min(100,*nrhs*))P+3(MB_A)+4(*nrhs*)).

  where, in the above formulas, P is the **actual** number of processes containing data.

  If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, you would substitute NB_A in place of MB_A in the formulas above.

  Note: In ScaLAPACK 1.5, PDPTSV requires *lwork* = 22P+3MB_A+2min(100,*nrhs*)P+4(*nrhs*). This value is greater than or equal to the value required by Parallel ESSL.

info
See On Return.

On Return

d
is overwritten; that is, the original input is not preserved.

e
is overwritten; that is, the original input is not preserved.

b
p is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 84 on page 573.

work
is the work area used by this subroutine if $lwork \ne 0$, where:

If $lwork \ne 0$ and $lwork \ne -1$, the size of *work* is (at least) of length *lwork*.

If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 84 on page 573, where:

- If $lwork \ge 1$, $work_1$ is set to the minimum *lwork* value needed.
- If *lwork* = −1, $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info
has the following meaning:

If *info* = 0, global submatrix ***A*** is positive definite, and the factorization completed successfully or the work area query completed successfully.

If $1 \le info \le p$, the portion of global submatrix ***A*** stored on process *info*−1 and factored locally, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If *info* $>$ *p*, the portion of global submatrix ***A*** stored on process *info*–*p*–1 representing interactions with other processes, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If *info* $>$ 0, the results of the computation are unpredictable.

Scope: **global**

Returned as: a fullword integer; *info* $\geq$ 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. | If *n* $>$ 0 and *nrhs* $=$ 0, only the factorization is completed.
3. ***d***, ***e***, ***B***, and *work* must have no common elements; otherwise, results are unpredictable.
4. In all cases, follow these rules:
   - *ia* $=$ *ib*
   - CTXT_A $=$ CTXT_B
   - If (the process grid is *p* $\times$ 1 and DTYPE_A $=$ 1) or DTYPE_A $=$ 502, MB_A $=$ MB_B.
   - If (the process grid is 1 $\times$ *p* and DTYPE_A $=$ 1) or DTYPE_A $=$ 501, NB_A $=$ MB_B.
   - If DTYPE_A=1, then:
     – For a *p* $\times$ 1 process grid (where *p*>1), N_A=1, NB_A≥1, and CSRC_A=0.

     – For a 1 $\times$ *p* process grid (where *p*>1), M_A=1, MB_A≥1, and RSRC_A=0.

     – For a 1 $\times$ 1 process grid:
       - If N_A=1, NB_A≥1 and CSRC_A=0.
       - If M_A=1, MB_A≥1 and RSRC_A=0.
   - If DTYPE_B=1, N_B≥*nrhs*, NB_B≥1, and CSRC_B=0.
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	DTYPE_B	Process Grid
501	502	*p* $\times$ 1 or 1 $\times$ *p*
502	502	*p* $\times$ 1 or 1 $\times$ *p*
501	1	*p* $\times$ 1
502	1	*p* $\times$ 1
1	502	*p* $\times$ 1 or 1 $\times$ *p*
1	1	*p* $\times$ 1

5. To determine the values of LOCp(*n*) used in the argument descriptions, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 for descriptor type-1 or “Determining the Number of Rows or Columns in Your Local Arrays” on page 30 for descriptor type-501 and type-502.
6. The global symmetric tridiagonal matrix ***A*** must be positive definite. This subroutine uses the *info* argument to provide information about ***A***, like

ScaLAPACK. However, this subroutine also issues an error message, which differs from ScaLAPACK.

7. The global positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in “Matrices” on page 40.

   For more information on using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.

8. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26. Also, see the section on distributing the right-hand side matrix in “Matrices” on page 40.
9. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
10. Although global matrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ia*, *ib*, MB_A (if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.
11. For global tridiagonal matrix ***A***, use of the type-1 array descriptor is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A = 1 and DTYPE_A = 1, a 1×1 process grid is treated as a $1 \times p$ process grid.
- If N_A = 1 and DTYPE_A = 1, a 1×1 process grid is treated as a $p \times 1$ process grid.

1. The process grid is not $1 \times p$ or $p \times 1$.
2. CTXT_A ≠ CTXT_B
3. $n < 0$
4. $ia < 1$
5. DTYPE_A = 1 and M_A ≠1 and N_A ≠ 1

If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501:

6. N_A < 0 and (n = 0); N_A < 1 otherwise
7. NB_A < 1
8. n > (NB_A)(p)−mod(ia,NB_A)
9. ia > N_A and ($n > 0$)
10. $ia+n-1$ > N_A and ($n > 0$)
11. CSRC_A < 0 or CSRC_A ≥ p
12. NB_A ≠ MB_B
13. CSRC_A ≠ RSRC_B

If the process grid is $1 \times p$ and DTYPE_A = 1:

14. M_A ≠ 1
15. MB_A < 1
16. RSRC_A ≠ 0

If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502:

17. M_A < 0 and (n = 0); M_A < 1 otherwise
18. MB_A < 1
19. n > (MB_A)(p)−mod(ia,MB_A)
20. ia > M_A and ($n > 0$)
21. $ia+n-1$ > M_A and ($n > 0$)
22. RSRC_A < 0 or RSRC_A ≥ p
23. MB_A ≠ MB_B
24. RSRC_A ≠ RSRC_B

If the process grid is $p \times 1$ and DTYPE_A = 1:

25. N_A ≠ 1
26. NB_A < 1
27. CSRC_A ≠ 0

In all cases:

28. $ia \neq ib$
29. DTYPE_B = 1 and the process grid is $1 \times p$ and $p > 1$
30. $nrhs < 0$
31. $ib < 1$
32. M_B < 0 and (n = 0); M_B < 1 otherwise
33. MB_B < 1
34. ib > M_B and ($n > 0$)
35. $ib+n-1$ > M_B and ($n > 0$)
36. RSRC_B ≤ 0 or RSRC_B ≥ p
37. LLD_B < max(1,LOCp(M_B))

If DTYPE_B = 1:

38. N_B $<$ 0 and (*nrhs* $=$ 0); N_B $<$ 1 otherwise
39. N_B $<$ *nrhs*
40. NB_B $<$ 1
41. CSRC_B $\neq$ 0

In all cases:
42. *lwork* $\neq$ 0, *lwork* $\neq$ −1, and *lwork* $<$ (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:
1. *n* differs.
2. *nrhs* differs.
3. *ia* differs.
4. *ib* differs.
5. DTYPE_A differs.

If DTYPE_A $=$ 1 on all processes:
6. M_A differs.
7. N_A differs.
8. MB_A differs.
9. NB_A differs.
10. RSRC_A differs.
11. CSRC_A differs.

If DTYPE_A $=$ 501 on all processes:
12. N_A differs.
13. NB_A differs.
14. CSRC_A differs.

If DTYPE_A $=$ 502 on all processes:
15. M_A differs.
16. MB_A differs.
17. RSRC_A differs.

In all cases:
18. DTYPE_B differs.

If DTYPE_B $=$ 1 on all processes:
19. M_B differs.
20. N_B differs.
21. MB_B differs.
22. NB_B differs.
23. RSRC_B differs.
24. CSRC_B differs.

If DTYPE_B $=$ 502 on all processes:
25. M_B differs.
26. MB_B differs.
27. RSRC_B differs.

Also:
28. *lwork* $=$ −1 on a subset of processes.

Example: This example shows a factorization of the positive definite symmetric tridiagonal matrix ***A*** of order 12:

$$\begin{bmatrix}
4.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0
\end{bmatrix}$$

Matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode and is distributed over a 1 × 3 process grid using block-cyclic distribution.

Notes:

1. On output, the vectors ***d*** and ***e*** are overwritten by this subroutine.
2. Notice **only one process grid was created**, even though, DTYPE_A = 501 and DTYPE_B = 502.
3. Because *lwork* = 0, this subroutine dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N  NRHS D   E  IA   DESC_A   B  IB   DESC_B   WORK LWORK INFO
              |   |   |   |   |      |     |   |      |      |     |     |
CALL PDPTSV( 12 , 3 , D , E , 1 , DESC_A , B , 1 , DESC_B , WORK , 0 , INFO)
```

	Desc_A
DTYPE_	501
CTXT_	*icontxt*[1]
N_	12
NB_	4
CSRC_	0
Not used	—
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
LLD_B	4
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global vector ***d*** with block size of 4:

```
B,D          0                       1                       2
     ┌                                                                ┐
 0   │ 4.0  5.0  5.0  5.0  |  5.0  5.0  5.0  5.0  |  5.0  5.0  5.0  5.0 │
     └                                                                ┘
```

Global vector ***e*** with block size of 4:

```
B,D          0                       1                      2
     ┌                                                               ┐
 0   │ 2.0  2.0  2.0  2.0  |  2.0  2.0  2.0  2.0  |  2.0  2.0  2.0   . │
     └                                                               ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local array D with block size of 4:

```
p,q  |          0           |           1           |          2
-----|----------------------|-----------------------|----------------------
 0   | 4.0  5.0  5.0  5.0   |  5.0  5.0  5.0  5.0   |  5.0  5.0  5.0  5.0
```

Local array E with block size of 4:

```
p,q  |          0           |           1           |         2
-----|----------------------|-----------------------|---------------------
 0   | 2.0  2.0  2.0  2.0   |  2.0  2.0  2.0  2.0   |  2.0  2.0  2.0   .
```

Global matrix ***B*** with a block size of 4:

```
p,q  |      0
-----|----------------
     | 70.0  8.0  6.0
     | 99.0 18.0  9.0
 0   | 90.0 27.0  9.0
     | 81.0 36.0  9.0
-----|----------------
     | 72.0 45.0  9.0
     | 63.0 54.0  9.0
 1   | 54.0 63.0  9.0
     | 45.0 72.0  9.0
-----|----------------
     | 36.0 81.0  9.0
     | 27.0 90.0  9.0
 2   | 18.0 99.0  9.0
     |  9.0 82.0  7.0
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local matrix ***B*** with a block size of 4:

```
p,q  |       0         |        1          |        2
-----|-----------------|-------------------|-----------------
     | 70.0  8.0  6.0  |  72.0 45.0  9.0   |  36.0 81.0  9.0
     | 99.0 18.0  9.0  |  63.0 54.0  9.0   |  27.0 90.0  9.0
 0   | 90.0 27.0  9.0  |  54.0 63.0  9.0   |  18.0 99.0  9.0
     | 81.0 36.0  9.0  |  45.0 72.0  9.0   |   9.0 82.0  7.0
```

Output:

Global matrix ***B*** with a block size of 4:

```
p,q  |      0
-----|----------------
     | 12.0  1.0  1.0
     | 11.0  2.0  1.0
 0   | 10.0  3.0  1.0
     |  9.0  4.0  1.0
-----|----------------
     |  8.0  5.0  1.0
     |  7.0  6.0  1.0
 1   |  6.0  7.0  1.0
     |  5.0  8.0  1.0
-----|----------------
     |  4.0  9.0  1.0
     |  3.0 10.0  1.0
 2   |  2.0 11.0  1.0
     |  1.0 12.0  1.0
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local matrix ***B*** with a block size of 4:

```
p,q  |       0         |        1         |        2
-----|-----------------|------------------|-----------------
     | 12.0  1.0  1.0  |   8.0  5.0  1.0  |   4.0  9.0  1.0
     | 11.0  2.0  1.0  |   7.0  6.0  1.0  |   3.0 10.0  1.0
  0  | 10.0  3.0  1.0  |   6.0  7.0  1.0  |   2.0 11.0  1.0
     |  9.0  4.0  1.0  |   5.0  8.0  1.0  |   1.0 12.0  1.0
```

The value of *info* is 0 on all processes.

PDPTTRF—Positive Definite Symmetric Tridiagonal Matrix Factorization

This subroutine factors the positive definite symmetric tridiagonal matrix ***A***, stored in parallel-symmetric-tridiagonal storage mode, where, in this description, ***A*** represents the global positive definite symmetric tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.

To solve a tridiagonal system of linear equations with multiple right-hand sides, follow the call to PDPTTRF with one or more calls to PDPTTRS. The output from this factorization subroutine should be used only as input to the solve subroutine PDPTTRS.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See reference [51].

Table 89. Data Types

d, ***e***, *af*, *work*	**Subroutine**
Long-precision real	PDPTTRF

Syntax

Fortran	CALL PDPTTRF *(n*, *d*, *e*, *ia*, *desc_a*, *af*, *laf*, *work*, *lwork*, *info)*
C and C++	pdpttrf *(n*, *d*, *e*, *ia*, *desc_a*, *af*, *laf*, *work*, *lwork*, *info);*

On Entry

n

is the order of the positive definite symmetric tridiagonal matrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \le n \le (\text{MB\_A})(p) - \text{mod}(ia-1,\text{MB\_A})$.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \le n \le (\text{NB\_A})(p) - \text{mod}(ia-1,\text{NB\_A})$.

where *p* is the number of processes in a process grid.

d

is the local part of the global vector ***d***. This identifies the **first element** of the local array D. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array D contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

The global vector ***d*** contains the main diagonal of the global positive definite symmetric tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*–1.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1). containing numbers of the data type indicated in Table 89. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, D is overwritten; that is, the original input is not preserved.

e

is the local part of the global vector ***e***. This identifies the **first element** of the local array E. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*−1) part of the local array E contains the local pieces of the leading *ia*+*n*−1 part of the global vector.

The global vector ***e*** contains the off-diagonal of the global positive definite symmetric tridiagonal submatrix ***A*** in elements *ia* through *ia*+*n*−2.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 89 on page 588. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

On output, E is overwritten; that is, the original input is not preserved.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $1 \le ia \le$ N_A and $ia+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid. The following tables describe three types of array descriptors.

Table 90 (Page 1 of 2). Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(*p*)−mod(*ia*−1,MB_A)	Global

Table 90 (Page 2 of 2). Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
5	RSRC_A	The process row over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 91. Type-1 Array Descriptor (p × 1 Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	
5	MB_A	Row block size	MB_A ≥ 1 and $0 \le n \le$ (MB_A)(p)−mod(ia−1,MB_A)	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 92 (Page 1 of 2). Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for $1 \times p$ or $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global

Table 92 (Page 2 of 2). Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
3	N_A	Number of columns in the global matrix	If $n = 0$: $N\_A \geq 0$ Otherwise: $N\_A \geq 1$	Global
4	NB_A	Column block size	$NB\_A \geq 1$ and $0 \leq n \leq (NB\_A)(p) - \text{mod}(ia-1, NB\_A)$	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq CSRC\_A < p$	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 93. Type-1 Array Descriptor (1 × p Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	$DTYPE\_A = 1$ for $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	$M\_A = 1$	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: $N\_A \geq 0$ Otherwise: $N\_A \geq 1$	Global
5	MB_A	Row block size	$MB\_A \geq 1$	Global
6	NB_A	Column block size	$NB\_A \geq 1$ and $0 \leq n \leq (NB\_A)(p) - \text{mod}(ia-1, NB\_A)$	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	$RSRC\_A = 0$	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	$0 \leq CSRC\_A < p$	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

af

See On Return.

laf

is the number of elements in array AF.

Scope: **local**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, *laf* ≥ 12P+3(MB_A).
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, *laf* ≥ 12P+3(NB_A).

where, in the formulas above, P is the **actual** number of processes containing data.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.
- If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 89 on page 588.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDPTTRF dynamically allocates the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = −1, PDPTTRF performs a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, *lwork* ≥ 8P, where P is the **actual** number of processes containing data.

info

See On Return.

On Return

d

d is the updated local part of the global vector ***d***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) length LOCp(*ia*+*n*−1), containing numbers of the data type indicated in Table 89 on page 588.

On output, `D` is overwritten; that is, the original input is not preserved.

e

e is the updated local part of the global vector ***e***, containing part of the factorization.

Scope: **local**

Returned as: a one-dimensional array of (at least) length LOCp(ia+n−1), containing numbers of the data type indicated in Table 89 on page 588.

On output, E is overwritten; that is, the original input is not preserved.

af

is a work area used by this subroutine and contains part of the factorization. Its size is specified by *laf*.

Scope: **local**

Returned as: a one-dimensional array of (at least) length *laf*, containing numbers of the data type indicated in Table 89 on page 588.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, the size of *work* is (at least) of length *lwork*.

If $lwork = -1$, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of data type indicated in Table 89 on page 588, where:

- If $lwork \geq 1$, the $work_1$ is set to the minimum *lwork* value needed.
- If $lwork = -1$, the $work_1$ is set to the optimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info

has the following meaning:

If $info = 0$, global submatrix ***A*** is positive definite, and the factorization completed successfully or the work area query completed successfully.

If $1 \leq info \leq p$, the portion of global submatrix ***A*** stored on process $info-1$ and factored locally, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info > p$, the portion of global submatrix ***A*** stored on process $info-p-1$ representing interactions with other processes, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info > 0$, the factorization is completed; however, if you call PDPTTRS with these factors, the results of the computation are unpredictable.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The output from these factorization subroutines should be used only as input to the solve subroutine PDPTTRS.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   The scalar data specified for input argument *n* must be the same for both PDPTTRF and PDPTTRS.

The global vectors for ***d***, ***e***, and *af* input to PDPTTRS must be the same as the corresponding output arguments for PDPTTRF; and thus, the scalar data specified for *ia*, *desc_a*, and *laf* must also be the same.

3. In all cases, follow these rules:
   - If DTYPE_A=1, then:
     - For a $p \times 1$ process grid (where p>1), N_A=1, NB_A≥1, and CSRC_A=0.
     - For a $1 \times p$ process grid (where p>1), M_A=1, MB_A≥1, and RSRC_A=0.
     - For a 1×1 process grid:
       - If N_A=1, NB_A≥1 and CSRC_A=0.
       - If M_A=1, MB_A≥1 and RSRC_A=0.
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	Process Grid
501	$p \times 1$ or $1 \times p$
502	$p \times 1$ or $1 \times p$
1	$p \times 1$ or $1 \times p$

4. To determine the values of LOCp(*n*) used in the argument descriptions, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 for descriptor type-1 or "Determining the Number of Rows or Columns in Your Local Arrays" on page 30 for descriptor type-501 and type-502.

5. ***d***, ***e***, *af*, and *work* must have no common elements; otherwise, results are unpredictable.

6. The global symmetric tridiagonal matrix ***A*** must be positive definite. This subroutine uses the *info* argument to provide information about ***A***, like ScaLAPACK. However, this subroutine also issues an error message, which differs from ScaLAPACK.

7. The global positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in "Matrices" on page 40.

   For more information on using block-cyclic data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26.

8. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.

9. Although global matrix ***A*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ia*, MB_A (if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of global submatrix ***A***.

10. For global tridiagonal matrix ***A***, use of the type-1 array descriptor is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: Matrix ***A*** is not positive definite. For details, see the description of the *info* argument.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A = 1 and DTYPE_A = 1, a 1 × 1 process grid is treated as a 1 × p process grid.
- If N_A = 1 and DTYPE_A = 1, a 1 × 1 process grid is treated as a p × 1 process grid.

1. The process grid is not 1 × p or p × 1.
2. $n < 0$
3. $ia < 1$
4. DTYPE_A = 1 and M_A ≠1 and N_A ≠ 1

If (the process grid is 1 × p and DTYPE_A = 1) or DTYPE_A = 501:

5. N_A < 0 and (n = 0); N_A < 1 otherwise
6. NB_A < 1
7. n > (NB_A)(p)−mod(ia−1,NB_A)
8. ia > N_A and (n > 0)
9. ia+n−1 > N_A and (n > 0)
10. CSRC_A < 0 or CSRC_A ≥ p

If the process grid is 1 × p and DTYPE_A = 1:

11. M_A ≠ 1
12. MB_A < 1
13. RSRC_A ≠ 0

If (the process grid is p × 1 and DTYPE_A = 1) or DTYPE_A = 502:

14. M_A < 0 and (n = 0); M_A < 1 otherwise
15. MB_A < 1
16. n > (MB_A)(p)−mod(ia−1,MB_A)
17. ia > M_A and (n > 0)
18. ia+n−1 > M_A and (n > 0)

19. RSRC_A $<$ 0 or RSRC_A $\geq$ *p*

If the process grid is *p* $\times$ 1 and DTYPE_A $=$ 1:

20. N_A $\neq$ 1
21. NB_A $<$ 1
22. CSRC_A $\neq$ 0

In all cases:

23. *laf* $<$ (minimum value) (For the minimum value, see the *laf* argument description.)
24. *lwork* $\neq$ 0, *lwork* $\neq$ −1, and *lwork* $<$ (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:

1. *n* differs.
2. *ia* differs.
3. DTYPE_A differs.

If DTYPE_A $=$ 1 on all processes:

4. M_A differs.
5. N_A differs.
6. MB_A differs.
7. NB_A differs.
8. RSRC_A differs.
9. CSRC_A differs.

If DTYPE_A $=$ 501 on all processes:

10. N_A differs.
11. NB_A differs.
12. CSRC_A differs.

If DTYPE_A $=$ 502 on all processes:

13. M_A differs.
14. MB_A differs.
15. RSRC_A differs.

Also:

16. *lwork* $=$ −1 on a subset of processes.

Example: This example shows a factorization of the positive definite symmetric tridiagonal matrix ***A*** of order 12.

$$\begin{bmatrix}
4.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0
\end{bmatrix}$$

Matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode and is distributed over a 3 × 1 process grid using block-cyclic distribution.

Notes:

1. The vectors ***d*** and ***e***, output from PDPTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDPTTRS.
2. The contents of the *af* vector, output from PDPTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine PDPTTRS.
3. Because *lwork* = 0, this subroutine dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N    D   E   IA   DESC_A   AF   LAF  WORK  LWORK INFO
              |    |   |   |      |      |     |     |     |    |
CALL PDPTTRF( 12 , D , E  , 1 , DESC_A , AF , 48 , WORK ,  0 , INFO )
```

	Desc_A
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
Not used	—
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global vector ***d*** with block size of 4:

```
B,D     0
      ┌     ┐
      | 4.0 |
      | 5.0 |
 0    | 5.0 |
      | 5.0 |
      | --- |
      | 5.0 |
      | 5.0 |
 1    | 5.0 |
      | 5.0 |
      | --- |
      | 5.0 |
      | 5.0 |
 2    | 5.0 |
      | 5.0 |
      └     ┘
```

Global vector ***e*** with block size of 4:

```
B,D     0
      ┌     ┐
      | 2.0 |
      | 2.0 |
 0    | 2.0 |
      | 2.0 |
      | --- |
      | 2.0 |
      | 2.0 |
 1    | 2.0 |
      | 2.0 |
      | --- |
      | 2.0 |
      | 2.0 |
 2    | 2.0 |
      |  .  |
      └     ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local array `D` with block size of 4:

```
p,q  |  0
-----|-----
     | 4.0
     | 5.0
 0   | 5.0
     | 5.0
-----|-----
     | 5.0
     | 5.0
 1   | 5.0
     | 5.0
-----|-----
     | 5.0
     | 5.0
 2   | 5.0
     | 5.0
```

Local array E with block size of 4:

```
p,q  |  0
-----|-----
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|-----
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|-----
     | 2.0
     | 2.0
 2   | 2.0
     |  .
```

Output:

Global vector ***d*** with block size of 4:

```
B,D     0
     ┌      ┐
     |  .25 |
     |  .25 |
 0   |  .25 |
     | 4.0  |
     | ---- |
     |  .2  |
     |  .24 |
 1   |  .25 |
     | 4.01 |
     | ---- |
     | 4.01 |
     |  .25 |
 2   |  .24 |
     |  .2  |
     └      ┘
```

Global vector ***e*** with block size of 4:

```
B,D     0
      ┌      ┐
      │ 2.0  │
      │ 2.0  │
 0    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │ 2.0  │
      │ 2.0  │
 1    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │  .49 │
      │  .48 │
 2    │  .4  │
      │  .   │
      └      ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local array D with block size of 4:

```
p,q  │  0
-----│------
     │  .25
     │  .25
 0   │  .25
     │ 4.0
-----│------
     │  .2
     │  .24
 1   │  .25
     │ 4.01
-----│------
     │ 4.01
     │  .25
 2   │  .24
     │  .2
```

Local array E with block size of 4:

```
p,q  |  0
-----|------
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|------
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|------
     |  .49
     |  .48
 2   |  .4
     |  .
```

The value of *info* is 0 on all processes.

PDPTTRS—Positive Definite Symmetric Tridiagonal Matrix Solve

This subroutine solves the following tridiagonal systems of linear equations for multiple right-hand sides, using the positive definite symmetric tridiagonal matrix ***A***, where ***A*** is stored in parallel-symmetric-tridiagonal storage mode:

$\boldsymbol{AX} = \boldsymbol{B}$

In this subroutine:

- ***A*** represents the global positive definite symmetric tridiagonal submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ia:ia+n-1}$.
- ***B*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the right-hand sides in its columns.
- ***X*** represents the global general submatrix $\boldsymbol{B}_{ib:ib+n-1,\ 1:nrhs}$ containing the output solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to PDPTTRF. The output from PDPTTRF should be used only as input to this solve subroutine.

If $n = 0$ or $nrhs = 0$, no computation is performed and the subroutine returns after doing some parameter checking. See reference [51].

Table 94. Data Types

d, ***e***, ***B***, *af*, *work*	**Subroutine**
Long-precision real	PDPTTRS

Syntax

Fortran	CALL PDPTTRS *(n*, *nrhs*, *d*, *e*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info)*
C and C++	pdpttrs *(n*, *nrhs*, *d*, *e*, *ia*, *desc_a*, *b*, *ib*, *desc_b*, *af*, *laf*, *work*, *lwork*, *info);*

On Entry

n

is the order of the positive definite symmetric tridiagonal submatrix ***A*** and the number of rows in the general submatrix ***B***, which contains the multiple right-hand sides.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $0 \le n \le (\text{MB\_A})(p) - \text{mod}(ia-1, \text{MB\_A})$.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $0 \le n \le (\text{NB\_A})(p) - \text{mod}(ia-1, \text{NB\_A})$.

where *p* is the number of processes in a process grid.

nrhs

is the number of right-hand sides; that is, the number of columns in submatrix ***B*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $nrhs \geq 0$.

d

is the local part of the global vector ***d***, containing part of the factorization produced from a preceding call to PDPTTRF. This identifies the **first element** of the local array `D`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array `D` contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 94 on page 602. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

e

is the local part of the global vector ***e***, containing part of the factorization produced from a preceding call to PDPTTRF. This identifies the **first element** of the local array `E`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, and *p*; therefore, the leading LOCp(*ia*+*n*–1) part of the local array `E` contains the local pieces of the leading *ia*+*n*–1 part of the global vector.

Scope: **local**

Specified as: a one-dimensional array of (at least) length LOCp(*ia*+*n*–1), containing numbers of the data type indicated in Table 94 on page 602. Details about block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row or column index of the global matrix ***A***, identifying the first row or column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, $1 \leq ia \leq$ N_A and $ia+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***. Because vectors are one-dimensional data structures, you may use a type-502, type-501, or type-1 array descriptor regardless of whether the process grid is $p \times 1$ or $1 \times p$. For a type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For a type-501 array descriptor, the process grid is used as if it is a $1 \times p$ process grid. For a type-1 array descriptor, the process grid is used as if it is either a $p \times 1$ process grid or a $1 \times p$ process grid. The following tables describe three types of array descriptors. For rules on using array descriptors, see “Notes and Coding Rules” on page 609.

Table 95. Type-502 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=502 for $p \times 1$ or $1 \times p$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	MB_A	Row block size	MB_A ≥ 1 and $0 \leq n \leq$ (MB_A)(p)−mod(ia−1,MB_A)	Global
5	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≥ RSRC_A < p	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 96. Type-1 Array Descriptor ($p \times 1$ Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	N_A = 1	
5	MB_A	Row block size	MB_A ≥ 1 and $0 \leq n \leq$ (MB_A)(p)−mod(ia−1,MB_A)	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_A < p	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	CSRC_A = 0	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

Table 97. Type-501 Array Descriptor

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor Type	DTYPE_A=501 for 1 × *p* or *p* × 1 where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	N_A	Number of columns in the global matrix	If *n* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
4	NB_A	Column block size	NB_A ≥ 1 and 0 ≤ *n* ≤ (NB_A)(*p*)−mod(*ia*−1,NB_A)	Global
5	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < *p*	Global
6	—	Not used by this subroutine.	—	—
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

Table 98. Type-1 Array Descriptor (1 × p Process Grid)

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A = 1 for 1 × *p* where *p* is the number of processes in a process grid.	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	M_A = 1	Global
4	N_A	Number of columns in the global matrix	If *n* = 0: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1 and 0 ≤ *n* ≤ (NB_A)(*p*)−mod(*ia*−1,NB_A)	Global
7	RSRC_A	The process row over which the first row of the global matrix is distributed	RSRC_A = 0	Global
8	CSRC_A	The process column over which the first column of the global matrix is distributed	0 ≤ CSRC_A < *p*	Global
9	—	Not used by this subroutine.	—	—

Specified as: an array of (at least) length 9, containing fullword integers.

b

is the local part of the global general matrix ***B***, containing the multiple right-hand sides of the system. This identifies the **first element** of the local array B. This subroutine computes the location of the first element of the local subarray used, based on *ib*, *desc_b*, and *p*; therefore, the leading LOCp(*ib*+*n*−1) by *nrhs* part of the local array B must contain the local pieces of the leading *ib*+*n*−1 by *nrhs* part of the global matrix.

Scope: **local**

Specified as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 94 on page 602. Details about the block-cyclic data distribution of global matrix ***B*** are stored in *desc_b*.

ib

is the row index of the global matrix ***B***, identifying the first row of the submatrix ***B***.

Scope: **global**

Specified as: a fullword integer; $1 \le ib \le$ M_B and $ib+n-1 \le$ M_B.

desc_b

is the array descriptor for global matrix ***B***, which may be type 502 or type 1, as described in the following tables. For type-502 array descriptor, the process grid is used as if it is a $p \times 1$ process grid. For rules on using array descriptors, see “Notes and Coding Rules” on page 609.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 502 for $p \times 1$ or $1 \times p$ where *p* is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	MB_B	Row block size	MB_B ≥ 1 and 0 ≤ *n* ≤ (MB_B)(*p*)−mod(*ib*−1,MB_B)	Global
5	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < *p*	Global
6	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**
7	—	Reserved	—	—

Specified as: an array of (at least) length 7, containing fullword integers.

desc_b	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_B	Descriptor type	DTYPE_B = 1 for $p \times 1$ where p is the number of processes in a process grid.	Global
2	CTXT_B	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_B	Number of rows in the global matrix	If $n = 0$: M_B ≥ 0 Otherwise: M_B ≥ 1	Global
4	N_B	Number of columns in the global matrix	N_B ≥ *nrhs*	Global
5	MB_B	Row block size	MB_B ≥ 1 and $0 \le n \le$ (MB_B)(p)−mod(ib−1,MB_B)	Global
6	NB_B	Column block size	NB_B ≥ 1	Global
7	RSRC_B	The process row over which the first row of the global matrix is distributed	0 ≤ RSRC_B < p	Global
8	CSRC_B	The process column over which the first column of the global matrix is distributed	CSRC_B = 0	Global
9	LLD_B	Leading dimension	LLD_B ≥ max(1, LOCp(M_B))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

af

is a work area used by this subroutine and contains part of the factorization produced on a preceding call to PDPTTRF. Its size is specified by *laf*.

Scope: **local**

Specified as: a one-dimensional array of (at least) length *laf*, containing numbers of the data type indicated in Table 94 on page 602.

laf

is the number of elements in array AF.

Scope: **local**

Specified as: a fullword integer, where:

- If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, *laf* ≥ 12P+3(MB_A).
- If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, *laf* ≥ 12P+3(NB_A).

where, in the above formulas, P is the **actual** number of processes containing data.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.

- If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 94 on page 602.

lwork
is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, this subroutine dynamically allocates the work area used by the subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = −1, PDPTTRS performs a work area query and return the optimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, *lwork* ≥ (10+2min(100,*nrhs*))P+4(*nrhs*), where P is the **actual** number of processes containing data.

info
See On Return.

On Return

b
b is the updated local part of the global matrix ***B***, containing the solution vectors.

Scope: **local**

Returned as: an LLD_B by (at least) *nrhs* array, containing numbers of the data type indicated in Table 94 on page 602.

work
is the work area used by this subroutine if *lwork* ≠ 0, where:

If *lwork* ≠ 0 and *lwork* ≠ −1, the size of *work* is (at least) of length *lwork*.

If *lwork* = −1, the size of *work* is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of data type indicated in Table 94 on page 602, where:

- If *lwork* = -1, the $work_1$ is set to the optimum *lwork* value needed.
- If *lwork* ≥ 1, the $work_1$ is set to the minimum *lwork* value needed.

Except for $work_1$, the contents of *work* are overwritten on return.

info
indicates that a successful computation or work area query occurred.

Scope: **global**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. The output from the PDPTTRF subroutine should be used only as input to the solve subroutine PDPTTRS.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   The scalar data specified for input argument *n* must be the same for both PDPTTRF and PDPTTRS.

   The global vectors for ***d***, ***e***, and *af* input to PDPTTRS must be the same as the corresponding output arguments for PDPTTRF; and thus, the scalar data specified for *ia*, *desc_a*, and *laf* must also be the same.
3. In all cases, follow these rules:
   - *ia* = *ib*
   - CTXT_A = CTXT_B
   - If (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502, MB_A = MB_B.
   - If (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501, NB_A = MB_B.
   - If DTYPE_A=1, then:
     – For a $p \times 1$ process grid (where p>1), N_A=1, NB_A≥1, and CSRC_A=0.

     – For a $1 \times p$ process grid (where p>1), M_A=1, MB_A≥1, and RSRC_A=0.

     – For a 1×1 process grid:

       - If N_A=1, NB_A≥1 and CSRC_A=0.
       - If M_A=1, MB_A≥1 and RSRC_A=0.
   - If DTYPE_B=1, N_B≥*nrhs*, NB_B≥1, and CSRC_B=0.
   - Following are the consistent combinations of array descriptor types and process grids, where *p* is the number of processes in the process grid:

DTYPE_A	DTYPE_B	Process Grid
501	502	$p \times 1$ or $1 \times p$
502	502	$p \times 1$ or $1 \times p$
501	1	$p \times 1$
502	1	$p \times 1$
1	502	$p \times 1$ or $1 \times p$
1	1	$p \times 1$

4. To determine the values of LOCp(*n*) used in the argument descriptions, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 for descriptor type-1 or “Determining the Number of Rows or Columns in Your Local Arrays” on page 30 for descriptor type-501 and type-502.
5. ***d***, ***e***, *af* and *work* must have no common elements; otherwise, results are unpredictable.

6. The global positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode and distributed over a one-dimensional process grid, using block-cyclic data distribution. See the section on block-cyclically distributing a tridiagonal matrix in “Matrices” on page 40.

   For more information on using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.

7. Matrix ***B*** must be distributed over a one-dimensional process grid, using block-cyclic data distribution. For more information using block-cyclic data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26. Also, see the section on distributing the right-hand side matrix in “Matrices” on page 40.
8. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.
9. Although global matrices ***A*** and ***B*** may be block-cyclically distributed on a $1 \times p$ or $p \times 1$ process grid, the values of *n*, *ia*, *ib*, MB_A (if (the process grid is $p \times 1$ and DTYPE_A = 1) or DTYPE_A = 502), NB_A (if (the process grid is $1 \times p$ and DTYPE_A = 1) or DTYPE_A = 501), must be chosen so that each process has at most one full or partial block of each of the global submatrices ***A*** and ***B***.
10. For global tridiagonal matrix ***A***, use of the type-1 array descriptor is an extension to ScaLAPACK 1.5. If your application needs to run with both Parallel ESSL and ScaLAPACK 1.5, it is suggested that you use either a type-501 or a type-502 array descriptor for the matrix ***A***.

Error Conditions

Computational Errors: None

Note: If the factorization performed by PDPTTRF failed because of a nonpositive definite matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDPTTRF.

Resource Errors: Unable to allocate workspace

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_B is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

Note: In the following error conditions:

- If M_A $=$ 1 and DTYPE_A $=$ 1, a 1×1 process grid is treated as a $1 \times p$ process grid.
- If N_A $=$ 1 and DTYPE_A $=$ 1, a 1×1 process grid is treated as a $p \times 1$ process grid.

1. The process grid is not $1 \times p$ or $p \times 1$.
2. CTXT_A $\neq$ CTXT_B
3. $n < 0$
4. $ia < 1$
5. DTYPE_A $=$ 1 and M_A $\neq$1 and N_A $\neq$ 1

If (the process grid is $1 \times p$ and DTYPE_A $=$ 1) or DTYPE_A $=$ 501:

6. N_A $<$ 0 and ($n = 0$); N_A $<$ 1 otherwise
7. NB_A $<$ 1
8. $n >$ (NB_A)(p)–mod(ia–1,NB_A)
9. $ia >$ N_A and ($n > 0$)
10. ia+n–1 $>$ N_A and ($n > 0$)
11. CSRC_A $<$ 0 or CSRC_A $\geq p$
12. NB_A $\neq$ MB_B
13. CSRC_A $\neq$ RSRC_B

If the process grid is $1 \times p$ and DTYPE_A $=$ 1:

14. M_A $\neq$ 1
15. MB_A $<$ 1
16. RSRC_A $\neq$ 0

If (the process grid is $p \times 1$ and DTYPE_A $=$ 1) or DTYPE_A $=$ 502:

17. M_A $<$ 0 and ($n = 0$); M_A $<$ 1 otherwise
18. MB_A $<$ 1
19. $n >$ (MB_A)(p)–mod(ia–1,MB_A)
20. $ia >$ M_A and ($n > 0$)
21. ia+n–1 $>$ M_A and ($n > 0$)
22. RSRC_A $<$ 0 or RSRC_A $\geq p$
23. MB_A $\neq$ MB_B
24. RSRC_A $\neq$ RSRC_B

If the process grid is $p \times 1$ and DTYPE_A $=$ 1:

25. N_A $\neq$ 1
26. NB_A $<$ 1
27. CSRC_A $\neq$ 0

In all cases:

28. $ia \neq ib$
29. DTYPE_B $=$ 1 and the process grid is $1 \times p$ and $p > 1$
30. $nrhs < 0$
31. $ib < 1$
32. M_B $<$ 0 and ($n = 0$); M_B $<$ 1 otherwise
33. MB_B $<$ 1
34. $ib >$ M_B and ($n > 0$)
35. ib+n–1 $>$ M_B and ($n > 0$)
36. RSRC_B $<$ 0 or RSRC_B $\geq p$
37. LLD_B $<$ max(1,LOCp(M_B))

If DTYPE_B $=$ 1:

38. N_B $<$ 0 and (*nrhs* = 0); N_B $<$ 1 otherwise
39. N_B $<$ *nrhs*
40. NB_B $<$ 1
41. CSRC_B $\neq$ 0

In all cases:

42. *laf* $<$ (minimum value) (For the minimum value, see the *laf* argument description.)
43. *lwork* $\neq$ 0, *lwork* $\neq$ −1, and *lwork* $<$ (minimum value) (For the minimum value, see the *lwork* argument description.)

Stage 5

Each of the following global input arguments are checked to determine whether its value is the same on all processes in the process grid:

1. *n* differs.
2. *nrhs* differs.
3. *ia* differs.
4. *ib* differs.
5. DTYPE_A differs.

If DTYPE_A = 1 on all processes:

6. M_A differs.
7. N_A differs.
8. MB_A differs.
9. NB_A differs.
10. RSRC_A differs.
11. CSRC_A differs.

If DTYPE_A = 501 on all processes:

12. N_A differs.
13. NB_A differs.
14. CSRC_A differs.

If DTYPE_A = 502 on all processes:

15. M_A differs.
16. MB_A differs.
17. RSRC_A differs.

In all cases:

18. DTYPE_B differs.

If DTYPE_B = 1 on all processes:

19. M_B differs.
20. N_B differs.
21. MB_B differs.
22. NB_B differs.
23. RSRC_B differs.
24. CSRC_B differs.

If DTYPE_B = 502 on all processes:

25. M_B differs.
26. MB_B differs.
27. RSRC_B differs.

Also:

28. *lwork* = −1 on a subset of processes.

Example: This example shows how to solve the system ***AX***=***B***, where matrix ***A*** is the same positive definite symmetric tridiagonal matrix factored in “Example” on page 596 for PDPTTRF.

Notes:

1. The vectors ***d*** and ***e***, output from PDPTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PDPTTRS.
2. The contents of the *af* vector, output from PDPTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine PDPTTRS.
3. Because *lwork* = 0, this subroutine dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 3
NPCOL = 1
CALL BLACS_GET (0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N  NRHS  D  E  IA   DESC_A   B  IB   DESC_B  AF   LAF  WORK LWORK INFO
              |    |   |  |  |       |     |  |       |    |     |    |     |     |
CALL PDPTTRS( 12 , 3 , D, E , 1 , DESC_A , B , 1 , DESC_B, AF , 48 , WORK , 0 , INFO)
```

	Desc_A
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
Not used	—
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

	Desc_B
DTYPE_	502
CTXT_	*icontxt*[1]
M_	12
MB_	4
RSRC_	0
LLD_B	4
Reserved	—
[1] *icontxt* is the output of the BLACS_GRIDINIT call.	

Global vector ***d*** with block size of 4:

```
B,D     0
      ┌      ┐
      │  .25 │
      │  .25 │
 0    │  .25 │
      │ 4.0  │
      │ ---- │
      │  .2  │
      │  .24 │
 1    │  .25 │
      │ 4.01 │
      │ ---- │
      │ 4.01 │
      │  .25 │
 2    │  .24 │
      │  .2  │
      └      ┘
```

Global vector ***e*** with block size of 4:

```
B,D     0
      ┌      ┐
      │ 2.0  │
      │ 2.0  │
 0    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │ 2.0  │
      │ 2.0  │
 1    │ 2.0  │
      │ 2.0  │
      │ ---- │
      │  .49 │
      │  .48 │
 2    │  .4  │
      │  .   │
      └      ┘
```

The following is the 3 × 1 process grid:

B,D	**0**
0	P_{00}
1	P_{10}
2	P_{20}

Local array `D` with block size of 4:

```
p,q  |  0
-----|------
     |  .25
     |  .25
 0   |  .25
     | 4.0
-----|------
     |  .2
     |  .24
 1   |  .25
     | 4.01
-----|------
     | 4.01
     |  .25
 2   |  .24
     |  .2
```

Local array E with block size of 4:

```
p,q  |  0
-----|------
     | 2.0
     | 2.0
 0   | 2.0
     | 2.0
-----|------
     | 2.0
     | 2.0
 1   | 2.0
     | 2.0
-----|------
     |  .49
     |  .48
 2   |  .4
     |  .
```

Global matrix ***B*** with a block size of 4:

```
p,q  |       0
-----|----------------
     | 70.0  8.0  6.0
     | 99.0 18.0  9.0
 0   | 90.0 27.0  9.0
     | 81.0 36.0  9.0
-----|----------------
     | 72.0 45.0  9.0
     | 63.0 54.0  9.0
 1   | 54.0 63.0  9.0
     | 45.0 72.0  9.0
-----|----------------
     | 36.0 81.0  9.0
     | 27.0 90.0  9.0
 2   | 18.0 99.0  9.0
     |  9.0 82.0  7.0
```

The following is the 3 × 1 process grid:

B,D	0
0	P_{00}
1	P_{10}
2	P_{20}

Local matrix ***B*** with block size of 4:

```
p,q  |       0
-----|----------------
     | 70.0  8.0  6.0
     | 99.0 18.0  9.0
 0   | 90.0 27.0  9.0
     | 81.0 36.0  9.0
-----|----------------
     | 72.0 45.0  9.0
     | 63.0 54.0  9.0
 1   | 54.0 63.0  9.0
     | 45.0 72.0  9.0
-----|----------------
     | 36.0 81.0  9.0
     | 27.0 90.0  9.0
 2   | 18.0 99.0  9.0
     |  9.0 82.0  7.0
```

Output:

Global matrix ***B*** with block size of 4:

```
B,D          0
     ┌                 ┐
     | 12.0  1.0  1.0  |
     | 11.0  2.0  1.0  |
 0   | 10.0  3.0  1.0  |
     |  9.0  4.0  1.0  |
     | --------------- |
     |  8.0  5.0  1.0  |
     |  7.0  6.0  1.0  |
 1   |  6.0  7.0  1.0  |
     |  5.0  8.0  1.0  |
     | --------------- |
     |  4.0   9.0  1.0 |
     |  3.0  10.0  1.0 |
 2   |  2.0  11.0  1.0 |
     |  1.0  12.0  1.0 |
     └                 ┘
```

The following is the 3 × 1 process grid:

B,D	0
0	P_{00}
1	P_{10}
2	P_{20}

Local matrix ***B*** with block size of 4:

```
p,q  |        0
-----|-----------------
     | 12.0  1.0  1.0
     | 11.0  2.0  1.0
 0   | 10.0  3.0  1.0
     |  9.0  4.0  1.0
-----|-----------------
     |  8.0  5.0  1.0
     |  7.0  6.0  1.0
 1   |  6.0  7.0  1.0
     |  5.0  8.0  1.0
-----|-----------------
     |  4.0   9.0  1.0
     |  3.0  10.0  1.0
 2   |  2.0  11.0  1.0
     |  1.0  12.0  1.0
```

The value of *info* is 0 on all processes.

Fortran 90 Sparse Linear Algebraic Equation Subroutines and Their Utility Subroutines

This section contains the sparse linear algebraic equation subroutine descriptions and their sparse utility subroutines.

PADALL—Allocates Space for an Array Descriptor for a General Sparse Matrix

This sparse utility subroutine allocates space for an array descriptor, which is needed to establish a mapping between the global general sparse matrix ***A*** and its corresponding distributed memory location. This subroutine also initializes the components of the array descriptor *desc_a*.

Syntax

Fortran	CALL PADALL (*n*, *parts*, *desc_a*, *icontxt*)

On Entry

n

is the order of the global general sparse matrix ***A*** and the size of the index space.

Scope: **global**

Type: **required**

Specified as: a fullword integer, where: $n > 0$.

parts

is a user-supplied subroutine that specifies a mapping between a global index for an element in the global general sparse matrix ***A*** and its corresponding storage location on one or more processes.

Sample *parts* subroutines for common types of data distributions are shown in “Sample PARTS Subroutine” on page 1084.

For details about how you must define the PARTS subroutine, see “Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)” on page 65.

Scope: **global**

Type: **required**

Specified as: *parts* must be declared as an external subroutine in your application program. It can be whatever name you choose.

desc_a

See On Return.

icontxt

is the BLACS context parameter.

Scope: **global**

Type: **required**

Specified as: a fullword integer that was returned in a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

On Return

desc_a

is the local space allocated for the array descriptor for the global general sparse matrix ***A***. This subroutine also initializes the components of the array descriptor *desc_a*. The components of *desc_a* are updated with subsequent calls to PSPINS and finalized with a call to PSPASB.

Table 25 on page 62 describes some of the elements of MATRIX_DATA, which is one component of the array descriptor, that you may want to reference. However, your application programs should not modify the components of the array descriptor directly. These components should only be updated with calls to PSPINS and PSPASB.

Type: **required**

Returned as: the derived data type DESC_TYPE.

Notes and Coding Rules

1. Before you call this subroutine, you must create a $np \times 1$ process grid, where *np* is the number of processes.
2. PADALL allocates *desc_a* as necessary. Prior to further calls to PADALL with the same *desc_a*, you must call PADFREE; otherwise, there will be a memory leak.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space.
2. Unable to allocate component(s) of *desc_a*

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. $n \le 0$

Stage 4

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   n differs.

Stage 5

1. *pv* or *nv*, output from the user-supplied *parts* subroutine, was not valid. For valid values, see the appropriate argument description in “Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)” on page 65.

PSPALL—Allocates Space for a General Sparse Matrix

This sparse utility subroutine allocates space for the local data of a general sparse matrix ***A***. It also initializes some values, which are only for internal use, of the general sparse matrix ***A***.

Syntax

Fortran	CALL PSPALL (*a*, *desc_a*) CALL PSPALL (*a*, *desc_a*, *nnz*)

On Entry

a
: See On Return.

desc_a
: is the array descriptor for a global general sparse matrix ***A*** that is produced on a preceding call to PADALL.

  Type: **required**

  Specified as: the derived data type DESC_TYPE.

nnz
: is an estimate of the number of non-zero elements in the local part of the global general sparse matrix ***A***. If the actual number of non-zero elements is greater than *nnz*, Parallel ESSL attempts to allocate additional space.

  If *nnz* is not present, Parallel ESSL estimates how many non-zero elements, *nnz*, are present based on the order of the global general sparse matrix ***A***.

  Scope: **local**

  Type: **optional**

  Specified as: a fullword integer, where $nnz > 0$.

On Return

a
: is the local space, which contains some internal values that are initialized by Parallel ESSL, allocated for the global general sparse matrix ***A***.

  Scope: **local**

  Type: **required**

  Returned as: the derived data type D_SPMAT.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADALL.
2. For details about some of the elements stored in DESC_A%MATRIX_DATA, see "Derived Data Type DESC_TYPE" on page 61.
3. PSPALL allocates matrix ***A*** as necessary. Prior to further calls to PSPALL with the same matrix ***A***, you must call PSPFREE; otherwise, there will be a memory leak.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate component(s) of ***A***.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. *desc_a* component(s) are not valid.
3. $nnz \le 0$

PGEALL—Allocates Space for a Dense Vector

This sparse utility subroutine allocates space for a dense vector.

Syntax

Fortran	CALL PGEALL (*x*, *desc_a*)

On Entry

x
See On Return.

desc_a
is the array descriptor that is produced on a preceding call to PADALL.

Type: **required**

Specified as: the derived data type DESC_TYPE.

On Return

x
is a pointer to the local space of the dense vector.

Scope: **local**

Type: **required**

Returned as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADALL.
2. You do not need a separate array descriptor for a dense vector because it must conform to the size of matrix ***A***. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.
3. This subroutine must be called for:
   - Vector ***b*** containing the right-hand side.
   - Vector ***x*** containing the initial guess to the solution.
4. PGEALL allocates the dense vector as necessary. Prior to further calls to PGEALL with the same dense vector, you must call PGEFREE; otherwise, there will be a memory leak.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate the dense vector.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not *np* × 1.
2. *desc_a* component(s) are not valid.

PSPINS—Inserts Local Data into a General Sparse Matrix

This sparse utility subroutine is used by each process to insert all blocks of data it owns into its local part of the general sparse matrix ***A***.

Syntax

Fortran	CALL PSPINS (*a*, *ia*, *ja*, *blck*, *desc_a*)

On Entry

a

is the local part of the global general sparse matrix ***A*** that is produced on a preceding call to PSPALL or previous call(s) to this subroutine.

Scope: **local**

Type: **required**

Specified as: the derived data type D_SPMAT.

ia

is the first global row index of the general sparse matrix ***A*** that receives data from the submatrix ***BLCK***.

Scope: **local**

Type: **required**

Specified as: a fullword integer; $1 \leq ia \leq$ DESC_A%MATRIX_DATA(M).

ja

is the first global column index of the general sparse matrix ***A*** that receives data from the submatrix ***BLCK***.

Scope: **local**

Type: **required**

Specified as: a fullword integer, where: $ja = 1$.

blck

is the local part of the submatrix ***BLCK*** to be inserted into the global general sparse matrix ***A***. Each call to this subroutine inserts one contiguous block of rows into the local part of the sparse matrix corresponding to the global submatrix $\boldsymbol{A}_{ia:ia+\text{BLCK\%M}-1, ja:ja+\text{BLCK\%N}-1}$. This subroutine only can insert blocks of data it owns into its local part of the general sparse matrix ***A***. ***BLCK*** contains the following components:

- BLCK%M is the number of local rows in the submatrix ***BLCK***. Scope: **local**.

  Specified as: a fullword integer;
  $1 \leq$ BLCK%M $\leq$ DESC_A%MATRIX_DATA(N_ROW).

- BLCK%N is an upper bound on the number of local columns in the submatrix ***BLCK***. Scope: **local**.

  Specified as: a fullword integer; $1 \leq$ BLCK%N $\leq n$, where *n* is the order of the global general sparse matrix ***A***.

- BLCK%FIDA is the storage mode for the submatrix ***BLCK***, where:

  If BLCK%FIDA='CSR', the submatrix ***BLCK*** is stored in the storage-by-rows storage mode. Scope: **global**.

Specified as: a character variable of length 5; BLCK%FIDA='CSR'.

If BLCK%FIDA='CSR', then you must specify the BLCK%AS, BLCK%IA1, and BLCK%IA2 components, as follows:

- BLCK%AS is a pointer to the submatrix ***BLCK*** that is stored by rows. See "Notes". Scope: **local**.

  Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

- BLCK%IA1 is a pointer to the column numbers of each non-zero element in the submatrix ***BLCK***. See "Notes". Scope: **local**.

  Specified as: a pointer to an assumed-shape array with shape (:), containing fullword integers; 1 ≤ BLCK%IA1(*i*) ≤ BLCK%N, where:

  i = 1, *nz* and *nz* is the **actual** number of non-zero elements in the submatrix ***BLCK***.

- BLCK%IA2 is a pointer to the starting positions of each row of the submatrix ***BLCK*** in BLCK%AS and one position past the end of BLCK%AS. See "Notes". Scope: **local**.

  Specified as: a pointer to an assumed-shape array with shape (:), containing fullword integers, where:

  BLCK%IA2(1) = 1
  BLCK%IA2(BLCK%M+1) = 1+*nz* and *nz* is the **actual** number of non-zero elements in the submatrix ***BLCK***.

Specified as: the derived data type D_SPMAT.

desc_a

is the descriptor vector for a global general sparse matrix ***A*** that is produced on a preceding call to PADALL or previous call(s) to this subroutine.

Type: **required**

Specified as: the derived data type DESC_TYPE.

On Return

a

is the updated local part of the global general sparse matrix ***A***, updated with data from the submatrix ***BLCK***.

Scope: **local**

Type: **required**

Returned as: the derived data type D_SPMAT.

desc_a

is the updated array descriptor for the global general sparse matrix ***A***.

Type: **required**

Returned as: the derived data type DESC_TYPE.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADALL and PSPALL.
2. This subroutine accepts mixed case letters for the BLCK%FIDA component.

3. Arguments ***BLCK*** and ***A*** must not have common elements; otherwise, results are unpredictable.
4. For details about some of the elements stored in DESC_A%MATRIX_DATA, see "Derived Data Type DESC_TYPE" on page 61.
5. The submatrix ***BLCK*** must be stored by rows; that is BLCK%FIDA = `'CSR'`. For information about the storage-by-rows storage mode, see the *ESSL Version 3 Guide and Reference*.
6. Once you declare BLCK of derived data type D_SPMAT, you must allocate the components of BLCK that point to an array. The following example shows how to code the allocate statement if each row of the submatrix ***BLCK*** contains no more than 20 elements:

   ```
   TYPE(D_SPMAT)  ::   BLCK                       !Declare the BLCK variable
           .
           .
           .
   ALLOCATE(BLCK%AS(20),BLCK%IA1(20),BLCK%IA2(2)) !Allocate array pointers
   ```

   When you are finished calling PSPINS, you should deallocate BLCK%AS, BLCK%IA1, and BLCK%IA2.
7. Each process has to call PSPINS as many times as necessary to insert the local rows it owns. It is also possible to call PSPINS multiple times to insert different or duplicate coefficients of the same local row it owns. For information on how duplicate coefficients are handled, see the *dupflag* argument description in PSPASB. For an example of inserting coefficients of the same local row, see "Example" on page 628.

Error Conditions:

Computational Errors: None

Resource Errors

1. Unable to allocate work space.
2. Unable to allocate component(s) of ***A***.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. $ia < 1$ or $ia >$ DESC_A%MATRIX_DATA(M)
3. $ja \neq 1$
4. *desc_a* component(s) are not valid.
5. The sparse matrix ***A*** is not valid.

6. BLCK%M < 1 or BLCK%M > DESC_A%MATRIX_DATA(N_ROW)
7. BLCK%N < 1 or BLCK%N > *n*
8. BLCK%FIDA ≠ 'CSR'
9. One or more rows to be inserted into submatrix ***A*** does not belong to the process.

Example: This piece of an example shows how to insert coefficients into the same GLOB_ROW row by calling PSPINS multiple times. It would be useful in finite element applications, where PSPINS inserts one element at a time into the global matrix, but more than one element may contribute to the same matrix row. In this case, PSPINS is called with the same value of *ia* by all the elements contributing to that row.

For a complete example, see “Example—Using the Fortran 90 Sparse Subroutines” on page 649.

```
           .
           .
           .
DO GLOB_ROW = 1, N

   ROW_MAT%DESCRA(1) = 'G'
   ROW_MAT%FIDA      = 'CSR'

   ROW_MAT%IA2(1) = 1
   ROW_MAT%IA2(2) = 1

   IA = GLOB_ROW

   !       (x-1,y,z)
   ROW_MAT%AS(1)  = COEFF(X-1,Y,Z,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X-1,Y,Z)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x,y-1,z)
   ROW_MAT%AS(1)  = COEFF(X,Y-1,Z,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X,Y-1,Z)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x,y,z-1)
   ROW_MAT%AS(1)  = COEFF(X,Y,Z-1,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X,Y,Z-1)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x,y,z)
   ROW_MAT%AS(1)  = COEFF(X,Y,Z,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X,Y,Z)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x,y,z+1)
   ROW_MAT%AS(1)  = COEFF(X,Y,Z+1,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X,Y,Z+1)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x,y+1,z)
   ROW_MAT%AS(1)  = COEFF(X,Y+1,Z,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X,Y+1,Z)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
   !       (x+1,y,z)
   ROW_MAT%AS(1)  = COEFF(X+1,Y,Z,X,Y,Z)
   ROW_MAT%IA1(1) = IDX(X+1,Y,Z)
   CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
END DO
            .
            .
            .
```

PGEINS—Inserts Local Data into a Dense Vector

This sparse utility subroutine is used by each process to insert all blocks of data it owns into its local part of the dense vector.

Syntax

Fortran	CALL PGEINS (*x*, *blck*, *desc_a*, *ix*)

On Entry

x

is a pointer to the local space for the dense vector that is produced by a preceding call to PGEALL or previous call(s) to this subroutine.

Scope: **local**

Type: **required**

Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

blck

is the local part of the submatrix ***BLCK*** to be inserted into the dense vector. Each call to this subroutine inserts one contiguous block of data into the local part of the dense vector corresponding to the global submatrix $\boldsymbol{X}_{ix:ix+\mathrm{size}(blck,1)-1}$. This subroutine only inserts a block of data it owns into its local part of the dense vector.

Scope: **local**

Type: **required**

Specified as: an assumed-shape array with shape (:), containing long-precision real numbers, where: $1 \le \mathrm{size(blck,1)} \le$ DESC_A%MATRIX_DATA(N_ROW)

desc_a

is the array descriptor that is produced by a preceding call to PADALL or PSPINS.

Type: **required**

Specified as: the derived data type DESC_TYPE.

ix

is the first global row index of the dense vector that receives data from the submatrix ***BLCK***.

Scope: **local**

Type: **optional**

Specified as: a fullword integer; $1 \le ix \le$ DESC_A%MATRIX_DATA(M). The default value is 1.

On Return

x

is a pointer to the local space for the dense vector, updated with local data from the submatrix ***BLCK***.

Scope: **local**

Type: **required**

Returned as: a pointer to an assumed-sized array with shape (:), containing long-precision real numbers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PGEALL and PADALL.
2. You do not need a separate array descriptor for a dense vector because it must conform to the size of matrix ***A***. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.
3. This subroutine must be called for:
   - Vector ***b*** containing the right-hand side.
   - Vector ***x*** containing the initial guess to the solution.
4. Each process has to call PGEINS as many times as necessary to insert the local elements it owns. It is also possible to call PGEINS multiple times to insert different coefficients of the same local row it owns. Duplicate coefficients are overwritten.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *desc_a* component(s) are not valid.
2. The process grid is not $np \times 1$.
3. $ix < 1$ or $ix >$ DESC_A%MATRIX_DATA(M)
4. size(x,1) $<$ max(1,DESC_A%MATRIX_DATA(N_ROW))
5. size($blck$,1) $<$ 1 or size($blck$,1) $>$ DESC_A%MATRIX_DATA(N_ROW)

Stage 5

1. One or more elements to be inserted into the dense vector does not belong to the process.

PSPASB—Assembles a General Sparse Matrix

This sparse utility subroutine uses the output from PSPINS to assemble the global general sparse matrix ***A*** and its array descriptor *desc_a*.

Syntax

Fortran	CALL PSPASB (*a*, *desc_a*) CALL PSPASB (*a*, *desc_a*, *mtype*, *stor*, *dupflag*, *info*)

On Entry

a

is the local part of the global general sparse matrix ***A*** that is produced by previous call(s) to PSPINS.

Scope: **local**

Type: **required**

Specified as: the derived data type D_SPMAT.

desc_a

is the array descriptor for the global general sparse matrix ***A*** that is produced by previous call(s) to PSPINS.

Type: **required**

Specified as: the derived data type DESC_TYPE.

mtype

indicates the form of the global sparse matrix ***A*** used, where:

If *mtype* = 'GEN', ***A*** is a general sparse matrix.

Scope: **global**

Type: **optional**

Specified as: a character variable of length 5; *mtype* = 'GEN'. The default value is 'GEN'.

stor

indicates the storage mode that the global general sparse matrix ***A*** is returned in, where:

If *stor* = 'DEF', this subroutine chooses an appropriate storage mode, which is an internal format accepted by the preconditioner and solver subroutines, for storing the global general sparse matrix ***A*** on output.

If *stor* = 'CSR', the global general sparse matrix ***A*** is stored in the storage-by-rows storage mode on output.

Scope: **global**

Type: **optional**

Specified as: a character variable of length 5; *stor* = 'DEF' or 'CSR'. The default value is 'DEF'.

dupflag

is a flag indicating how to use coefficients that are specified more than once on the same process; that is, duplicate coefficients within the same local part of the matrix ***A***:

If *dupflag* = 0, this subroutine uses the first of the duplicate coefficients.

If *dupflag* = 1, this subroutine adds all the duplicate coefficients with the same indices.

If *dupflag* = 2, this subroutine raises an error condition indicating that there are unexpected duplicate coefficients.

Scope: **global**

Type: **optional**

Specified as: a fullword integer; *dupflag* = 0, 1, or 2. The default value is 0.

info

See On Return.

On Return

a

is the updated local part of the global general sparse matrix ***A***, where:

If *stor* = `'DEF'`, this subroutine chooses an appropriate storage mode, which is an internal format accepted by the preconditioner and solver subroutines, for storing the global general sparse matrix ***A*** on output.

If *stor* = `'CSR'`, the global general sparse matrix ***A*** is stored in the storage-by-rows storage mode on output.

Scope: **local**

Type: **required**

Returned as: the derived data type D_SPMAT.

desc_a

is the final updated array descriptor for the global general sparse matrix ***A***.

Type: **required**

Returned as: the derived data type DESC_TYPE.

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then one or more of the following computational errors occurred and the appropriate error messages were issued, indicating an error exit, where:

- If *info* = 1, the sparse matrix ***A*** contains duplicate coefficients.
- If *info* = 2, the sparse matrix ***A*** contains empty row(s).

Scope: **global**

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PSPINS as many times as needed; that is, you must have completed building the matrix with call(s) to PSPINS before you place a call to this subroutine.

2. This subroutine accepts mixed case letters for the *mtype* and *stor* arguments.
3. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.

Error Conditions

Computational Errors: The sparse matrix ***A*** contains duplicate coefficients or empty row(s). For details, see the description of the *info* argument.

Resource Errors

1. Unable to allocate work space.
2. Unable to allocate component(s) of *desc_a*.
3. Unable to allocate component(s) of ***A***.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. *desc_a* component(s) are not valid.
2. The process grid is not $np \times 1$.
3. The sparse matrix ***A*** is not valid.
4. *mtype* ≠ `'GEN'`
5. *stor* ≠ `'DEF'` or `'CSR'`
6. *dupflag* ≠ 0, 1, or 2
7. Some local rows in the sparse matrix ***A*** are missing.

Stage 5

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   mtype differs.
   stor differs.
   dupflag differs.

PGEASB—Assembles a Dense Vector

This sparse utility subroutine assembles a dense vector.

Syntax

Fortran	CALL PGEASB (*x*, *desc_a*)

On Entry

x

is a pointer to the local part of the dense vector that is produced by previous call(s) to PGEINS.

Scope: **local**

Type: **required**

Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

desc_a

is the array descriptor, which was finalized in a preceding call to PSPASB.

Type: **required**

Specified as: the derived data type DESC_TYPE.

On Return

x

is a pointer to the local part of the global dense vector.

Scope: **local**

Type: **required**

Returned as: a pointer to an assumed-sized array with shape (:), containing long-precision real numbers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PGEINS as many times as needed; that is, you must have completed building the dense vectors with call(s) to PGEINS before you place a call to this subroutine.

   Before you call this subroutine, you must have called PSPASB.

2. You do not need a separate array descriptor for a dense vector because it must conform to the size of matrix ***A***. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.

3. This subroutine must be called for:

   - Vector ***b*** containing the right-hand side.
   - Vector ***x*** containing the initial guess to the solution.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. *desc_a* component(s) are not valid.
3. size(*x*,1) $<$ DESC_A%MATRIX_DATA(N_ROW)

PSPGPR—Preconditioner for a General Sparse Matrix

This subroutine computes a preconditioner for a global general sparse matrix ***A*** that should be passed unchanged to the PSPGIS subroutine. The preconditioners include diagonal scaling or an incomplete LU factorization.

Syntax

Fortran	CALL PSPGPR (*iprec*, *a*, *prcs*, *desc_a*) CALL PSPGPR (*iprec*, *a*, *prcs*, *desc_a*, *info*)

On Entry

iprec
is a flag that determines the type of preconditioning, where:

If *iprec* = 0, which is referred to as *none*, indicates the local part of the submatrix ***A*** is not preconditioned. PSPGIS will not be effective in this case, unless the coefficient matrix is well conditioned; if your input matrix is not well conditioned, you should consider using *iprec* = 1 or 2.

If *iprec* = 1, which is referred to as *diagsc*, indicates the local part of the submatrix ***A*** is preconditioned by a local diagonal submatrix.

If *iprec* = 2, which is referred to as *ilu*, indicates the local part of the submatrix ***A*** is preconditioned by a local incomplete LU factorization.

It is suggested that you use a preconditioner. For an explanation, see “Notes and Coding Rules” on page 638.

Scope: **global**

Type: **required**

Specified as: a fullword integer, where: *iprec* = 0, 1, or 2.

a
is the local part of the global general sparse matrix ***A***, finalized on a preceding call to PSPASB.

Scope: **local**

Type: **required**

Specified as: the derived data type D_SPMAT.

prcs
See On Return.

desc_a
is the array descriptor for the global general sparse matrix ***A*** that was finalized in a call to PSPASB.

Type: **required**

Specified as: the derived data type DESC_TYPE.

info
See On Return.

On Return

prcs
is the preconditioner data structure *prcs* that must be passed unchanged to PSPGIS.

Scope: **local**

Type: **required**

Returned as: the derived data type D_PRECN.

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, the value stored in *info* indicates the row index in the global general sparse matrix ***A*** where the preconditioner failed.

Scope: **global**

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PSPASB and PGEASB.
2. PSPGPR allocates *prcs*, as necessary. Prior to further calls to PSPGPR with the same *prcs*, you must call PSPFREE; otherwise, there will be a memory leak.
3. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.
4. Parallel ESSL builds the preconditioner, *prcs*, which is specified as derived data type D_PRECN, and its components. All the components of derived data type D_PRECN are used for internal use only.
5. The convergence rate of an iterative method as applied to a given system of linear equations depends on the spectral properties of the coefficient matrix of the linear system; therefore it is often convenient to apply a linear transformation to the system such that the solution of the transformed system is the same (in exact arithmetic) as that of the original, but the spectral properties and the convergence behavior are more favorable. Such a transformation is called preconditioning. If a matrix ***M*** approximates ***A***, then:

   $(\boldsymbol{M}^{-1})\boldsymbol{Ax} = (\boldsymbol{M}^{-1})\boldsymbol{b}$

   is a preconditioned system and ***M*** is called a preconditioner. In practice, the new coefficient matrix $(\boldsymbol{M}^{-1})\boldsymbol{A}$ is almost never formed explicitly, but rather its action is computed during the application of the iterative method. The effectiveness of the preconditioning operation depends on a trade-off between how well ***M*** approximates ***A*** and how costly it is to compute and invert it; no single preconditioner will give best overall performance under all situations. Note finally that it is quite rare for a linear system to behave well enough so as not to require preconditioning; indeed most linear systems originating from the discretization of difficult physical problems require preconditioning to have any convergence at all.

   See references [9] and [37].

Error Conditions

Computational Errors

1. The preconditioner for the sparse matrix ***A*** is unstable. For details, see the *info* output argument for this subroutine.

Resource Errors

1. Unable to allocate work space.
2. Unable to allocate component(s) of *prcs*.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. *desc_a* component(s) are not valid.
3. *iprec* $\neq$ 0, 1, or 2
4. The storage format for ***A*** is not supported.

Stage 5

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   iprec differs.

PSPGIS—Iterative Linear System Solver for a General Sparse Matrix

This subroutine solves a general sparse linear system of equations, using an iterative algorithm, with or without preconditioning. The methods include the more smoothly converging variant of the CGS method (Bi-CGSTAB), conjugate gradient squared (CGS), or transpose-free quasi-minimal residual method (TFQMR).

See references [7], [9], [12], and [35].

Syntax

Fortran	CALL PSPGIS (*a*, *b*, *x*, *prcs*, *desc_a*) CALL PSPGIS (*a*, *b*, *x*, *prcs*, *desc_a*, *iparm*, *rparm*, *info*)

On Entry

a

is the local part of the coefficient matrix ***A***, produced on a previous call to PSPASB.

Scope: **local**

Type: **required**

Specified as: the derived data type D_SPMAT.

b

is a pointer to the local part of the global vector ***b***, containing the right-hand side of the matrix problem and produced on a previous call to PGEASB.

Scope: **local**

Type: **required**

Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

x

is a pointer to the local part of the global vector ***x***, containing the initial guess to the solution of the linear system and produced on a previous call to PGEASB.

Scope: **local**

Type: **required**

Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

prcs

is the preconditioner data structure *prcs*, produced on a previous call to PSPGPR.

Scope: **local**

Type: **required**

Specified as: the derived data type D_PRECN.

desc_a

is the array descriptor, produced on a previous call to PSPASB, for the global general sparse matrix ***A***.

Type: **required**

Specified as: the derived data type DESC_TYPE.

iparm

is an array of parameters, `IPARM(`*i*`)`, where:

- `IPARM(1)` is the flag, referred to as *methd*, used to select the iterative procedure used, where:

  If *methd* = 1, the more smoothly converging variant of the CGS method, referred to as Bi-CGSTAB, is used.

  If *methd* = 2, the conjugate gradient squared method, referred to as CGS, is used.

  If *methd* = 3, the transpose-free quasi-minimal residual method, referred to as TFQMR, is used.

- `IPARM(2)` is the flag, *istopc*, used to select the stopping criterion used in the computation, where the following items are used in the definitions of the stopping criteria below:

  - ε is the desired relative accuracy and is stored in *eps*
  - x_j is the solution found at the *j*-th iteration.
  - r_j and r_0 are the preconditioned residuals obtained at iterations *j* and 0, respectively. (The residual at iteration *j* is defined as $\boldsymbol{b}-\boldsymbol{A}x_j$.)

  If *istopc* = 1, the iterative method is stopped when:

  $$\|r_j\|_2 \,/\, \|x_j\|_2 < \varepsilon$$

  If *istopc* = 2, the iterative method is stopped when:

  $$\|r_j\|_2 \,/\, \|r_0\|_2 < \varepsilon$$

  If *istopc* = 3, the iterative method is stopped when:

  $$\|x_j - x_{j-1}\|_2 \,/\, \|x_j\|_2 < \varepsilon$$

  Note: Stopping criterion 3 performs poorly with the TFQMR method; therefore, if you specify TFQMR (*methd* = 3), you should not specify stopping criterion 3.

- `IPARM(3)` is the maximum number of iterations *itmax* allowed.
- `IPARM(4)`, referred to as *itrace*, has the following meaning:

  If *itrace* = 0, then *itrace* is ignored.

  If *itrace* > 0, an informational message about the convergence, which is based on the stopping criterion described in *istopc*, is issued at every *itrace*-th iteration and upon exit.

- `IPARM(5)`, see On Return.
- `IPARM(6)` through `IPARM(20)` are reserved.

Scope: **global**

Type: **optional**

Default:

methd = 1
istopc = 1
itmax = 500
itrace = 0

Specified as: an array of length 20, containing fullword integers, where:
methd = 1, 2, or 3
istopc = 1, 2, or 3
itmax ≥ 0
itrace ≥ 0
`IPARM(6)` through `IPARM(20)` should be set to zero.

rparm
is an array of parameters, `RPARM(`*i*`)`, where:

- `RPARM(1)`, referred to as *eps*, is the relative accuracy ε used in the stopping criterion.
- `RPARM(2)`, see On Return.
- `RPARM(3)` through `RPARM(20)` are reserved.

Scope: **global**

Type: **optional**

Default: *eps* = 10^{-8}

Specified as: an array of length 20, containing long-precision real numbers, where:
eps ≥ 0.
`RPARM(3)` through `RPARM(20)` should be set to zero.

info
See On Return.

On Return

x
is a pointer to the local part of the solution vector ***x***

Scope: **local**

Type: **required**

Returned as: a pointer to an assumed-shape array of shape (:), containing long-precision real numbers.

iparm
has the following meaning, when *iparm* is **present**:

`IPARM(5)` is the number of iterations, *iter*, performed by this subroutine.

Scope: **global**

Type: **optional**

Returned as: an array of length 20, containing fullword integers, where *iter* ≥ 0.

rparm
has the following meaning, when *rparm* is **present**:

`RPARM(2)` contains the estimate of the error, *err*, of the solution, according to the stopping criterion, *istopc*, in use. For details, see the *istopc* argument description.

Scope: **global**

Type: **optional**

Returned as: an array of length 20, containing long-precision real numbers, where *err* ≥ 0.

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then this subroutine exceeded *itmax* iterations without converging. You may want to try the following to get your matrix to converge:

- You can increase the number of iterations and call this subroutine again without making any other changes to your program.
- You can change the requested precision and/or the stopping criterion; your original precision requirement may be too stringent under a given stopping criterion.
- You can use a preconditioner if you were not already doing so, or to change the one you were using. Note also that the efficiency of the preconditioner may depend on the data distribution strategy adopted. See “Notes and Coding Rules” on page 638.

Scope: **global**

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PSPGPR.
2. For details about some of the elements stored in DESC_A%MATRIX_DATA, see “Derived Data Type DESC_TYPE” on page 61.
3. Parallel ESSL builds the preconditioner, *prcs*, which is specified as derived data type D_PRECN, and its components. All the components of derived data type D_PRECN are used for internal use only.

Error Conditions

Computational Errors: This subroutine exceeded *itmax* iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.

Note: If the preconditioner computed by PSPGPR failed because the sparse matrix ***A*** is unstable, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PSPGPR.

You may want to try the following to get your matrix to converge:

- You can increase the number of iterations and call this subroutine again without making any other changes to your program.

- You can change the requested precision and/or the stopping criterion; your original precision requirement may be too stringent under a given stopping criterion.
- You can use a preconditioner if you were not already doing so, or to change the one you were using. Note also that the efficiency of the preconditioner may depend on the data distribution strategy adopted. See “Notes and Coding Rules” on page 638.

Resource Errors

1. Unable to allocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. *desc_a* component(s) are not valid.
3. The sparse matrix ***A*** is not valid.
4. size(*iparm*) $<$ 20
5. size(*rparm*) $<$ 20
6. *eps* $<$ 0.0
7. *methd* $\neq$ 1, 2, or 3
8. *iprec* $\neq$ 0, 1, or 2
9. *istopc* $\neq$ 1, 2, or 3
10. *itmax* $<$ 0
11. *itrace* $<$ 0
12. The storage format for the sparse matrix ***A*** is not supported.

Stage 5

1. size(*x*) $<$ DESC_A%MATRIX_DATA(N_ROW)
2. size(*b*) $<$ DESC_A%MATRIX_DATA(N_ROW)
3. The preconditioner data structure *prcs* is not valid.

Stage 6

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   eps differs.
   methd differs.
   istopc differs.
   itmax differs.
   itrace differs.
   Component(s) of *prcs* differ.

PGEFREE—Deallocates Space for a Dense Vector

This sparse utility subroutine deallocates space that is used for a dense vector.

Syntax

Fortran	CALL PGEFREE (*x*, *desc_a*)

On Entry

x

is a pointer to the dense vector ***x***.

Scope: **local**

Type: **required**

Specified as: a pointer to an assumed-shape array with shape (:), containing long-precision real numbers.

desc_a

is the array descriptor for the sparse matrix ***A***.

Type: **required**

Specified as: the derived data type DESC_TYPE.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PGEALL.
2. You must deallocate ***b***, ***x***, sparse matrix ***A***, and preconditioner data structure *prcs* before you deallocate the array descriptor *desc_a*.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. The pointer *x* is not associated and therefore cannot be deallocated.

PSPFREE—Deallocates Space for a General Sparse Matrix

This sparse utility subroutine deallocates space that is used for a global general sparse matrix ***A*** or a preconditioner data structure *prcs*.

Syntax

Fortran	CALL PSPFREE (*a*, *desc_a*) CALL PSPFREE (*prcs*, *desc_a*)

On Entry

a

is the general sparse matrix ***A***.

Scope: **local**

Type: **required**

Specified as: the derived data type D_SPMAT.

prcs

is the preconditioner data structure *prcs*.

Scope: **local**

Type: **required**

Specified as: the derived data type D_PRECN.

desc_a

is the array descriptor for the sparse matrix ***A***.

Type: **required**

Specified as: the derived data type DESC_TYPE.

Notes and Coding Rules

1. Before you call this subroutine to deallocate the sparse matrix ***A***, you must have called PSPALL.

   Before you call this subroutine to deallocate the preconditioner data structure *prcs*, you must have called PSPGPR.
2. You must deallocate ***b***, ***x***, sparse matrix ***A***, and preconditioner data structure *prcs* before you deallocate the array descriptor *desc_a*.
3. PSPGPR allocates components of *prcs* as necessary. Prior to further calls to PSPGPR with the same *prcs* you must call PSPFREE; otherwise, there will be a memory leak.
4. PSPALL allocates matrix ***A*** as necessary. Prior to further calls to PSPALL with the same matrix ***A***, you must call PSPFREE; otherwise, there will be a memory leak.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.
2. The preconditioner data structure *prcs* is not valid.
3. The pointer components of ***A*** or *prcs* are not associated and therefore cannot be deallocated.

PADFREE—Deallocates Space for an Array Descriptor for a General Sparse Matrix

This sparse utility subroutine deallocates space that is used for the array descriptor for a global general sparse matrix ***A***.

Syntax

Fortran	CALL PADFREE (*desc_a*)

On Entry

desc_a
is the array descriptor for the sparse matrix ***A***.

Type: **required**

Specified as: the derived data type DESC_TYPE.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADALL.
2. You must deallocate ***b***, ***x***, sparse matrix ***A***, and preconditioner data structure *prcs* before you deallocate the array descriptor *desc_a*.
3. PADALL allocates *desc_a* as necessary. Prior to further calls to PADALL with the same *desc_a*, you must call PADFREE; otherwise, there will be a memory leak.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. *desc_a* has not been initialized.

Stage 2

1. The BLACS context is invalid.

Stage 3

1. This subroutine was called from outside the process grid.

Stage 4

1. The process grid is not $np \times 1$.

Example—Using the Fortran 90 Sparse Subroutines

This example finds the solution to the linear system ***Ax*** = ***b***. It also contains an application program that shows how you can use the Fortran 90 sparse linear algebraic equation subroutines and their utilities to solve this example.

The following is the general sparse matrix ***A***:

$$\begin{bmatrix}
2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 0.0 & 0.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0 & -1.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 2.0
\end{bmatrix}$$

The following is the dense vector ***b***, containing the right-hand side:

$$\begin{bmatrix}
2.0 \\ 1.0 \\ 3.0 \\ 2.0 \\ 2.0 \\ 2.0 \\ 2.0 \\ 2.0 \\ 3.0
\end{bmatrix}$$

The following is the dense vector ***x***, containing the initial guess to the solution:

$$\begin{bmatrix}
0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0
\end{bmatrix}$$

Output: Global vector ***x***:

```
B,D    0
       ┌     ┐
       | 1.0 |
 0     | 1.0 |
       | 1.0 |
       | --- |
       | 1.0 |
 1     | 1.0 |
       | 1.0 |
       | --- |
       | 1.0 |
 2     | 1.0 |
       | 1.0 |
       └     ┘
```

The following is the 3 × 1 process grid:

B,D	0
0	P_{00}
1	P_{10}
2	P_{20}

Local vector ***x***:

```
  p,q |   0
------|------
      |  1.0
   0  |  1.0
      |  1.0
------|------
      |  1.0
   1  |  1.0
      |  1.0
------|------
      |  1.0
   2  |  1.0
      |  1.0
```

`ITER` = 4

`ERR` = 0.4071D-15

The value of *info* is 0 on all processes.

Application Program: This application program illustrates how to use the Fortran 90 sparse linear algebraic equation subroutines and their utilities.

```
@process init(f90ptr)
!
! This program illustrates how to use the PESSL F90 Sparse Iterative
! Solver and its supporting utility subroutines.  A very simple problem
! (DSRIS Example 1 from the ESSL Guide and Reference) using an
! HPF BLOCK data distribution is solved.
!
      PROGRAM EXAMPLE90

! Interface module required to use the PESSL F90 Sparse Iterative Solver

      USE F90SPARSE
      IMPLICIT NONE

! Interface definition for the PARTS subroutine PART_BLOCK

      INTERFACE PART_BLOCK
       SUBROUTINE PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
       IMPLICIT NONE
       INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
       INTEGER, INTENT(OUT) :: NV
       INTEGER, INTENT(OUT) :: PV(*)

       END SUBROUTINE PART_BLOCK
      END INTERFACE

! Parameters
      CHARACTER, PARAMETER          :: ORDER='R'
      INTEGER, PARAMETER            :: IZERO=0, IONE=1

! Sparse Matrices
      TYPE(D_SPMAT)                 :: A, BLCK
! Preconditioner Data Structure
      TYPE(D_PRECN)                 :: PRC

! Dense Vectors
      REAL(KIND(1.D0)), POINTER     :: B(:), X(:)

! Communications data structure
      TYPE(DESC_TYPE)               :: DESC_A

! BLACS parameters
      INTEGER            :: NPROW, NPCOL, ICTXT, IAM, NP, MYROW, MYCOL

! Solver parameters
      INTEGER            :: ITER, ITMAX, IERR, ITRACE,
     &                      IPREC, METHD, ISTOPC, IPARM(20)
      REAL(KIND(1.D0))   :: ERR, EPS, RPARM(20)

! Other variables
      CHARACTER*5        :: AFMT, ATYPE
      INTEGER            :: IRCODE, IRCODE1, IRCODE2, IRCODE3
      INTEGER            :: I,J
      INTEGER            :: N,NNZERO
      INTEGER, POINTER   :: PV(:)
      INTEGER            :: LPROCS, NROW, NCOL
      INTEGER            :: GLOBAL_INDX, NV_COUNT
      INTEGER            :: GLOBAL_INDX_OWNER, NV
      INTEGER            :: LOCAL_INDX
!
!     Global Problem
!      DSRIS Example 1 from the ESSL Guide and Reference
!
      REAL*8          :: A_GLOBAL(22),B_GLOBAL(9),XINIT_GLOBAL(9)
      INTEGER         :: JA(22),IA(10)
      DATA A_GLOBAL     /2.D0,2.D0,-1.D0,1.D0,2.D0,1.D0,2.D0,-1.D0,
     $                   1.D0,2.D0,-1.D0,1.D0,2.D0,-1.D0,1.D0,2.D0,
     $                  -1.D0,1.D0,2.D0,-1.D0,1.D0,2.D0/
      DATA JA           /1,2,3,2,3,1,4,5,4,5,6,5,6,7,6,7,8,
     $                   7,8,9,8,9/
      DATA IA           /1,2,4,6,9,12,15,18,21,23/

      DATA B_GLOBAL     /2.D0,1.D0,3.D0,2.D0,2.D0,2.D0,2.D0,2.D0,
     $                   3.D0/
      DATA XINIT_GLOBAL /0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,
     $                   0.D0/

! Initialize BLACS
! Define a NP x 1 Process Grid

      CALL BLACS_PINFO(IAM, NP)
      CALL BLACS_GET(IZERO, IZERO, ICTXT)
      CALL BLACS_GRIDINIT(ICTXT, ORDER, NP, IONE)
      CALL BLACS_GRIDINFO(ICTXT, NPROW, NPCOL, MYROW, MYCOL)

!
! Initialize the global problem size
!
      N = SIZE(IA)-1

!
! Guess for the local number of nonzeros
!
      NNZERO = SIZE(A_GLOBAL)

!
! Allocate and initialize some elements of the sparse matrix A
! its descriptor vector, DESC_A, the rhs vector B, and the
! solution vector X.
!
      CALL PADALL(N,PART_BLOCK,DESC_A,ICTXT)
      CALL PSPALL(A,DESC_A,NNZ=NNZERO)
      CALL PGEALL(B,DESC_A)
      CALL PGEALL(X,DESC_A)

!
! Allocate an integer work area to be used as an argument for
! the PART_BLOCK PARTS subroutine
!
      NROW   = N
      NCOL   = NROW
      LPROCS = MAX(NPROW, NROW + NCOL)
      ALLOCATE(PV(LPROCS), STAT = IRCODE)
      IF (IRCODE /= 0) THEN
       WRITE(6,*) 'PV Allocation failed'
       CALL BLACS_ABORT(ICTXT,-1)
```

```
        STOP
       ENDIF


! SETUP BLCK

       BLCK%M  = 1
       BLCK%N  = NCOL
       BLCK%FIDA = 'CSR'

       ALLOCATE(BLCK%AS(BLCK%N),STAT=IRCODE1)
       ALLOCATE(BLCK%IA1(BLCK%N),STAT=IRCODE2)
       ALLOCATE(BLCK%IA2(BLCK%M+1),STAT=IRCODE3)
       IRCODE = IRCODE1 + IRCODE2 + IRCODE3
       IF (IRCODE /= 0) THEN
        WRITE(6,*) 'Error allocating BLCK'
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       ENDIF

!
! In this simple example, all processes have a copy of
! the global sparse matrix, A, the global rhs vector, B,
! and the global initial guess vector, X.
!
! Each process will call PSPINS as many times as necessary
! to insert the local rows it owns.
!
! Each process will call PGEINS as many times as necessary
! to insert the local elements it owns.
!
       DO GLOBAL_INDX = 1, NROW
        CALL PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
!
! In this simple example, NV will always be 1
! since there will not be duplicate coefficients
!
        DO NV_COUNT = 1, NV
         GLOBAL_INDX_OWNER = PV(NV_COUNT)
         IF (GLOBAL_INDX_OWNER == MYROW) THEN
          BLCK%IA2(1) = 1
          BLCK%IA2(2) = 1
          DO J = IA(GLOBAL_INDX), IA(GLOBAL_INDX+1)-1
           BLCK%AS(BLCK%IA2(2)) = A_GLOBAL(J)
           BLCK%IA1(BLCK%IA2(2)) = JA(J)
           BLCK%IA2(2) =BLCK%IA2(2) + 1
          ENDDO
          CALL PSPINS(A,GLOBAL_INDX,1,BLCK,DESC_A)
          CALL PGEINS(B,B_GLOBAL(GLOBAL_INDX:GLOBAL_INDX),
     &                 DESC_A,GLOBAL_INDX)
          CALL PGEINS(X,XINIT_GLOBAL(GLOBAL_INDX:GLOBAL_INDX),
     &                 DESC_A,GLOBAL_INDX)
         ENDIF
        END DO
       END DO

! Assemble A and DESC_A
       AFMT = 'DEF'
```

```
      ATYPE = 'GEN'
      CALL PSPASB(A,DESC_A,MTYPE=ATYPE,
     &            STOR=AFMT,DUPFLAG=2,INFO=IERR)

      IF (IERR /= 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in assembly :',IERR
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

! Assemble B and X

      CALL PGEASB(B,DESC_A)
      CALL PGEASB(X,DESC_A)

!
! Deallocate BLCK
!

      IF (ASSOCIATED(BLCK%AS))    DEALLOCATE(BLCK%AS)
      IF (ASSOCIATED(BLCK%IA1))   DEALLOCATE(BLCK%IA1)
      IF (ASSOCIATED(BLCK%IA2))   DEALLOCATE(BLCK%IA2)
!
! Deallocate Work vector
!
      IF (ASSOCIATED(PV))      DEALLOCATE(PV)

!
!  Preconditioning
!
!  We are using ILU for the preconditioner; PESSL
!  will allocate PRC.
!
      IPREC = 2
      CALL PSPGPR(IPREC,A,PRC,DESC_A,INFO=IERR)

      IF (IERR /= 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in preconditioner :',IERR
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

!
!  Iterative Solver - use the BICGSTAB method
!
      ITMAX = 1000
      EPS   = 1.D-8
      METHD = 1
      ISTOPC = 1
      ITRACE = 0
      IPARM    = 0
      IPARM(1) = METHD
      IPARM(2) = ISTOPC
      IPARM(3) = ITMAX
      IPARM(4) = ITRACE
      RPARM    = 0.0D0
      RPARM(1) = EPS

      CALL PSPGIS(A,B,X,PRC,DESC_A,IPARM=IPARM,RPARM=RPARM,
     &            INFO=IERR)


      IF (IERR /= 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in solver :',IERR
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

      ITER = IPARM(5)
      ERR  = RPARM(2)
      IF (IAM.EQ.0) THEN
       WRITE(6,*) 'Number of iterations : ',ITER
       WRITE(6,*) 'Error on exit        : ',ERR
      END IF

!
!  Each process prints their local piece of the solution vector
!
      IF (IAM.EQ.0) THEN
       Write(6,*) 'Solution Vector X'
      END IF

      LOCAL_INDX = 1
      Do GLOBAL_INDX = 1, NROW
       CALL PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
!
! In this simple example, NV will always be 1
! since there will not be duplicate coefficients
!
       DO NV_COUNT = 1, NV
        GLOBAL_INDX_OWNER = PV(NV_COUNT)
        IF (GLOBAL_INDX_OWNER == MYROW) THEN
         Write(6,*) GLOBAL_INDX, X(LOCAL_INDX)
         LOCAL_INDX = LOCAL_INDX +1
        ENDIF
       END DO
      END DO

!
!  Deallocate the vectors, the sparse matrix, and
!  the preconditioner data structure.
!  Finally, deallocate the descriptor vector
!
       CALL PGEFREE(B, DESC_A)
       CALL PGEFREE(X, DESC_A)
       CALL PSPFREE(A, DESC_A)
       CALL PSPFREE(PRC, DESC_A)
       CALL PADFREE(DESC_A)
```

```
!
!  Terminate the process grid and the BLACS
!
       CALL BLACS_GRIDEXIT(ICTXT)
       CALL BLACS_EXIT(0)

       END PROGRAM EXAMPLE90
```

Fortran 77 Sparse Linear Algebraic Equation Subroutines and Their Utility Subroutines

This section contains the Fortran 77 sparse linear algebraic equation subroutine descriptions and their sparse utility subroutines.

PADINIT—Initializes an Array Descriptor for a General Sparse Matrix

This sparse utility subroutine initializes an array descriptor, which is needed to establish a mapping between the global general sparse matrix ***A*** and its corresponding distributed memory location.

Syntax

Fortran	CALL PADINIT (*n*, *parts*, *desc_a*, *icontxt*)
C and C++	padinit (*n*, *parts*, *desc_a*, *icontxt*);

On Entry

n

is the order of the global general sparse matrix ***A*** and the size of the index space.

Scope: **global**

Specified as: a fullword integer, where: $n > 0$.

parts

is a user-supplied subroutine that specifies a mapping between a global index for an element in the global general sparse matrix ***A*** and its corresponding storage location on one or more processes.

Sample *parts* subroutines for common types of data distributions are shown in “Sample PARTS Subroutine” on page 1084.

For details about how you must define the PARTS subroutine, see “Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)” on page 65.

Scope: **global**

Specified as: *parts* must be declared as an external subroutine in your application program. It can be whatever name you choose.

desc_a

is the array descriptor for the global general sparse matrix ***A***. DESC_A(11), which is the length of the array descriptor, DLEN, is the only element that you must specify. To determine a sufficient value, see “Array Descriptor” on page 63.

Specified as: an array of length DLEN, containing fullword integers.

icontxt

is the BLACS context parameter.

Scope: **global**

Specified as: a fullword integer that was returned in a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

On Return

desc_a

is the array descriptor for the global general sparse matrix ***A***. This subroutine initializes the remaining elements in the array descriptor *desc_a*. The elements of *desc_a* are updated with subsequent calls to PDSPINS and finalized with a call to PDSPASB.

Table 28 on page 64 describes some of the elements of the array descriptor that you may want to reference. Your application programs should not modify the elements of the array descriptor directly. The elements should only be updated with calls to PDSPINS and PDSPASB.

Returned as: an array of length DLEN, containing fullword integers.

Notes and Coding Rules

1. Before you call this subroutine, you must create a $np \times 1$ process grid, where *np* is the number of processes.
2. N_ROW is stable after you have placed a call to this subroutine. N_COL is stable after you have placed a call to PDSPASB. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.

Error Conditions

Computational Errors: None.

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. $n \leq 0$

Stage 4

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   n differs.

Stage 5

1. *pv* or *nv*, output from the user-supplied *parts* subroutine, was not valid. For valid values, see the appropriate argument description in “Programming Considerations for the Parts Subroutine (Fortran 90 and Fortran 77)” on page 65.
2. DLEN is too small. For valid values, see “Array Descriptor” on page 63.

PDSPINIT—Initializes a General Sparse Matrix

This sparse utility subroutine initializes the local part of a general sparse matrix ***A***.

Syntax

Fortran	CALL PDSPINIT (*as*, *ia1*, *ia2*, *infoa*, *desc_a*)
C and C++	pdspinit (*as*, *ia1*, *ia2*, *infoa*, *desc_a*);

On Entry

as
: See On Return.

ia1
: See On Return.

ia2
: See On Return.

infoa
: is an array, referred to as `INFOA`, providing more information about the general sparse matrix ***A***. You must specify `INFOA(1)` through `INFOA(3)`, as follows:

- `INFOA(1)` is the length of the array `AS`, where: `INFOA(1)` ≥ max(2,*nnze*).
- `INFOA(2)` is the length of the array `IA1`, where: `INFOA(2)` ≥ max(3,(*nnze*+N_ROW)).
- `INFOA(3)` is the length of the array `IA2`, where: `INFOA(3)` ≥ max(3,(*nnze*+N_COL)).
- `INFOA(4)` through `INFOA(30)` are reserved for internal use.

nnze is the number of non-zero elements (including duplicate coefficients) in the local part of the global general sparse matrix ***A***.

Specified as: an array of length 30, containing fullword integers.

desc_a
: is the array descriptor for a global general sparse matrix ***A*** that is produced on a preceding call to PADINIT.

Specified as: an array of length DLEN, containing fullword integers.

On Return

as
: is the local part, containing some internal values that are initialized by Parallel ESSL, of the global general sparse matrix ***A***.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1
: is the local part, containing some internal values that are initialized by Parallel ESSL, of the sparse matrix indices.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2
is the local part, containing some internal values that are initialized by Parallel ESSL, of the sparse matrix indices.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa
is the array `INFOA` updated with some internal values that are set by Parallel ESSL.

Returned as: an array of length 30, containing fullword integers.

desc_a
is the updated array descriptor for the global general sparse matrix ***A***.

Returned as: an array of length DLEN, containing fullword integers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADINIT.
2. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
3. For details about some of the elements stored in *infoa*, see Table 27 on page 63.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. `INFOA(1)` $<$ 2; that is, the size of `AS` $<$ 2
3. `INFOA(2)` $<$ 3; that is, the size of `IA1` $<$ 3
4. `INFOA(3)` $<$ 3; that is, the size of `IA2` $<$ 3

PDSPINS—Inserts Local Data into a General Sparse Matrix

This sparse utility subroutine is used by each process to insert all blocks of data it owns into its local part of the general sparse matrix ***A***.

Syntax

Fortran	CALL PDSPINS (*as*, *ia1*, *ia2*, *infoa*, *desc_a*, *ia*, *ja*, *blcks*, *ib1*, *ib2*, *infob*)
C and C++	pdspins (*as*, *ia1*, *ia2*, *infoa*, *desc_a*, *ia*, *ja*, *blcks*, *ib1*, *ib2*, *infob*);

On Entry

as

is the local part of the global general sparse matrix ***A*** that is produced on a preceding call to PDSPINIT or previous call(s) to this subroutine.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1

is the local part of array `IA1` that is produced on a preceding call to PDSPINIT or previous call(s) to this subroutine.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2

is the local part of the array `IA2` that is produced on a preceding call to PDSPINIT or previous call(s) to this subroutine.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa

is the array `INFOA` that is produced on a preceding call to PDSPINIT or previous call(s) to this subroutine.

Specified as: an array of length 30, containing fullword integers.

desc_a

is the array descriptor for a global general sparse matrix ***A*** that is produced on a preceding call to PDSPINIT or previous call(s) to this subroutine.

Specified as: an array of length DLEN, containing fullword integers.

ia

is the first global row index of the general sparse matrix ***A*** that receives data from the submatrix ***BLCK***.

Scope: **local**

Specified as: a fullword integer; $1 \le ia \le M$.

ja

is the first global column index of the general sparse matrix ***A*** that receives data from the submatrix ***BLCK***.

Scope: **local**

Specified as: a fullword integer, where: $ja = 1$.

blcks

is the local part of the sparse submatrix ***BLCK***, referred to as `BLCKS`, to be inserted into the global general sparse matrix ***A***. Each call to this subroutine inserts one contiguous block of rows into the local part of the sparse matrix corresponding to the global submatrix $\boldsymbol{A}_{ia:ia+\mathtt{INFOB(6)}-1,\ ja:ja+\mathtt{INFOB(7)}-1}$. This subroutine only can insert blocks of data it owns into its local part of the general sparse matrix ***A***.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOB(1)`, containing long-precision real numbers.

ib1

is an array, referred to as `IB1`, containing column numbers of each non-zero element in the submatrix ***BLCK***.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOB(2)`, containing fullword integers.

ib2

is the array, referred to as `IB2`, containing the starting positions of each row of the submatrix ***BLCK*** in array `BLCKS` and one position past the end of `BLCKS`.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOB(3)`, containing fullword integers:

IB2(1) = 1

IB2(`INFOB(6)`+1) = 1+*nz* and *nz* is the **actual** number of non-zero elements in the submatrix ***BLCK***.

infob

is an array, referred to as `INFOB`, providing information about the submatrix ***BLCK***. You must specify `INFOB(1)` through `INFOB(7)`, as follows:

- `INFOB(1)` is the length of the array `BLCKS`, where: `INFOB(1)` ≥ *nz*.
- `INFOB(2)` is the length of the array `IB1`, where: `INFOB(2)` ≥ *nz*.
- `INFOB(3)` is the length of the array `IB2`, where: `INFOB(3)` ≥ (`INFOB(6)`+1).
- `INFOB(4)` is the storage format of submatrix ***BLCK***, where:

  If `INFOB(4)` = 1, submatrix ***BLCK*** is stored by rows.
- `INFOB(5)` indicates the matrix type, where:

  If `INFOB(5)` = 1, ***BLCK*** is a general sparse matrix.
- `INFOB(6)` is the number of local rows in the submatrix ***BLCK***, where: 1 ≤ `INFOB(6)` ≤ N_ROW.
- `INFOB(7)` is an upper bound on the number of local columns in the submatrix ***BLCK***, where: 1 ≤ `INFOB(7)` ≤ *n*. *n* is the order of the global general sparse matrix ***A***.
- `INFOB(8)` through `INFOB(30)` are reserved.

Specified as: an array of length 30, containing fullword integers.

On Return

as

is the updated local part of the global general sparse matrix ***A***, updated with data from the submatrix ***BLCK***.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1

is the updated local part of array `IA1`.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2

is the updated local part of the array `IA2`.

Scope: **local**

Returned as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa

is the updated local part of array `INFOA`.

Returned as: an array of length 30, containing fullword integers.

desc_a

is the updated array descriptor for the global general sparse matrix ***A***.

Returned as: an array of length DLEN, containing fullword integers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADINIT and PDSPINIT.
2. Arguments ***BLCK*** and ***A*** must not have common elements; otherwise, results are unpredictable.
3. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
4. For details about some of the elements stored in *infoa* or *infob*, see Table 27 on page 63.
5. The submatrix ***BLCK*** must be stored by rows; that is `INFOB(4) = 1`. For information about the storage-by-rows storage mode, see the *ESSL Version 3 Guide and Reference*.
6. Each process has to call PDSPINS as many times as necessary to insert the local rows it owns. It is also possible to call PDSPINS multiple times to insert different or duplicate coefficients of the same local row it owns. For information on how duplicate coefficients are handled, see the *dupflag* argument description in PDSPASB. For an example of inserting coefficients of the same local row, see “Example” on page 665.

Error Conditions

Computational Errors: None.

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not *np* × 1.
2. *ja* ≠ 1
3. *desc_a* is not valid.
4. The sparse matrix ***A*** is not valid.
5. INFOB(4) ≠ 1
6. INFOB(5) ≠ 1
7. INFOB(6) < 1 or INFOB(6) > N_ROW
8. INFOB(7) < 1 or INFOB(7) > *n*
9. *ia* < 1 or *ia* > M
10. One or more rows to be inserted into submatrix ***A*** does not belong to the process.
11. DLEN is too small. For valid values, see “Array Descriptor” on page 63.
12. INFOB(1) < *nz*; that is, the size of BLCKS < *nz*
13. INFOB(2) < *nz*; that is, the size of IB1 < *nz*
14. INFOB(3) < (INFOB(6)+1); that is, the size of IB2 < (INFOB(6)+1)
15. INFOA(1) < max(2,*nnze*); that is, the size of AS < max(2,*nnze*)
16. INFOA(2) < max(3,(*nnze*+N_ROW)); that is, the size of IA1 < max(3,(*nnze*+N_ROW))
17. INFOA(3) < max(3,(*nnze*+N_COL)); that is, the size of IA2 < max(3,(*nnze*+N_COL))

Example: This piece of an example shows how to insert coefficients into the same GLOB_ROW row by calling PDSPINS multiple times. This example would be useful in finite element applications, where PDSPINS inserts one element at a time into the global matrix, but more than one element may contribute to the same matrix row. In this case, PDSPINS is called with the same value of *ia* by all the elements contributing to that row.

For a complete example, see “Example—Using the Fortran 77 Sparse Subroutines” on page 685.

```
            .
            .
            .
      DO GLOB_ROW = 1, N

         RINFOA(1) = 20
         RINFOA(2) = 20
         RINFOA(3) = 20
         RINFOA(4) = 1
         RINFOA(5) = 1
         RINFOA(6) = 1
         RINFOA(7) = N
         RIA2(1)   = 1
         RIA2(2)   = 2
         IA = GLOB_ROW
```

```
C          !       (x-1,y)
         RAS(1)  = COEFF(X-1,Y,X,Y)
         RIA1(1) = IDX(X-1,Y)
         CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +      IA,1,RAS,RIA1,RIA2,RINFOA)
C          !       (x,y-1)
         RAS(1)  = COEFF(X,Y-1,X,Y)
         RIA1(1) = IDX(X,Y-1)
         CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +      IA,1,RAS,RIA1,RIA2,RINFOA)
C          !       (x,y)
         RAS(1)  = COEFF(X,Y,X,Y)
         RIA1(1) = IDX(X,Y)
         CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +      IA,1,RAS,RIA1,RIA2,RINFOA)

C          !       (x,y+1)
         RAS(1)  = COEFF(X,Y+1,X,Y)
         RIA1(1) = IDX(X,Y+1)
         CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +      IA,1,RAS,RIA1,RIA2,RINFOA)

C          !       (x+1,y)
         RAS(1)  = COEFF(X+1,Y,X,Y)
         RIA1(1) = IDX(X+1,Y)
         CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +      IA,1,RAS,RIA1,RIA2,RINFOA)

      END DO
            .
            .
            .
```

PDGEINS—Inserts Local Data into a Dense Vector

This sparse utility subroutine is used by each process to insert all blocks of data it owns into its local part of the dense vector.

Syntax

Fortran	CALL PDGEINS (*nx*, *x*, *ldx*, *ix*, *jx*, *mb*, *nb*, *blcks*, *ldb*, *desc_a*)
C and C++	pdgeins (*nx*, *x*, *ldx*, *ix*, *jx*, *mb*, *nb*, *blcks*, *ldb*, *desc_a*);

On Entry

nx
is the number of columns in the local dense vector.

Scope: **local**

Specified as: fullword integer; *nx* = 1.

x
See On Return.

ldx
is the local leading dimension of the local array.

Scope: **local**

Specified as: fullword integer; $ldx \geq \max(1,N\_ROW)$.

ix
is the first global row index of the dense vector that receives data from the submatrix ***BLCK***.

Scope: **local**

Specified as: a fullword integer; $1 \leq ix \leq M$.

jx
is the first global column index of the dense vector that receives data from the submatrix ***BLCK***.

Scope: **local**

Specified as: fullword integer; *jx* = 1.

mb
is the number of local rows to be inserted into the dense vector.

Scope: **local**

Specified as: fullword integer; $1 \leq mb \leq \min(N\_ROW, ldb)$.

nb
is the number of local columns to be inserted into the dense vector.

Scope: **local**

Specified as: fullword integer; *nb* = 1.

blcks
is the local part, referred to as BLCKS, of the submatrix ***BLCK***, containing the coefficients to be inserted into the dense vector. Each call to this subroutine inserts one contiguous block of data into the local part of the dense vector corresponding to the global submatrix $\boldsymbol{X}_{ix:ix+mb-1,jx:jx+nb-1}$.

Scope: **local**

Specified as: an *ldb* by (at least) *nb* array, containing long-precision real numbers.

ldb

is the local leading dimension for the local array `BLCKS`.

Scope: **local**

Specified as: fullword integer; *ldb* ≥ max(1,*mb*).

desc_a

is the array descriptor that is produced on a preceding call to PADINIT, PDSPINIT, or PDSPINS.

Specified as: an array of length DLEN, containing fullword integers.

On Return

x

is the updated local part of the dense vector.

Scope: **local**

Returned as: an *ldx* by (at least) *nx* array, containing long-precision real numbers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PADINIT.
2. You do not need a separate array descriptor for a dense vector because it must conform to the size of matrix ***A***. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
3. This subroutine must be called for:
   - Vector ***b*** containing the right-hand side.
   - Vector ***x*** containing the initial guess to the solution.
4. Each process has to call PDGEINS as many times as necessary to insert the local elements it owns. It is also possible to call PDGEINS multiple times to insert different coefficients of the same local row it owns. Duplicate coefficients are overwritten.

Error Conditions

Computational Errors: None

Resource Errors

1. None.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not *np* $\times$ 1.
2. *nb* $\neq$ 1
3. *nx* $\neq$ 1
4. *jx* $\neq$ 1
5. *desc_a* is not valid.

Stage 4

1. *ldx* $<$ max(1,N_ROW)
2. 1 $<$ *mb* or *mb* $>$ N_ROW
3. *ldb* $<$ max(1,*mb*)
4. *ix* $<$ 1 or *ix* $>$ M

Stage 5

1. One or more elements to be inserted into the submatrix ***BLCK*** does not belong to the process.

PDSPASB—Assembles a General Sparse Matrix

This sparse utility subroutine uses the output from PDSPINS to assemble the global general sparse matrix ***A*** and its array descriptor *desc_a*.

Syntax

Fortran	CALL PDSPASB (*as*, *ia1*, *ia2*, *infoa*, *desc_a*, *mtype*, *stor*, *dupflag*, *info*)
C and C++	pdspasb (*as*, *ia1*, *ia2*, *infoa*, *desc_a*, *mtype*, *stor*, *dupflag*, *info*);

On Entry

as

is the local part of the global general sparse matrix ***A*** that is produced by previous call(s) to PDSPINS.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1

is the local part of array `IA1` that is produced by previous call(s) to PDSPINS.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2

is the local part of array `IA2` that is produced by previous call(s) to PDSPINS.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa

is the array `INFOA` that is produced by previous call(s) to PDSPINS.

Specified as: an array of length 30, containing fullword integers.

desc_a

is the array descriptor for the global general sparse matrix ***A*** that is produced by previous call(s) to PDSPINS.

Specified as: an array of length DLEN, containing fullword integers.

mtype

indicates the the form of the global sparse matrix ***A*** used, where:

If *mtype* = `'GEN'`, ***A*** is a general sparse matrix.

Scope: **global**

Specified as: a character variable of length 5; *mtype* = `'GEN'`.

stor

indicates the storage mode that the global general sparse matrix ***A*** is returned in, where:

If *stor* = `'DEF'`, this subroutine chooses an appropriate storage mode, which is an internal format accepted by the preconditioner and solver subroutines, for storing the global general sparse matrix ***A*** on output.

If *stor* = `'CSR'`, the global general sparse matrix ***A*** is stored in the storage-by-rows storage mode on output.

Scope: **global**

Specified as: a character variable of length 5; *stor* = `'DEF'` or `'CSR'`.

dupflag
: is a flag indicating how to use coefficients that are specified more than once on the same process; that is, duplicate coefficients within the same local part of the matrix ***A***:

  If *dupflag* = 0, this subroutine uses the first of the duplicate coefficients.

  If *dupflag* = 1, this subroutine adds all the duplicate coefficients with the same indices.

  If *dupflag* = 2, this subroutine raises an error condition indicating that there are unexpected duplicate coefficients.

  Scope: **global**

  Specified as: a fullword integer; *dupflag* = 0, 1, or 2.

info
: See On Return.

On Return

as
: is the updated local part of array `AS` of the global general sparse matrix ***A***, where:

  If *stor* = `'DEF'`, this subroutine chooses an appropriate storage mode, which is an internal format accepted by the preconditioner and solver subroutines, for storing the global general sparse matrix ***A*** on output.

  If *stor* = `'CSR'`, the global general sparse matrix ***A*** is stored in the storage-by-rows storage mode on output.

  Scope: **local**

  Returned as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1
: is the updated local part of array `IA2`.

  Scope: **local**

  Returned as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2
: is the updated local part of array `IA2`.

  Scope: **local**

  Returned as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa
: is the updated array `INFOA`.

  Returned as: an array of length 30, containing fullword integers.

desc_a
: is the final updated array descriptor for the global general sparse matrix ***A***.

  Returned as: an array of length DLEN, containing fullword integers.

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then one or more of the following computational errors occurred and the appropriate error messages were issued, indicating an error exit, where:

- If *info* = 1, the sparse matrix ***A*** contains duplicate coefficients.
- If *info* = 2, the sparse matrix ***A*** contains empty row(s).

Scope: **global**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. In your C program, *info* must be passed by reference.
2. This subroutine accepts mixed case letters for the *mtype* and *stor* arguments.
3. Before you call this subroutine, you must have called PDSPINS as many times as needed; that is, you must have completed building the matrix with call(s) to PDSPINS before you place a call to this subroutine.
4. Your program must declare *mtype* and *stor* to be characters of length 5 with blanks padded to the right. C programs can use the fifth character for the null terminator.
5. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
6. For details about some of the elements stored in *infoa*, see Table 27 on page 63.

Error Conditions

Computational Errors: The sparse matrix ***A*** contains duplicate coefficients or empty row(s). For details, see the description of the *info* argument.

Resource Errors

1. Unable to allocate work space.
2. Unable to deallocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not *np* × 1.
2. *desc_a* is not valid.
3. The sparse matrix ***A*** is not valid.
4. *mtype* ≠ `'GEN'`
5. *stor* ≠ `'DEF'` or `'CSR'`
6. *dupflag* ≠ 0, 1, or 2
7. Some local rows in the sparse matrix ***A*** are missing.

Stage 4

1. DLEN is too small. For valid values, see "Array Descriptor" on page 63.
2. `INFOA(1)` < max(2,*nnze*); that is, the size of `AS` < max(2,*nnze*)
3. `INFOA(2)` < max(3,(*nnze*+N_ROW)); that is, the size of `IA1` < max(3,(*nnze*+N_ROW))
4. `INFOA(3)` < max(3,(*nnze*+N_COL)); that is, the size of `IA2` < max(3,(*nnze*+N_COL))

Stage 5

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   mtype differs.
   stor differs.
   dupflag differs.

PDGEASB—Assembles a Dense Vector

This sparse utility subroutine assembles a dense vector.

Syntax

Fortran	CALL PDGEASB (*nx*, *x*, *ldx*, *desc_a*)
C and C++	pdgeasb (*nx*, *x*, *ldx*, *desc_a*);

On Entry

nx
is the number of columns in the local dense vector.

Scope: **local**

Specified as: fullword integer; *nx* = 1.

x
is the local part of the dense matrix ***x*** produced by previous call(s) to PDGEINS.

Scope: **local**

Specified as: an *ldx* by (at least) *nx* array, containing long-precision real numbers.

ldx
is the local leading dimension of the dense matrix.

Scope: **local**

Specified as: fullword integer; *ldx* ≥ max(1,N_ROW).

desc_a
is the array descriptor, which was finalized in a preceding call to PDSPASB.

Specified as: an array of length DLEN, containing fullword integers.

On Return

x
is the updated local part of the dense matrix.

Scope: **local**

Returned as: an *ldx* by (at least) length *nx*, containing long-precision real numbers.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PDGEINS as many times as needed; that is, you must have completed building the dense vectors with call(s) to PDGEINS before you place a call to this subroutine.

   Before you call this subroutine, you must have called PDSPASB.

2. You do not need a separate array descriptor for a dense vector because it must conform to the size of matrix ***A***. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.

3. This subroutine must be called for:

   - Vector ***b*** containing the right-hand side.
   - Vector ***x*** containing the initial guess to the solution.

Error Conditions

Computational Errors: None

Resource Errors: None.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. *desc_a* is not valid.

Stage 4

1. $ldx < \max(1,N\_ROW)$

PDSPGPR—Preconditioner for a General Sparse Matrix

This subroutine computes a preconditioner for the global general sparse matrix ***A*** that should be passed unchanged to the PDSPGIS subroutine. The preconditioners include diagonal scaling or an incomplete LU factorization.

Syntax

Fortran	CALL PDSPGPR (*iprec*, *as*, *ia1*, *ia2*, *infoa*, *prcs*, *lprcs*, *desc_a*, *info*)
C and C++	pdspgpr (*iprec*, *as*, *ia1*, *ia2*, *infoa*, *prcs*, *lprcs*, *desc_a*, *info*);

On Entry

iprec

is a flag that determines the type of preconditioning, where:

If *iprec* = 0, which is referred to as *none*, indicates the local part of the submatrix ***A*** is not preconditioned. PDSPGIS may not be effective in this case, unless the coefficient matrix is well conditioned; if your input matrix is not well conditioned, you should consider using *iprec* = 1 or 2.

If *iprec* = 1, which is referred to as *diagsc*, indicates the local part of the submatrix ***A*** is preconditioned by a local diagonal submatrix.

If *iprec* = 2, which is referred to as *ilu*, indicates the local part of the submatrix ***A*** is preconditioned by a local incomplete LU factorization.

It is suggested that you use a preconditioner. For an explanation, see "Notes and Coding Rules" on page 677.

Scope: **global**

Specified as: a fullword integer, where: *iprec* = 0, 1, or 2.

as

is the local part of the global general sparse matrix ***A***, finalized on a preceding call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1

is the local part of array `IA1` produced by a previous call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2

is the local part of array `IA2` produced by a previous call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa

is the array `INFOA` produced by a previous call to PDSPASB.

Specified as: an array of length 30, containing fullword integers.

prcs

See On Return.

lprcs
is the length of array PRCS.

Scope: **local**

Specified as: fullword integer, where:
If $iprec = 0$, $lprcs \geq 10$.
If $iprec = 1$, $lprcs \geq 10+\text{N\_ROW}$.
If $iprec = 2$, $lprcs \geq 10+2(nnz)+\text{N\_ROW}+\text{N\_COL}+31$

nnz is the number of non-zero elements (without duplicate coefficients) in the local part of the global general sparse matrix ***A***.

desc_a
is the array descriptor for the global general sparse matrix ***A*** that was finalized in a call to PDSPASB.

Specified as: an array of length DLEN, containing fullword integers.

info
See On Return.

On Return

prcs
is the preconditioner data structure that must be pass unchanged to PDSPGIS.

Scope: **local**

Returned as: a one-dimensional array of (at least) length *lprcs*, containing long-precision real numbers.

info
has the following meaning, when *info* is **present**:

If $info = 0$, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If $info > 0$, the value stored in *info* indicates the row index in the global general sparse matrix ***A*** where the preconditioner failed.

Scope: **global**

Returned as: a fullword integer; $info \geq 0$.

Notes and Coding Rules

1. Before you call this subroutine, you must have called PDGEASB and PDSPASB.
2. In your C program, *info* must be passed by reference.
3. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
4. For details about some of the elements stored in *infoa* see Table 27 on page 63.
5. The convergence rate of an iterative method as applied to a given system of linear equations depends on the spectral properties of the coefficient matrix of the linear system; therefore it is often convenient to apply a linear

transformation to the system such that the solution of the transformed system is the same (in exact arithmetic) as that of the original, but the spectral properties and the convergence behavior are more favorable. Such a transformation is called preconditioning. If a matrix ***M*** approximates ***A***, then:

$(\boldsymbol{M}^{-1})\boldsymbol{Ax} = (\boldsymbol{M}^{-1})\boldsymbol{b}$

is a preconditioned system and ***M*** is called a preconditioner. In practice, the new coefficient matrix $(\boldsymbol{M}^{-1})\boldsymbol{A}$ is almost never formed explicitly, but rather its action is computed during the application of the iterative method. The effectiveness of the preconditioning operation depends on a trade-off between how well ***M*** approximates ***A*** and how costly it is to compute and invert it; no single preconditioner will give best overall performance under all situations. Note finally that it is quite rare for a linear system to behave well enough so as not to require preconditioning; indeed most linear systems originating from the the discretization of difficult physical problems require preconditioning to have any convergence at all.

Error Conditions

Computational Errors

1. The preconditioner for the sparse matrix ***A*** is unstable. For details, see the *info* output argument for this subroutine.

Resource Errors

1. Unable to allocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. *desc_a* is not valid.
3. *iprec* $\neq$ 0, 1, or 2
4. *iprec* $=$ 0 and *lprcs* $<$ 10
5. *iprec* $=$ 1 and *lprcs* $<$ 10+N_ROW
6. *iprec* $=$ 2 and *lprcs* $<$ 10+2(*nnz*)+N_ROW+N_COL+31
7. The storage format for ***A*** is not supported.

Stage 4

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   iprec differs.

PDSPGIS—Iterative Linear System Solver for a General Sparse Matrix

This subroutine solves a general sparse linear system of equations, using an iterative algorithm, with or without preconditioning. The methods include the more smoothly converging variant of the CGS method (Bi-CGSTAB), conjugate gradient squared (CGS), or transpose-free quasi-minimal residual method (TFQMR).

See references [7], [9], [12], and [35].

Syntax

Fortran	CALL PDSPGIS (*as*, *ia1*, *ia2*, *infoa*, *nrhs*, *b*, *ldb*, *x*, *ldx*, *prcs*, *desc_a*, *iparm*, *rparm*, *info*)
C and C++	pdspgis (*as*, *ia1*, *ia2*, *infoa*, *nrhs*, *b*, *ldb*, *x*, *ldx*, *prcs*, *desc_a*, *iparm*, *rparm*, *info*);

On Entry

as

is the local part of the global general sparse matrix ***A***, finalized on a preceding call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(1)`, containing long-precision real numbers.

ia1

is the local part of array `IA1` produced by a previous call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(2)`, containing fullword integers.

ia2

is the local part of array `IA2` produced by a previous call to PDSPASB.

Scope: **local**

Specified as: a one-dimensional array of (at least) length `INFOA(3)`, containing fullword integers.

infoa

is the array `INFOA` produced by a previous call to PDSPASB.

Specified as: an array of length 30, containing fullword integers.

nrhs

the number of right-hand sides.

Scope: **global**

Specified as: a fullword integer; *nrhs* = 1.

b

is the local part of the matrix ***b***, containing the right-hand side of the matrix problem produced on a previous call to PDGEASB.

Scope: **local**

Specified as: an *ldb* by (at least) length *nrhs* array, containing long-precision real numbers.

ldb

is the leading dimension of the local array `B`.

Scope: **local**

Specified as: a fullword integer; *ldb* ≥ max(1,N_ROW)

x

is the local part of the global vector ***x***, containing the initial guess to the solution of the linear system and produced on a previous call to PDGEASB.

Scope: **local**

Specified as: an *ldx* by (at least) *nrhs* array, containing long-precision real numbers.

ldx

is the leading dimension of the local array X.

Scope: **local**

Specified as: a fullword integer; *ldx* ≥ max(1,N_ROW)

prcs

is the preconditioner data structure *prcs* produced by a previous call to PDSPGPR.

Scope: **local**

Specified as: a one-dimensional array of (at least) length *lprcs*, containing long-precision real numbers.

desc_a

is the array descriptor for the global general sparse matrix ***A*** that was finalized in a call to PDSPASB.

Specified as: an array of length DLEN, containing fullword integers.

iparm

is an array of parameters, IPARM(*i*), where:

- IPARM(1) is the flag, *methd*, used to select the iterative procedure used, where:

  If IPARM(1) = 0, the following defaults are used:

  methd = 1
  istopc = 1
  itmax = 500
  itrace = 0
  eps = 10^{-8}

  If *methd* = 1, the more smoothly converging variant of the CGS method, referred to as Bi-CGSTAB, is used.

  If *methd* = 2, the conjugate gradient squared method, referred to as CGS, is used.

  If *methd* = 3, the transpose-free quasi-minimal residual method, referred to as TFQMR, is used.

- IPARM(2) is the flag, *istopc* used to select the stopping criterion used in the computation, where the following items are used in the definitions of the stopping criteria below:

  – ε is the desired relative accuracy and is stored in *eps*

  – x_j is the solution found at the *j*-th iteration.

  – r_j and r_0 are the preconditioned residuals obtained at iterations *j* and 0, respectively. (The residual at iteration *j* is defined as $\boldsymbol{b}-\boldsymbol{A}x_j$.)

  If *istopc* = 1, the iterative method is stopped when:

$\|r_j\|_2 / \|x_j\|_2 < \varepsilon$

If *istopc* = 2, the iterative method is stopped when:

$\|r_j\|_2 / \|r_0\|_2 < \varepsilon$

If *istopc* = 3, the iterative method is stopped when:

$\|x_j - x_{j-1}\|_2 / \|x_j\|_2 < \varepsilon$

Note: Stopping criterion 3 performs poorly with the TFQMR method; therefore, if you specify TFQMR (*methd* = 3), you should not specify stopping criterion 3.

- IPARM(3) is the maximum number, *itmax*, of iterations allowed.
- IPARM(4), referred to as *itrace*, has the following meaning:

  If *itrace* = 0, then *itrace* is ignored.

  If *itrace* > 0, an informational message about convergence, which is based on the stopping criterion described in *istopc*, is issued at every *itrace*-th iteration and upon exit.
- IPARM(5), see On Return.
- IPARM(6) through IPARM(20) are reserved.

Scope: **global**

Specified as: an array of length 20, containing fullword integers, where:
methd = 1, 2, or 3
istopc = 1, 2, or 3.
itmax ≥ 0.
itrace ≥ 0.
IPARM(6) through IPARM(20) should be set to zero.

rparm

is an array of parameters, RPARM(*i*), where:

- RPARM(1) is the relative accuracy ϵ, referred to as *eps*, used in the stopping criterion.
- RPARM(2), see On Return.
- RPARM(3) through RPARM(20) are reserved.

Scope: **global**

Specified as: an array of length 20, containing long-precision real numbers, where:
eps ≥ 0.
RPARM(3) through RPARM(20) should be set to zero.

info

See On Return.

On Return

x

is the local part of the solution vector ***x***

Scope: **local**

Returned as: an array of (at least) length N_ROW, containing long-precision real numbers.

iparm

is an array of parameters, `IPARM(`*i*`)`, where:

- `IPARM(2)` is the number of iterations, *iter*, performed by this subroutine.

Scope: **global**

Returned as: an array of length 20, containing fullword integers, where:
iter ≥ 0

rparm

is an array of parameters, `RPARM(`*i*`)`, where:

- `RPARM(2)` contains the estimate of the error, *err*, of the solution, according to the stopping criterion, *istopc*, in use. For details, see the *istopc* argument description.

Scope: **global**

Returned as: an array of length 20, containing long-precision real numbers, where:
err ≥ 0

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then this subroutine exceeded *itmax* iterations without converging. You may want to try the following to get your matrix to converge:

1. You can increase the number of iterations and call this subroutine again without making any other changes to your program.
2. You can change the requested precision and/or the stopping criterion; your original precision requirement may be too stringent under a given stopping criterion.
3. You can use a preconditioner if you were not already doing so, or to change the one you were using. Note also that the efficiency of the preconditioner may depend on the data distribution strategy adopted. See “Notes and Coding Rules” on page 677.

Scope: **global**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. In your C program, *info* must be passed by reference.
2. Before you call this subroutine, you must have called PDSPGPR.
3. For more details about N_ROW, N_COL, and other elements of *desc_a*, see Table 28 on page 64.
4. For details about some of the elements stored in *infoa* see Table 27 on page 63.

Error Conditions

Computational Errors

1. This subroutine exceeded *itmax* iterations without converging. Vector ***x*** contains the approximate solution computed at the last iteration.

   Note: If the preconditioner computed by PDSPGPR failed because the sparse matrix ***A*** is unstable, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PDSPGPR.

   You may want to try the following to get your matrix to converge:

   a. You can increase the number of iterations and call this subroutine again without making any other changes to your program.
   b. You can change the requested precision and/or the stopping criterion; your original precision requirement may be too stringent under a given stopping criterion.
   c. You can use a preconditioner if you were not already doing so, or to change the one you were using. Note also that the efficiency of the preconditioner may depend on the data distribution strategy adopted. See “Notes and Coding Rules” on page 677.

Resource Errors

1. Unable to allocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. The BLACS context is invalid.

Stage 2

1. This subroutine was called from outside the process grid.

Stage 3

1. The process grid is not $np \times 1$.
2. *desc_a* is not valid.
3. $nrhs \neq 1$
4. $eps < 0.0$
5. $methd \neq 1, 2,$ or 3
6. The preconditioner data structure *prcs* is not valid.
7. $istopc \neq 1, 2,$ or 3
8. $itmax < 0$
9. $itrace < 0$

10. The sparse matrix ***A*** is not valid.
11. The storage format for the sparse matrix ***A*** is not supported.

Stage 4

1. *ldb* $<$ max(1,N_ROW)
2. *ldx* $<$ max(1,N_ROW)
3. The preconditioner data structure *prcs* is not valid.

Stage 5

1. Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:
   iparm differs.
   rparm differs.
   eps differs.
   methd differs.
   istopc differs.
   itmax differs.
   itrace differs.
   Some element(s) of *prcs* differ.

Example—Using the Fortran 77 Sparse Subroutines

This example finds the solution to the linear system ***Ax*** = ***b***. It also contains an application program that shows how you can use the Fortran 77 sparse linear algebraic equation subroutines and their utilities to solve the problem shown in "Example—Using the Fortran 90 Sparse Subroutines" on page 649.

Application Program: This application program illustrates how to use the Fortran 77 sparse linear algebraic equation subroutines and their utilities.

```
!
! This program illustrates how to use the PESSL F77 Sparse Iterative
! Solver and its supporting utility subroutines.  A very simple problem
! (DSRIS Example 1 from the ESSL Guide and Reference) using an
! HPF BLOCK data distribution is solved.
!
      PROGRAM EXAMPLE77

      IMPLICIT NONE

! Interface definition for the PARTS subroutine PART_BLOCK

      INTERFACE PART_BLOCK
       SUBROUTINE PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
       IMPLICIT NONE
       INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
       INTEGER, INTENT(OUT) :: NV
       INTEGER, INTENT(OUT) :: PV(*)

       END SUBROUTINE PART_BLOCK
      END INTERFACE

! External declaration for the PARTS subroutine PART_BLOCK
      EXTERNAL              PART_BLOCK

! Parameters
      CHARACTER*1           ORDER
      CHARACTER*5           STOR
      CHARACTER*5           MTYPE
      INTEGER*4             IZERO, IONE, DUPFLAG, N, NNZ
      PARAMETER (ORDER='R')
      PARAMETER (STOR='DEF')
      PARAMETER (MTYPE='GEN')
      PARAMETER (IZERO=0)
      PARAMETER (IONE=1)
      PARAMETER (N=9)
      PARAMETER (NNZ=22)
      PARAMETER (DUPFLAG=2)

! Descriptor Vector
      INTEGER*4, ALLOCATABLE :: DESC_A(:)

! Sparse Matrices and related information
      REAL*8                AS(NNZ)
      INTEGER*4             IA1(NNZ+N), IA2(NNZ+N)
      INTEGER*4             INFOA(30)

      REAL*8                BS(NNZ)
      INTEGER*4            IB1(N+1), IB2(NNZ)
      INTEGER*4            INFOB(30)

! Preconditioner Data Structure
      REAL*8               PRCS(2*NNZ+2*N+41)

! Dense Vectors
      REAL*8               B(N), X(N)

! BLACS parameters
      INTEGER*4            NPROW, NPCOL, ICTXT, IAM, NP, MYROW, MYCOL

! Solver parameters
      INTEGER*4            ITER, ITMAX, INFO, ITRACE,
     &                     IPREC, METHD, ISTOPC, IPARM(20)
      REAL*8               ERR, EPS, RPARM(20)

! We will not have duplicates so PV used by the PARTS subroutine
! PART_BLOCK only needs to be of length 1.

      INTEGER              PV(1)

! Other variables
      INTEGER              IERR
      INTEGER              NB, LDB, LDBG
      INTEGER              NX, LDX, LDXG
      INTEGER              NRHS
      INTEGER              I,J
      INTEGER              GLOBAL_INDX, NV_COUNT
      INTEGER              GLOBAL_INDX_OWNER, NV
      INTEGER              LOCAL_INDX
!
!     Global Problem
!      DSRIS Example 1 from the ESSL Guide and Reference
!
      REAL*8              A_GLOBAL(NNZ),B_GLOBAL(N),XINIT_GLOBAL(N)
      INTEGER             JA(NNZ),IA(N+1)
      DATA A_GLOBAL      /2.D0,2.D0,-1.D0,1.D0,2.D0,1.D0,2.D0,-1.D0,
     $                    1.D0,2.D0,-1.D0,1.D0,2.D0,-1.D0,1.D0,2.D0,
     $                   -1.D0,1.D0,2.D0,-1.D0,1.D0,2.D0/
      DATA JA            /1,2,3,2,3,1,4,5,4,5,6,5,6,7,6,7,8,
     $                    7,8,9,8,9/
      DATA IA            /1,2,4,6,9,12,15,18,21,23/

      DATA B_GLOBAL      /2.D0,1.D0,3.D0,2.D0,2.D0,2.D0,2.D0,2.D0,
     $                    3.D0/
      DATA XINIT_GLOBAL /0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,0.D0,
     $                    0.D0/

! Initialize BLACS
! Define a NP x 1 Process Grid

      CALL BLACS_PINFO(IAM, NP)
      CALL BLACS_GET(IZERO, IZERO, ICTXT)
      CALL BLACS_GRIDINIT(ICTXT, ORDER, NP, IONE)
      CALL BLACS_GRIDINFO(ICTXT, NPROW, NPCOL, MYROW, MYCOL)
!
```

```
! Allocate the descriptor vector
!
      ALLOCATE(DESC_A(30 + 3*NP + 4*N + 3),STAT=IERR)
      IF (IERR .NE. 0) THEN
        WRITE(6,*) 'Error allocating DESC_A :',IERR
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
      END IF

! Initialize some elements of the sparse matrix A
! and its descriptor vector, DESC_A
!

      DESC_A(11) = SIZE(DESC_A)
      CALL PADINIT(N,PART_BLOCK,DESC_A,ICTXT)

      INFOA(1) = SIZE(AS)
      INFOA(2) = SIZE(IA1)
      INFOA(3) = SIZE(IA2)
      CALL PDSPINIT(AS,IA1,IA2,INFOA,DESC_A)

!
! In this simple example, all processes have a copy of
! the global sparse matrix, A, the global rhs vector B,
! and the global initial guess vector, X
!
! Each process will call PDSPINS as many times as necessary
! to insert the local rows it owns.
!
! Each process will call PDGEINS as many times as necessary
! to insert the local elements it owns.
!
      NB = 1
      LDB = SIZE(B,1)
      LDBG = SIZE(B_GLOBAL,1)
      NX = 1
      LDX = SIZE(X,1)
      LDXG = SIZE(XINIT_GLOBAL,1)

      DO GLOBAL_INDX = 1, N
       CALL PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
!
! In this simple example, NV will always be 1
! since there will not be duplicate coefficients
!
       DO NV_COUNT = 1, NV
        GLOBAL_INDX_OWNER = PV(NV_COUNT)
        IF (GLOBAL_INDX_OWNER == MYROW) THEN
         IB2(1) = 1
         IB2(2) = 1
         DO J = IA(GLOBAL_INDX), IA(GLOBAL_INDX+1)-1
          BS(IB2(2)) = A_GLOBAL(J)
          IB1(IB2(2)) = JA(J)
          IB2(2) = IB2(2) + 1
         ENDDO
         INFOB(1) = IB2(2) - 1
         INFOB(2) = IB2(2) - 1
         INFOB(3) = 2
```

```
            INFOB(4) = 1
            INFOB(5) = 1
            INFOB(6) = 1
            INFOB(7) = N
            CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,GLOBAL_INDX, 1,
     &                   BS,IB1,IB2,INFOB)
            CALL PDGEINS(NB,B,LDB,GLOBAL_INDX,1,1,1,
     &                   B_GLOBAL(GLOBAL_INDX),LDBG,DESC_A)
            CALL PDGEINS(NX,X,LDX,GLOBAL_INDX,1,1,1,
     &                   XINIT_GLOBAL(GLOBAL_INDX),LDXG,DESC_A)
           ENDIF
          END DO
         END DO

! Assemble A and DESC_A
      CALL PDSPASB(AS,IA1,IA2,INFOA,DESC_A,
     &             MTYPE,STOR,DUPFLAG,INFO)

      IF (INFO .NE. 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in assembly :',INFO
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

! Assemble B and X

      CALL PDGEASB(NB,B,LDB,DESC_A)
      CALL PDGEASB(NX,X,LDX,DESC_A)

!
!  Preconditioning
!
!  We are using ILU for the preconditioner
!
      IPREC = 2

      CALL PDSPGPR(IPREC,AS,IA1,IA2,INFOA,
     &             PRCS,SIZE(PRCS),DESC_A,INFO)

      IF (INFO .NE. 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in preconditioner :',INFO
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

!
!  Iterative Solver - use the BICGSTAB method
!
      NRHS  = 1
      ITMAX = 1000
      EPS   = 1.D-8
      METHD = 1
      ISTOPC = 1
      ITRACE = 0
      IPARM = 0
      IPARM(1) = METHD
      IPARM(2) = ISTOPC
      IPARM(3) = ITMAX
      IPARM(4) = ITRACE
      RPARM = 0.0D0
      RPARM(1) = EPS

      CALL PDSPGIS(AS,IA1,IA2,INFOA,NRHS,B,LDB,X,LDX,PRCS,DESC_A,
     &             IPARM,RPARM,INFO)

      IF (INFO .NE. 0) THEN
       IF (IAM.EQ.0) THEN
        WRITE(6,*) 'Error in solver :',INFO
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
       END IF
      END IF

      ERR  = RPARM(2)
      ITER = IPARM(5)
      IF (IAM.EQ.0) THEN
       WRITE(6,*) 'Number of iterations : ',ITER
       WRITE(6,*) 'Error on exit        : ',ERR
      END IF

!
!  Each process prints their local piece of the solution vector
!
      IF (IAM.EQ.0) THEN
       Write(6,*) 'Solution Vector X'
      END IF

      LOCAL_INDX = 1
      Do GLOBAL_INDX = 1, N
       CALL PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
!
! In this simple example, NV will always be 1
! since there will not be duplicate coefficients
!
       DO NV_COUNT = 1, NV
        GLOBAL_INDX_OWNER = PV(NV_COUNT)
        IF (GLOBAL_INDX_OWNER == MYROW) THEN
         Write(6,*) GLOBAL_INDX, X(LOCAL_INDX)
         LOCAL_INDX = LOCAL_INDX +1
        ENDIF
       END DO
      END DO

!
! Deallocate the descriptor vector
!
      DEALLOCATE(DESC_A, STAT=IERR)
      IF (IERR .NE. 0) THEN
        WRITE(6,*) 'Error deallocating DESC_A :',IERR
        CALL BLACS_ABORT(ICTXT,-1)
        STOP
      END IF
!
!  Terminate the process grid and the BLACS
!
       CALL BLACS_GRIDEXIT(ICTXT)
       CALL BLACS_EXIT(0)

       END PROGRAM EXAMPLE77
```

Chapter 9. Eigensystem Analysis and Singular Value Analysis (Message Passing)

This chapter describes the eigensystem analysis and singular value analysis subroutines.

Overview of the Eigensystem Analysis and Singular Value Analysis Subroutines

The eigensystem analysis and singular value analysis subroutines include a subset of the ScaLAPACK subroutines. See references [19] and [20].

Note: These subroutines are designed in accordance with the proposed ScaLAPACK standard. If these subroutines do no comply with the standard as approved, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 99. List of Eigensystem Analysis and Singular Value Analysis Subroutines (Message Passing)

Descriptive Name	**Long-Precision Subroutine**	**Page**
Selected Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix	PDSYEVX	693
Reduce a Real Symmetric Matrix to Tridiagonal Form	PDSYTRD	711
Reduce a General Matrix to Upper Hessenberg Form	PDGEHRD	722
Reduce a General Matrix to Bidiagonal Form	PDGEBRD	732

Eigensystem Analysis and Singular Value Analysis Subroutines

This section contains the eigensystem analysis subroutine descriptions.

PDSYEVX—Selected Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix

This subroutine computes selected eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A***, where ***A*** represents the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$. Eigenvalues and eigenvectors can be selected by specifying a range of values or a range of indices for the eigenvalues.

If *n* = 0, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13], [24], [25], and [26].

Table 100. Data Types

A, *vl*, *vu*, *abstol*, *orfac*, ***Z***, ***w***, *work*, ***gap***	*iwork*, ***ifail***, ***iclustr***	**Subroutine**
Long-precision real	Integer	PDSYEVX

Syntax

Fortran	CALL PDSYEVX (*jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *desc_z*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*)
C and C++	pdsyevx (*jobz*, *range*, *uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *vl*, *vu*, *il*, *iu*, *abstol*, *m*, *nz*, *w*, *orfac*, *z*, *iz*, *jz*, *desc_z*, *work*, *lwork*, *iwork*, *liwork*, *ifail*, *iclustr*, *gap*, *info*);

On Entry

jobz
indicates the type of computation to be performed, where:

If *jobz* = `'N'`, eigenvalues only are computed.

If *jobz* = `'V'`, eigenvalues and eigenvectors are computed.

Scope: **global**

Specified as: a single character; *jobz* = `'N'` or `'V'`.

range
indicates which eigenvalues to compute, where:

If *range* = `'A'`, all eigenvalues are to be found.

If *range* = `'V'`, all eigenvalues in the interval [*vl*, *vu*] are to be found.

If *range* = `'I'`, the *il*-th through *iu*-th eigenvalues are to be found.

Scope: **global**

Specified as: a single character; *range* = `'A'`, `'V'`, or `'I'`.

uplo
indicates whether the upper or lower triangular part of the global symmetric submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the order of submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

a

is the local part of the global symmetric matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ia+n-1$) by LOCq($ja+n-1$) part of the local array A must contain the local pieces of the leading $ia+n-1$ by $ja+n-1$ part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the symmetric matrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part of the symmetric matrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 100 on page 693. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global

desc_a	**Name**	**Description**	**Limits**	**Scope**
6	NB_A	Column block size	NB_A $\geq$ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

vl

has the following meaning:

If *range* = 'V', it is the lower bound of the interval to be searched for eigenvalues.

If *range* ≠ 'V', this argument is ignored.

Scope: **global**

Specified as: a number of the data type indicated in Table 100 on page 693. If *range* = 'V', $vl < vu$.

vu

has the following meaning:

If *range* = 'V', it is the upper bound of the interval to be searched for eigenvalues.

If *range* ≠ 'V', this argument is ignored.

Scope: **global**

Specified as: a number of the data type indicated in Table 100 on page 693. If *range* = 'V', $vl < vu$.

il

has the following meaning:

If *range* = 'I', it is the index (from smallest to largest) of the smallest eigenvalue to be returned.

If *range* ≠ 'I', this argument is ignored.

Scope: **global**

Specified as: a fullword integer; $il \geq 1$.

iu

has the following meaning:

If *range* = 'I', it is the index (from smallest to largest) of the largest eigenvalue to be returned.

If *range* ≠ 'I', this argument is ignored.

Scope: **global**

Specified as: a fullword integer; $\min(il, n) \leq iu \leq n$.

abstol

is the absolute tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*, *b*] of width less than or equal to:

abstol+ε(max(|*a*|, |*b*|))

where ε is the machine precision. If *abstol* is less than or equal to zero, then ε(norm(***T***)) is used in its place, where norm(***T***) is the 1-norm of the tridiagonal matrix obtained by reducing ***A*** to tridiagonal form. For most problems, this is the appropriate level of accuracy to request.

For certain strongly graded matrices, greater accuracy can be obtained in very small eigenvalues by setting *abstol* to a very small positive number. However, if *abstol* is less than:

$$\sqrt{unfl}$$

where *unfl* is the underflow threshold, then:

$$\sqrt{unfl}$$

is used in its place.

Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold—that is, (2)(*unfl*).

If *jobz* = `'V'`, then setting *abstol* to *unfl*, the underflow threshold, yields the most orthogonal eigenvectors.

Note:

- ε is approximately $0.222044604925031308\text{E}-15$
- $unfl$ is approximately $0.222507385850720138\text{E}-307$
- $\sqrt{unfl}$ is approximately $0.149166814624004135\text{E}-153$

Scope: **global**

Specified as: a number of the data type indicated in Table 100 on page 693.

m

See On Return.

nz

See On Return.

w

See On Return.

orfac

specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within:

ortol = (*orfac*)(norm(***A***))

of each other (where norm(***A***) is the 1-norm of ***A***) are to be reorthogonalized.

However, if the workspace is insufficient (see *lwork*), *ortol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process.

If *orfac* is zero, no reorthogonalization is done.

If *orfac* is less than zero, a default value of 10^{-3} is used.

Scope: **global**

Specified as: a number of the data type indicated in Table 100 on page 693.

z

See On Return.

iz

is the row index of the global matrix ***Z***, identifying the first row of the submatrix ***Z***.

Scope: **global**

Specified as: a fullword integer; $1 \le iz \le$ M_Z and $iz+n-1 \le$ M_Z.

jz

is the column index of the global matrix ***Z***, identifying the first column of the submatrix ***Z***.

Scope: **global**

Specified as: a fullword integer; $1 \le jz \le$ N_Z and $jz+n-1 \le$ N_Z.

desc_z

is the array descriptor for global matrix ***Z***, described in the following table:

desc_z	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_Z	Descriptor type	DTYPE_Z=1	Global
2	CTXT_Z	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_Z	Number of rows in the global matrix	If $n = 0$: M_Z ≥ 0 Otherwise: M_Z ≥ 1	Global
4	N_Z	Number of columns in the global matrix	If $n = 0$: N_Z ≥ 0 Otherwise: N_Z ≥ 1	Global
5	MB_Z	Row block size	MB_Z ≥ 1	Global
6	NB_Z	Column block size	NB_Z ≥ 1	Global
7	RSRC_Z	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_Z $< p$	Global
8	CSRC_Z	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_Z $< q$	Global
9	LLD_Z	The leading dimension of the local array	LLD_Z $\ge$ max(1,LOCp(M_Z))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is a work area used by this subroutine, where:

- If *lwork* ≠ -1, then its size is (at least) of length *lwork*.
- If *lwork* = -1, then its size is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 100 on page 693.

lwork
is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**.
- If *lwork* = -1, *lwork* is **global**.

Specified as: a fullword integer; where:

- If *lwork* = 0, PDSYEVX dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = -1, PDSYEVX performs a work area query and returns the minimum required size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, use the following rules to determine the value to specify:
  - If *jobz* = `'N'`, it must have the following value:

    $lwork \geq 3(n+ja-1)+2n+\max(5nn, (nb)(np+1))$
  - If *jobz* = `'V'`, then the amount of workspace required to guarantee that all eigenvectors are computed is:

    $lwork \geq 3(n+ja-1)+2n+\max(5nn, (np0)(mq0)+2(nb)(nb)) + \text{iceil}(neig, (nprow)(npcol))(nn)$

    where:

    nn = max(*n*, *nb*, 2)
    neig is the number of eigenvectors requested
    nb = MB_A = NB_A = MB_Z = NB_Z
    np = NUMROC(*nn*, *nb*, *myrow*, *iarow*, *nprow*)
    *np*0 = NUMROC(*nn*+*iroffz*, *nb*, *izrow*, *izrow*, *nprow*)
    *mq*0 = NUMROC(max(*neig*, *nb*, 2)+*icoffz*, *nb*, *izcol*, *izcol*, *npcol*)
    iarow = mod(RSRC_A + (*ia*–1)/*nb*, *nprow*)
    izrow = mod(RSRC_Z + (*iz*–1)/*nb*, *nprow*)
    izcol = mod(CSRC_Z + (*jz*–1)/*nb*, *npcol*)
    iroffz = mod(*iz*–1, MB_Z)
    icoffz = mod(*jz*–1, NB_Z)

  The computed eigenvectors may not be orthogonal if the minimum workspace is supplied and *ortol* is too small; therefore, if you want to guarantee orthogonality (at the cost of potentially compromising performance), you should add the following to *lwork*:

  $(clustersize-1)(n)$

  where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:

$\{w_k, ..., w_{k+iclustrsz-1} \mid w_{j+1} \le w_j + orfac(2)(\text{norm}(\mathbf{A}))\}$

Note: This subroutine does **not** add this amount when dynamically allocating this workspace. You must use static allocation if you want to guarantee orthogonality.

When *lwork* is too small:

- If *lwork* is too small to guarantee orthogonality, this subroutine attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.
- If *lwork* is too small to compute all the eigenvectors requested, no computation is performed, except that if *range* = 'V', this subroutine does not know how many eigenvectors are requested until the eigenvalues are computed. Therefore, if *range* = 'V' and *lwork* is large enough to allow this subroutine to compute the eigenvalues, this subroutine computes the eigenvalues and as many eigenvectors as it can.

Relationship between workspace, orthogonality, and performance:

If *clustersize* is:

$$clustersize \ge n / \sqrt{(nprow)(npcol)}$$

then providing enough space to compute all the eigenvectors orthogonally causes serious degradation in performance. In the limit (*clustersize* = *n*–1), performance may be no better than using one process.
If *clustersize* is:

$$clustersize = n / \sqrt{(nprow)(npcol)}$$

then reorthogonalizing all eigenvectors increases the total execution time by a factor of 2 or more.
If *clustersize* is:

$$clustersize > n / \sqrt{(nprow)(npcol)}$$

then execution time grows as the square of the cluster size, assuming all other factors remain equal and there is enough workspace. Less workspace means less reorthogonalization, but faster execution.

iwork
has the following meaning:

If *liwork* = 0, *iwork* is ignored.

If *liwork* ≠ 0, *iwork* is a work area used by this subroutine, where:

- If *liwork* ≠ -1, then its size is (at least) of length *liwork*.
- If *liwork* = -1, then its size is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing fullword integers.

liwork

is the number of elements in array `IWORK`.

Scope:

- If *liwork* ≥ 0, *liwork* is **local**.
- If *liwork* = -1, *liwork* is **global**.

Specified as: a fullword integer; where:

- If *liwork* = 0, PDSYEVX dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *liwork* = -1, PDSYEVX performs a work area query and returns the minimum required size of *iwork* in $iwork_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  $liwork \geq \max(isizestein, isizestebz)+2n$

  where:

  isizestein must have the following value:

  – If *jobz* = `'N'`, $isizestein = 3n+nprocs+1$

  – If *jobz* = `'V'`, $isizestein = (n+jz-1) +2n+nprocs+1$

  $isizestebz = \max(4n, 14, nprocs)$
  $nprocs = (nprow)(npcol)$

ifail

See On Return.

iclustr

See On Return.

gap

See On Return.

info

See On Return.

On Return

a

a is the local part of the global symmetric matrix ***A***, where:

If *uplo* = `'U'`, the upper triangle and diagonal of submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ are overwritten; that is, the original input is not preserved.

If *uplo* = `'L'`, the lower triangle and diagonal of submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ are overwritten; that is, the original input is not preserved.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 100 on page 693.

m

is the number of eigenvalues found.

Scope: **global**

Returned as: a fullword integer; $0 \leq m \leq n$.

nz

has the following meaning:

If *jobz* ≠ `'V'`, then *nz* is ignored.

If *jobz* = `'V'`, then *nz* is the number of eigenvectors computed—that is, the number of columns of ***Z*** used in the computation. On output, *nz* = *m* unless you provide insufficient space. To get all the eigenvectors requested, you must supply both sufficient space to hold the eigenvectors in ***Z*** and sufficient workspace to compute them (see *lwork*).

If *range* = `'A'` or `'I'`, PDSYEVX does not perform any computations if the work space supplied is insufficient. In this case, an input-argument error is issued and your job is terminated. For *range* = `'V'`, the number of requested eigenvectors is unknown until the eigenvalues are found. In this case, PDSYEVX computes as many eigenvectors as space allows. Then, if *nz* ≠ *m*, a computational error message is issued.

Scope: **global**

Returned as: a fullword integer; $0 \leq nz \leq m$.

w

On normal exit (see *info*), it is the vector ***w***, containing the selected eigenvalues in ascending order in the first *m* elements of ***w***.

Scope: **global**

Returned as: a one-dimensional array of (at least) length *n*, containing numbers of the data type indicated in Table 100 on page 693.

z

has the following meaning:

If *jobz* = `'N'`, then *z* is ignored.

If *jobz* = `'V'` and there is a normal exit (see *info*), then this is the updated local part of the global matrix ***Z***, where columns *jz* to *jz*+*m*–1 of the global matrix ***Z*** contain the orthonormal eigenvectors of the global matrix ***A***, corresponding to the selected eigenvalues. If an eigenvector fails to converge, then the corresponding column of the global matrix ***Z*** contains the last approximation to the eigenvector, and the index of the eigenvector is returned in ***ifail***.

This identifies the **first element** of the local array `Z`. This subroutine computes the location of the first element of the local subarray used, based on *iz*, *jz*, *desc_z*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*iz*+*n*–1) by LOCq(*jz*+*n*–1) part of the local array `Z` must contain the local pieces of the leading *iz*+*n*–1 by *jz*+*n*–1 part of the global matrix ***Z***.

Scope: **local**

Returned as: an LLD_Z by (at least) LOCq(N_Z) array, containing numbers of the data type indicated in Table 100 on page 693.

work

is the work area used by this subroutine if *lwork* ≠ 0,where:

If *lwork* ≠ 0 and *lwork* ≠ -1, its size is (at least) of length *lwork*.

If *lwork* = -1, its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, containing numbers of the data type indicated in Table 100 on page 693, where:

If *lwork* $\geq$ 1 or *lwork* = -1, then:

- If *jobz* = `'N'`, then $work_1$ is set to the minimum *lwork* value needed.
- If *jobz* = `'V'`, then $work_1$ is set to the minimum *lwork* value needed to compute all eigenvectors, but not necessarily sufficient to guarantee orthogonality of the eigenvectors. Except for $work_1$, the contents of *work* are overwritten on return.

iwork

is the work area used by this subroutine if *liwork* $\neq$ 0, where:

If *liwork* $\neq$ 0 and *liwork* $\neq$ -1, then its size is (at least) of length *liwork*.

If *liwork* = -1, then its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, where:

If *liwork* $\geq$ 1 or *liwork* = -1, then $iwork_1$ is set to the minimum *liwork* value and contains numbers of the data type indicated in Table 100 on page 693. Except for $iwork_1$, the contents of *iwork* are overwritten on return.

ifail

has the following meaning:

If *jobz* = `'N'`, then *ifail* is ignored.

If *jobz* = `'V'`, it is vector ***ifail***, where:

- If there is a normal exit (see *info*), the first *m* elements of ***ifail*** are zero.
- If there is an error exit (where one or more eigenvectors failed to converge—see *info*), ***ifail*** contains the indices of the eigenvectors that failed to converge.

Scope: **global**

Returned as: a one-dimensional array of (at least) length *n*, containing fullword integers; $0 \leq ifail_i \leq n$.

iclustr

has the following meaning:

If *jobz* = `'N'`, then *iclustr* is ignored.

If *jobz* = `'V'`, it is vector ***iclustr***, containing the indices of the eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace. Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr_{2i-1}$ to $iclustr_{2i}$ could not be reorthogonalized due to lack of workspace. **Hence, the eigenvectors corresponding to these clusters may not be orthogonal.**

iclustr is a zero-terminated vector; that is, the last element of ***iclustr*** is set to zero. Assuming that *k* is the number of clusters, then:

$iclustr_{2k} \neq 0$ and $iclustr_{2k+1} = 0$

Scope: **global**

Returned as: a one-dimensional array of (at least) length 2(*nprow*)(*npcol*), containing fullword integers; $0 \le iclustr_i \le n$.

gap

has the following meaning:

If *jobz* = `'N'`, then *gap* is ignored.

If *jobz* = `'V'`, it is vector ***gap***, containing the gap between the eigenvalues whose eigenvectors could not be reorthogonalized. The values in this vector correspond to the clusters indicated by ***iclustr***. As a result, the dot product between the eigenvectors corresponding to the *i*-th cluster may be as high as $(C)(n)/gap_i$, where *C* is a small constant.

Scope: **global**

Returned as: a one-dimensional array of (at least) length (*nprow*)(*npcol*), containing numbers of the data type indicated in Table 100 on page 693.

info

has the following meaning:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: One use of *info* in ScaLAPACK is to identify whether input-argument errors occurred. Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then one or more of the following computational errors occurred and the appropriate error messages were issued, indicating an error exit, where:

- If mod(*info*, 2) ≠ 0, then one or more eigenvectors failed to converge. Their indices are stored in ***ifail***. (Ensure that *abstol* = (2)(*unfl*).)
- If mod(*info*/2, 2) ≠ 0, then the eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in ***iclustr***.
- If mod(*info*/4, 2) ≠ 0, then all the eigenvectors between *vl* and *vu* could not be computed because of insufficient space. The number of eigenvectors computed is returned in *nz*.
- If mod(*info*/8, 2) ≠ 0, then one of more eigenvalues were not computed. (Ensure that *abstol* = (2)(*unfl*).)

Scope: **global**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. This subroutine accepts lowercase letters for the *jobz*, *range*, and *uplo* arguments.
2. In your C program, argument *info* must be passed by reference.
3. ***A***, ***Z***, ***w***, ***ifail***, ***iclustr***, ***gap***, *work*, and *iwork* must have no common elements; otherwise, results are unpredictable.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on

page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.

5. The global symmetric matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.

6. The global symmetric matrix ***A*** must be aligned on a block boundary; that is:

   - *ia*–1 must be a multiple of MB_A
   - *ja*–1 must be a multiple of NB_A

7. If *jobz* = `'V'`, then also follow these rules:

   - In the process grid, the process row containing the first row of the submatrix ***A*** must also contain the first row of the submatrix ***Z***; that is: *iarow* = *izrow*

     where:

     – *iarow* = mod(RSRC_A+(*ia*–1)/MB_A, *p*)

     – *izrow* = mod(RSRC_Z+(*iz*–1)/MB_Z, *p*)
   - M_A = M_Z
   - MB_A = MB_Z
   - NB_A = NB_Z
   - RSRC_A = RSRC_Z
   - CSRC_A = CSRC_Z
   - CTXT_A = CTXT_Z
   - The block row offset of the global symmetric matrix ***A*** must be equal to the block row offset of the global general matrix ***Z***; that is:

     – mod((*ia*–1, MB_A) = mod(*iz*–1, MB_Z))

8. Eigenvectors associated with tightly clustered eigenvalues may not be orthogonal.

9. Eigenvectors that are on different processes are not reorthogonalized. For details, see the *lwork* argument.

10. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See “Program Main (Message Passing)” on page 1006.

11. If *lwork* = –1 on any process, it must equal –1 on all processes. That is, if a subset of the processes specifies –1 for the work area size, they must all specify –1.

12. If *liwork* = –1 on any process, it must equal –1 on all processes. That is, if a subset of the processes specifies –1 for the work area size, they must all specify –1.

Function: This subroutine computes selected eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A***. Eigenvalues and eigenvectors can be selected by specifying a range of values or a range of indices for the eigenvalues. The computation involves the following steps:

1. Reduce the real symmetric matrix ***A*** to real symmetric tridiagonal form.

2. Compute the requested eigenvalues of the real symmetric tridiagonal matrix using bisection.

3. If requested, compute the eigenvectors of the real symmetric tridiagonal matrix using inverse iteration, and then back transform the eigenvectors to obtain the eigenvectors of the real symmetric matrix ***A***.

Error Conditions

Computational Errors

Note: For more details, see output argument *info*.

1. Bisection failed to converge for some eigenvalues. The eigenvalues may not be as accurate as the absolute and relative tolerances.
2. The number of eigenvalues computed does not match the number of eigenvalues requested.
3. No eigenvalues were computed, because the Gershgorin interval initially used was incorrect.
4. Some eigenvectors failed to converge. The indices are stored in ***ifail***.
5. Eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in ***iclustr***.
6. All the eigenvectors between *vl* and *vu* could not be computed due to insufficient workspace. The number of eigenvectors computed is returned in *nz*.

Resource Errors

1. (*lwork* = 0 or *liwork* = 0) and unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.
2. DTYPE_Z is invalid and *jobz* = 'V'

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDSYEVX has been called from outside the process grid.

Stage 4

1. *jobz* ≠ 'N' or 'V'
2. *range* ≠ 'U', 'V', or 'I'
3. *uplo* ≠ 'U' or 'L'
4. *n* < 0
5. M_A < 0 and *n* = 0; M_A < 1 otherwise
6. N_A < 0 and *n* = 0; N_A < 1 otherwise
7. MB_A < 1
8. NB_A < 1
9. RSRC_A < 0 or RSRC_A ≥ *p*
10. CSRC_A < 0 or CSRC_A ≥ *q*
11. *ia* < 1
12. *ja* < 1

If *jobz* = 'V':

13. M_Z $<$ 0 and *n* = 0; M_Z $<$ 1 otherwise
14. N_Z $<$ 0 and *n* = 0; N_Z $<$ 1 otherwise
15. MB_Z $<$ 1
16. NB_Z $<$ 1
17. RSRC_Z $<$ 0 or RSRC_Z $\geq$ *p*
18. CSRC_Z $<$ 0 or CSRC_Z $\geq$ *q*
19. *iz* $<$ 1
20. *jz* $<$ 1
21. CTXT_A $\neq$ CTXT_Z

Stage 5

1. *vu* $\leq$ *vl* and *range* = 'V' and *n* $\neq$ 0
2. *il* $<$ 1 and *range* = 'I'
3. (*iu* $<$ min(*n*, *il*) or *iu* $>$ *n*) and *range* = 'I'

If *n* $\neq$ 0:

4. *ia* $>$ M_A
5. *ja* $>$ N_A
6. *ia*+*n*–1 $>$ M_A
7. *ja*+*n*–1 $>$ N_A

If *n* $\neq$ 0 and *jobz* = 'V':

8. *iz* $>$ M_Z
9. *jz* $>$ N_Z
10. *iz*+*n*–1 $>$ M_Z
11. *jz*+*n*–1 $>$ N_Z

In all cases:

12. MB_A $\neq$ NB_A
13. mod(*ia*–1, MB_A) $\neq$ 0
14. mod(*ja*–1, NB_A) $\neq$ 0

If *jobz* = 'V':

15. M_A $\neq$ M_Z
16. MB_A $\neq$ MB_Z
17. NB_A $\neq$ NB_Z
18. mod(*iz*–1, MB_Z) $\neq$ mod(*ia*–1, MB_A)
19. In the process grid, the process row containing the first row of the submatrix ***A*** does not contain the first row of the submatrix ***Z***; that is, *iarow* $\neq$ *izrow*, where:
    a. *iarow* = mod(RSRC_A + (*ia*–1)/MB_A, *p*)
    b. *izrow* = mod(RSRC_Z + (*iz*–1)/MB_Z, *p*)
20. RSRC_A $\neq$ RSRC_Z
21. CSRC_A $\neq$ CSRC_Z

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. *lwork* $\neq$ 0, *lwork* $\neq$ -1, and *lwork* $<$ (*minimum value*). (For the minimum value, see the *lwork* argument description.)
3. *liwork* $\neq$ 0, *liwork* $\neq$ -1, and *liwork* $<$ (*minimum value*). (For the minimum value, see the *liwork* argument description.)

If *jobz* = 'V':

4. LLD_Z < max(1, LOCp(M_Z))

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *jobz* differs.
2. *range* differs.
3. *uplo* differs.
4. *n* differs.
5. *ia* differs.
6. *ja* differs.
7. DTYPE_A differs.
8. M_A differs.
9. N_A differs.
10. MB_A differs.
11. NB_A differs.
12. RSRC_A differs.
13. CSRC_A differs.
14. ABSTOL differs.

Also:

15. *lwork* = −1 on a subset of processes.
16. *liwork* = −1 on a subset of processes.

Stage 8

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

If *range* = `'V'`:

1. *vl* differs.
2. *vu* differs.

If *range* = `'I'`:

3. *il* differs.
4. *iu* differs.

If *jobz* = `'V'`:

5. *iz* differs.
6. *jz* differs.
7. DTYPE_Z differs.
8. M_Z differs.
9. N_Z differs.
10. MB_Z differs.
11. NB_Z differs.
12. RSRC_Z differs.
13. CSRC_Z differs.
14. ORFAC differs.

Example: This example shows how to find all the eigenvalues and eigenvectors of a real symmetric matrix ***A*** of order 4 using a 2 × 2 process grid.

Notes:

1. Because *range* = 'A', arguments *vl*, *vu*, *il*, and *iu* are not referenced.
2. Because *lwork* = 0 and *liwork* = 0, PDSYEVX dynamically allocates the work areas used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              JOBZ RANGE UPLO  N  A IA JA  DESC_A   VL     VU   IL IU  ABSTOL  M  NZ  W
               |     |     |   |  |  |  |    |       |      |    |  |     |    |   |  |
CALL PDSYEVX( 'V',  'A',  'U', 4, A, 1, 1, DESC_A, 0.0D0, 0.0D0, 0, 0, -1.0D0, M, NZ, W,

              ORFAC  Z IZ JZ  DESC_Z  WORK LWORK IWORK LIWORK IFAIL  ICLUSTR  GAP  INFO
+              |     |  |  |    |      |     |     |      |     |       |      |    |
             -1.0D0, Z, 1, 1, DESC_Z, WORK , 0 , IWORK ,  0 , IFAIL, ICLUSTR, GAP, INFO)
```

	DESC_A	**DESC_Z**
DTYPE_	1	1
CTXT_	*icontxt*[1]	*icontxt*[1]
M_	4	4
N_	4	4
MB_	1	1
NB_	1	1
RSRC_	0	0
CSRC_	0	0
LLD_	See below[2]	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
LLD_Z = MAX(1,NUMROC(M_Z, MB_Z, MYROW, RSRC_Z, NPROW))
```

In this example, LLD_A = LLD_Z = 2 on all processes.

Global symmetric matrix ***A*** of order 4 with block sizes 1 × 1:

```
B,D     0        1        2        3
       ┌                                ┐
 0     │  5.0  │  4.0   │  1.0   │  1.0 │
       │ ------|--------|--------|------ │
 1     │   .   │  5.0   │  1.0   │  1.0 │
       │ ------|--------|--------|------ │
 2     │   .   │   .    │  4.0   │  2.0 │
       │ ------|--------|--------|------ │
 3     │   .   │   .    │   .    │  4.0 │
       └                                ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │     0      │      1
-----|------------|------------
 0   │  5.0  1.0  │   4.0  1.0
     │   .   4.0  │    .   2.0
-----|------------|------------
 1   │   .   1.0  │   5.0  1.0
     │   .    .   │    .   4.0
```

Output:

The upper triangle, including the diagonal, of the global symmetric matrix ***A*** is overwritten; that is, the original input is not preserved.

On all processes, *m* = 4 and *nz* = 4.

Global vector ***w*** of length 4 is the same on all processes:

w = (1.00, 2.00, 5.00, 10.00)

Global general matrix ***Z*** of order 4 with block sizes 1 × 1:

```
B,D      0           1           2           3
       ┌                                                ┐
 0     │  0.7071  │   0.0000  │  -0.3162  │  -0.6325 │
       │ ---------|-----------|-----------|--------- │
 1     │ -0.7071  │   0.0000  │  -0.3162  │  -0.6325 │
       │ ---------|-----------|-----------|--------- │
 2     │  0.0000  │  -0.7071  │   0.6325  │  -0.3162 │
       │ ---------|-----------|-----------|--------- │
 3     │  0.0000  │   0.7071  │   0.6325  │  -0.3162 │
       └                                                ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1 3
0 2	P_{00}	P_{01}
1 3	P_{10}	P_{11}

Local arrays for ***Z***:

```
p,q  |         0         |         1
-----|-------------------|-------------------
 0   |  0.7071  -0.3162  |   0.0000  -0.6325
     |  0.0000   0.6325  |  -0.7071  -0.3162
-----|-------------------|-------------------
 1   | -0.7071  -0.3162  |   0.0000  -0.6325
     |  0.0000   0.6325  |   0.7071  -0.3162
```

Global vector ***ifail*** of length 4 is the same on all processes:

ifail = (0, 0, 0, 0)

Global vector ***iclustr*** of length 8 (= 2(*nprow*)(*npcol*)) is the same on all processes:

iclustr = (0, 0, 0, 0, 0, 0, 0, 0)

Global vector ***gap*** of length 4 (= (*nprow*)(*npcol*)) is the same on all processes:

gap = (–1.0, –1.0, –1.0, –1.0)

The value of *info* is 0 on all processes.

PDSYTRD—Reduce a Real Symmetric Matrix to Tridiagonal Form

This subroutine reduces a real symmetric matrix ***A*** to symmetric tridiagonal form ***T*** by an orthogonal similarity transformation:

$$\boldsymbol{T} = \boldsymbol{Q}^{T}\boldsymbol{A}\boldsymbol{Q}$$

where ***A*** represents the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.

If $n = 0$, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13] and [21].

Table 101. Data Types

A, ***d***, ***e***, τ, *work*	**Subroutine**
Long-precision real	PDSYTRD

Syntax

Fortran	CALL PDSYTRD (*uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *d*, *e*, *tau*, *work*, *lwork*, *info*)
C and C++	pdsytrd (*uplo*, *n*, *a*, *ia*, *ja*, *desc_a*, *d*, *e*, *tau*, *work*, *lwork*, *info*);

On Entry

uplo

indicates whether the upper or lower triangular part of the global symmetric submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Scope: **global**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

n

is the order of submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \ge 0$.

a

is the local part of the global symmetric matrix ***A***. This identifies the **first element** of the local array `A`. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp($ia+n-1$) by LOCq($ja+n-1$) part of the local array `A` must contain the local pieces of the leading $ia+n-1$ by $ja+n-1$ part of the global matrix, and:

- If *uplo* = `'U'`, the leading $n \times n$ upper triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the upper triangular part of the symmetric matrix, and the strictly lower triangular part is not referenced.
- If *uplo* = `'L'`, the leading $n \times n$ lower triangular part of the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must contain the lower triangular part

of the symmetric matrix, and the strictly upper triangular part is not referenced.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 101 on page 711. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+n-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

d

See On Return.

e

See On Return.

tau

See On Return.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, its size is (at least) of length *lwork*.
- If *lwork* = −1, its size is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 101 on page 711.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDSYTRD dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = −1, PDSYTRD performs a work area query and returns the minimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  $lwork \geq \max(nb(np{+}1),\ 3nb)$

  where:

  nb = MB_A = NB_A
  $iarow = \mathrm{mod}(\mathrm{RSRC\_A}+(ia-1)/nb,\ nprow)$.
  $np = \mathrm{NUMROC}(n, nb, myrow, iarow, nprow)$

info

See On Return.

On Return

a

is the updated local part of the global symmetric matrix ***A***, containing the results of the computation, where:

- If *uplo* = `'U'`, the diagonal and first superdiagonal of $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ are overwritten by the corresponding elements of the tridiagonal matrix ***T***. The elements above the first superdiagonal are overwritten with $\boldsymbol{v}_{1:i-1}$. These elements with τ represent the orthogonal matrix ***Q*** as a product of of elementary reflectors.
- If *uplo* = `'L'`, the diagonal and first subdiagonal of $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ are overwritten by the corresponding elements of the tridiagonal matrix ***T***. The elements below the first subdiagonal are overwritten with $\boldsymbol{v}_{i+2:n}$. These elements with τ represent the orthogonal matrix ***Q*** as a product of elementary reflectors.

See “Function” on page 716, for more information.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 101 on page 711. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

d

is the updated local part of the global matrix ***D***, where $\boldsymbol{d}_{ja:ja+n-1}$ contains the diagonal elements of the tridiagonal matrix ***T***.

This identifies the **first element** of the local array D. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*n*−1) part of the local array D must contain the local pieces of the leading 1 by *ja*+*n*−1 part of the global matrix ***D***.

A copy of the vector ***d***, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of ***d*** is distributed is CSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 101 on page 711.

e

is the updated local part of the global matrix ***E***, containing the off-diagonal elements of the tridiagonal matrix ***T***, where:

If *uplo* = 'U', then $\boldsymbol{e}_{ja} = 0$ and $\boldsymbol{e}_{ja+1:ja+n-1}$ contains the superdiagonal elements of the tridiagonal matrix ***T***.

If *uplo* = 'L', then $\boldsymbol{e}_{ja:ja+n-2}$ contains the subdiagonal elements of the tridiagonal matrix ***T***, and $\boldsymbol{e}_{ja+n-1} = 0$.

This identifies the **first element** of the local array E. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*n*−1) part of the local array E must contain the local pieces of the leading 1 by *ja*+*n*−1 part of the global matrix ***E***.

A copy of the vector ***e***, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of E is distributed is CSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 101 on page 711.

tau

is the updated local part of the global matrix τ, containing the scalar factors of the elementary reflectors, where:

If *uplo* = 'U', then $\tau_{ja} = 0$ and $\tau_{ja+1:ja+n-1}$ contains the scalar factors of the elementary reflectors.

If *uplo* = 'L', then $\tau_{ja:ja+n-2}$ contains the scalar factors of the elementary reflectors and $\tau_{ja+n-1} = 0$

This identifies the **first element** of the local array τ. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*n*−1) part of the local array τ must contain the local pieces of the leading 1 by *ja*+*n*−1 part of the global matrix τ.

A copy of the vector τ, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of τ is distributed is CSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 101 on page 711.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, its size is (at least) of length *lwork*.

If $lwork = -1$, its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, where:

If $lwork \geq 1$ or $lwork = -1$, then $work_1$ is set to the minimum *lwork* value and contains numbers of the data type indicated in Table 101 on page 711. Except for $work_1$, the contents of *work* are overwritten on return.

info

indicates that a successful computation occurred.

Scope: **global**

Returned as: a fullword integer; $info = 0$.

Notes and Coding Rules

1. This subroutine accepts lowercase letters for the *uplo* argument.
2. In your C program, argument *info* must be passed by reference.
3. Matrix ***A***, ***d***, ***e***, τ, and *work* must have no common elements; otherwise, results are unpredictable.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.
5. The global symmetric matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
6. The global symmetric matrix ***A*** must be aligned on a block boundary; that is:
   - *ia*−1 must be a multiple of MB_A
   - *ja*−1 must be a multiple of NB_A
7. There are no array descriptors for ***d, e,*** and τ. These are all row distributed vectors with block size NB_A, local arrays of dimension 1 by LOCq(N_A), and global index *ja*. A copy of these vectors exist on each row of the process grid, and the process column over which the first column of ***D, E,*** and τ is distributed is CSRC_A.
8. For suggested block sizes, see “Coding Tips for Optimizing Parallel Performance” on page 83.

9. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.

Function: This subroutine reduces a real symmetric matrix $\boldsymbol{A}$ to symmetric tridiagonal form $\boldsymbol{T}$ by an orthogonal similarity transformation:

$$\boldsymbol{T} = \boldsymbol{Q}^T\boldsymbol{A}\boldsymbol{Q}$$

where:

- $\boldsymbol{A}$ represents the global symmetric submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$.
- Matrix $\boldsymbol{Q}$ represents the following:
  - For *uplo* = `'U'`, the matrix $\boldsymbol{Q}$ is the product of elementary reflectors: $\boldsymbol{Q} = \boldsymbol{H}_{n-1} \ldots \boldsymbol{H}_2\ \boldsymbol{H}_1$,

    where:

    For each *i*: $\boldsymbol{H}_i = \boldsymbol{I}-\tau\boldsymbol{v}\boldsymbol{v}^T$
    τ is a real scalar
    $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{i+1:n} = 0$ and $\boldsymbol{v}_i = 1$
    $\boldsymbol{v}_{1:i-1}$ is stored on return in submatrix $\boldsymbol{A}_{1+(ia-1):i-1+(ia-1),\ i+1+(ja-1)}$
    τ is stored on return in $\tau_{i+(ja-1)}$
    $\boldsymbol{I}$ is the identity matrix

    If *uplo* = `'U'`, then the following example shows the contents of $\boldsymbol{A}$ on return with *n* = 5 and *ia* = *ja* = 1:

$$\begin{bmatrix} d & e & v_2 & v_3 & v_4 \\ . & d & e & v_3 & v_4 \\ . & . & d & e & v_4 \\ . & . & . & d & e \\ . & . & . & . & d \end{bmatrix}$$

    where:

    d represents the diagonal elements of $\boldsymbol{T}$
    e represents the superdiagonal elements of $\boldsymbol{T}$
    v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.

  - For *uplo* = `'L'`, the matrix $\boldsymbol{Q}$ is the product of elementary reflectors: $\boldsymbol{Q} = \boldsymbol{H}_1\boldsymbol{H}_2 \ldots \boldsymbol{H}_{n-1}$,

    where:

    For each *i*: $\boldsymbol{H}_i = \boldsymbol{I}-\tau\boldsymbol{v}\boldsymbol{v}^T$
    τ is a real scalar
    $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{1:i} = 0$ and $\boldsymbol{v}_{i+1} = 1$.
    $\boldsymbol{v}_{i+2:n}$ is stored on return in submatrix $\boldsymbol{A}_{i+2+(ia-1):n+(ia-1),\ i+(ja-1)}$.
    τ is stored on return in $\tau_{i+(ja-1)}$
    $\boldsymbol{I}$ is the identity matrix.

    If *uplo* = `'L'`, then the following example shows the contents of $\boldsymbol{A}$ on return with *n* = 5 and *ia* = *ij* = 1:

$$\begin{bmatrix} d & . & . & . & . \\ e & d & . & . & . \\ v_1 & e & d & . & . \\ v_1 & v_2 & e & d & . \\ v_1 & v_2 & v_3 & e & d \end{bmatrix}$$

where:

d represents the diagonal elements of ***T***
e represents the subdiagonal elements of ***T***
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.

Error Conditions

Computational Errors: None

Resource Errors

1. *lwork* = 0 and unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDSYTRD has been called from outside the process grid.

Stage 4

1. *uplo* ≠ `'U'` or `'L'`
2. $n < 0$
3. M_A < 0 and *n* = 0; M_A < 1 otherwise
4. N_A < 0 and *n* = 0; N_A < 1 otherwise
5. MB_A < 1
6. NB_A < 1
7. RSRC_A < 0 or RSRC_A ≥ *p*
8. CSRC_A < 0 or CSRC_A ≥ *q*
9. *ia* < 1
10. *ja* < 1

Stage 5: If $n \neq 0$:

1. *ia* > M_A
2. *ja* > N_A
3. *ia*+*n*−1 > M_A
4. *ja*+*n*−1 > N_A

In all cases:

1. MB_A ≠ NB_A

2. mod(ia–1, MB_A) $\neq$ 0
3. mod(ja–1, NB_A) $\neq$ 0

Stage 6

1. LLD_A $<$ max(1, LOCp(M_A))
2. *lwork* $\neq$ 0, *lwork* $\neq$ –1, and *lwork* $<$ max(*nb*(*np*+1), 3*nb*)

   where:
   - *nb* = MB_A = NB_A
   - *iarow* = mod(RSRC_A+(*ia*–1)/*nb*, *nprow*).
   - *np* = NUMROC(*n, nb, myrow, iarow, nprow*)

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *uplo* differs.
2. *n* differs.
3. *ia* differs.
4. *ja* differs.
5. DTYPE_A differs.
6. M_A differs.
7. N_A differs.
8. MB_A differs.
9. NB_A differs.
10. RSRC_A differs.
11. CSRC_A differs.

Also:
12. *lwork* = –1 on a subset of processes.

Example: This example shows the reduction of a symmetric matrix of order 4 to symmetric tridiagonal form, using a 2 $\times$ 2 process grid.

Note: Because *lwork* = 0, PDSYTRD dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              UPLO  N   A   IA  JA   DESC_A   D   E   TAU   WORK  LWORK   INFO
               |    |   |    |   |     |      |   |    |     |      |      |
CALL PDSYTRD( 'U' , 4 , A  , 1 , 1 , DESC_A , D , E , TAU , WORK ,  0   , INFO )
```

	DESC_A
DTYPE_	1
CTXT_	*icontxt*1
M_	4

	DESC_A
N_	4
MB_	1
NB_	1
RSRC_	0
CSRC_	0
LLD_	See below[2]
[1] *icontxt* is the output of the BLACS_GRIDINIT call. [2] Each process should set the LLD_ as follows: `LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))` In this example, LLD_A = 2 on all processes.	

Global symmetric matrix ***A*** of order 4 with block sizes 1 × 1:

```
B,D     0       1       2       3
      ┌                            ┐
 0    |  5.0 |   4.0 |   1.0 |   1.0 |
      | ------|--------|--------|------ |
 1    |   .  |   5.0 |   1.0 |   1.0 |
      | ------|--------|--------|------ |
 2    |   .  |    .  |   4.0 |   2.0 |
      | ------|--------|--------|------ |
 3    |   .  |    .  |    .  |   4.0 |
      └                            ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |      0      |      1
-----|------------|------------
 0   |  5.0  1.0  |   4.0  1.0
     |   .   4.0  |    .   2.0
-----|------------|------------
 1   |   .   1.0  |   5.0  1.0
     |   .    .   |    .   4.0
```

Output:

Global symmetric matrix ***A*** of order 4 with block sizes 1 × 1:

```
B,D      0         1         2         3
     ┌                                   ┐
 0   │  1.00  │   0.00  │   0.41  │   0.22 │
     │ -------|---------|---------|------- │
 1   │   .    │   6.00  │   2.83  │   0.22 │
     │ -------|---------|---------|------- │
 2   │   .    │    .    │   7.00  │  -2.45 │
     │ -------|---------|---------|------- │
 3   │   .    │    .    │    .    │   4.00 │
     └                                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
0 **2**	P_{00}	P_{01}
1 **3**	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │      0       │      1
-----|--------------|--------------
 0   │  1.00  0.41  │  0.00  0.22
     │   .    7.00  │   .   -2.45
-----|--------------|--------------
 1   │   .    2.83  │  6.00  0.22
     │   .     .    │   .    4.00
```

Global row vector ***D*** of length 4 with block sizes 1 × 1:

```
B,D      0         1         2         3
     ┌                                   ┐
 0   │  1.00  │   6.00  │   7.00  │   4.00 │
     └                                   ┘
```

Note: A copy of ***D*** is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	**0 2**	**1 3**
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for ***D***:

```
p,q  │      0       │      1
-----|--------------|--------------
 0   │  1.00  7.00  │  6.00  4.00
-----|--------------|--------------
 1   │  1.00  7.00  │  6.00  4.00
```

Global row vector ***E*** of length 4 with block sizes 1 × 1:

```
B,D      0         1         2         3
     ┌                                    ┐
 0   | 0.00  |   0.00  |   2.83  |  -2.45 |
     └                                    ┘
```

Note: A copy of ***E*** is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	0 2	1 3
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for ***E***:

```
p,q  |      0       |       1
-----|--------------|--------------
 0   | 0.00  2.83   |   0.00 -2.45
-----|--------------|--------------
 1   | 0.00  2.83   |   0.00 -2.45
```

Global row vector τ of length 4 with block sizes 1 × 1:

```
B,D      0         1         2         3
     ┌                                    ┐
 0   | 0.00  |   0.00  |   1.71  |   1.82 |
     └                                    ┘
```

Note: A copy of τ is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	0 2	1 3
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for τ:

```
p,q  |      0       |       1
-----|--------------|--------------
 0   | 0.00  1.71   |   0.00  1.82
-----|--------------|--------------
 1   | 0.00  1.71   |   0.00  1.82
```

The value of *info* is 0 on all processes.

PDGEHRD—Reduce a General Matrix to Upper Hessenberg Form

This subroutine reduces a real general matrix $\boldsymbol{A}$ to upper Hessenberg form $\boldsymbol{H}$ by an orthogonal similarity transformation:

$$\boldsymbol{H} = \boldsymbol{Q}^{T}\boldsymbol{A}\boldsymbol{Q}$$

where $\boldsymbol{A}$ represents the global general submatrix $\boldsymbol{A}_{ia+ilo-1:\ ia+ihi-1,\ ja+ilo-1:\ ja+ihi-1}$.

If $n = 0$, no computation is performed, and the subroutine returns after doing some parameter checking. Then, if $ihi = ilo$, the subroutine returns after doing some parameter checking and setting $\tau_{ja:ja+ilo-2}$ and $\tau_{ja+ihi-1:ja+n-2}$ to zero.

See references [13] and [21].

Table 102. Data Types

A, τ, *work*	**Subroutine**
Long-precision real	PDGEHRD

Syntax

Fortran	CALL PDGEHRD (*n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desc_a*, *tau*, *work*, *lwork*, *info*)
C and C++	pdgehrd (*n*, *ilo*, *ihi*, *a*, *ia*, *ja*, *desc_a*, *tau*, *work*, *lwork*, *info*);

On Entry

n

is the order of submatrix $\boldsymbol{A}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

ilo

lower range of the rows or columns in the global general submatrix $\boldsymbol{A}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $1 \leq ilo \leq \max(1, n)$.

ihi

upper range of the rows or columns in the global general submatrix $\boldsymbol{A}$ used in the computation.

Scope: **global**

Specified as: a fullword integer; $\min(ilo, n) \leq ihi \leq n$.

a

is the local part of the global general matrix $\boldsymbol{A}$. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*n*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 102. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ia \leq$ M_A and $ia+n-1 \leq$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \leq ja \leq$ N_A and $ja+n-1 \leq$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \leq$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \leq$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\geq$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

tau

See On Return.

work

has the following meaning:

If $lwork = 0$, *work* is ignored.

If $lwork \neq 0$, *work* is the work area used by this subroutine, where:

- If $lwork \neq -1$, its size is (at least) of length *lwork*.
- If $lwork = -1$, its size is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 102 on page 722.

lwork

is the number of elements in array `WORK`.

Scope:

- If $lwork \geq 0$, *lwork* is **local**
- If $lwork = -1$, *lwork* is **global**

Specified as: a fullword integer; where:

- If $lwork = 0$, PDGEHRD dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If $lwork = -1$, PDGEHRD performs a work area query and returns the minimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  $lwork \geq (nb \times nb)+nb \times \max(ihip+1, ihlp+inlq)$

  where:

  $nb = MB\_A = NB\_A$
  $ioff = \mathrm{mod}(ia+ilo-2, nb)$
  $iroffa = \mathrm{mod}(ia-1, nb)$
  $iarow = \mathrm{mod}(RSRC\_A+(ia-1)/nb, nprow)$
  $ilrow = \mathrm{mod}(RSRC\_A+(ia+ilo-2)/nb, nprow)$
  $ilcol = \mathrm{mod}(CSRC\_A+(ja+ilo-2)/nb, npcol)$
  $ihip = \mathrm{NUMROC}(ihi+iroffa, nb, myrow, iarow, nprow)$
  $ihlp = \mathrm{NUMROC}(ihi-ilo+ioff+1, nb, myrow, ilrow, nprow)$
  $inlq = \mathrm{NUMROC}(n-ilo+ioff+1, nb, mycol, ilcol, npcol)$

info

See On Return.

On Return

a

is the updated local part of the global general matrix ***A***, containing the results of the computation.

The upper triangle and the first subdiagonal of $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ are overwritten by the corresponding elements of the upper Hessenberg matrix ***H***. The elements below the first subdiagonal are overwritten with $\boldsymbol{v}_{i+2:ihi}$. These elements with τ represent the orthogonal matrix ***Q*** as a product of elementary reflectors.

See "Function" on page 726, for more information.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 102 on page 722. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

tau

is the updated local part of the global matrix τ, where:

- $\tau_{ja+ilo-1:ja+ihi-2}$ contains the scalar factors of the elementary reflectors.
- $\tau_{ja:ja+ilo-2}$ are set to zero.
- $\tau_{ja+ihi-1:ja+n-2}$ are set to zero.

This identifies the **first element** of the local array τ. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*n*−2) part of the local array τ must contain the local pieces of the leading 1 by *ja*+*n*−2 part of the global matrix τ.

A copy of the vector τ, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of τ is distributed is CSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(N_A−1) array, containing numbers of the data type indicated in Table 102 on page 722.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, its size is (at least) of length *lwork*.

If $lwork = -1$, its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, where:

If $lwork \geq 1$ or $lwork = -1$, then $work_1$ is set to the minimum *lwork* value and contains numbers of the data type indicated in Table 102 on page 722. Except for $work_1$, the contents of *work* are overwritten on return.

info

indicates that a successful computation occurred.

Scope: **global**

Returned as: a fullword integer; $info = 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. Matrix ***A***, τ, and *work* must have no common elements; otherwise, results are unpredictable.
3. On entry, the general submatrix $\boldsymbol{A}_{ia:ia+n-1,\ ja:ja+n-1}$ must already be upper triangular in rows (*ia*:*ia*+*ilo*−2) and (*ia*+*ihi*:*ia*+*n*−1), and upper triangular in columns (*ja*:*ja*+*ilo*−2) and (*ja*+*ihi*:*ja*+*n*−1). If this is not the case, you should set $ilo = 1$ and $ihi = n$.

   If $n = 0$, you should set $ilo = 1$ and $ihi = 0$. If $n > 0$, you should set $1 \leq ilo \leq ihi \leq n$.
4. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see “Determining the Number of Rows and Columns in Your Local Arrays” on page 25 and “NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process” on page 803.

5. The global general matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
6. The global general matrix ***A*** must be aligned on a block boundary; that is:
   - *ia*–1 must be a multiple of MB_A
   - *ja*–1 must be a multiple of NB_A
7. There is no array descriptor for τ. τ is a row-distributed vector with block size NB_A, local arrays of dimension 1 by LOCq(N_A-1), and global index *ja*. A copy of τ exists on each row of the process grid, and the process column over which the first column of τ is distributed is CSRC_A.
8. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
9. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.

Function: This subroutine reduces a real general matrix ***A*** to upper Hessenberg form ***H*** by an orthogonal similarity transformation:

$$\boldsymbol{H} = \boldsymbol{Q}^{T}\boldsymbol{A}\boldsymbol{Q}$$

where:

- ***A*** represents the global general submatrix $\boldsymbol{A}_{ia+ilo-1:ia+ihi-1,\ ja+ilo-1:ja+ihi-1}$
- Matrix ***Q*** is represented as a product of (*ihi*–*ilo*) elementary reflectors:

  $\boldsymbol{Q} = \boldsymbol{H}_{ilo}\, \boldsymbol{H}_{ilo+1} \dots \boldsymbol{H}_{ihi-1}$

  where:

  For each *i*: $\boldsymbol{H}_i = \boldsymbol{I} - \tau \boldsymbol{v}\boldsymbol{v}^T$
  τ is a real scalar
  v is a real vector with $\boldsymbol{v}_{1:i} = 0$, $\boldsymbol{v}_{i+1} = 1$, and $\boldsymbol{v}_{ihi+1:n} = 0$
  $\boldsymbol{v}_{i+2:ihi}$ is stored on return in in submatrix $\boldsymbol{A}_{i+ilo+1+(ia-1):ihi+(ia-1),\ ilo+i-1+(ja-1)}$
  τ is stored on return in $\tau_{i+ilo-1+(ja-1)}$
  I is the identity matrix

The following example shows the contents of the general submatrix ***A*** on entry with *n* = 7, *ia* = *ja* = 1, *ilo* = 2, and *ihi* = 6:

$$\begin{bmatrix} a & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & . & . & . & . & . & a \end{bmatrix}$$

Following is the general submatrix ***A*** on return:

$$\begin{bmatrix} a & a & h & h & h & h & a \\ . & a & h & h & h & h & a \\ . & h & h & h & h & h & h \\ . & v_2 & h & h & h & h & h \\ . & v_2 & v_3 & h & h & h & h \\ . & v_2 & v_3 & v_4 & h & h & h \\ . & . & . & . & . & . & a \end{bmatrix}$$

where:

a represents an element of the original submatrix ***A***.
h represents a updated element of the upper Hessenberg matrix ***H***.
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_{ilo+i-1+(ja-1)}$.

Error Conditions

Computational Errors: None

Resource Errors

1. *lwork* = 0 and unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDGEHRD has been called from outside the process grid.

Stage 4

1. $n < 0$
2. M_A < 0 and $n = 0$; M_A < 1 otherwise
3. N_A < 0 and $n = 0$; N_A < 1 otherwise
4. MB_A < 1
5. NB_A < 1
6. RSRC_A < 0 or RSRC_A ≥ p
7. CSRC_A < 0 or CSRC_A ≥ q
8. $ia < 1$
9. $ja < 1$

Stage 5

1. $ilo < 1$ or $ilo > \max(1, n)$
2. $ihi < \min(ilo, n)$ or $ihi > n$

If $n \neq 0$:

1. $ia >$ M_A
2. $ja >$ N_A
3. $ia+n-1 >$ M_A

4. $ja+n-1 > N\_A$

In all cases:

1. $MB\_A \neq NB\_A$
2. $\mathrm{mod}(ia-1, MB\_A) \neq 0$
3. $\mathrm{mod}(ja-1, NB\_A) \neq 0$

Stage 6

1. $LLD\_A < \max(1, LOCp(M\_A))$
2. $lwork \neq 0$, $lwork \neq -1$, and $lwork < (nb \times nb)+nb \times \max(ihip+1, ihlp+inlq)$

   where:

   $nb = MB\_A = NB\_A$
   $ioff = \mathrm{mod}(ia+ilo-2, nb)$
   $iroffa = \mathrm{mod}(ia-1, nb)$
   $iarow = \mathrm{mod}(RSRC\_A+(ia-1)/nb, nprow)$
   $ilrow = \mathrm{mod}(RSRC\_A+(ia+ilo-2)/nb, nprow)$
   $ilcol = \mathrm{mod}(CSRC\_A+(ja+ilo-2)/nb, npcol)$
   $ihip = \mathrm{NUMROC}(ihi+iroffa, nb, myrow, iarow, nprow)$
   $ihlp = \mathrm{NUMROC}(ihi-ilo+ioff+1, nb, myrow, ilrow, nprow)$
   $inlq = \mathrm{NUMROC}(n-ilo+ioff+1, nb, mycol, ilcol, npcol)$

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *n* differs.
2. *ilo* differs.
3. *ihi* differs.
4. *ia* differs.
5. *ja* differs.
6. DTYPE_A differs.
7. M_A differs.
8. N_A differs.
9. MB_A differs.
10. NB_A differs.
11. RSRC_A differs.
12. CSRC_A differs.

Also:

13. $lwork = -1$ on a subset of processes.

Example: This example shows the reduction of a general matrix of order 3 to upper Hessenberg form using a 2 × 2 process grid.

Note: Because $lwork = 0$, PDGEHRD dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              N   ILO   IHI   A  IA  JA   DESC_A   TAU   WORK   LWORK   INFO
              |    |     |    |   |   |     |       |     |       |      |
CALL PDGEHRD( 3 ,  1  ,  3  , A , 1 , 1 , DESC_A , TAU , WORK  ,  0   , INFO)
```

	DESC_A
DTYPE_	1
CTXT_	*icontxt*[1]
M_	3
N_	3
MB_	1
NB_	1
RSRC_	0
CSRC_	0
LLD_	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
```

In this example, LLD_A = 2 on P_{00} and P_{01}, and LLD_A = 1 on P_{10} and P_{11}.

Global general matrix ***A*** of order 3 with block sizes 1 × 1:

```
B,D      0         1         2
     ┌                           ┐
 0   │  33.0  │   16.0  │   72.0 │
     │ -------|---------|------- │
 1   │ -24.0  │  -10.0  │  -57.0 │
     │ -------|---------|------- │
 2   │  -8.0  │   -4.0  │  -17.0 │
     └                           ┘
```

The following is the 2 × 2 process grid:

B,D	**0 2**	**1**
0 **2**	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |      0       |    1
-----|--------------|--------
 0   |  33.0  72.0  |   16.0
     |  -8.0 -17.0  |   -4.0
-----|--------------|--------
 1   | -24.0 -57.0  |  -10.0
```

Output:

Global general matrix ***A*** of order 3 with block sizes 1 × 1:

```
B,D      0          1          2
     ┌                              ┐
 0   |  33.00  |  -37.95  |   63.25 |
     | --------|----------|-------- |
 1   |  25.30  |  -29.00  |   53.00 |
     | --------|----------|-------- |
 2   |   0.16  |    0.00  |    2.00 |
     └                              ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |       0        |    1
-----|----------------|---------
 0   |  33.0   63.25  |  -37.95
     |   0.16   2.00  |    0.00
-----|----------------|---------
 1   |  25.30  53.00  |  -29.00
```

Global row vector τ of length 2 with block sizes of 1:

```
B,D      0         1
     ┌                 ┐
 0   |  1.95  |   0.00 |
     └                 ┘
```

Note: A copy of τ is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	0	1
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for τ:

```
p,q  |    0    |    1
-----|---------|--------
 0   |  1.95   |   0.00
-----|---------|--------
 1   |  1.95   |   0.00
```

The value of *info* is 0 on all processes.

PDGEBRD—Reduce a General Matrix to Bidiagonal Form

This subroutine reduces a real general matrix ***A*** of order *m* by *n* to upper or lower bidiagonal form ***B*** by an orthogonal transformation:

$$\boldsymbol{B} = \boldsymbol{Q}^{T}\boldsymbol{A}\boldsymbol{P}$$

where:

- ***A*** represents the global general submatrix $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$.
- If $m \geq n$, then ***B*** is upper bidiagonal.
- If $m < n$, then ***B*** is lower bidiagonal.

If min(*m*, *n*) = 0, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13] and [21].

Table 103. Data Types

A, ***d***, ***e***, τ_q, τ_p, *work*	**Subroutine**
Long-precision real	PDGEBRD

Syntax

Fortran	CALL PDGEBRD (*m*, *n*, *a*, *ia*, *ja*, *desc_a*, *d*, *e*, *tauq*, *taup*, *work*, *lwork*, *info*)
C and C++	pdgebrd (*m*, *n*, *a*, *ia*, *ja*, *desc_a*, *d*, *e*, *tauq*, *taup*, *work*, *lwork*, *info*);

On Entry

m

is the number of rows of submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $m \geq 0$

n

is the number of columns of submatrix ***A*** used in the computation.

Scope: **global**

Specified as: a fullword integer; $n \geq 0$.

a

is the local part of the global general matrix ***A***. This identifies the **first element** of the local array A. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*m*–1) by LOCq(*ja*+*n*–1) part of the local array A must contain the local pieces of the leading *ia*+*m*–1 by *ja*+*n*–1 part of the global matrix.

Scope: **local**

Specified as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 103. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

ia

is the row index of the global matrix ***A***, identifying the first row of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ia \le$ M_A and $ia+m-1 \le$ M_A.

ja

is the column index of the global matrix ***A***, identifying the first column of the submatrix ***A***.

Scope: **global**

Specified as: a fullword integer; $1 \le ja \le$ N_A and $ja+n-1 \le$ N_A.

desc_a

is the array descriptor for global matrix ***A***, described in the following table:

desc_a	**Name**	**Description**	**Limits**	**Scope**
1	DTYPE_A	Descriptor type	DTYPE_A=1	Global
2	CTXT_A	BLACS context	Valid value, as returned by BLACS_GRIDINIT or BLACS_GRIDMAP	Global
3	M_A	Number of rows in the global matrix	If $m = 0$ or $n = 0$: M_A ≥ 0 Otherwise: M_A ≥ 1	Global
4	N_A	Number of columns in the global matrix	If $m = 0$ or $n = 0$: N_A ≥ 0 Otherwise: N_A ≥ 1	Global
5	MB_A	Row block size	MB_A ≥ 1	Global
6	NB_A	Column block size	NB_A ≥ 1	Global
7	RSRC_A	The process row of the $p \times q$ grid over which the first row of the global matrix is distributed	$0 \le$ RSRC_A $< p$	Global
8	CSRC_A	The process column of the $p \times q$ grid over which the first column of the global matrix is distributed	$0 \le$ CSRC_A $< q$	Global
9	LLD_A	The leading dimension of the local array	LLD_A $\ge$ max(1,LOCp(M_A))	**Local**

Specified as: an array of (at least) length 9, containing fullword integers.

d

See On Return.

e

See On Return.

tauq

See On Return.

taup

See On Return.

work

has the following meaning:

If *lwork* = 0, *work* is ignored.

If *lwork* ≠ 0, *work* is the work area used by this subroutine, where:

- If *lwork* ≠ −1, its size is (at least) of length *lwork*.
- If *lwork* = −1, its size is (at least) of length 1.

Scope: **local**

Specified as: an area of storage containing numbers of data type indicated in Table 103 on page 732.

lwork

is the number of elements in array `WORK`.

Scope:

- If *lwork* ≥ 0, *lwork* is **local**
- If *lwork* = -1, *lwork* is **global**

Specified as: a fullword integer; where:

- If *lwork* = 0, PDGEBRD dynamically allocates the work area used by this subroutine. The work area is deallocated before control is returned to the calling program. This option is an extension to the ScaLAPACK standard.
- If *lwork* = −1, PDGEBRD performs a work area query and return the minimum size of *work* in $work_1$. No computation is performed and the subroutine returns after error checking is complete.
- Otherwise, it must have the following value:

  $lwork \geq nb(mp0+nq0+1)+nq0$

  where:

  nb = MB_A = NB_A
  $iroffa$ = mod(ia–1, nb)
  $icoffa$ = mod(ja–1, nb)
  $iarow$ = mod(RSRC_A+(ia–1)/nb, $nprow$).
  $iacol$ = mod(CSRC_A+(ja–1)/nb, $npcol$).
  $mp0$ = NUMROC(m+$iroffa$, nb, $myrow$, $iarow$, $nprow$)
  $nq0$ = NUMROC(n+$icoffa$, nb, $mycol$, $iacol$, $npcol$)

info

See On Return.

On Return

a

is the updated local part of the global general matrix ***A***, containing the results of the computation, where:

- If $m \geq n$, the diagonal and first superdiagonal of $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$ are overwritten by the corresponding elements of the upper bidiagonal matrix ***B***. The elements below the diagonal are overwritten with $\boldsymbol{v}_{i+1:m}$. These elements with τ_q represent the orthogonal matrix ***Q*** as a product of of elementary reflectors. The elements above the first superdiagonal are overwritten with $\boldsymbol{u}_{i+2:n}$. These elements with τ_p represent the orthogonal matrix ***P*** as a product of of elementary reflectors.
- If $m < n$, the diagonal and first subdiagonal of $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$ are overwritten by the corresponding elements of the lower bidiagonal matrix ***B***. The elements below the first subdiagonal are overwritten with $\boldsymbol{v}_{i+2:m}$. These elements with τ_q represent the orthogonal matrix ***Q*** as a product of elementary reflectors. The elements above the diagonal are overwritten

with $\boldsymbol{u}_{i+1:n}$. These elements with τ_p represent the orthogonal matrix ***P*** as a product of elementary reflectors.

See “Function” on page 738, for more information.

Scope: **local**

Returned as: an LLD_A by (at least) LOCq(N_A) array, containing numbers of the data type indicated in Table 103 on page 732. Details about the square block-cyclic data distribution of global matrix ***A*** are stored in *desc_a*.

d

is the updated local part of the global matrix ***D***, where:

- If $m \geq n$, then $\boldsymbol{d}_{ja:ja+n-1}$ contains the diagonal elements of the bidiagonal matrix ***B***.

  This identifies the **first element** of the local array D. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*n*–1) part of the local array D must contain the local pieces of the leading 1 by *ja*+*n*–1 part of the global matrix ***D***.

  A copy of the vector ***d***, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of ***d*** is distributed is CSRC_A.

- If $m < n$, then $\boldsymbol{d}_{ia:ia+m-1}$ contains the diagonal elements of the bidiagonal matrix ***B***.

  This identifies the **first element** of the local array D. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*m*–1) by 1 part of the local array D must contain the local pieces of the leading *ia*+*m*–1 by 1 part of the global matrix ***D***.

  A copy of the vector ***d***, with a block size of MB_A and global index *ia*, is returned to each column of the process grid. The process row over which the first row of ***d*** is distributed is RSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(N_A) array if $m \geq n$, and a LOCp(M_A) by 1 array if $m < n$, containing numbers of the data type indicated in Table 103 on page 732.

e

is the updated local part of the global matrix ***E***, where:

- If $m \geq n$, then $\boldsymbol{e}_{ia:ia+n-2}$ contains the superdiagonal elements of the bidiagonal matrix ***B***.

  This identifies the **first element** of the local array E. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+*n*–2) by 1 part of the local array E must contain the local pieces of the leading *ia*+*n*–2 by 1 part of the global matrix ***E***.

  A copy of the vector ***e***, with a block size of MB_A and global index *ia*, is returned to each column of the process grid. The process row over which the first row of ***e*** is distributed is RSRC_A.

- If $m < n$, then $\boldsymbol{e}_{ja:ja+m-2}$ contains the subdiagonal elements of the bidiagonal matrix ***B***.

  This identifies the **first element** of the local array `D`. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+*m*−2) part of the local array `E` must contain the local pieces of the leading 1 by *ja*+*m*−2 part of the global matrix ***E***.

  A copy of the vector ***e***, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of ***e*** is distributed is CSRC_A.

Scope: **local**

Returned as: an (at least) LOCp(N_A−1) by 1 array if $m \geq n$ and a 1 by (at least) LOCq(M_A−1) array if $m < n$, containing numbers of the data type indicated in Table 103 on page 732.

tauq

is the updated local part of the global matrix τ_q, where:

$$\tau_{q_{ja:ja+\min(m,n)-1}}$$

contains the scalar factors of the elementary reflectors which represent the orthogonal matrix ***Q***. See “Function” on page 738 for more details.

This identifies the **first element** of the local array τ_q. This subroutine computes the location of the first element of the local subarray used, based on *ja*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading 1 by LOCq(*ja*+min(*m*, *n*)−1) part of the local array τ_q must contain the local pieces of the leading 1 by *ja*+min(*m*, *n*)−1 part of the global matrix τ_q.

A copy of the vector τ_q, with a block size of NB_A and global index *ja*, is returned to each row of the process grid. The process column over which the first column of τ_q is distributed is CSRC_A.

Scope: **local**

Returned as: a 1 by (at least) LOCq(min(M_A, N_A)) array, containing numbers of the data type indicated in Table 103 on page 732.

taup

is the updated local part of the global matrix τ_p, where:

$$\tau_{p_{ia:ia+\min(m,n)-1}}$$

contains the scalar factors of the elementary reflectors which represent the orthogonal matrix ***P***. See “Function” on page 738 for more details.

This identifies the **first element** of the local array τ_p. This subroutine computes the location of the first element of the local subarray used, based on *ia*, *desc_a*, *p*, *q*, *myrow*, and *mycol*; therefore, the leading LOCp(*ia*+min(*m*, *n*)−1) by 1 part of the local array τ_p must contain the local pieces of the leading *ia*+min(*m*, *n*)−1 by 1 part of the global matrix τ_p.

A copy of the vector τ_p, with a block size of MB_A and global index *ia*, is returned to each column of the process grid. The process row over which the first row of τ_p is distributed is RSRC_A.

Scope: **local**

Returned as: an (at least) LOCp(min(M_A, N_A)) by 1 array, containing numbers of the data type indicated in Table 103 on page 732.

work

is the work area used by this subroutine if $lwork \neq 0$, where:

If $lwork \neq 0$ and $lwork \neq -1$, its size is (at least) of length *lwork*.

If $lwork = -1$, its size is (at least) of length 1.

Scope: **local**

Returned as: an area of storage, where:

If $lwork \geq 1$ or $lwork = -1$, then $work_1$ is set to the minimum *lwork* value and contains numbers of the data type indicated in Table 103 on page 732. Except for $work_1$, the contents of *work* are overwritten on return.

info

indicates that a successful computation occurred.

Scope: **global**

Returned as: a fullword integer; $info = 0$.

Notes and Coding Rules

1. In your C program, argument *info* must be passed by reference.
2. Matrix ***A***, ***d***, ***e***, τ_q, τ_p, and *work* must have no common elements; otherwise, results are unpredictable.
3. The NUMROC utility subroutine can be used to determine the values of LOCp(M_) and LOCq(N_) used in the argument descriptions above. For details, see "Determining the Number of Rows and Columns in Your Local Arrays" on page 25 and "NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process" on page 803.
4. The global general matrix ***A*** must be distributed using a square block-cyclic distribution; that is, MB_A = NB_A.
5. For the global general matrix ***A***, the block row offset must be equal to the block column offset; that is, mod(*ia*–1, MB_A) = mod(*ja*–1, NB_A)
6. For suggested block sizes, see "Coding Tips for Optimizing Parallel Performance" on page 83.
7. There is no array descriptor for ***d***, where:
   - If $m \geq n$, then ***d*** is a row-distributed vector with block size NB_A, local array of dimension 1 by LOCq(N_A), and global index *ja*. A copy of ***d*** exists on each row of the process grid, and the process column over which the first column of ***d*** is distributed is CSRC_A.
   - If $m < n$, then ***d*** is a column-distributed vector with block size MB_A, local array of dimension LOCp(M_A) by 1, and global index *ia*. A copy of ***d*** exists on each column of the process grid, and the process row over which the first row of ***d*** is distributed is RSRC_A.
8. There is no array descriptor for ***e***, where:

- If $m \geq n$, then ***e*** is a column-distributed vector with block size MB_A, local array of dimension LOCp(N_A-1) by 1, and global index *ia*. A copy of ***e*** exists on each column of the process grid, and the process row over which the first row of ***e*** is distributed is RSRC_A.
- If $m < n$, then ***e*** is a row-distributed vector with block size NB_A, local array of dimension 1 by LOCq(M_A-1), and global index *ja*. A copy of ***e*** exists on each row of the process grid, and the process column over which the first column of ***e*** is distributed is CSRC_A.

9. There is no array descriptor for τ_q. τ_q is a row-distributed vector with block size NB_A, local array of dimension 1 by LOCq(min(M_A, N_A), and global index *ja*. A copy of τ_q exists on each row of the process grid, and the process column over which the first column of τ_q is distributed is CSRC_A.
10. There is no array descriptor for τ_p. τ_p is a column-distributed vector with block size MB_A, local array of dimension LOCp(min(M_A, N_A) by 1, and global index *ia*. A copy of τ_p exists on each column of the process grid, and the process row over which the first row of τ_p is distributed is RSRC_A.
11. If *lwork* = −1 on any process, it must equal −1 on all processes. That is, if a subset of the processes specifies −1 for the work area size, they must all specify −1.

Function: This subroutine reduces a real general matrix ***A*** of order *m* by *n* to upper or lower bidiagonal form ***B*** by an orthogonal transformation:

$\boldsymbol{B} = \boldsymbol{Q}^T\boldsymbol{AP}$

where:

- ***A*** represents the global general submatrix $\boldsymbol{A}_{ia:ia+m-1,\ ja:ja+n-1}$.
- If $m \geq n$, then ***B*** is upper bidiagonal, and matrices ***Q*** and ***P*** are represented as the product of elementary reflectors:

  $\boldsymbol{Q} = \boldsymbol{H}_1 \boldsymbol{H}_2 \ldots \boldsymbol{H}_n$
  $\boldsymbol{P} = \boldsymbol{G}_1 \boldsymbol{G}_2 \ldots \boldsymbol{G}_{n-1}$

  where:

  For each i: $\boldsymbol{H}_i = \boldsymbol{I} - \tau_q \boldsymbol{vv}^T$
  For each i: $\boldsymbol{G}_i = \boldsymbol{I} - \tau_p \boldsymbol{uu}^T$
  τ_q is a real scalar and is stored on return in:

$$\tau_{q_{i+(ja-1)}}$$

  τ_p is a real scalar and is stored on return in:

$$\tau_{p_{i+(ia-1)}}$$

  $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{1:i-1} = 0$ and $\boldsymbol{v}_i = 1$
  $\boldsymbol{v}_{i+1:m}$ is stored on return in submatrix $\boldsymbol{A}_{i+1+(ia-1):m+(ia-1),\ i+(ja-1)}$
  $\boldsymbol{u}$ is a real vector with $\boldsymbol{u}_{1:i} = 0$ and $\boldsymbol{u}_{i+1} = 1$
  $\boldsymbol{u}_{i+2:n}$ is stored on return in submatrix $\boldsymbol{A}_{i+(ia-1),\ i+2+(ja-1):n+(ja-1)}$
  I is the identity matrix

The following example shows the contents of ***A*** on return with *ia* = *ja* = 1, *m* = 6, and *n* = 5:

$$\begin{bmatrix} d & e & u_1 & u_1 & u_1 \\ v_1 & d & e & u_2 & u_2 \\ v_1 & v_2 & d & e & u_3 \\ v_1 & v_2 & v_3 & d & e \\ v_1 & v_2 & v_3 & v_4 & d \\ v_1 & v_2 & v_3 & v_4 & v_5 \end{bmatrix}$$

where:

d represents the diagonal elements of ***B***
e represents the off-diagonal elements of ***B***
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.
u_i represents the corresponding elements of the vector defining $\boldsymbol{G}_i$.

- If $m < n$, then ***B*** is lower bidiagonal, and matrices ***Q*** and ***P*** are represented as the product of elementary reflectors:

  $\boldsymbol{Q} = \boldsymbol{H}_1 \boldsymbol{H}_2 \ldots \boldsymbol{H}_{m-1}$
  $\boldsymbol{P} = \boldsymbol{G}_1 \boldsymbol{G}_2 \ldots \boldsymbol{G}_m$

  where:

  For each *i*: $\boldsymbol{H}_i = \boldsymbol{I} - \tau_q \boldsymbol{v}\boldsymbol{v}^T$
  For each *i*: $\boldsymbol{G}_i = \boldsymbol{I} - \tau_p \boldsymbol{u}\boldsymbol{u}^T$
  τ_q and τ_p are real scalars
  τ_q is stored on return in:

$$\tau_{q_{i+(ja-1)}}$$

  τ_p is stored on return in:

$$\tau_{p_{i+(ia-1)}}$$

  v is a real vector with $\boldsymbol{v}_{1:i} = 0$ and $\boldsymbol{v}_{i+1} = 1$
  $\boldsymbol{v}_{i+2:m}$ is stored on return in submatrix $\boldsymbol{A}_{i+2+(ia-1):m+(ia-1),\ i+(ja-1)}$
  u is a real vector with $\boldsymbol{u}_{1:i-1} = 0$ and $\boldsymbol{u}_i = 1$
  $\boldsymbol{u}_{i+1:n}$ is stored on return in submatrix $\boldsymbol{A}_{i+(ia-1),\ i+1+(ja-1):n+(ja-1)}$
  I is the identity matrix

The following example shows the contents of ***A*** on return with *ia* = *ja* = 1, *m* = 5, and *n* = 6:

$$\begin{bmatrix} d & u_1 & u_1 & u_1 & u_1 & u_1 \\ e & d & u_2 & u_2 & u_2 & u_2 \\ v_1 & e & d & u_3 & u_3 & u_3 \\ v_1 & v_2 & e & d & u_4 & u_4 \\ v_1 & v_2 & v_3 & e & d & u_5 \end{bmatrix}$$

where:

d represents the diagonal elements of ***B***
e represents the off-diagonal elements of ***B***
v_i represents the corresponding elements of the vector defining $\boldsymbol{H_i}$.
u_i represents the corresponding elements of the vector defining $\boldsymbol{G_i}$.

Error Conditions

Computational Errors: None

Resource Errors

1. *lwork* = 0 and unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. DTYPE_A is invalid.

Stage 2

1. CTXT_A is invalid.

Stage 3

1. PDGEBRD has been called from outside the process grid.

Stage 4

1. $m < 0$
2. $n < 0$
3. M_A < 0 if $m = 0$ or $n = 0$; M_A < 1 otherwise
4. N_A < 0 if $m = 0$ or $n = 0$; N_A < 1 otherwise
5. MB_A < 1
6. NB_A < 1
7. RSRC_A < 0 or RSRC_A ≥ *p*
8. CSRC_A < 0 or CSRC_A ≥ *q*
9. $ia < 1$
10. $ja < 1$

Stage 5: If $m \neq 0$ and $n \neq 0$:

1. *ia* > M_A
2. *ja* > N_A
3. *ia*+*m*–1 > M_A
4. *ja*+*n*–1 > N_A

In all cases:

1. MB_A ≠ NB_A

Stage 6

1. mod(*ia*–1, MB_A) ≠ mod(*ja*–1, NB_A)
2. LLD_A < max(1, LOCp(M_A))
3. *lwork* ≠ 0, *lwork* ≠ –1, and *lwork* < *nb*(*mp0*+*nq0*+1)+*nq0*

   where:
      nb = MB_A = NB_A

iroffa = mod(*ia*–1, *nb*)
icoffa = mod(*ja*–1, *nb*)
iarow = mod(RSRC_A+(*ia*–1)/*nb*, *nprow*).
iacol = mod(CSRC_A+(*ja*–1)/*nb*, *npcol*).
mp0 = NUMROC(*m*+*iroffa*, *nb*, *myrow*, *iarow*, *nprow*)
nq0 = NUMROC(*n*+*icoffa*, *nb*, *mycol*, *iacol*, *npcol*)

Stage 7

Each of the following global input arguments are checked to determine whether its value differs from the value specified on process P_{00}:

1. *m* differs.
2. *n* differs.
3. *ia* differs.
4. *ja* differs.
5. M_A differs.
6. N_A differs.
7. DTYPE_A differs.
8. MB_A differs.
9. NB_A differs.
10. RSRC_A differs.
11. CSRC_A differs.

Also:
12. *lwork* = –1 on a subset of processes.

Example: This example shows the reduction of a general matrix of order 4 by 3 to bidiagonal form using a 2 × 2 process grid.

Note: Because *lwork* = 0, PDGEBRD dynamically allocates the work area used by this subroutine.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

              M   N   A  IA  JA   DESC_A   D   E   TAUQ   TAUP   WORK  LWORK INFO
              |   |   |   |   |     |      |   |    |      |      |      |    |
CALL PDGEBRD( 4 , 3 , A , 1 , 1 , DESC_A , D , E , TAUQ , TAUP , WORK ,  0 , INFO )
```

	DESC_A
DTYPE_	1
CTXT_	*icontxt*[1]
M_	4
N_	3
MB_	2
NB_	2
RSRC_	0

	DESC_A
CSRC_	0
LLD_	See below[2]

[1] *icontxt* is the output of the BLACS_GRIDINIT call.

[2] Each process should set the LLD_ as follows:

```
LLD_A = MAX(1,NUMROC(M_A, MB_A, MYROW, RSRC_A, NPROW))
```

In this example, LLD_A = 2 on all processes.

Global general matrix ***A*** of order 4 × 3 with block sizes 2 × 2:

```
B,D       0         1
     ┌                   ┐
 0   │ 10.0  5.0  │   9.0 │
     │  2.0 16.0  │  10.0 │
     │ -----------|------ │
 1   │  3.0  7.0  │  21.0 │
     │  4.0  8.0  │  12.0 │
     └                   ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  │     0      │  1
-----│------------│-------
 0   │ 10.0  5.0  │   9.0
     │  2.0 16.0  │  10.0
-----│------------│-------
 1   │  3.0  7.0  │  21.0
     │  4.0  8.0  │  12.0
```

Output:

Global general matrix ***A*** of order 4 × 3 with block sizes 2 × 2:

```
B,D         0            1
     ┌                         ┐
 0   │ -11.36  22.80  │    0.56 │
     │   0.09  23.32  │    1.67 │
     │ ---------------|-------- │
 1   │   0.14   0.46  │   -9.68 │
     │   0.19   0.22  │    0.08 │
     └                         ┘
```

The following is the 2 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***A***:

```
p,q  |       0        |    1
-----|----------------|---------
 0   | -11.36  22.80  |    0.56
     |   0.09  23.32  |    1.67
-----|----------------|---------
 1   |   0.14   0.46  |   -9.68
     |   0.19   0.22  |    0.08
```

Global row vector ***D*** of length 3 with block size 2:

```
B,D          0             1
     ┌                          ┐
 0   | -11.36  23.32  |   -9.68 |
     └                          ┘
```

Note: A copy of ***D*** is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	0	1
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for ***D***:

```
p,q  |       0        |    1
-----|----------------|---------
 0   | -11.36  23.32  |   -9.68
-----|----------------|---------
 1   | -11.36  23.32  |   -9.68
```

Global column vector ***E*** of length 2 with block size 2:

```
B,D      0
     ┌       ┐
 0   | 22.80 |
     |  1.67 |
     └       ┘
```

Note: A copy of ***E*** is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D		
0	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for ***E***:

```
p,q  |   0    |    1
-----|--------|--------
 0   | 22.80  |  22.80
     |  1.67  |   1.67
-----|--------|--------
 1   |  .     |   .
```

Global row vector τ_q of length 3 with block size 2:

```
B,D        0          1
     ┌                    ┐
 0   |  1.88  1.59  |   1.99 |
     └                    ┘
```

Note: A copy of τ_q is distributed across each row of the process grid.

The following is the 2 × 2 process grid:

B,D	0	1
	P_{00}	P_{01}
	P_{10}	P_{11}

Local arrays for τ_q:

```
p,q  |      0       |    1
-----|--------------|--------
 0   |  1.88  1.59  |   1.99
-----|--------------|--------
 1   |  1.88  1.59  |   1.99
```

Global column vector τ_p of length 3 with block size 2:

```
B,D     0
     ┌       ┐
 0   |  1.52 |
     |  0.00 |
     | ----- |
 1   |  0.00 |
     └       ┘
```

Note: A copy of τ_p is distributed across each column of the process grid.

The following is the 2 × 2 process grid:

B,D		
0	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for τ_p:

```
p,q  |   0    |    1
-----|--------|--------
 0   |  1.52  |   1.52
     |  0.00  |   0.00
-----|--------|--------
 1   |  0.00  |   0.00
```

The value of *info* is 0 on all processes.

Chapter 10. Fourier Transforms (Message Passing)

This chapter describes the Fourier Transforms subroutines.

Overview of the Fourier Transforms Subroutines

The Fourier transform subroutines perform mixed-radix transforms in two and three dimensions. See references [1] and [3].

Table 104. List of Fourier Transform Subroutines (Message Passing)

Descriptive Name	**Short-Precision Subroutine**	**Long-Precision Subroutine**	**Page**
Complex Fourier Transforms in Two Dimensions	PSCFT2	PDCFT2	748
Real-to-Complex Fourier Transforms in Two Dimensions	PSRCFT2	PDRCFT2	755
Complex-to-Real Fourier Transforms in Two Dimensions	PSCRFT2	PDCRFT2	761
Complex Fourier Transforms in Three Dimensions	PSCFT3	PDCFT3	767
Real-to-Complex Fourier Transforms in Three Dimensions	PSRCFT3	PDRCFT3	776
Complex-to-Real Fourier Transforms in Three Dimensions	PSCRFT3	PDCRFT3	783

Acceptable Lengths for the Transforms

Use the following formula to determine acceptable transform lengths:

$$n = (2^h)(3^i)(5^j)(7^k)(11^m) \quad \text{for } n \leq 37748736$$

where:

$h = 1, 2, ..., 25$
$i = 0, 1, 2$
$j, k, m = 0, 1$

Figure 12 on page 746 lists all the acceptable values for transform lengths in the Fourier transform subroutines.

2	4	6	8	10	12	14	16	18
20	22	24	28	30	32	36	40	42
44	48	56	60	64	66	70	72	80
84	88	90	96	110	112	120	126	128
132	140	144	154	160	168	176	180	192
198	210	220	224	240	252	256	264	280
288	308	320	330	336	352	360	384	396
420	440	448	462	480	504	512	528	560
576	616	630	640	660	672	704	720	768
770	792	840	880	896	924	960	990	1008
1024	1056	1120	1152	1232	1260	1280	1320	1344
1386	1408	1440	1536	1540	1584	1680	1760	1792
1848	1920	1980	2016	2048	2112	2240	2304	2310
2464	2520	2560	2640	2688	2772	2816	2880	3072
3080	3168	3360	3520	3584	3696	3840	3960	4032
4096	4224	4480	4608	4620	4928	5040	5120	5280
5376	5544	5632	5760	6144	6160	6336	6720	6930
7040	7168	7392	7680	7920	8064	8192	8448	8960
9216	9240	9856	10080	10240	10560	10752	11088	11264
11520	12288	12320	12672	13440	13860	14080	14336	14784
15360	15840	16128	16384	16896	17920	18432	18480	19712
20160	20480	21120	21504	22176	22528	23040	24576	24640
25344	26880	27720	28160	28672	29568	30720	31680	32256
32768	33792	35840	36864	36960	39424	40320	40960	42240
43008	44352	45056	46080	49152	49280	50688	53760	55440
56320	57344	59136	61440	63360	64512	65536	67584	71680
73728	73920	78848	80640	81920	84480	86016	88704	90112
92160	98304	98560	101376	107520	110880	112640	114688	118272
122880	126720	129024	131072	135168	143360	147456	147840	157696
161280	163840	168960	172032	177408	180224	184320	196608	197120
202752	215040	221760	225280	229376	236544	245760	253440	258048
262144	270336	286720	294912	295680	315392	322560	327680	337920
344064	354816	360448	368640	393216	394240	405504	430080	443520
450560	458752	473088	491520	506880	516096	524288	540672	573440
589824	591360	630784	645120	655360	675840	688128	709632	720896
737280	786432	788480	811008	860160	887040	901120	917504	946176
983040	1013760	1032192	1048576	1081344	1146880	1179648	1182720	1261568
1290240	1310720	1351680	1376256	1419264	1441792	1474560	1572864	1576960
1622016	1720320	1774080	1802240	1835008	1892352	1966080	2027520	2064384
2097152	2162688	2293760	2359296	2365440	2523136	2580480	2621440	2703360
2752512	2838528	2883584	2949120	3145728	3153920	3244032	3440640	3548160
3604480	3670016	3784704	3932160	4055040	4128768	4194304	4325376	4587520
4718592	4730880	5046272	5160960	5242880	5406720	5505024	5677056	5767168
5898240	6291456	6307840	6488064	6881280	7096320	7208960	7340032	7569408
7864320	8110080	8257536	8388608	8650752	9175040	9437184	9461760	10092544
10321920	10485760	10813440	11010048	11354112	11534336	11796480	12582912	12615680
12976128	13762560	14192640	14417920	14680064	15138816	15728640	16220160	16515072
16777216	17301504	18350080	18874368	18923520	20185088	20643840	20971520	21626880
22020096	22708224	23068672	23592960	25165824	25231360	25952256	27525120	28385280
28835840	29360128	30277632	31457280	32440320	33030144	33554432	34603008	36700160
37748736								

Figure 12. Table of Acceptable Lengths for the Transforms

Fourier Transforms Subroutines

This section contains the Fourier transform subroutine descriptions.

PSCFT2 and PDCFT2—Complex Fourier Transforms in Two Dimensions

These subroutines compute the mixed-radix two-dimensional discrete Fourier transform of complex data:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}$$

for:

$k1 = 0, 1, ..., n1-1$
$k2 = 0, 1, ..., n2-1$

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
Isign is + or − (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/((*n1*)(*n2*)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 105. Data Types

X, Y	*scale*	**Subroutine**
Short-precision complex	Short-precision real	PSCFT2
Long-precision complex	Long-precision real	PDCFT2

Syntax

Fortran	CALL PSCFT2 | PDCFT2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	pscft2 | pdcft2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array X, containing the two-dimensional data to be transformed that has been block-column distributed over a 1 × *q* process grid, where *q* is the number of processes. (The value of *ldx* is set in the IP array.)

Scope: **local**

Specified as: an array of (at least) length $ldx \times$ LOCq($n2$), containing numbers of the data type indicated in Table 105 on page 748. This array must be aligned on a doubleword boundary.

y

See On Return.

n1

is the length of the first dimension of the two-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n1 \leq 37748736$ and must be one of the values listed in Figure 12 on page 746.

n2

is the length of the second dimension of the two-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n2 \leq 37748736$ and must be one of the values listed in Figure 12 on page 746.

isign

controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If $isign$ = positive value, $Isign = +$ (transforming time to frequency).

If $isign$ = negative value, $Isign = -$ (transforming frequency to time).

Scope: **global**

Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale

is the scaling constant *scale*.

Scope: **global**

Specified as: a number of the data type indicated in Table 105 on page 748, where $scale > 0.0$ or $scale < 0.0$.

icontxt

is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip

is an array of parameters, `IP(`*i*`)`, where:

- `IP(1)` indicates whether the default values for *ip* are used or you set the values for *ip*.

  If `IP(1) = 0`, then the following default values are used:

  – *y* is returned in transposed form; that is, global *y* has dimensions $n2 \times n1$
  – *ldx*, the leading dimension of the array specified for `X`, equals *n1*
  – *ldy*, the leading dimension of the array specified for `Y`, equals *n2*

  The remaining parameters of the array `IP` are ignored.

If `IP(1)` ≠ 0, then you set the remaining values of *ip* to indicate whether *y* is stored in normal or transposed form, and indicate values for *ldx* and *ldy*.

- `IP(2)` indicates whether *y* is to be stored in normal or transposed form.

  If `IP(2)` = 0, then *y* is to be stored in transposed form on output.

  If `IP(2)` = 1, then *y* is to be stored in normal form on output.

- `IP(3-19)` are reserved.
- `IP(20)` indicates the value of the leading dimension, *ldx*, of the array specified for `X`, where:

  If `IP(20)` = 0, then *ldx* = *n1*.

  If `IP(20)` ≠ 0, then *ldx* is this value of `IP(20)`.

- `IP(21)` indicates the value of the leading dimension, *ldy*, of the array specified for `Y`, where:

  If `IP(21)` = 0 and *y* is to be stored in normal form, then *ldy* = *n1*.

  If `IP(21)` = 0 and *y* is to be stored in transposed form, then *ldy* = *n2*.

  If `IP(21)` ≠ 0, then *ldy* is this value of `IP(21)`.

- `IP(22-40)` are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:
`IP(1)` is any integer
`IP(2)` = 0 or 1
`IP(20)` ≥ *n1* or `IP(20)` = 0
`IP(21)` ≥ *n1* (for normal form) or `IP(21)` = 0
`IP(21)` ≥ *n2* (for transposed form) or `IP(21)` = 0

On Return

y

is the local array `Y` that is block-column distributed and contains the results of the computation, where:

If `IP(1)` = 0, the local array `Y` is stored in transposed form and has dimensions *n2* × LOCq(*n1*).

If `IP(1)` ≠ 0 and `IP(2)` = 0, the local array `Y` is stored in transposed form and has dimensions *ldy* × LOCq(*n1*).

If `IP(1)` ≠ 0 and `IP(2)` = 1, the local array `Y` is stored in normal form and has dimensions *ldy* × LOCq(*n2*).

Scope: **local**

Returned as: an *ldy* × LOCq(*n2*) array (for normal form) or an *ldy* × LOCq(*n1*) array (for transposed form), containing the numbers of the data type indicated in Table 105 on page 748. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. You may specify the same array for both X and Y. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable.
2. For the output array Y, these subroutines may use any extra space available when *ldy* is greater than its minimum value.
3. For more information on LOCq(_) and how sequences are block-column distributed, see "Two-Dimensional Sequence" on page 66.

   In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.
4. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See subroutine fourier in "Module Fourier (Message Passing)" on page 1017.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not $1 \times q$
2. The subroutine was called from outside the process grid.

Stage 3

1. $n1 > 37748736$
2. $n2 > 37748736$
3. The length of *n1* or *n2* is not an allowable transform length.
4. $isign = 0$
5. $scale = 0.0$
6. IP(1) ≠ 0 and IP(2) ≠ 0 or 1
7. IP(1) ≠ 0 and IP(20) ≠ 0 and IP(20) < *n1* (that is, *ldx* < *n1*)
8. IP(1) ≠ 0 and IP(2) = 1 (for normal mode) and IP(21) ≠ 0 and IP(21) < *n1* (that is, *ldy* < *n1*)
9. IP(1) ≠ 0 and IP(2) = 0 (for transpose mode) and IP(21) ≠ 0 and IP(21) < *n2* (that is, *ldy* < *n2*)

Example 1: This example shows how to compute a two-dimensional transform. In this example, the IP array is set to 0, which means array Y is returned in transposed form, *ldx*=*n1*, and *ldy*=*n2*. The data is block-column distributed over a 1×2 process grid. The arrays are declared as follows:

```
  COMPLEX*16 X(0:7,0:2), Y(0:5,0:3)
  INTEGER*4  IP(40)
  REAL*8     SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 0

             X   Y   N1  N2  ISIGN     SCALE       ICONTXT   IP
             |   |   |   |     |         |            |      |
CALL PDCFT2( X , Y , 8 , 6 ,  -1  , 1.0D0/48.0D0 , ICONTXT , IP)
```

Global matrix ***X*** of order 8 × 6:

```
B,D                   0                                     1
     ┌                                                                        ┐
     │  (48.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
 0   │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0) │
     └                                                                        ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***X***:

```
p,q  │                 0                  │                 1
-----│------------------------------------│-----------------------------------
     │  (48.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
 0   │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
     │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  │   (0.0,0.0)  (0.0,0.0)  (0.0,0.0)
```

Output: Global matrix for ***Y***:

```
B,D                    0                                                 1
    ┌                                                                                                  ┐
    │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
    │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
    │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
 0  │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
    │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
    │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
    └                                                                                                  ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local matrix for ***Y***:

```
p,q  │                    0                         │                         1
-----│----------------------------------------------│-----------------------------------------------
     │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
 0   │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     │   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  │    (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
```

Example 2: This example shows how to compute a two-dimensional transform. This is an example of uneven block-column distribution over a 1 × 3 process grid. In this example, the `IP` array is set to 0, which means array `Y` is returned in transposed form, *ldx*=*n1*, and *ldy*=*n2*. The arrays are declared as follows:

```
  COMPLEX*16 X(0:7,0:2), Y(0:7,0:2)
  INTEGER*4  IP(40)
  REAL*8     SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 0

             X   Y   N1  N2  ISIGN     SCALE       ICONTXT   IP
             |   |   |   |     |         |            |      |
CALL PDCFT2( X , Y , 8 , 8 ,   1  , 1.0D0/16.0D0 , ICONTXT , IP)
```

Global matrix ***X*** of order 8 × 8:

```
B,D                 0                                   1                                 2
   ┌                                                                                                       ┐
   │  (0.0,98.0) (67.0,27.0) (67.0,82.0) │ (84.0,99.0) (26.0,41.0) (24.0,15.0) │ (27.0,55.0)  (48.0,9.0) │
   │ (13.0,49.0) (93.0,91.0)  (0.0,12.0) │ (52.0,88.0)  (4.0,84.0) (98.0,57.0) │ (43.0,89.0) (89.0,27.0) │
   │ (75.0,26.0) (38.0,52.0)  (38.0,1.0) │  (9.0,23.0) (73.0,26.0) (72.0,80.0) │ (76.0,62.0)  (90.0,0.0) │
   │  (45.0,9.0) (51.0,46.0)  (6.0,68.0) │ (65.0,30.0) (32.0,41.0)  (75.0,3.0) │ (47.0,84.0)  (6.0,41.0) │
 0 │ (53.0,94.0) (83.0,94.0) (41.0,86.0) │ (41.0,35.0) (63.0,53.0) (65.0,53.0) │ (23.0,15.0)  (90.0,2.0) │
   │  (21.0,7.0)   (3.0,5.0) (68.0,62.0) │ (70.0,51.0) (75.0,46.0)  (7.0,49.0) │ (27.0,21.0) (50.0,70.0) │
   │  (4.0,50.0)  (5.0,76.0) (58.0,73.0) │ (91.0,59.0) (99.0,28.0) (63.0,95.0) │ (35.0,71.0) (51.0,93.0) │
   │ (67.0,38.0) (52.0,77.0) (93.0,72.0) │ (76.0,84.0) (36.0,17.0) (88.0,74.0) │ (16.0,13.0) (31.0,23.0) │
   └                                                                                                       ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local arrays for ***X***:

```
p,q  |                 0                  |                 1                     |            2
-----|------------------------------------|---------------------------------------|--------------------------
     | (0.0,98.0) (67.0,27.0) (67.0,82.0) |  (84.0,99.0)  (26.0,41.0)  (24.0,15.0) | (27.0,55.0)   (48.0,9.0)
     |(13.0,49.0) (93.0,91.0)  (0.0,12.0) |  (52.0,88.0)   (4.0,84.0)  (98.0,57.0) | (43.0,89.0)  (89.0,27.0)
     |(75.0,26.0) (38.0,52.0)  (38.0,1.0) |   (9.0,23.0)  (73.0,26.0)  (72.0,80.0) | (76.0,62.0)   (90.0,0.0)
     | (45.0,9.0) (51.0,46.0)  (6.0,68.0) |  (65.0,30.0)  (32.0,41.0)   (75.0,3.0) | (47.0,84.0)   (6.0,41.0)
 0   |(53.0,94.0) (83.0,94.0) (41.0,86.0) |  (41.0,35.0)  (63.0,53.0)  (65.0,53.0) | (23.0,15.0)   (90.0,2.0)
     | (21.0,7.0)   (3.0,5.0) (68.0,62.0) |  (70.0,51.0)  (75.0,46.0)   (7.0,49.0) | (27.0,21.0)  (50.0,70.0)
     | (4.0,50.0)  (5.0,76.0) (58.0,73.0) |  (91.0,59.0)  (99.0,28.0)  (63.0,95.0) | (35.0,71.0)  (51.0,93.0)
     |(67.0,38.0) (52.0,77.0) (93.0,72.0) |  (76.0,84.0)  (36.0,17.0)  (88.0,74.0) | (16.0,13.0)  (31.0,23.0)
```

Output: Global matrix for ***Y***:

```
B,D                      0                                     1                                  2
   ┌                                                                                                                  ┐
   │(198.6,200.1)  (-10.6,9.8)     (0.8,7.2)  │   (5.8,-5.2)   (11.2,9.1) (-38.3,-18.7) │(-10.2,-1.9)   (14.0,12.6) │
   │  (-0.3,-6.8)  (19.3,-18.7)  (28.7,-3.6)  │   (-7.2,2.5)   (1.5,14.6) (-22.0,-20.7) │(29.8,-15.0)   (-10.7,0.8) │
   │  (11.3,-6.2)  (-24.0,-8.1)   (8.6,11.6)  │  (-29.9,6.5)  (13.7,13.5)  (-16.7,-4.4) │(-26.6,-0.8)    (-3.3,9.5) │
   │   (5.7,17.1)    (3.7,-7.0)  (-2.5,13.9)  │(-19.5,-15.9) (-18.4,20.1)   (11.6,-1.8) │ (-0.3,-8.2)   (26.8,30.0) │
0  │ (-29.8,-3.4)    (-0.5,7.4) (-17.1,27.5)  │  (18.5,32.6)    (9.4,9.6)    (7.6,-8.0) │(-13.1,13.9) (-26.6,-16.5) │
   │  (-10.2,1.6)   (-5.0,28.8)  (-5.0,25.0)  │   (5.0,12.1)  (-13.5,9.9)     (2.5,0.6) │  (0.0,-5.6)  (-11.8,-8.3) │
   │ (-8.7,-13.6)   (10.0,11.1)    (0.6,9.4)  │ (12.2,-21.2)  (-9.3,-0.9)  (14.5,-15.6) │  (2.4,11.1)   (-22.7,0.2) │
   │ (-27.7,-3.1) (-21.8,-21.3)  (-22.6,6.0)  │   (0.2,11.6)   (-1.6,6.6)   (-7.2,-0.4) │  (0.5,25.6)   (20.3,23.8) │
   └                                                                                                                  ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local matrix for ***Y***:

```
p,q|                      0                     |                   1                     |             2
---|--------------------------------------------|-----------------------------------------|-------------------------
   |(198.6,200.1)  (-10.6,9.8)     (0.8,7.2)    |   (5.8,-5.2)   (11.2,9.1) (-38.3,-18.7) |(-10.2,-1.9)   (14.0,12.6)
   |  (-0.3,-6.8)  (19.3,-18.7)  (28.7,-3.6)    |   (-7.2,2.5)   (1.5,14.6) (-22.0,-20.7) |(29.8,-15.0)   (-10.7,0.8)
   |  (11.3,-6.2)  (-24.0,-8.1)   (8.6,11.6)    |  (-29.9,6.5)  (13.7,13.5)  (-16.7,-4.4) |(-26.6,-0.8)    (-3.3,9.5)
   |   (5.7,17.1)    (3.7,-7.0)  (-2.5,13.9)    |(-19.5,-15.9) (-18.4,20.1)   (11.6,-1.8) | (-0.3,-8.2)   (26.8,30.0)
0  | (-29.8,-3.4)    (-0.5,7.4) (-17.1,27.5)    |  (18.5,32.6)    (9.4,9.6)    (7.6,-8.0) |(-13.1,13.9) (-26.6,-16.5)
   |  (-10.2,1.6)   (-5.0,28.8)  (-5.0,25.0)    |   (5.0,12.1)  (-13.5,9.9)     (2.5,0.6) |  (0.0,-5.6)  (-11.8,-8.3)
   | (-8.7,-13.6)   (10.0,11.1)    (0.6,9.4)    | (12.2,-21.2)  (-9.3,-0.9)  (14.5,-15.6) |  (2.4,11.1)   (-22.7,0.2)
   | (-27.7,-3.1) (-21.8,-21.3)  (-22.6,6.0)    |   (0.2,11.6)   (-1.6,6.6)   (-7.2,-0.4) |  (0.5,25.6)   (20.3,23.8)
```

PSRCFT2 and PDRCFT2—Real-to-Complex Fourier Transforms in Two Dimensions

These subroutines compute the mixed-radix two-dimensional complex conjugate even discrete Fourier transform of real data:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2}$$

for:

$k1 = 0, 1, ..., n1-1$
$k2 = 0, 1, ..., n2-1$

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array `X`.
$y_{k1,k2}$ are elements of array `Y`.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/((*n1*)(*n2*)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 106. Data Types

X, *scale*	Y	**Subroutine**
Short-precision real	Short-precision complex	PSRCFT2
Long-precision real	Long-precision complex	PDRCFT2

Syntax

Fortran	CALL PSRCFT2 | PDRCFT2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	psrcft2 | pdrcft2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array `X`, containing the two-dimensional data to be transformed that has been block-column distributed over a 1 × *q* process grid, where *q* is the number of processes. (The value of *ldx* is set in the `IP` array.)

Scope: **local**

Specified as: an array of (at least) length *ldx* × LOCq(*n2*), containing numbers of the data type indicated in Table 106 on page 755. This array must be aligned on a doubleword boundary.

y

See On Return.

n1

is the length of the first dimension of the two-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n1 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

n2

is the length of the second dimension of the two-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n2 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

isign

controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Scope: **global**

Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale

is the scaling constant *scale*.

Scope: **global**

Specified as: a number of the data type indicated in Table 106 on page 755, where $scale > 0.0$ or $scale < 0.0$.

icontxt

is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip

is an array of parameters, `IP(`*i*`)`, where:

- `IP(1)` indicates whether the default values for *ip* are used or you set the values for *ip*.

  If `IP(1)` = 0, then the following default values are used:

  – *ldx*, the leading dimension of the array specified for `X`, equals *n1*
  – *ldy*, the leading dimension of the array specified for `Y`, equals *n2*

  The remaining parameters of the array `IP` are ignored.

  If `IP(1)` ≠ 0, then you set the remaining values of *ip* to indicate values for *ldx* and *ldy*.

- IP(2-19) are reserved.
- IP(20) indicates the value of the leading dimension, *ldx*, of the array specified for X, where:

  If IP(20) = 0, then *ldx* = *n1*.

  If IP(20) ≠ 0, then *ldx* is this value of IP(20).
- IP(21) indicates the value of the leading dimension, *ldy*, of the array specified for Y, where:

  If IP(21) = 0, then *ldy* = *n2*.

  If IP(21) ≠ 0, then *ldy* is this value of IP(21).
- IP(22-40) are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:

IP(1) is any integer

IP(20) ≥ *n1* or IP(20) = 0

IP(21) ≥ *n2* or IP(21) = 0

On Return

y

is the local array Y, stored in FFT-packed storage mode, containing the results of the computation that are block-column distributed, where:

If IP(1) = 0, the local array Y has dimensions *n2* × LOCq(*n1/2*).

If IP(1) ≠ 0 and IP(21) = 0, the local array Y has dimensions *n2* × LOCq(*n1/2*).

If IP(1) ≠ 0 and IP(21) ≠ 0, the local array Y has dimensions *ldy* × LOCq(*n1/2*).

Scope: **local**

Returned as: an *ldy* × LOCq(*n1/2*) array, containing the numbers of the data type indicated in Table 106 on page 755. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. These subroutines always return Y in transposed form.
2. For the output array Y, these subroutines may use any extra space available when *ldy* is greater than its minimum value.
3. You may specify the same array for X and Y. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable.
4. For more information on LOCq(_), and how sequences are block-column distributed and stored in FFT-packed storage mode, see “Two-Dimensional Sequence” on page 66.

In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not 1 × *q*
2. The subroutine was called from outside the process grid.

Stage 3

1. *n1* > 37748736
2. *n2* > 37748736
3. The length of *n1* or *n2* is not an allowable transform length.
4. *isign* = 0
5. *scale* = 0.0
6. IP(1) ≠ 0 and IP(20) ≠ 0 and IP(20) < *n1* (that is, *ldx* < *n1*)
7. IP(1) ≠ 0 and IP(21) ≠ 0 and IP(21) < *n2* (that is, *ldy* < *n2*)

Example: This example shows how to compute a two-dimensional transform. The data is block-column distributed over a 1 × 2 process grid. The arrays are declared as follows:

```
REAL*8 X(0:11,0:1)
COMPLEX*16 Y(0:6,0:1)
INTEGER*4 IP(40)
REAL*8 SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 1
IP(20) = 12 (that is, ldx = 12)
IP(21) = 7  (that is, ldy = 7)

              X   Y   N1  N2  ISIGN  SCALE   ICONTXT   IP
              |   |   |   |     |      |        |       |
CALL PDRCFT2( X , Y , 8 , 4 ,   1   , 1.0D0 , ICONTXT , IP)
```

Global matrix ***X*** of order 8 × 4:

```
B,D       0            1
        ┌                     ┐
        | 1.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
 0      | 0.0   0.0  |  0.0  0.0 |
        | 0.0   0.0  |  0.0  0.0 |
        |  .     .   |   .    .  |
        |  .     .   |   .    .  |
        |  .     .   |   .    .  |
        |  .     .   |   .    .  |
        └                     ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***X***:

```
p,q  |     0      |     1
-----|------------|------------
     | 1.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
 0   | 0.0  0.0   |  0.0  0.0
     | 0.0  0.0   |  0.0  0.0
     |  .    .    |   .    .
     |  .    .    |   .    .
     |  .    .    |   .    .
     |  .    .    |   .    .
```

Output: The following global matrix ***Y*** is returned in transposed form and stored in FFT-packed storage mode:

```
B,D            0                        1
      ┌                                               ┐
      | (1.0,1.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0) |
      | (1.0,0.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0) |
      | (1.0,1.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0) |
 0    | (1.0,0.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0) |
      |     .          .      |       .          .     |
      |     .          .      |       .          .     |
      |     .          .      |       .          .     |
      └                                               ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

The following local arrays for ***Y*** are returned in transposed form and stored in FFT-packed storage mode:

```
p,q  |          0             |           1
-----|------------------------|------------------------
     | (1.0,1.0)   (1.0,0.0)  |  (1.0,0.0)   (1.0,0.0)
     | (1.0,0.0)   (1.0,0.0)  |  (1.0,0.0)   (1.0,0.0)
     | (1.0,1.0)   (1.0,0.0)  |  (1.0,0.0)   (1.0,0.0)
 0   | (1.0,0.0)   (1.0,0.0)  |  (1.0,0.0)   (1.0,0.0)
     |     .           .      |      .           .
     |     .           .      |      .           .
     |     .           .      |      .           .
```

PSCRFT2 and PDCRFT2—Complex-to-Real Fourier Transforms in Two Dimensions

These subroutines compute the mixed-radix two-dimensional real discrete Fourier transform of complex conjugate even data:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}$$

for:

$k1$ = 0, 1, ..., $n1$–1
$k2$ = 0, 1, ..., $n2$–1

where:

$$W_{n1} = e^{-2\pi(\sqrt{-1})/n1}$$
$$W_{n2} = e^{-2\pi(\sqrt{-1})/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/((*n1*)(*n2*)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 107. Data Types

X	Y, *scale*	**Subroutine**
Short-precision complex	Short-precision real	PSCRFT2
Long-precision complex	Long-precision real	PDCRFT2

Syntax

Fortran	CALL PSCRFT2 | PDCRFT2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	pscrft2 | pdcrft2 (*x*, *y*, *n1*, *n2*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array X, containing the two-dimensional data to be transformed that has been block-column distributed over a 1 × *q* process grid, where *q* is the number of processes. (The value of *ldx* is set in the IP array.) Array X is stored in FFT-packed storage mode.

Scope: **local**

Specified as: an array of (at least) length $ldx \times$ LOCq($n1/2$), containing numbers of the data type indicated in Table 107 on page 761. This array must be aligned on a doubleword boundary.

y
See On Return.

n1
is the length of the second dimension of the two-dimensional data of the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n1 \leq 37748736$ and must be one of the values listed in Figure 12 on page 746.

n2
is the length of the first dimension of two-dimensional data of the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n2 \leq 37748736$ and must be one of the values listed in Figure 12 on page 746.

isign
controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Scope: **global**

Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale
is the scaling constant *scale*.

Scope: **global**

Specified as: a number of the data type indicated in Table 107 on page 761, where $scale > 0.0$ or $scale < 0.0$.

icontxt
is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip
is an array of parameters, `IP(`*i*`)`, where:

- `IP(1)` indicates whether the default values for *ip* are used or you set the values for *ip*.

  If `IP(1)` = 0, then the following default values are used:

  – *ldx*, the leading dimension of the array specified for `X`, equals *n2*
  – *ldy*, the leading dimension of the array specified for `Y`, equals *n1*

  The remaining parameters of the array `IP` are ignored.

  If `IP(1)` ≠ 0, then you set the remaining values of *ip* to indicate values for *ldx* and *ldy*.

- IP(2-19) are reserved.
- IP(20) indicates the value of the leading dimension, *ldx*, of the array specified for X, where:

  If IP(20) = 0, then *ldx* = *n2*.

  If IP(20) ≠ 0, then *ldx* is this value of IP(20).
- IP(21) indicates the value of the leading dimension, *ldy*, of the array specified for Y, where:

  If IP(21) = 0 then *ldy* = *n1*.

  If IP(21) ≠ 0, then *ldy* is this value of IP(21).
- IP(22-40) are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:

IP(1) is any integer

IP(20) ≥ *n2* or IP(20) = 0

IP(21) ≥ *n1* or IP(21) = 0

On Return

y

is the local array Y that is block-column distributed and contains the results of the computation, where:

If IP(1) = 0, the local array Y is stored in normal form and has dimensions *n1* × LOCq(*n2*).

If IP(1) ≠ 0 and IP(21) = 0, the local array Y is stored in normal form and has dimensions *n1* × LOCq(*n2*).

If IP(1) ≠ 0 and IP(21) ≠ 0, the local array Y is stored in normal form and has dimensions *ldy* × LOCq(*n2*).

Scope: **local**

Returned as: an *ldy* × LOCq(*n2*) array, containing the numbers of the data type indicated in Table 107 on page 761. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. These subroutines always return Y in normal form.
2. For the output array Y, these subroutines may use any extra space available when *ldy* is greater than its minimum value.
3. For more information on LOCq(_), and how sequences are block-column distributed and stored in FFT-packed storage mode, see “Two-Dimensional Sequence” on page 66.

   In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

4. You may specify the same array for X and Y. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not 1 × *q*
2. The subroutine was called from outside the process grid.

Stage 3

1. *n1* > 37748736
2. *n2* > 37748736
3. The length of *n1* or *n2* is not an allowable transform length.
4. *isign* = 0
5. *scale* = 0.0
6. IP(1) ≠ 0 and IP(20) ≠ 0 and IP(20) < *n2* (that is, *ldx* < *n2*)
7. IP(1) ≠ 0 and IP(21) ≠ 0 and IP(21) < *n1* (that is, *ldy* < *n1*)

Example: This example shows how to compute a two-dimensional transform. The data is block-column distributed over a 1 × 2 process grid. The arrays are declared as follows:

```
COMPLEX*16 X(0:6,0:1)
REAL*8 Y(0:11,0:1)
INTEGER*4 IP(40)
REAL*8 SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 1
IP(20) = 7 (that is, ldx = 7)
IP(21) = 12 (that is, ldy = 12)

              X   Y   N1  N2  ISIGN     SCALE       ICONTXT   IP
              |   |   |   |     |         |            |      |
CALL PDCRFT2( X , Y , 8 , 4 ,  -1  , 1.0D0/32.0D0 , ICONTXT , IP)
```

The following global matrix ***X*** is stored in FFT-packed storage mode:

```
B,D           0                        1
    ┌                                            ┐
    | (1.0,1.0)  (1.0,0.0)  |    (1.0,0.0)  (1.0,0.0) |
    | (1.0,0.0)  (1.0,0.0)  |    (1.0,0.0)  (1.0,0.0) |
    | (1.0,1.0)  (1.0,0.0)  |    (1.0,0.0)  (1.0,0.0) |
 0  | (1.0,0.0)  (1.0,0.0)  |    (1.0,0.0)  (1.0,0.0) |
    |     .          .      |        .          .     |
    |     .          .      |        .          .     |
    |     .          .      |        .          .     |
    └                                            ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

The following local arrays for ***X*** are stored in FFT-packed storage mode:

```
p,q  |           0            |           1
-----|------------------------|-----------------------
     | (1.0,1.0)  (1.0,0.0)   |  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)   |  (1.0,0.0)  (1.0,0.0)
     | (1.0,1.0)  (1.0,0.0)   |  (1.0,0.0)  (1.0,0.0)
 0   | (1.0,0.0)  (1.0,0.0)   |  (1.0,0.0)  (1.0,0.0)
     |     .          .       |      .          .
     |     .          .       |      .          .
     |     .          .       |      .          .
```

Output: Global matrix ***Y***:

```
B,D       0          1
    ┌                     ┐
    | 1.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
 0  | 0.0   0.0  |  0.0   0.0 |
    | 0.0   0.0  |  0.0   0.0 |
    |  .     .   |   .     .  |
    |  .     .   |   .     .  |
    |  .     .   |   .     .  |
    |  .     .   |   .     .  |
    └                     ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***Y***:

```
p,q  |     0      |      1
-----|------------|------------
     |  1.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
 0   |  0.0  0.0  |   0.0  0.0
     |  0.0  0.0  |   0.0  0.0
     |   .    .   |    .    .
     |   .    .   |    .    .
     |   .    .   |    .    .
     |   .    .   |    .    .
```

PSCFT3 and PDCFT3—Complex Fourier Transforms in Three Dimensions

These subroutines compute the mixed-radix three-dimensional discrete Fourier transform of complex data:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}\ W_{n3}^{(Isign)j3k3}$$

for:

$k1 = 0, 1, ..., n1-1$
$k2 = 0, 1, ..., n2-1$
$k3 = 0, 1, ..., n3-1$

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array `X`.
$y_{k1,k2,k3}$ are elements of array `Y`.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/((*n1*)(*n2*)(*n3*)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 108. Data Types

`X`, `Y`	*scale*	**Subroutine**
Short-precision complex	Short-precision real	PSCFT3
Long-precision complex	Long-precision real	PDCFT3

Syntax

Fortran	CALL PSCFT3 | PDCFT3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	pscft3 | pdcft3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array X, containing the three-dimensional data to be transformed that has been block-plane distributed over a $1 \times q$ process grid, where *q* is the number of processes. (The values of *ldx1* and *ldx2* are set in the IP array.)

Scope: **local**

Specified as: an array of (at least) length $ldx1 \times ldx2 \times$ LOCq(*n3*), containing numbers of the data type indicated in Table 108 on page 767. This array must be aligned on a doubleword boundary.

y

See On Return.

n1

is the length of the first dimension of the three-dimensional data of the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n1 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

n2

is the length of the second dimension of the three-dimensional data of the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n2 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

n3

is the length of the third dimension of the three-dimensional data of the array to be transformed.

Scope: **global**

Specified as: a fullword integer; $n3 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

isign

controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Scope: **global**

Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale

is the scaling constant *scale*.

Scope: **global**

Specified as: a number of the data type indicated in Table 108 on page 767, where $scale > 0.0$ or $scale < 0.0$.

icontxt

is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip

is an array of parameters, IP(*i*), where:

- IP(1) indicates whether the default values for *ip* are used or you set the values for *ip*.

  If IP(1) = 0, then the following default values are used:

  – *y* is returned in transposed form; that is global *y* has dimensions $n3 \times n2 \times n1$.
  – *ldx1* and *ldx2*, the leading dimensions of the array specified for X, equal *n1* and *n2*, respectively.
  – *ldy1* and *ldy2*, the leading dimensions of the array specified for Y, equal *n3* and *n2*, respectively.

    The remaining parameters of array IP are ignored.

  If IP(1) ≠ 0, then you set the remaining values of *ip* to indicate whether *y* is stored in normal or transposed form, and indicate values for the leading dimensions.

- IP(2) indicates whether *y* is to be stored in normal or transposed form.

  If IP(2)=0, then *y* is to be stored in transposed form on output.

  If IP(2)=1, then *y* is to be stored in normal form on output.

- IP(3-19) are reserved.

- IP(20) indicates the values of the leading dimension, *ldx1*, for the array specified for X, where:

  If IP(20) = 0, then *ldx1* = *n1*.

  If IP(20) ≠ 0, then *ldx1* is this value of IP(20).

- IP(21) indicates the values of the leading dimension, *ldx2*, for the array specified for X, where:

  If IP(21) = 0, then *ldx2* = *n2*.

  If IP(21) ≠ 0, then *ldx2* is this value of IP(21).

- IP(22) indicates the values of the leading dimension, *ldy1*, for the array specified for Y, where:

  If IP(22) = 0 and IP(2) = 1, then *ldy1* = *n1*.

  If IP(22) = 0 and IP(2) = 0, then *ldy1* = *n3*.

  If IP(22) ≠ 0, then *ldy1* is this value of IP(22).

- IP(23) indicates the values of the leading dimension, *ldy2*, for the array specified for Y, where:

  If IP(23) = 0, then *ldy2* = *n2*.

  If IP(23) ≠ 0, then *ldy2* is this value of IP(23).

- IP(24-40) are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:

IP(1) is any integer
IP(2) = 0 or 1
IP(20) ≥ *n1* or IP(20)=0
IP(21) ≥ *n2* or IP(21)=0
IP(22) ≥ *n1* (for normal form) or IP(22)=0
IP(22) ≥ *n3* (for transposed form) or IP(22) = 0
IP(23) ≥ *n2* or IP(23)=0

On Return

y

is the local array Y that is block-plane distributed and contains the results of the computation, where:

If IP(1) = 0, the local array Y is stored in transposed form and has dimensions *n3* × *n2* × LOCq(*n1*).

If IP(1) ≠ 0 and IP(2)=0, then the local array Y is stored in transposed form and has dimensions *ldy1* × *ldy2* × LOCq(*n1*).

If IP(1) ≠ 0 and IP(2)=1, then the local array Y is stored in normal form and has dimensions *ldy1* × *ldy2* × LOCq(*n3*).

Scope: **local**

Returned as: an *ldy1* × *ldy2* × LOCq(*n3*) array (for normal form) or an *ldy1* × *ldy2* × LOCq(*n1*) array (for transposed form), containing the numbers of the data type indicated in Table 108 on page 767. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. For the output array Y, these subroutines may use any extra space available when *ldy1* and *ldy2* are greater than their minimum value.
2. You may specify the same array for X and Y. In this case, output overwrites input. If you specify different arrays X and Y, they must have no common elements; otherwise, results are unpredictable.
3. For more information on LOCq(_) and how sequences are block-plane distributed, see “Three-Dimensional Sequences” on page 71.

   In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space.

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not 1 × *q*
2. The subroutine was called from outside the process grid.

Stage 3

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. The length of *n1*, *n2*, or *n3* is not an allowable transform length.
5. *isign* = 0
6. *scale* = 0.0
7. IP(1) ≠ 0 and IP(2) ≠ 0 or 1
8. IP(1) ≠ 0 and IP(20) ≠ 0 and IP(20) < *n1* (that is, *ldx1* < *n1*)
9. IP(1) ≠ 0 and IP(21) ≠ 0 and IP(21) < *n2* (that is, *ldx2* < *n2*)
10. IP(1) ≠ 0 and IP(2)=0 (for transpose mode) and IP(22) ≠ 0 and IP(22) < *n3* (that is, *ldy1* < *n3*)
11. IP(1) ≠ 0 and IP(2)=1 (for normal mode) and IP(22) ≠ 0 and IP(22) < *n1* (that is, *ldy1* < *n1*)
12. IP(1) ≠ 0 and IP(23) ≠ 0 and IP(23) < *n2* (that is, *ldy2* < *n2*)

Example 1: This example shows how to compute a three-dimensional transform. The data is block-plane distributed over a 1 × 2 process grid. The arrays are declared as follows:

```
  COMPLEX*16 X(0:3,0:3,0)
  COMPLEX*16 Y(0:3,0:3,0)
  INTEGER*4 IP(40)
  REAL*8 SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 1
IP(2) = 1
IP(20) = 4
IP(21) = 4
IP(22) = 4
IP(23) = 4

             X   Y   N1  N2  N3  ISIGN  SCALE   ICONTXT   IP
             |   |   |   |   |     |      |        |      |
CALL PDCFT3( X , Y , 4 , 4 , 2  ,  1  , 1.0D0 , ICONTXT , IP)
```

Global matrix ***X***:

Plane 0:

```
B,D                     0
     ┌                                               ┐
     | (1.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
 0   | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     └                                               ┘
```

Plane 1:

```
B,D                     1
     ┌                                               ┐
     | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
 0   | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     | (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
     └                                               ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***X***:

```
p,q  |                   0                      |                   1
-----|------------------------------------------|------------------------------------------
     | (1.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0) | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0)
     | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0) | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0)
 0   | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0) | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0)
     | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0) | (0.0,0.0) (0.0,0.0) (0.0,0.0) (0.0,0.0)
```

Output: Global matrix ***Y***:

Plane 0:

```
B,D                     0
     ┌                                               ┐
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
 0   | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     └                                               ┘
```

Plane 1:

```
B,D                     1
     ┌                                               ┐
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
 0   | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
     └                                               ┘
```

The following is the 1 × 2 process grid:

B,D	0	1
0	P_{00}	P_{01}

Local arrays for ***Y***:

```
p,q  |                   0                        |                    1
-----|--------------------------------------------|--------------------------------------------
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
 0   | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
```

Example 2: This example shows how to compute a three-dimensional transform. In this example, the `IP` array is set to 0, which means array `Y` is returned in transposed form, *ldx*=*n1*, and *ldy*=*n3*. This is an example of uneven block-plane distribution over a 1 × 3 process grid. The arrays are declared as follows:

```
  COMPLEX*16 X(0:3,0:1,0:1), Y(0:5,0:1,0:1)
  INTEGER*4  IP(40)
  REAL*8     SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 3
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 0

             X   Y   N1  N2  N3  ISIGN     SCALE     ICONTXT   IP
             |   |   |   |   |     |         |          |      |
CALL PDCFT3( X , Y , 4 , 2 , 6  ,  1  , 1.0D0/8.0D0, ICONTXT , IP)
```

Global matrix ***X***:

```
          Plane 0:             Plane 1:
----------------------------------------------------
B,D                         0
----------------------------------------------------
     ┌                                             ┐
     | (4.9,3.9)  (5.9,3.1) | (2.2,5.1)  (5.9,1.5) |
0    | (6.8,4.6)  (6.7,9.8) | (2.1,0.5)  (3.5,2.9) |
     | (4.9,1.9)  (1.6,4.9) | (7.0,6.8)  (6.4,1.1) |
     | (2.9,7.6)  (7.5,5.5) | (0.6,4.6)  (9.0,7.6) |
     └                                             ┘
```

```
          Plane 2:             Plane 3:
----------------------------------------------------
B,D                        1
----------------------------------------------------
     ┌                                             ┐
     | (8.3,2.3)  (7.3,7.3) | (4.6,5.9)  (3.3,3.0) |
0    | (4.5,8.9)  (0.2,0.9) | (3.6,5.0)  (3.4,0.4) |
     | (1.8,6.5)  (2.5,1.7) | (8.5,9.3)  (0.2,1.7) |
     | (6.4,5.2)  (7.8,5.3) | (8.7,9.1)  (9.1,5.5) |
     └                                             ┘
```

```
          Plane 4:             Plane 5:
----------------------------------------------------
B,D                        2
----------------------------------------------------
     ┌                                             ┐
     | (1.0,1.0)  (2.2,8.2) | (5.4,3.6)  (4.8,4.0) |
0    | (8.8,1.0)  (8.9,5.8) | (2.0,2.6)  (5.2,9.5) |
     | (3.3,0.0)  (8.9,7.5) | (3.1,9.7)  (5.4,7.5) |
     | (8.7,8.3)  (6.3,5.4) | (1.5,9.6)  (4.4,4.4) |
     └                                             ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local arrays for ***X***:

```
p,q|                   0                   |                   1                   |                   2
---|---------------------------------------|---------------------------------------|---------------------------------------
   |(4.9,3.9) (5.9,3.1) (2.2,5.1) (5.9,1.5) |(8.3,2.3) (7.3,7.3) (4.6,5.9) (3.3,3.0) |(1.0,1.0) (2.2,8.2) (5.4,3.6) (4.8,4.0)
   |(6.8,4.6) (6.7,9.8) (2.1,0.5) (3.5,2.9) |(4.5,8.9) (0.2,0.9) (3.6,5.0) (3.4,0.4) |(8.8,1.0) (8.9,5.8) (2.0,2.6) (5.2,9.5)
 0 |(4.9,1.9) (1.6,4.9) (7.0,6.8) (6.4,1.1) |(1.8,6.5) (2.5,1.7) (8.5,9.3) (0.2,1.7) |(3.3,0.0) (8.9,7.5) (3.1,9.7) (5.4,7.5)
   |(2.9,7.6) (7.5,5.5) (0.6,4.6) (9.0,7.6) |(6.4,5.2) (7.8,5.3) (8.7,9.1) (9.1,5.5) |(8.7,8.3) (6.3,5.4) (1.5,9.6) (4.4,4.4)
```

Output: Global matrix ***Y***:

```
          Plane 0:                    Plane 1:
--------------------------------------------------------
B,D                            0
--------------------------------------------------------
     ┌                                                  ┐
     | (29.8,29.7)  (-1.9,1.1) |  (-3.0,0.9) (-3.0,-3.8) |
     |  (-3.3,1.0)  (0.4,-2.5) |  (4.1,-3.9) (-4.6,-1.4) |
0    | (-1.7,-1.2)   (0.1,3.4) |   (0.9,5.0)  (-0.6,1.6) |
     |  (2.3,-0.5)  (1.0,-4.6) |   (3.1,0.8)   (1.7,0.4) |
     |   (3.0,1.9)   (4.5,0.6) |  (0.5,-3.8) (-3.0,-0.1) |
     |   (1.0,0.1) (-5.7,-1.9) | (-1.4,-1.2)   (0.8,2.6) |
     └                                                  ┘

          Plane 2:                      Plane 3:
--------------------------------------------------------
B,D                             1
--------------------------------------------------------
     ┌                                                  ┐
     | (-2.4,-2.8)   (2.0,0.1) |  (3.6,-3.4)   (1.4,0.0) |
     |  (2.1,-3.5)   (2.8,1.3) | (-1.9,-0.1)   (2.3,6.3) |
0    |  (-1.7,0.7)   (2.8,1.0) |   (2.0,1.4)  (-0.6,1.0) |
     | (-3.3,-2.2) (-3.2,-5.1) |  (-0.3,3.3)   (0.8,0.5) |
     | (-1.5,-3.1)   (1.4,0.1) | (-1.3,-1.7)  (-2.7,0.7) |
     |   (1.8,0.6)  (-0.7,3.2) |   (0.2,2.9)  (1.0,-2.1) |
     └                                                  ┘
```

The following is the 1 × 3 process grid:

B,D	**0**	**1**	**2**
0	P_{00}	P_{01}	P_{02}

Local arrays for ***Y***:

```
p,q  |                        0                       |                       1
-----|------------------------------------------------|------------------------------------------------
     | (29.8,29.7)  (-1.9,1.1)  (-3.0,0.9) (-3.0,-3.8) | (-2.4,-2.8)   (2.0,0.1)  (3.6,-3.4)   (1.4,0.0)
     |  (-3.3,1.0)  (0.4,-2.5)  (4.1,-3.9) (-4.6,-1.4) |  (2.1,-3.5)   (2.8,1.3) (-1.9,-0.1)   (2.3,6.3)
 0   | (-1.7,-1.2)   (0.1,3.4)   (0.9,5.0)  (-0.6,1.6) |  (-1.7,0.7)   (2.8,1.0)   (2.0,1.4)  (-0.6,1.0)
     |  (2.3,-0.5)  (1.0,-4.6)   (3.1,0.8)   (1.7,0.4) | (-3.3,-2.2) (-3.2,-5.1)  (-0.3,3.3)   (0.8,0.5)
     |   (3.0,1.9)   (4.5,0.6)  (0.5,-3.8) (-3.0,-0.1) | (-1.5,-3.1)   (1.4,0.1) (-1.3,-1.7)  (-2.7,0.7)
     |   (1.0,0.1) (-5.7,-1.9) (-1.4,-1.2)   (0.8,2.6) |   (1.8,0.6)  (-0.7,3.2)   (0.2,2.9)  (1.0,-2.1)
```

There is not any data located on P_{02}.

PSRCFT3 and PDRCFT3—Real-to-Complex Fourier Transforms in Three Dimensions

These subroutines compute the mixed-radix three-dimensional complex conjugate even discrete Fourier transform of real data:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}\ W_{n3}^{(Isign)j3k3}$$

for:

$k1$ = 0, 1, ..., $n1$–1
$k2$ = 0, 1, ..., $n2$–1
$k3$ = 0, 1, ..., $n3$–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array X.
$y_{k1,k2,k3}$ are elements of array Y.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

See references [1] and [3].

Table 109. Data Types

X, *scale*	Y	**Subroutine**
Short-precision real	Short-precision complex	PSRCFT3
Long-precision real	Long-precision complex	PDRCFT3

Syntax

Fortran	CALL PSRCFT3 | PDRCFT3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	psrcft3 | pdrcft3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array X, containing the three-dimensional data to be transformed that has been block-plane distributed over a 1 × *q* process grid, where *q* is the number of processes. (The value of *ldx1* and *ldx2* are set in the IP array.)

Scope: **local**

Specified as: an array of (at least) length *ldx1* × *ldx2* × LOCq(*n3*), containing numbers of the data type indicated in Table 109. This array must be aligned on a doubleword boundary.

y
: See On Return.

n1
: is the length of the first dimension of the three-dimensional data in the array to be transformed.

  Scope: **global**

  Specified as: a fullword integer; $n1 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

n2
: is the length of the second dimension of the three-dimensional data in the array to be transformed.

  Scope: **global**

  Specified as: a fullword integer; $n2 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

n3
: is the length of the third dimension of the three-dimensional data in the array to be transformed.

  Scope: **global**

  Specified as: a fullword integer; $n3 \le 37748736$ and must be one of the values listed in Figure 12 on page 746.

isign
: controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

  If *isign* = positive value, *Isign* = + (transforming time to frequency).

  If *isign* = negative value, *Isign* = − (transforming frequency to time).

  Scope: **global**

  Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale
: is the scaling constant *scale*.

  Scope: **global**

  Specified as: a number of the data type indicated in Table 109 on page 776, where $scale > 0.0$ or $scale < 0.0$.

icontxt
: is the BLACS context parameter.

  Scope: **global**

  Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip
: is an array of parameters, `IP(`*i*`)`, where:

  - `IP(1)` indicates whether the default values for *ip* are used or you set the values for *ip*.

    If `IP(1)` = 0, then the following default values are used:

- *ldx1* and *ldx2*, the leading dimensions of the array specified for `X`, equal *n1* and *n2*, respectively.
- *ldy1* and *ldy2*, the leading dimensions of the array specified for `Y`, equal *n3* and *n2*, respectively.

The remaining parameters of the array `IP` are ignored.

If `IP(1)` ≠ 0, then you set the remaining values of *ip* to indicate values for the leading dimensions.

- `IP(2-19)` are reserved.
- `IP(20)` indicates the value of the leading dimension, *ldx1*, of the array specified for `X`, where:

  If `IP(20)` = 0, then *ldx1* = *n1*.

  If `IP(20)` ≠ 0, then *ldx1* is this value of `IP(20)`.
- `IP(21)` indicates the value of the leading dimension, *ldx2*, of the array specified for `X`, where:

  If `IP(21)` = 0, then *ldx2* = *n2*.

  If `IP(21)` ≠ 0, then *ldx2* is this value of `IP(21)`.
- `IP(22)` indicates the value of the leading dimension, *ldy1*, of the array specified for `Y`, where:

  If `IP(22)` = 0, then *ldy1* = *n3*.

  If `IP(22)` ≠ 0, then *ldy1* is this value of `IP(22)`.
- `IP(23)` indicates the value of the leading dimension, *ldy2*, of the array specified for `Y`, where:

  If `IP(23)` = 0, then *ldy2* = *n2*.

  If `IP(23)` ≠ 0, then *ldy2* is this value of `IP(23)`.
- `IP(24-40)` are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:
`IP(1)` is any integer
`IP(20)` ≥ *n1* or `IP(20)` = 0
`IP(21)` ≥ *n2* or `IP(21)` = 0
`IP(22)` ≥ *n3* or `IP(22)` = 0
`IP(23)` ≥ *n2* or `IP(23)` = 0

On Return

y

is the local array `Y`, stored in FFT-packed storage mode, containing the results of the computation that are block-plane distributed, where:

If `IP(1)` = 0, the local array `Y` has dimensions *n3* × *n2* × LOCq(*n1/2*).

If `IP(1)` ≠ 0, the local array `Y` has dimensions *ldy1* × *ldy2* × LOCq(*n1/2*).

Scope: **local**

Returned as: an *ldy1* × *ldy2* × LOCq(*n1/2*) array, containing the numbers of the data type indicated in Table 109 on page 776. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. These subroutines always return `Y` in transposed form.
2. For the output array `Y`, these subroutines may use any extra space available when *ldy1* and *ldy2* are greater than their minimum value.
3. You may specify the same array for `X` and `Y`. In this case, output overwrites input. If you specify different arrays `X` and `Y`, they must have no common elements; otherwise, results are unpredictable.
4. For more information on LOCq(_), and how sequences are blocked-plane distributed and stored in FFT-packed storage mode, see "Three-Dimensional Sequences" on page 71.

   In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not 1 × *q*
2. The subroutine was called from outside the process grid.

Stage 3

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. The length of *n1*, *n2*, or *n3* is not an allowable transform length.
5. *isign* = 0
6. *scale* = 0.0
7. `IP(1)` ≠ 0 and `IP(20)` ≠ 0 and `IP(20)` < *n1* (that is, *ldx1* < *n1*)
8. `IP(1)` ≠ 0 and `IP(21)` ≠ 0 and `IP(21)` < *n2* (that is, *ldx2* < *n2*)
9. `IP(1)` ≠ 0 and `IP(22)` ≠ 0 and `IP(22)` < *n3* (that is, *ldy1* < *n3*)

10. `IP(1)` ≠ 0 and `IP(23)` ≠ 0 and `IP(23)` < *n2* (that is, *ldy2* < *n2*)

Example: This example shows how to compute a three-dimensional transform. The data is block-plane distributed over a 1 × 2 process grid. The arrays are declared as follows:

```
  REAL*8 X(0:8,0:3,0:1)
  COMPLEX*16 Y(0:4,0:3,0)
  INTEGER*4 IP(40)
  REAL*8 SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
IP(1) = 1
IP(20) = 9
IP(21) = 4
IP(22) = 5
IP(23) = 4
```

```
              X   Y   N1  N2  N3  ISIGN  SCALE   ICONTXT   IP
              |   |   |   |   |     |      |        |      |
CALL PDRCFT3( X , Y , 4 , 4 , 4  ,  1  , 1.0D0 , ICONTXT , IP)
```

Global matrix ***X***:

```
          Plane 0:                Plane 1:
----------------------------------------------------
B,D                          0
----------------------------------------------------
     ┌                                            ┐
     | 1.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
 0   |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     └                                            ┘
```

```
          Plane 2:                Plane 3:
----------------------------------------------------
B,D                          1
----------------------------------------------------
     ┌                                            ┐
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
     | 0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0 |
 0   |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     |  .    .    .    .   |   .    .    .    .  |
     └                                            ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***X***:

```
p,q  |                   0                     |                    1
-----|-----------------------------------------|-----------------------------------------
     |  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0   |   .    .    .    .    .    .    .    .   |   .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .   |   .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .   |   .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .   |   .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .   |   .    .    .    .    .    .    .    .
```

Output: The following global matrix ***Y*** is returned in transposed form and stored in FFT-packed storage mode:

Plane 0:

```
B,D                     0
     ┌                                                 ┐
     |  (1.0,1.0)   (1.0,0.0)   (1.0,1.0)   (1.0,0.0) |
     |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
 0   |  (1.0,1.0)   (1.0,0.0)   (1.0,1.0)   (1.0,0.0) |
     |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
     |      .           .           .           .     |
     └                                                 ┘
```

Plane 1:

```
B,D                     1
     ┌                                                 ┐
     |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
     |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
 0   |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
     |  (1.0,0.0)   (1.0,0.0)   (1.0,0.0)   (1.0,0.0) |
     |      .           .           .           .     |
     └                                                 ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

The following local arrays for ***Y*** are returned in transposed form and stored in FFT-packed storage mode:

```
p,q  |                     0                     |                      1
-----|-------------------------------------------|-------------------------------------------
     | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
 0   | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     |     .          .          .          .     |      .          .          .          .
```

PSCRFT3 and PDCRFT3—Complex-to-Real Fourier Transforms in Three Dimensions

These subroutines compute the mixed-radix three-dimensional real discrete Fourier transform of complex conjugate even data:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}\ W_{n3}^{(Isign)j3k3}$$

for:

$k1$ = 0, 1, ..., $n1$–1
$k2$ = 0, 1, ..., $n2$–1
$k3$ = 0, 1, ..., $n3$–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array X.
$y_{k1,k2,k3}$ are elements of array Y.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

See references [1] and [3].

Table 110. Data Types

X	Y, *scale*	**Subroutine**
Short-precision complex	Short-precision real	PSCRFT3
Long-precision complex	Long-precision real	PDCRFT3

Syntax

Fortran	CALL PSCRFT3 | PDCRFT3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*)
C and C++	pscrft3 | pdcrft3 (*x*, *y*, *n1*, *n2*, *n3*, *isign*, *scale*, *icontxt*, *ip*);

On Entry

x

is the local array X, containing the three-dimensional data to be transformed that has been block-plane distributed over a 1 × *q* process grid, where *q* is the number of processes. (The value of *ldx1* and *ldx2* are set in the IP array.) Array X is stored in FFT-packed storage mode.

Scope: **local**

Specified as: an array of (at least) length *ldx1* × *ldx2* × LOCq(*n1/2*), containing numbers of the data type indicated in Table 110 on page 783. This array must be aligned on a doubleword boundary.

y
See On Return.

n1
is the length of the first dimension of the three-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; *n1* ≤ 37748736 and must be one of the values listed in Figure 12 on page 746.

n2
is the length of the second dimension of the three-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; *n2* ≤ 37748736 and must be one of the values listed in Figure 12 on page 746.

n3
is the length of the third dimension of the three-dimensional data in the array to be transformed.

Scope: **global**

Specified as: a fullword integer; *n3* ≤ 37748736 and must be one of the values listed in Figure 12 on page 746.

isign
controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Scope: **global**

Specified as: a fullword integer; where *isign* > 0 or *isign* < 0.

scale
is the scaling constant *scale*.

Scope: **global**

Specified as: a number of the data type indicated in Table 110 on page 783, where *scale* > 0.0 or *scale* < 0.0.

icontxt
is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

ip
is an array of parameters, `IP(`*i*`)`, where:

- `IP(1)` indicates whether the default values for *ip* are used or you set the values for *ip*.

  If `IP(1)` = 0, then the following default values are used:

– *ldx1* and *ldx2*, the leading dimensions of the array specified for X, equal *n3* and *n2*, respectively.
– *ldy1* and *ldy2*, the leading dimensions of the array specified for Y, equal *n1* and *n2*, respectively.

The remaining parameters of the array IP are ignored.

If IP(1) ≠ 0, then you set the remaining values of *ip* to indicate values for the leading dimensions.

- IP(2-19) are reserved.
- IP(20) indicates the value of the leading dimension, *ldx1*, of the array specified for X, where:

  If IP(20) = 0, then *ldx1* = *n3*.

  If IP(20) ≠ 0, then *ldx1* is this value of IP(20).
- IP(21) indicates the value of the leading dimension, *ldx2*, of the array specified for X, where:

  If IP(21) = 0, then *ldx2* = *n2*.

  If IP(21) ≠ 0, then *ldx2* is this value of IP(21).
- IP(22) indicates the value of the leading dimension, *ldy1*, of the array specified for Y, where:

  If IP(22) = 0, then *ldy1* = *n1*.

  If IP(22) ≠ 0, then *ldy1* is this value of IP(22).
- IP(23) indicates the value of the leading dimension, *ldy2*, of the array specified for Y, where:

  If IP(23) = 0, then *ldy2* = *n2*.

  If IP(23) ≠ 0, then *ldy2* is this value of IP(23).
- IP(24-40) are reserved.

Scope: **global**

Specified as: a one-dimensional array of (at least) length 40, containing fullword integers, where:
IP(1) is any integer
IP(20) ≥ *n3* or IP(20) = 0
IP(21) ≥ *n2* or IP(21) = 0
IP(22) ≥ *n1* or IP(22) = 0
IP(23) ≥ *n2* or IP(23) = 0

On Return

y

is the local array Y that is block-plane distributed and contains the results of the computation, where:

If IP(1) = 0, the local array Y has dimensions *n1* × *n2* × LOCq(*n3*).

If IP(1) ≠ 0, the local array Y has dimensions *ldy1* × *ldy2* × LOCq(*n3*).

Scope: **local**

Returned as: an *ldy1* × *ldy2* × LOCq(*n3*) array, containing the numbers of the data type indicated in Table 110 on page 783. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. These subroutines always return `Y` in normal form.
2. For the output array `Y`, these subroutines may use any extra space available when *ldy1* and *ldy2* are greater than their minimum value.
3. You may specify the same array for `X` and `Y`. In this case, output overwrites input. If you specify different arrays `X` and `Y`, they must have no common elements; otherwise, results are unpredictable.
4. For more information on LOCq(_), and how sequences are block-plane distributed and stored in FFT-packed storage mode, see “Three-Dimensional Sequences” on page 71.

   In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

Error Conditions

Computational Errors: None

Resource Errors

1. Unable to allocate work space

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. Process grid is not 1 × *q*
2. The subroutine was called from outside the process grid.

Stage 3

1. *n1* > 37748736
2. *n2* > 37748736
3. *n3* > 37748736
4. The length of *n1*, *n2*, *n3* is not an allowable transform length.
5. *isign* = 0
6. *scale* = 0.0
7. `IP(1)` ≠ 0 and `IP(20)` ≠ 0 and `IP(20)` < *n3* (that is, *ldx1* < *n3*)
8. `IP(1)` ≠ 0 and `IP(21)` ≠ 0 and `IP(21)` < *n2* (that is, *ldx2* < *n2*)
9. `IP(1)` ≠ 0 and `IP(22)` ≠ 0 and `IP(22)` < *n1* (that is, *ldy1* < *n1*)
10. `IP(1)` ≠ 0 and `IP(23)` ≠ 0 and `IP(23)` < *n2* (that is, *ldy2* < *n2*)

Example: This example shows how to compute a three-dimensional transform. The data is block-plane distributed over a 1 × 2 process grid. The arrays are declared as follows:

```
  COMPLEX*16 X(0:4,0:3,0)
  REAL*8 Y(0:8,0:3,0:1)
  INTEGER*4 IP(40)
  REAL*8 SCALE
```

Call Statements and Input

```
ORDER = 'R'
NPROW = 1
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
SCALE = 1.0D0/4*4*4
IP(1) = 1
IP(20) = 5
IP(21) = 4
IP(22) = 9
IP(23) = 4

              X   Y   N1  N2  N3  ISIGN      SCALE      ICONTXT   IP
              |   |   |   |   |     |          |           |      |
CALL PDCRFT3( X , Y , 4 , 4 , 4  , -1  , 1.0D0/64.0D0 , ICONTXT , IP)
```

The following global matrix ***X*** is stored in FFT-packed storage mode:

Plane 0:

```
B,D                         0
          ┌                                                ┐
          | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |
          | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
 0        | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |
          | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
          |     .          .          .          .     |
          └                                                ┘
```

Plane 1:

```
B,D                         1
          ┌                                                ┐
          | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
          | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
 0        | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
          | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
          |     .          .          .          .     |
          └                                                ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

The following local arrays for ***X*** are stored in FFT-packed storage mode:

```
p,q  |                     0                       |                     1
-----|---------------------------------------------|---------------------------------------------
     | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
 0   | (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     | (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  |   (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)
     |     .          .          .          .      |       .          .          .          .
```

Output:

Global matrix ***Y***:

```
          Plane 0:                Plane 1:
----------------------------------------------------
B,D                        0
----------------------------------------------------
     ┌                                            ┐
     |  1.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
 0   |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     └                                            ┘

          Plane 2:                Plane 3:
----------------------------------------------------
B,D                        1
----------------------------------------------------
     ┌                                            ┐
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
     |  0.0  0.0  0.0  0.0  |   0.0  0.0  0.0  0.0 |
 0   |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     |   .    .    .    .   |    .    .    .    .  |
     └                                            ┘
```

The following is the 1 × 2 process grid:

B,D	**0**	**1**
0	P_{00}	P_{01}

Local arrays for ***Y***:

```
p,q  |                   0                    |                   1
-----|----------------------------------------|----------------------------------------
     |  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
     |  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0   |   .    .    .    .    .    .    .    .  |    .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .  |    .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .  |    .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .  |    .    .    .    .    .    .    .    .
     |   .    .    .    .    .    .    .    .  |    .    .    .    .    .    .    .    .
```

Chapter 11. Random Number Generation (Message Passing)

This chapter describes the random number generation subroutines.

Overview of the Random Number Generation Subroutines

The random number generation subroutine generates uniformly distributed random numbers.

Table 111. List of Random Number Generation Subroutines (Message Passing)

Descriptive Name	**Long-Precision Subroutine**	**Page**
Uniform Random Number Generator	PDURNG	793

Random Number Generation Subroutines

This section contains the random number generation subroutine description.

PDURNG—Uniform Random Number Generator

This subroutine generates a global vector ***x*** of *n* uniform pseudo-random numbers in the ranges (0,1) or (−1,1), depending on the *iopt* argument. The random numbers are generated using the multiplicative congruential method with a user-specified seed, as follows:

$s_i = (a(s_{i-1})) \bmod (m) = (a^i s_0) \bmod (m)$
$x_i = s_i/m$ if $iopt = 0$
$x_i = (2s_i/m)-1$ if $iopt = 1$
for $i = 1, 2, ..., n$

where:

s_0 is the initial seed provided by the caller.
s_i for $i = 1, n$ is a random sequence.
x_i for $i = 1, n$ are the random numbers.
$a = 44485709377909.0$
$m = 2.0^{48}$
n is the number of random numbers to be generated.

If *n* is 0, no computation is performed, and the initial seed is unchanged.

The global output vector ***x*** is distributed across the *np* processes, using block-cyclic distribution with a block size *nb*. (The processor grid can be one- or two-dimensions. For two dimensions, processes are selected in row-major order.) The length *n* of vector ***x*** must be a multiple of (*np*)(*nb*).

Table 112. Data Types

x, *seed*	**Subroutine**
Long-precision real	PDURNG

Syntax

Fortran	CALL PDURNG (*seed*, *n*, *nb*, *x*, *iopt*, *icontxt*)
C and C++	pdurng (*seed*, *n*, *nb*, *x*, *iopt*, *icontxt*);

On Entry

seed
is the initial value s_0 used to generate the random numbers.

Scope: **global**

Specified as: a number of the data type indicated in Table 112. You should specify *seed* to be an **odd**, **whole** number; otherwise, PDURNG sets it to an odd, whole number and continues with the computation. The value of *seed* must be $1.0 \le seed < 2.0^{48}$.

n
is the global number of random numbers to be generated.

Scope: **global**

Specified as: a fullword integer; $n \ge 0$ and *n* must be divisible by (*nb*)(*np*).

nb
is the block size for vector ***x***, used in the block-cyclic distribution.

Scope: **global**

Specified as: a fullword integer; $nb > 0$.

x
See On Return.

iopt
indicates the range of uniform random numbers to generate, where:

If $iopt = 0$, the range is (0,1).

If $iopt = 1$, the range is (−1,1).

Scope: **global**

Specified as: a fullword integer; $iopt = 0$ or 1.

icontxt
is the BLACS context parameter.

Scope: **global**

Specified as: the fullword integer that was returned by a prior call to BLACS_GRIDINIT or BLACS_GRIDMAP.

On Return

seed
is the new seed that is to be used to generate additional random numbers in subsequent invocations of PDURNG, having a value of $seed = (a^n s_0) \bmod (m)$.

Scope: **global**

Returned as: a number of the data type indicated in Table 112 on page 793. It is an **odd**, **whole** number, where $1.0 \le seed < 2.0^{48}$.

x
is the local vector ***x*** of size n/np, containing the uniform pseudo-random numbers, where:

If $iopt = 0$, they are in the range (0,1).

If $iopt = 1$, they are in the range (−1,1).

Scope: **local**

Returned as: a one-dimensional array of (at least) length n/np, with a block size of *nb*, containing numbers of the data type indicated in Table 112 on page 793.

Notes and Coding Rules

1. In your C program, argument *seed* must be passed by reference.
2. The suggested block size is (data cache size)/2, where, the data cache size can be obtained by entering:
   ```
   lsattr -E -H -l sys0
   ```
3. There is no performance impact for $nb = 1$.
4. If you want n/np random numbers generated on each process, just set the block size to $nb = n/np$.

5. To generate more than $(2^{31}-1)$ random numbers, you should make multiple calls to PDURNG.
6. The local vector ***x*** of length *n*/*np* can have sequential correlations. For details, see references [42], [47], [48], [49], and [50].

Error Conditions

Computational Errors: None

Input-Argument and Miscellaneous Errors

Stage 1

1. *icontxt* is invalid

Stage 2

1. PDURNG was called from outside the process grid.

Stage 3

1. $seed < 1.0$ or $seed \geq 2.0^{48}$
2. $n < 0$
3. *n* is not divisible by (*nb*)(*np*)
4. $nb \leq 0$
5. $iopt \neq 0$ or 1

Example: This example generates 30 random numbers in global vector ***x*** with a block size of 3, using block-cyclic distribution over a 5×1 process grid.

Call Statements and Input

```
ORDER = 'R'
NPROW = 5
NPCOL = 1
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)

             SEED   N     NB  X   IOPT  ICONTXT
              |     |     |   |    |       |
CALL PDURNG( SEED , 30 ,  3 , X ,  0 ,  ICONTXT)

SEED    = 31415926535897.0
```

Note: *icontxt* is the output of the BLACS_GRIDINIT call.

Output: SEED = $(a^{30}s_0) \bmod (m)$ = 6316434292705.0

Global vector ***x*** with block size 3:

```
B,D          0
     ┌                       ┐
     | 0.683821516135299845 |
 0   | 0.058874407800946215 |
     | 0.391855250856924187 |
     | -------------------- |
     | 0.755994653022330709 |
 1   | 0.557764301423606668 |
     | 0.001333801764989317 |
     | -------------------- |
     | 0.056855932753212101 |
 2   | 0.331063036202269956 |
     | 0.347339794409027292 |
     | -------------------- |
     | 0.649429020370863697 |
 3   | 0.386144876217390021 |
     | 0.457224855098420591 |
     | -------------------- |
     | 0.892518134165118937 |
 4   | 0.074548748224632532 |
     | 0.912379366805073033 |
     | -------------------- |
     | 0.112809499110515077 |
 5   | 0.857547605095465570 |
     | 0.756480901897081282 |
     | -------------------- |
     | 0.046993364463578046 |
 6   | 0.889457684002341153 |
     | 0.167775766106718294 |
     | -------------------- |
     | 0.504952722600595649 |
 7   | 0.999725924546471134 |
     | 0.696269487398215148 |
     | -------------------- |
     | 0.671896598019703362 |
 8   | 0.271472156040264423 |
     | 0.566418406688985243 |
     | -------------------- |
     | 0.464684865759100063 |
 9   | 0.982442539763031419 |
     | 0.022440482512937620 |
     └                       ┘
```

The following is the 5 × 1 process grid:

B,D	0
0 5	P_{00}
1 6	P_{10}
2 7	P_{20}
3 8	P_{30}
4 9	P_{40}

Local arrays for ***x***:

```
p,q  |         0
-----|----------------------
     | 0.683821516135299845
     | 0.058874407800946215
     | 0.391855250856924187
 0   | 0.112809499110515077
     | 0.857547605095465570
     | 0.756480901897081282
-----|----------------------
     | 0.755994653022330709
     | 0.557764301423606668
     | 0.001333801764989317
 1   | 0.046993364463578046
     | 0.889457684002341153
     | 0.167775766106718294
-----|----------------------
     | 0.056855932753212101
     | 0.331063036202269956
     | 0.347339794409027292
 2   | 0.504952722600595649
     | 0.999725924546471134
     | 0.696269487398215148
-----|----------------------
     | 0.649429020370863697
     | 0.386144876217390021
     | 0.457224855098420591
 3   | 0.671896598019703362
     | 0.271472156040264423
     | 0.566418406688985243
-----|----------------------
     | 0.892518134165118937
     | 0.074548748224632532
     | 0.912379366805073033
 4   | 0.464684865759100063
     | 0.982442539763031419
     | 0.022440482512937620
```

Chapter 12. Utilities (Message Passing)

This chapter describes the utility subroutines.

Overview of the Utility Subroutines

The utility subroutines perform general service functions that support Parallel ESSL, rather than mathematical computations.

Table 113. List of Utility Subroutines (Message Passing)

Descriptive Name	**Integer Subroutine**	**Page**
Determine the Level of Parallel ESSL Installed on Your System	IPESSL	801
Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process	NUMROC	803

Utility Subroutines

This section contains the utility subroutine descriptions.

IPESSL—Determine the Level of Parallel ESSL Installed on Your System

This function returns the current level of Parallel ESSL installed on your system, where the level consists of a version number, release number, and modification number, plus the fix number of the most recent PTF installed.

Note: This subroutine is useful to you in those instances where your program is using a subroutine or feature that exists only in certain levels of Parallel ESSL. It is also useful when your program is dependent upon certain PTFs being applied to Parallel ESSL.

Syntax

Fortran	IPESSL ()
C and C++	ipessl ();

On Return

Function value
is the level of Parallel ESSL installed on your system. It is provided as a fullword integer in the form *vvrrmmff*, where each two digits represents a part of the level:

- *vv* is the version number.
- *rr* is the release number.
- *mm* is the modification number.
- *ff* is the fix number of the most recent PTF installed.

Scope: **global**

Returned as: a fullword integer; *vvrrmmff* > 0.

Notes

1. To use IPESSL effectively, you must install your Parallel ESSL PTFs in their proper sequential order. As part of the result, IPESSL returns the value *ff* of the **most recent** PTF installed, rather than the **highest number** PTF installed. Therefore, if you do not install your PTFs sequentially, the *ff* value returned by IPESSL does not reflect the actual level of ESSL.
2. Declare the IPESSL function in your program as returning a fullword integer value.
3. For the first release of Parallel ESSL for AIX Version 4, *vv* = 01 and *rr* = 01, and IPESSL returns 1010000.

Example: This example shows several ways to use the IPESSL function. Most typically, you use IPESSL for checking the version and release level of Parallel ESSL. Suppose you are dependent on a new capability in Parallel ESSL, such as a new subroutine or feature, provided for the first time in **(fictitious)** Parallel ESSL Version 3 Release 2. You can add the following check in your program before using the new capability:

```
IF IPESSL() ≥ 3020000
```

By specifying 0000 for *mmff*, the modification and fix level, you are independent of the order in which your modifications and PTFs are installed.

Less typically, you use IPESSL for checking the PTF level of Parallel ESSL. Suppose you are dependent on **(fictitious)** PTF 24 being installed on your Parallel ESSL Version 1 Release 0 system You want to know whether to call a different user-callable subroutine to set up your array data. You can add the following check in your program before making the call:

```
IF IPESSL() ≥ 1000024
```

If your system support group installed the Parallel ESSL PTFs in their proper sequential order, this test works properly; otherwise, it is unpredictable.

NUMROC—Compute the Number of Rows or Columns of a Block-Cyclically Distributed Matrix Contained in a Process

This function computes the local number of rows or columns of a block-cyclically distributed matrix contained in a process row or process column, respectively, indicated by the calling sequence argument *iproc*.

See references [14] and [15].

Syntax

Fortran	NUMROC (*n*, *nb*, *iproc*, *isrcproc*, *nprocs*)
C and C++	numroc (*n*, *nb*, *iproc*, *isrcproc*, *nprocs*);

On Entry

n
is the number of rows M_ or columns N_ in a global matrix that has been block-cyclically distributed.

Scope: **global**

Specified as: a fullword integer; $n \ge 0$.

nb
is the row block size MB_ or the column block size NB_.

Scope: **global**

Specified as: a fullword integer; $nb > 0$.

iproc
is process row index *myrow* or the process column index *mycol*.

Scope: **local**

Specified as: a fullword integer; $0 \le iproc < nprocs$.

isrcproc
is the process row RSRC_ or the process column CSRC_ over which the first row or column, respectively, of the global matrix is distributed.

Scope: **global**

Specified as: a fullword integer; $0 \le isrcproc < nprocs$.

nprocs
is the number of rows *nprow* or the number of columns *npcol* in the process grid.

Scope: **global**

Specified as: a fullword integer; $nprocs > 0$.

On Return

Function value
is the local number of rows or columns of a block-cyclically distributed matrix contained in a process row or process column, respectively, indicated by the calling sequence argument *iproc*.

Scope: **local**

Returned as: a fullword integer.

Note: The variables *p* and *nprow* are used interchangeably to indicate the number of rows in a process grid. The variables *q* and *npcol* are used interchangeably to indicate the number of columns in a process grid.

Error Conditions

Computational Errors: None

Resource Errors: None

Input-Argument and Miscellaneous Errors

Stage 1

1. $nb \le 0$
2. $nprocs \le 0$

Example: This example shows the local invocations of NUMROC from four processes in a 2 × 2 process grid, using a global symmetric matrix of order 9.

Call Statements and Input

```
ORDER = 'R'
NPROW = 2
NPCOL = 2
CALL BLACS_GET(0, 0, ICONTXT)
CALL BLACS_GRIDINIT(ICONTXT, ORDER, NPROW, NPCOL)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYROW, MYCOL)
```

The NUMROC function invocations and associated data on each process are shown in “Local Invocations of NUMROC.”

The pertinent array descriptor values for global matrix ***C*** are shown below:

	Desc_C
M_	9
N_	9
MB_	4
NB_	4
RSRC_	0
CSRC_	0

Global symmetric matrix ***C*** of order 9 is stored in upper storage mode with block sizes 4 × 4:

```
B,D           0                       1              2
     ┌                                                     ┐
     │ -6.0  0.0  0.0  0.0  │   0.0 -2.0 -2.0  0.0  │   -2.0 │
     │   .  -6.0 -2.0  0.0  │  -2.0 -4.0  0.0 -4.0  │   -2.0 │
 0   │   .    .  -6.0 -2.0  │  -2.0  0.0  2.0  0.0  │    6.0 │
     │   .    .    .  -6.0  │   2.0  0.0  2.0  0.0  │    2.0 │
     │ ---------------------|-----------------------|------- │
     │   .    .    .    .   │  -8.0 -4.0  0.0 -2.0  │    0.0 │
     │   .    .    .    .   │    .  -6.0  0.0 -4.0  │   -6.0 │
 1   │   .    .    .    .   │    .    .  -4.0  0.0  │    0.0 │
     │   .    .    .    .   │    .    .    .  -4.0  │   -4.0 │
     │ ---------------------|-----------------------|------- │
 2   │   .    .    .    .   │    .    .    .    .   │  -16.0 │
     └                                                     ┘
```

The following is the 2 × 2 process grid:

B,D	0 2	1
0 2	P_{00}	P_{01}
1	P_{10}	P_{11}

Local arrays for ***C***:

```
p,q  │             0              │          1
-----|----------------------------|---------------------
     │ -6.0  0.0  0.0  0.0  -2.0  │   0.0 -2.0 -2.0  0.0
     │   .  -6.0 -2.0  0.0  -2.0  │  -2.0 -4.0  0.0 -4.0
 0   │   .    .  -6.0 -2.0   6.0  │  -2.0  0.0  2.0  0.0
     │   .    .    .  -6.0   2.0  │   2.0  0.0  2.0  0.0
     │   .    .    .    .  -16.0  │    .    .    .    .
-----|----------------------------|---------------------
     │   .    .    .    .    0.0  │  -8.0 -4.0  0.0 -2.0
     │   .    .    .    .   -6.0  │    .  -6.0  0.0 -4.0
 1   │   .    .    .    .    0.0  │    .    .  -4.0  0.0
     │   .    .    .    .   -4.0  │    .    .    .  -4.0
```

Local Invocations of NUMROC:

```
p,q  │                      0                      │                      1
-----|---------------------------------------------|---------------------------------------------
     │              M_C  MB_C  MYROW  RSRC_C  NPROW │              M_C  MB_C  MYROW  RSRC_C  NPROW
     │               |    |     |       |      |   │               |    |     |       |      |
     │ MP = NUMROC( 9,   4,    0,      0,     2)   │ MP = NUMROC( 9,   4,    0,      0,     2)
 0   │                                             │
     │              N_C  NB_C  MYCOL  CSRC_C  NPCOL │              N_C  NB_C  MYCOL  CSRC_C  NPCOL
     │               |    |     |       |      |   │               |    |     |       |      |
     │ NQ = NUMROC( 9,   4,    0,      0,     2)   │ NQ = NUMROC( 9,   4,    1,      0,     2)
-----|---------------------------------------------|---------------------------------------------
     │              M_C  MB_C  MYROW  RSRC_C  NPROW │              M_C  MB_C  MYROW  RSRC_C  NPROW
     │               |    |     |       |      |   │               |    |     |       |      |
     │ MP = NUMROC( 9,   4,    1,      0,     2)   │ MP = NUMROC( 9,   4,    1,      0,     2)
 1   │                                             │
     │              N_C  NB_C  MYCOL  CSRC_C  NPCOL │              N_C  NB_C  MYCOL  CSRC_C  NPCOL
     │               |    |     |       |      |   │               |    |     |       |      |
     │ NQ = NUMROC( 9,   4,    0,      0,     2)   │ NQ = NUMROC( 9,   4,    1,      0,     2)
```

Output:

The local number of rows MP and columns NQ of the block-cyclically distributed matrix returned by NUMROC on each process:

```
p,q  |   0    |    1
-----|--------|--------
 0   |  MP=5  |   MP=5
     |  NQ=5  |   NQ=4
-----|--------|--------
 1   |  MP=4  |   MP=4
     |  NQ=5  |   NQ=4
```

Part 3. Reference Information (HPF)

This part of the book is organized into five areas, providing reference information for coding the Parallel ESSL calling sequences in a High Performance Fortran (HPF) program. It is organized as follows:

- PBLAS
- Linear Algebraic Equations
- Eigensystem Analysis and Singular Value Analysis
- Fourier Transforms
- Random Number Generation

Chapter 13. PBLAS (HPF)

This chapter describes the Level 2 and 3 PBLAS subroutines that can be called from an HPF program.

Overview of the PBLAS Subroutines

The Level 2 and 3 PBLAS include a subset of the standard set of distributed memory parallel versions of the Level 2 and 3 BLAS.

Note: These subroutines are designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Level 2 PBLAS

Table 114. List of Level 2 PBLAS (HPF)

Descriptive Name	**Long-Precision Subprogram**	**Page**
Matrix-Vector Product for a General Matrix or Its Transpose	GEMM	811
Matrix-Vector Product for a Real Symmetric Matrix	SYMM	822
Rank-One Update of a General Matrix	GEMM	811
Rank-One Update of a Real Symmetric Matrix	SYRK	840
Rank-Two Update of a Real Symmetric Matrix	SYR2K	846
Matrix-Vector Product for a Triangular Matrix or Its Transpose	TRMM	828
Solution of Triangular System of Equations with a Single Right-Hand Side	TRSM	834

Level 3 PBLAS

Table 115. List of Level 3 PBLAS (HPF)

Descriptive Name	**Long-Precision Subprogram**	**Page**
Matrix-Matrix Product for a General Matrix, Its Transpose, or Its Conjugate Transpose	GEMM	811
Matrix-Matrix Product Where One Matrix is Real Symmetric	SYMM	822
Triangular Matrix-Matrix Product	TRMM	828
Solution of Triangular System of Equations with Multiple Right-Hand Sides	TRSM	834
Rank-K Update of a Real Symmetric Matrix	SYRK	840
Rank-2K Update of a Real Symmetric Matrix	SYR2K	846
Matrix Transpose for a General Matrix	TRAN	852

PBLAS Subroutines

This section contains the PBLAS subroutine descriptions.

GEMM—Matrix-Matrix Product for a General Matrix, Its Transpose, or Its Conjugate Transpose

This subroutine performs any one of the following combined matrix computations:

1. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}+\beta\boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}^{\mathrm{T}}+\beta\boldsymbol{C}$
3. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}+\beta\boldsymbol{C}$
4. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\beta\boldsymbol{C}$
5. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}+\beta\boldsymbol{C}$
6. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{T}}+\beta\boldsymbol{C}$
7. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{AB}^{\mathrm{H}}+\beta\boldsymbol{C}$
8. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{H}}+\beta\boldsymbol{C}$
9. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}^{\mathrm{H}}+\beta\boldsymbol{C}$
10. $\boldsymbol{c} \leftarrow \alpha\boldsymbol{Ab}+\beta\boldsymbol{c}$
11. $\boldsymbol{c} \leftarrow \alpha\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b}+\beta\boldsymbol{c}$
12. $\boldsymbol{C} \leftarrow \alpha\boldsymbol{ab}^{\mathrm{T}}+\boldsymbol{C}$

where, in the formulas above:

A, ***B***, and ***C*** are general matrices.
a, ***b***, and ***c*** are vectors.
α and β are scalars.

Note: No data should be moved to form the matrix transposes or matrix conjugate transposes; that is, the matrices should always be stored in their untransposed forms.

In the following cases, no computation is performed and the subroutine returns after doing some parameter checking:

- For equations 1–9:
  - The assumed-shape array for ***C*** has a size of zero.
  - α is zero and β is one.
  - β is one, and the assumed-shape arrays for ***A*** and ***B*** have a size of zero.

  Assuming the above conditions do not exist, if β is not one and the assumed-shape arrays for ***A*** and ***B*** have a size of zero, then β***C*** is returned.
- For equations 10 and 11:
  - Any of the assumed-shape arrays have a size of zero.
  - α is zero and β is one.
- For equation 12:
  - Any of the assumed-shape arrays have a size of zero.
  - α is zero.

See references [17], [30], [31], and [44].

Table 116. Data Types

α, β, ***A***, ***B***, ***C***, ***a***, ***b***, ***c***	**Subroutine**
Long-precision real	GEMM
Long-precision complex	GEMM

Syntax

HPF	**Equations 1–9**	CALL GEMM (*alpha*, *a*, *b*, *beta*, *c*) CALL GEMM (*alpha*, *a*, *b*, *beta*, *c*, *transa*, *transb*)
HPF	**Equations 10 and 11**	CALL GEMM (*alpha*, *a*, *b*, *beta*, *c*) CALL GEMM (*alpha*, *a*, *b*, *beta*, *c*, *transa*)
HPF	**Equation 12**	CALL GEMM (*alpha*, *a*, *b*, *c*)

On Entry

alpha
is the scalar α.

Type: **required**

Specified as: a number of the data type indicated in Table 116 on page 811.

a
is the general matrix $\boldsymbol{A}$ or the vector $\boldsymbol{a}$, where:

If *transa* = `'N'`, $\boldsymbol{A}$ is used in the computation.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation.

Note: No data should be moved to form $\boldsymbol{A}^T$ or $\boldsymbol{A}^H$; that is, the matrix $\boldsymbol{A}$ should always be stored in its untransposed form.

Type: **required**

Specified as: an assumed-shape array with shape (`:,:`) or (`:`), containing numbers of the data type indicated in Table 116 on page 811.

b
is the general matrix $\boldsymbol{B}$ or the vector $\boldsymbol{b}$, where:

If *transb* = `'N'`, $\boldsymbol{B}$ is used in the computation.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation.

If *transb* = `'C'`, $\boldsymbol{B}^H$ is used in the computation.

Type: **required**

Note: No data should be moved to form $\boldsymbol{B}^T$ or $\boldsymbol{B}^H$; that is, the matrix $\boldsymbol{B}$ should always be stored in its untransposed form.

Specified as: an assumed-shape array with shape (`:,:`) or (`:`), containing numbers of the data type indicated in Table 116 on page 811.

beta
is the scalar β.

Type: **required** (equations 1–11); **not present** (equation 12)

Specified as: a number of the data type indicated in Table 116 on page 811.

c
is the general matrix $\boldsymbol{C}$ or the vector $\boldsymbol{c}$. When β is zero, *c* need not be set on input.

Type: **required**

Specified as: an assumed-shape array with shape (`:,:`) or (`:`), containing numbers of the data type indicated in Table 116 on page 811.

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in equation 1, 2, 7, or 10.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation, resulting in equation 3, 4, 8, or 11.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation, resulting in equation 5, 6, or 9.

Type: **optional** (equations 1–11); **not present** (equation 12)

Default: *transa* = `'N'`

Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

transb

indicates the form of matrix ***B*** to use in the computation, where:

If *transb* = `'N'`, ***B*** is used in the computation, resulting in equation 1, 3, or 5.

If *transb* = `'T'`, $\boldsymbol{B}^T$ is used in the computation, resulting in equation 2, 4, or 6.

If *transb* = `'C'`, $\boldsymbol{B}^H$ is used in the computation, resulting in equation 7, 8, or 9.

Type: **optional** (equations 1–9); **not present** (equations 10–12)

Default: *transb* = `'N'`

Specified as: a single character; *transb* = `'N'` or `'T'`.

On Return

c

is the updated matrix ***C*** or vector ***c***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 116 on page 811.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For equations 1 through 9:
     - If *transa* = `'N'` and *transb* = `'N'`:
       - size(*a*,1) = size(*c*,1)
       - size(*b*,2) = size(*c*,2)
       - size(*a*,2) = size(*b*,1)
     - If *transa* = `'N'` and *transb* = `'T'` or `'C'`:
       - size(*a*,1) = size(*c*,1)
       - size(*b*,1) = size(*c*,2)
       - size(*a*,2) = size(*b*,2)
     - If *transa* = `'T'` or `'C'` and *transb* = `'N'`:
       - size(*a*,2) = size(*c*,1)
       - size(*b*,2) = size(*c*,2)
       - size(*a*,1) = size(*b*,1)
     - If *transa* = `'T'` or `'C'` and *transb* = `'T'` or `'C'`:
       - size(*a*,2) = size(*c*,1)
       - size(*b*,1) = size(*c*,2)
       - size(*a*,1) = size(*b*,2)

- For equations 10 and 11:
  - If *transa* = `'N'`:
    - size(*a*,1) = size(*c*)
    - size(*a*,2) = size(*b*)
  - If *transa* = `'T'`:
    - size(*a*,1) = size(*b*)
    - size(*a*,2) = size(*c*)
- For equation 12:
  - size(*c*,1) = size(*a*)
  - size(*c*,2) = size(*b*)

2. This subroutine accepts lowercase letters for the *transa* and *transb* arguments.
3. If you are using long-precision real data and specify `'C'` for the *transa* or *transb* argument, it is interpreted as though you specified `'T'`.
4. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
5. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
6. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
7. The restrictions given in “Notes and Coding Rules” on page 287, “Notes and Coding Rules” on page 169, and “Notes and Coding Rules” on page 209 also apply to this subroutine.
8. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See “Program Main (HPF)” on page 1036.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 290, “Error Conditions” on page 171, and “Error Conditions” on page 210 also apply to this subroutine.

Input-Argument Errors for Equations 1–9

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2

1. *transa* is present, and *transa* ≠`'N'`, `'T'`, or `'C'`.
2. *transb* is present, and *transb* ≠`'N'`, `'T'`, or `'C'`.

Stage 3: The process grid is not the same for *a*, *b*, and *c*.

Stage 4: The data distribution is inconsistent for *a*, *b*, and *c*.

Stage 5: The shape of the assumed-shape arrays *a*, *b*, and *c* is incompatible:

1. *transa* = `'N'` and *transb* = `'N'`:
   - size(*a*,1) ≠ size(*c*,1) or
   - size(*b*,2) ≠ size(*c*,2) or
   - size(*a*,2) ≠ size(*b*,1)
2. *transa* = `'N'` and *transb* = `'T'` or `'C'`:
   - size(*a*,1) ≠ size(*c*,1) or
   - size(*b*,1) ≠ size(*c*,2) or
   - size(*a*,2) ≠ size(*b*,2)
3. *transa* = `'T'` or `'C'` and *transb* = `'N'`:
   - size(*a*,2) ≠ size(*c*,1) or
   - size(*b*,2) ≠ size(*c*,2) or
   - size(*a*,1) ≠ size(*b*,1)
4. *transa* = `'T'` or `'C'` and *transb* = `'T'` or `'C'`:
   - size(*a*,2) ≠ size(*c*,1) or
   - size(*b*,1) ≠ size(*c*,2) or
   - size(*a*,1) ≠ size(*b*,2)

Stage 6: The data distribution for *a*, *b*, or *c* is unsupported.

Input-Argument Errors for Equations 10 and 11

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: *transa* is present, and *transa* ≠`'N'`, `'T'`, or `'C'`.

Stage 3: The process grid is not the same for *a*, *b*, and *c*.

Stage 3

1. The vector for *b* or *c* is replicated.
2. The data distribution for *a* is unsupported.

Stage 5

1. Vector distribution error for *b* or *c*
2. The shape of the assumed-shape arrays *a*, *b*, and *c* is incompatible:
   a. *transa* = `'N'`:
      - size(*a*,1) ≠ size(*c*) or
      - size(*a*,2) ≠ size(*b*)

   b. *transa* = `'T'`:
      - size(*a*,1) ≠ size(*b*) or
      - size(*a*,2) ≠ size(*c*)

Stage 6: The data distribution for *a*, *b*, or *c* is unsupported.

Input-Argument Errors for Equation 12

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.

3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: The process grid is not the same for *a*, *b*, and *c*.

Stage 3: The data distribution for *c* is unsupported.

Stage 4: The vector for *a* or *b* is replicated.

Stage 5: The data distribution is inconsistent for *a* and *b*.

Stage 6

1. Vector distribution error for *a* or *b*.
2. The shape of the assumed-shape arrays *a*, *b*, and *c* is incompatible:
   size(*c*,1) ≠ size(*a*) or
   size(*c*,2) ≠ size(*b*)

Stage 7: The data distribution for *a*, *b*, or *c* is unsupported.

Example 1: This example computes ***C*** = α***AB***+β***C***. As in "Example 1" on page 292, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL GEMM( 1.0D0 , A , B , 2.0D0 , C )
-or-
CALL GEMM( 1.0D0 , A , B , 2.0D0 , C , TRANSA='N' , TRANSB='N' )
```

Input: General 6 × 5 matrix ***A***:

```
┌                          ┐
|  1.0   2.0  -1.0  -1.0   4.0 |
|  2.0   0.0   1.0   1.0  -1.0 |
|  1.0  -1.0  -1.0   1.0   2.0 |
| -3.0   2.0   2.0   2.0   0.0 |
|  4.0   0.0  -2.0   1.0  -1.0 |
| -1.0  -1.0   1.0  -3.0   2.0 |
└                          ┘
```

General 5 × 4 matrix ***B***:

```
┌                    ┐
|  1.0  -1.0   0.0   2.0 |
|  2.0   2.0  -1.0  -2.0 |
|  1.0   0.0  -1.0   1.0 |
| -3.0  -1.0   1.0  -1.0 |
|  4.0   2.0  -1.0   1.0 |
└                    ┘
```

General 6 × 4 matrix ***C***:

```
┌                 ┐
| 0.5  0.5  0.5  0.5 |
| 0.5  0.5  0.5  0.5 |
| 0.5  0.5  0.5  0.5 |
| 0.5  0.5  0.5  0.5 |
| 0.5  0.5  0.5  0.5 |
| 0.5  0.5  0.5  0.5 |
└                 ┘
```

Output: General 6 × 4 matrix ***C***:

```
┌                        ┐
| 24.0  13.0  -5.0   3.0 |
| -3.0  -4.0   2.0   4.0 |
|  4.0   1.0   2.0   5.0 |
| -2.0   6.0  -1.0  -9.0 |
| -4.0  -6.0   5.0   5.0 |
| 16.0   7.0  -4.0   7.0 |
└                        ┘
```

Example 2: This example computes ***C*** = α***AB***+β***C***. As in "Example 2" on page 295, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL GEMM( (1.0D0,0.0D0) , A , B , (2.0D0,0.0D0) , C )
-or-
CALL GEMM( (1.0D0,0.0D0) , A , B , (2.0D0,0.0D0) , C , TRANSA='N' , TRANSB='N' )
```

Input: General 6 × 3 matrix ***A***:

```
┌                                      ┐
|  (1.0,5.0)   (9.0,2.0)  (1.0,9.0)   |
|  (2.0,4.0)   (8.0,3.0)  (1.0,8.0)   |
|  (3.0,3.0)   (7.0,5.0)  (1.0,7.0)   |
|  (4.0,2.0)   (4.0,7.0)  (1.0,5.0)   |
|  (5.0,1.0)   (5.0,1.0)  (1.0,6.0)   |
|  (6.0,6.0)   (3.0,6.0)  (1.0,4.0)   |
└                                      ┘
```

General 3 × 2 matrix ***B***:

```
┌                          ┐
|  (1.0,8.0)  (2.0,7.0)   |
|  (4.0,4.0)  (6.0,8.0)   |
|  (6.0,2.0)  (4.0,5.0)   |
└                          ┘
```

General 6 × 2 matrix ***C***:

```
┌                         ┐
| (0.5,0.0)  (0.5,0.0)   |
| (0.5,0.0)  (0.5,0.0)   |
| (0.5,0.0)  (0.5,0.0)   |
| (0.5,0.0)  (0.5,0.0)   |
| (0.5,0.0)  (0.5,0.0)   |
| (0.5,0.0)  (0.5,0.0)   |
└                         ┘
```

Output: General 6 × 2 matrix ***C***:

```
┌                                    ┐
|  (-22.0,113.0)  (-35.0.142.0)     |
|  (-19.0,114.0)  (-35.0.141.0)     |
|  (-20.0,119.0)  (-43.0.146.0)     |
|  (-27.0,110.0)  (-58.0.131.0)     |
|    (8.0,103.0)    (0.0.112.0)     |
|  (-55.0,116.0)  (-75.0.135.0)     |
└                                    ┘
```

Example 3: This example computes ***c*** = α***Ab***+β***c***. The input matrices ***A***, ***B***, and ***C***, used here, are the same as the matrices used in “Example 1” on page 816. The updated portion of ***C*** is also the same, as this computation is equivalent to a portion of the computation.

Array sections are specified for arguments *a*, *b*, and *c*, resulting in the computation using a submatrix ***A*** starting at row 3 and column 1 in an array, a column vector ***b*** starting at row 1 and column 2 in an array, and a column vector ***c***, starting at row 3 and column 2 in an array.

As in “Example 1” on page 174, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL GEMM( 1.0D0 , A(3:6,1:5) , B(1:5,2:2) , 2.0D0 , C(3:6,2:2) )
-or-
CALL GEMM( 1.0D0 , A(3:6,1:5) , B(1:5,2:2) , 2.0D0 , C(3:6,2:2) , TRANSA='N' )
```

Input: Only a portion of the data structure is used—that is, submatrix ***A***. Following is the 4 × 5 submatrix ***A***, starting at row 3 and column 1 in the 6 × 5 array:

```
┌                             ┐
|    .     .     .     .     . |
|    .     .     .     .     . |
|  1.0  -1.0  -1.0   1.0   2.0 |
| -3.0   2.0   2.0   2.0   0.0 |
|  4.0   0.0  -2.0   1.0  -1.0 |
| -1.0  -1.0   1.0  -3.0   2.0 |
└                             ┘
```

Only a portion of the data structure is used—that is, vector ***b***, which is a column vector. Following is the vector ***b*** of size 5, starting at row 1 and column 2 in the 5 × 4 array:

```
┌                      ┐
|  .   -1.0    .     .  |
|  .    2.0    .     .  |
|  .    0.0    .     .  |
|  .   -1.0    .     .  |
|  .    2.0    .     .  |
└                      ┘
```

Only a portion of the data structure is used—that is, vector ***c***, which is a column vector. Following is the vector ***c*** of size 4, starting at row 3 and column 2 in the 6 × 4 array:

```
┌                   ┐
|  .    .    .    .  |
|  .    .    .    .  |
|  .   0.5   .    .  |
|  .   0.5   .    .  |
|  .   0.5   .    .  |
|  .   0.5   .    .  |
└                   ┘
```

Output: Only a portion of the data structure is used—that is, vector ***c***, which is a column vector. Following is the vector ***c*** of size 4, starting at row 3 and column 2 in the 6 × 4 array:

```
┌                      ┐
|  .     .     .     . |
|  .     .     .     . |
|  .    1.0    .     . |
|  .    6.0    .     . |
|  .   -6.0    .     . |
|  .    7.0    .     . |
└                      ┘
```

Example 4: This example computes ***c*** = α***Ab***+β***c***. The input matrices ***A***, ***B***, and ***C***, used here, are the same as ***A***, ***B***, and ***C***, used in "Example 1" on page 816.

Array sections are specified for arguments *a*, *b*, and *c*, resulting in the computation using a submatrix ***A*** starting at row 2 and column 2 in an array, a row vector ***b*** starting at row 4 and column 2 in an array, and a column vector ***c*** starting at row 2 and column 3 in an array.

As in "Example 2" on page 177, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL GEMM( 1.0D0 , A(2:5,2:4) , B(4:4,2:4) , 2.0D0 , C(2:5,3:3) )
-or-
CALL GEMM( 1.0D0 , A(2:5,2:4) , B(4:4,2:4) , 2.0D0 , C(2:5,3:3) , TRANSA='N' )
```

Input: Only a portion of the data structure is used—that is, submatrix ***A***. Following is the 4 × 3 submatrix ***A***, starting at row 2 and column 2 in the 6 × 5 array:

```
┌                            ┐
|  .     .     .     .     . |
|  .    0.0   1.0   1.0    . |
|  .   -1.0  -1.0   1.0    . |
|  .    2.0   2.0   2.0    . |
|  .    0.0  -2.0   1.0    . |
|  .     .     .     .     . |
└                            ┘
```

Only a portion of the data structure is used—that is, vector ***b***, which is a row vector. Following is the vector ***b*** of size 3, starting at row 4 and column 2 in the 5 × 4 array:

```
┌                      ┐
|  .     .     .     . |
|  .     .     .     . |
|  .     .     .     . |
|  .   -1.0   1.0  -1.0|
|  .     .     .     . |
└                      ┘
```

Only a portion of the data structure is used—that is, vector ***c***, which is a column vector. Following is the vector ***c*** of size 4, starting at row 2 and column 3 in the 6 × 4 array:

```
┌                 ┐
|  .    .    .    .  |
|  .    .   0.5   .  |
|  .    .   0.5   .  |
|  .    .   0.5   .  |
|  .    .   0.5   .  |
|  .    .    .    .  |
└                 ┘
```

Output: Only a portion of the data structure is used—that is, vector ***c***, which is a column vector. Following is the vector ***c*** of size 4, starting at row 2 and column 3 in the 6 × 4 array:

```
┌                        ┐
|  .     .     .      .  |
|  .     .    1.0     .  |
|  .     .    0.0     .  |
|  .     .   -1.0     .  |
|  .     .   -2.0     .  |
|  .     .     .      .  |
└                        ┘
```

Example 5: This example computes ***C*** = α***ab***^{T}+***C***.

Array sections are specified for arguments *a*, *b*, and *c*, resulting in the computation using a submatrix ***C*** starting at row 2 and column 2 in an array, a column vector ***a***, starting at element 2 in an array, and a row vector ***b*** starting at element 2 in an array.

As in "Example 1" on page 212, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN A(:) WITH C(:,1)
!HPF$ ALIGN B(:) WITH C(1,:)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: C

CALL GEMM( 1.0D0 , A(2:10) , B(2:10) , C(2:10,2:10) )
```

Input: Only a portion of the data structure is used—that is, submatrix ***C***. Following is the 9 × 9 submatrix ***C***, starting at row 2 and column 2 in the 10 × 10 array:

```
┌                                                                   ┐
|  .     .      .      .      .      .      .      .      .      .  |
|  .   12.0   22.0   32.0   42.0   52.0   62.0   72.0   82.0   92.0 |
|  .   13.0   23.0   33.0   43.0   53.0   63.0   73.0   83.0   93.0 |
|  .   14.0   24.0   34.0   44.0   54.0   64.0   74.0   84.0   94.0 |
|  .   15.0   25.0   35.0   45.0   55.0   65.0   75.0   85.0   95.0 |
|  .   16.0   26.0   36.0   46.0   56.0   66.0   76.0   86.0   96.0 |
|  .   17.0   27.0   37.0   47.0   57.0   67.0   77.0   87.0   97.0 |
|  .   18.0   28.0   38.0   48.0   58.0   68.0   78.0   88.0   98.0 |
|  .   19.0   29.0   39.0   49.0   59.0   69.0   79.0   89.0   99.0 |
|  .   20.0   30.0   40.0   50.0   60.0   70.0   80.0   90.0  100.0 |
└                                                                   ┘
```

Only a portion of the data structure is used—that is, vector ***a***, which is a column vector. Following is the vector ***a*** of size 9, starting at element 2 in the array of size 11:

```
┌     ┐
|  .  |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
|  .  |
└     ┘
```

Only a portion of the data structure is used—that is, vector ***b***, which is a row vector. Following is the vector ***b*** of size 9, starting at element 2 in the array of size 11:

```
┌                                                           ┐
|  .    2.0   3.0   4.0   5.0   6.0   7.0   8.0   9.0  10.0    .  |
└                                                           ┘
```

Output: Only a portion of the data structure is used—that is, submatrix ***C***. Following is the 9 × 9 submatrix ***C***, starting at row 2 and column 2 in the 10 × 10 array:

```
┌                                                                  ┐
|  .     .      .      .      .      .      .      .      .      .  |
|  .   14.0   25.0   36.0   47.0   58.0   69.0   80.0   91.0  102.0 |
|  .   15.0   26.0   37.0   48.0   59.0   70.0   81.0   92.0  103.0 |
|  .   16.0   27.0   38.0   49.0   60.0   71.0   82.0   93.0  104.0 |
|  .   17.0   28.0   39.0   50.0   61.0   72.0   83.0   94.0  105.0 |
|  .   18.0   29.0   40.0   51.0   62.0   73.0   84.0   95.0  106.0 |
|  .   19.0   30.0   41.0   52.0   63.0   74.0   85.0   96.0  107.0 |
|  .   20.0   31.0   42.0   53.0   64.0   75.0   86.0   97.0  108.0 |
|  .   21.0   32.0   43.0   54.0   65.0   76.0   87.0   98.0  109.0 |
|  .   22.0   33.0   44.0   55.0   66.0   77.0   88.0   99.0  110.0 |
└                                                                  ┘
```

SYMM—Matrix-Matrix Product Where One Matrix is Real Symmetric

This subroutine computes one of the following matrix-matrix products:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{AB}+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{BA}+\beta \boldsymbol{C}$
3. $\boldsymbol{c} \leftarrow \alpha \boldsymbol{Ab}+\beta \boldsymbol{c}$

where, in the formulas above:

A is a symmetric matrix.
B and ***C*** are general matrices.
b and ***c*** are vectors.
α and β are scalars.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- Any of the assumed-shape arrays have a size of zero.
- α is zero and β is one.

See references [17], [30], [31], and [44].

Table 117. Data Types

α, β, ***A***, ***B***, ***C***, ***b***, ***c***	**Subprogram**
Long-precision real	SYMM

Syntax

HPF	**Equations 1 and 2**	CALL SYMM (*alpha*, *a*, *b*, *beta*, *c*, *uplo*, *side*)
HPF	**Equation 3**	CALL SYMM (*alpha*, *a*, *b*, *beta*, *c*, *uplo*)

On Entry

alpha
is the scalar α.

Type: **required**

Specified as: a number of the data type indicated in Table 117.

a
is the symmetric matrix ***A***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 117, where size(*a*,1) = size(*a*,2).

b
is the general matrix ***B*** or the vector ***b***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 117.

beta

is the scalar β.

Type: **required**

Specified as: a number of the data type indicated in Table 117 on page 822.

c

is the general matrix ***C*** or the vector ***c***. When β is zero, *c* need not be set on input.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 117 on page 822.

uplo

indicates whether the upper or lower triangular part of the symmetric matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

side

indicates whether ***A*** is located to the left or right of ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in equation 1.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in equation 2.

Type: **required** (equations 1 and 2); **not present** (equation 3)

Specified as: a single character; *side* = `'L'` or `'R'`.

On Return

c

is the updated matrix ***C*** or vector ***c***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 117 on page 822.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For equations 1 and 2:
     - If *side* = `'L'`:
       - size(*b*,1) = size(*c*,1) = size(*a*,1) = size(*a*,2)
       - size(*b*,2) = size(*c*,2)
     - If *side* = `'R'`:
       - size(*b*,1) = size(*c*,1)
       - size(*b*,2) = size(*c*,2) = size(*a*,1) = size(*a*,2)
   - For equation 3: size(*a*,1) = size(*a*,2) = size(*b*) = size(*c*)

2. For migration purposes, note that the *side* and *uplo* arguments appear in reverse order from the corresponding BLAS and PBLAS subroutines.
3. This subroutine accepts lowercase letters for the *side* and *uplo* arguments.
4. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
5. The restrictions given in “Notes and Coding Rules” on page 304 and “Notes and Coding Rules” on page 194 also apply to this subroutine.
6. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
7. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 306 and “Error Conditions” on page 195 also apply to this subroutine.

Input-Argument Errors for Equations 1 and 2

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: The process grid is not the same for *a*, *b*, and *c*.

Stage 3: The data distribution is inconsistent for *a*, *b*, and *c*.

Stage 4

1. *side* is present, and *side* ≠ `'L'` or `'R'`.
2. *side* = `'L'` or `'R'`, and the shape of the assumed-shape arrays *a*, *b*, and *c* is incompatible:
   a. *side* = `'L'` and:
      size(*b*,1) ≠ size(*c*,1) or
      size(*c*,1) ≠ size(*a*,1) or
      size(*a*,1) ≠ size(*a*,2) or
      size(*b*,2) ≠ size(*c*,2)
   b. *side* = `'R'` and:
      size(*b*,1) ≠ size(*c*,1) or
      size(*b*,2) ≠ size(*c*,2) or
      size(*c*,2) ≠ size(*a*,1) or
      size(*a*,1) ≠ size(*a*,2)
3. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a*, *b*, or *c* is unsupported.

Input-Argument Errors for Equation 3

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: The process grid is not the same for *a*, *b*, and *c*.

Stage 3

1. The vector for *b* or *c* is replicated.
2. The data distribution for *a* is unsupported.

Stage 4

1. Vector distribution error for *b* or *c*.
2. The shape of the assumed-shape arrays *a*, *b*, and *c* is incompatible:
   size(*a*,1) ≠ size(*a*,2) or
   size(*a*,1) ≠ size(*b*) or
   size(*a*,1) ≠ size(*c*)
3. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a*, *b*, or *c* is unsupported.

Example 1: This example computes $\boldsymbol{C} = \alpha\boldsymbol{BA}+\beta\boldsymbol{C}$. Because $\beta = 0$, ***C*** need not be set on input. As in "Example 1" on page 309, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL SYMM( 1.0D0, A, B, 0.0D0, C, 'U', 'R' )
```

Input: Symmetric matrix ***A*** of order 8:

```
┌                                               ┐
| 0.0  -1.0  -1.0   0.0   0.0   0.0   0.0   0.0 |
|  .    1.0   0.0   1.0   0.0   1.0   0.0   1.0 |
|  .     .   -1.0  -1.0   0.0   0.0   1.0   0.0 |
|  .     .     .   -1.0   1.0   1.0   0.0   1.0 |
|  .     .     .     .   -1.0   0.0   0.0   0.0 |
|  .     .     .     .     .    1.0   0.0   0.0 |
|  .     .     .     .     .     .    0.0   0.0 |
|  .     .     .     .     .     .     .    0.0 |
└                                               ┘
```

General 16 × 8 matrix ***B***:

```
┌                                               ┐
│ -1.0   0.0   1.0  -1.0   1.0   1.0  -1.0  -1.0 │
│ -1.0  -1.0   1.0   0.0   1.0  -1.0  -1.0   1.0 │
│  1.0   1.0  -1.0   0.0  -1.0   0.0   1.0   0.0 │
│  0.0  -1.0   0.0   0.0   0.0   0.0   0.0  -1.0 │
│  0.0   1.0   0.0   1.0   0.0   1.0   1.0   0.0 │
│  0.0   0.0   1.0   0.0  -1.0  -1.0   0.0   0.0 │
│  1.0   1.0   0.0   0.0   1.0   1.0   0.0  -1.0 │
│  0.0   0.0  -1.0   0.0   0.0   1.0   0.0   1.0 │
│  0.0   0.0   0.0  -1.0   1.0   1.0   0.0   1.0 │
│ -1.0  -1.0   1.0   0.0   0.0  -1.0   0.0   1.0 │
│  0.0   0.0   0.0   1.0   1.0   0.0   0.0   0.0 │
│  0.0   0.0   1.0   1.0   0.0  -1.0   0.0   0.0 │
│  1.0   1.0  -1.0   0.0  -1.0  -1.0   1.0   1.0 │
│  0.0   0.0   0.0   0.0   1.0   0.0   0.0  -1.0 │
│  0.0   1.0   0.0   0.0   0.0   0.0   0.0   0.0 │
│ -1.0   0.0  -1.0   0.0   0.0   1.0   1.0   0.0 │
└                                               ┘
```

Output: General 16 × 8 matrix ***C***:

```
┌                                               ┐
│ -1.0   0.0   0.0   1.0  -2.0   0.0   1.0  -1.0 │
│  0.0   0.0  -1.0  -1.0  -1.0  -2.0   1.0  -1.0 │
│  0.0   0.0   1.0   1.0   1.0   1.0  -1.0   1.0 │
│  1.0  -2.0   0.0  -2.0   0.0  -1.0   0.0  -1.0 │
│ -1.0   3.0   0.0   1.0   1.0   3.0   0.0   2.0 │
│ -1.0  -1.0  -1.0  -3.0   1.0  -1.0   1.0   0.0 │
│ -1.0   0.0  -1.0   2.0  -1.0   2.0   0.0   1.0 │
│  1.0   2.0   1.0   3.0   0.0   1.0  -1.0   0.0 │
│  0.0   1.0   1.0   4.0  -2.0   0.0   0.0  -1.0 │
│  0.0   0.0   0.0  -2.0   0.0  -2.0   1.0  -1.0 │
│  0.0   1.0  -1.0   0.0   0.0   1.0   0.0   1.0 │
│ -1.0   0.0  -2.0  -3.0   1.0   0.0   1.0   1.0 │
│  0.0   0.0   1.0   1.0   1.0   0.0  -1.0   1.0 │
│  0.0  -1.0   0.0   0.0  -1.0   0.0   0.0   0.0 │
│ -1.0   1.0   0.0   1.0   0.0   1.0   0.0   1.0 │
│  1.0   2.0   3.0   2.0   0.0   1.0  -1.0   0.0 │
└                                               ┘
```

Example 2: This example computes $\boldsymbol{c} = \alpha \boldsymbol{Ab} + \beta \boldsymbol{c}$. As in “Example 1” on page 197, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN B(:) WITH A(:,1)
!HPF$ ALIGN C(:) WITH A(:,1)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL SYMM( 1.0D0, A, B, 0.0D0, C, 'U' )
```

Input: Symmetric matrix ***A*** of order 8:

```
┌                                               ┐
| 0.0  -1.0  -1.0   0.0   0.0   0.0   0.0   0.0 |
|  .    1.0   0.0   1.0   0.0   1.0   0.0   1.0 |
|  .     .   -1.0  -1.0   0.0   0.0   1.0   0.0 |
|  .     .     .   -1.0   1.0   1.0   0.0   1.0 |
|  .     .     .     .   -1.0   0.0   0.0   0.0 |
|  .     .     .     .     .    1.0   0.0   0.0 |
|  .     .     .     .     .     .    0.0   0.0 |
|  .     .     .     .     .     .     .    0.0 |
└                                               ┘
```

Vector ***b*** of size 8:

```
┌     ┐
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
└     ┘
```

Output: Vector ***c*** of size 8:

```
┌      ┐
| -2.0 |
|  3.0 |
| -2.0 |
|  2.0 |
|  0.0 |
|  3.0 |
|  1.0 |
|  2.0 |
└      ┘
```

TRMM—Triangular Matrix-Matrix Product

This subroutine computes one of the following matrix-matrix products:

1. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{AB}$
2. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{A}^{T}\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{BA}^{T}$
5. $\boldsymbol{b} \leftarrow \boldsymbol{Ab}$
6. $\boldsymbol{b} \leftarrow \boldsymbol{A}^{T}\boldsymbol{b}$

where, in the formulas above:

$\boldsymbol{A}$ is a triangular matrix.
$\boldsymbol{B}$ is a general matrix.
$\boldsymbol{b}$ is a vector.
α is a scalar.

Note: No data should be moved to form the matrix transpose; that is, the matrix should always be stored in its untransposed form.

If any of the assumed-shape arrays have a size of zero, no computation is performed, and the subroutine returns after doing some parameter checking.

See references [17], [30], [31], and [44].

Table 118. Data Types

α, ***A***, ***B***, ***b***	**Subprogram**
Long-precision real	TRMM

Syntax

HPF	**Equations 1–4**	CALL TRMM (*alpha*, *a*, *b*, *uplo*, *side*) CALL TRMM (*alpha*, *a*, *b*, *uplo*, *side*, *transa*, *diag*)
HPF	**Equations 5 and 6**	CALL TRMM (*a*, *b*, *uplo*) CALL TRMM (*a*, *b*, *uplo*, *transa*, *diag*)

On Entry

alpha
is the scalar α.

Type: **required** (equations 1–4); **not present** (equations 5 and 6)

Specified as: a number of the data type indicated in Table 118.

a
is the triangular matrix ***A***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the triangular matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the triangular matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 118, where size(*a*,1) = size(*a*,2).

b

is the general matrix ***B*** or the vector ***b***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 118 on page 828.

uplo

indicates whether the upper or lower triangular part of the triangular matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

side

indicates whether ***A*** is located to the left or right of ***B*** in the equation used for this computation, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in equation 1 or 2.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in equation 3 or 4.

Type: **required** (equations 1–4); **not present** (equations 5 and 6)

Specified as: a single character; *side* = `'L'` or `'R'`.

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in equation 1, 3, or 5.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation, resulting in equation 2, 4, or 6.

Type: **optional**

Default: *transa* = `'N'`

Specified as: a single character; *transa* = `'N'` or `'T'`.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Type: **optional**

Default: *diag* = `'N'`

Specified as: a single character; *diag* = `'U'` or `'N'`.

On Return

b

is the updated matrix ***B*** or vector ***b***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 118 on page 828.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For equations 1 through 4:
     - If *side* = `'L'`, size(*b*,1) = size(*a*,1) = size(*a*,2)
     - If *side* = `'R'`, size(*b*,2) = size(*a*,1) = size(*a*,2)
   - For equations 5 and 6: size(*b*) = size(*a*,1) = size(*a*,2)
2. For migration purposes, note that the *side* and *uplo* arguments appear in reverse order from the corresponding BLAS and PBLAS subroutines.
3. This subroutine accepts lowercase letters for the *side*, *uplo*, *transa*, and *diag* arguments.
4. If you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
5. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
6. This subroutine assumes certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the strictly lower or upper triangular part, respectively, are assumed to be zero.
7. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
8. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
9. The restrictions given in “Notes and Coding Rules” on page 325 and “Notes and Coding Rules” on page 256 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 326 and “Error Conditions” on page 257 also apply to this subroutine.

Input-Argument Errors for Equations 1–4

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *b*.
2. The process rank is not the same for *a* and *b*.
3. The process rank is not 1 or 2 for *a* or *b*.

Stage 2: The process grid is not the same for *a* and *b*.

Stage 3: The data distribution is inconsistent for *a* and *b*.

Stage 4

1. *side* is present, and *side* ≠`'L'` or `'R'`.
2. *side* = `'L'` or `'R'`, and the shape of the assumed-shape arrays *a* and *b* is incompatible:
   a. *side* = `'L'` and:
      size(*b*,1) ≠ size(*a*,2) or
      size(*a*,1) ≠ size(*a*,2)
   b. *side* = `'R'` and:
      size(*b*,2) ≠ size(*a*,1) or
      size(*a*,1) ≠ size(*a*,2)
3. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a* or *b* is unsupported.

Input-Argument Errors for Equations 5 and 6

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *b*.
2. The process rank is not the same for *a* and *b*.
3. The process rank is not 1 or 2 for *a* or *b*.

Stage 2: The process grid is not the same for *a* and *b*.

Stage 3: The data distribution is inconsistent for *a* and *b*.

Stage 4: The shape of the assumed-shape array for *a* is invalid:
size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a* or *b* is unsupported.

Stage 6: The shape of the assumed-shape arrays *a* and *b* is incompatible:
size(*a*,1) ≠ size(*b*)

Example 1: This example computes ***B*** = α***AB***. As in “Example 1” on page 328, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL TRMM( 1.0D0 , A , B , 'U' , 'L' )
-or-
CALL TRMM( 1.0D0 , A , B , 'U' , 'L' , TRANSA='N' , DIAG='N' )
```

Input: Triangular matrix ***A*** of order 5 is upper triangular:

```
┌                            ┐
| 3.0  -1.0   2.0   2.0   1.0 |
|  .   -2.0   4.0  -1.0   3.0 |
|  .     .   -3.0   0.0   2.0 |
|  .     .     .    4.0  -2.0 |
|  .     .     .     .    1.0 |
└                            ┘
```

Rectangular 5 × 3 matrix ***B***:

```
┌                  ┐
│  2.0   3.0   1.0 │
│  5.0   5.0   4.0 │
│  0.0   1.0   2.0 │
│  3.0   1.0  -3.0 │
│ -1.0   2.0   1.0 │
└                  ┘
```

Output: Rectangular 5 × 3 matrix ***B***:

```
┌                     ┐
│   6.0   10.0   -2.0 │
│ -16.0   -1.0    6.0 │
│  -2.0    1.0   -4.0 │
│  14.0    0.0  -14.0 │
│  -1.0    2.0    1.0 │
└                     ┘
```

Example 2: This example computes ***b*** = ***Ab***, where ***A*** is not a unit triangular matrix, and ***b*** is a column vector.

Array sections are specified for arguments *a* and *b*, resulting in the computation using a submatrix ***A*** starting at row 2 and column 2 in an array and a column vector ***b*** starting at element 2 in an array.

As in "Example 1" on page 258, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN B(:) WITH A(:,1)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL TRMM( A(2:13,2:13) , B(2:13) , 'U' )
-or-
CALL TRMM( A(2:13,2:13) , B(2:13) , 'U' , TRANSA='N' , DIAG='N' )
```

Input: Only a portion of the data structure is used—that is, submatrix ***A***. Following is the triangular submatrix ***A*** of order 12, starting at row 2 and column 2 in the array of order 13:

```
┌                                                                   ┐
│  .    .    .    .    .    .    .    .    .    .    .    .    .   │
│  .   1.0  2.0  1.0  2.0  1.0  1.0  3.0  1.0  1.0  2.0  3.0  2.0  │
│  .    .   3.0  2.0  3.0  1.0  2.0  3.0  1.0  1.0  2.0  3.0  3.0  │
│  .    .    .   3.0  1.0  3.0  2.0  1.0  2.0  1.0  2.0  3.0  1.0  │
│  .    .    .    .   1.0  2.0  2.0  1.0  1.0  1.0  2.0  3.0  2.0  │
│  .    .    .    .    .   2.0  1.0  2.0  2.0  1.0  2.0  3.0  3.0  │
│  .    .    .    .    .    .   1.0  2.0  1.0  1.0  2.0  3.0  1.0  │
│  .    .    .    .    .    .    .   2.0  1.0  1.0  2.0  3.0  2.0  │
│  .    .    .    .    .    .    .    .   2.0  1.0  2.0  3.0  3.0  │
│  .    .    .    .    .    .    .    .    .   3.0  1.0  3.0  1.0  │
│  .    .    .    .    .    .    .    .    .    .   2.0  2.0  2.0  │
│  .    .    .    .    .    .    .    .    .    .    .   1.0  3.0  │
│  .    .    .    .    .    .    .    .    .    .    .    .   1.0  │
└                                                                   ┘
```

Only a portion of the data structure is used—that is, vector ***b***, which is a column vector. Following is the vector ***b*** of size 12, starting at element 2 in the array of size 13:

```
┌     ┐
|  .  |
| 2.0 |
| 3.0 |
| 1.0 |
| 2.0 |
| 3.0 |
| 1.0 |
| 2.0 |
| 3.0 |
| 1.0 |
| 2.0 |
| 3.0 |
| 1.0 |
└     ┘
```

Output: Only a portion of the data structure is used—that is, vector ***b***, which is a column vector. Following is the vector ***b*** of size 12, starting at element 2 in the array of size 13:

```
┌      ┐
|   .  |
| 42.0 |
| 48.0 |
| 39.0 |
| 31.0 |
| 34.0 |
| 23.0 |
| 23.0 |
| 23.0 |
| 15.0 |
| 12.0 |
|  6.0 |
|  1.0 |
└      ┘
```

TRSM—Solution of Triangular System of Equations with Multiple Right-Hand Sides

This subroutine performs one of the following solves for a triangular system of equations with multiple right-hand sides:

Solution	Equation
1. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$	$\boldsymbol{AX} = \alpha\boldsymbol{B}$
2. $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-T})\boldsymbol{B}$	$\boldsymbol{A}^T\boldsymbol{X} = \alpha\boldsymbol{B}$
3. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-1})$	$\boldsymbol{XA} = \alpha\boldsymbol{B}$
4. $\boldsymbol{B} \leftarrow \alpha\boldsymbol{B}(\boldsymbol{A}^{-T})$	$\boldsymbol{XA}^T = \alpha\boldsymbol{B}$
5. $\boldsymbol{b} \leftarrow (\boldsymbol{A}^{-1})\boldsymbol{b}$	$\boldsymbol{Ax} = \boldsymbol{b}$
6. $\boldsymbol{b} \leftarrow (\boldsymbol{A}^{-T})\boldsymbol{b}$	$\boldsymbol{A}^T\boldsymbol{x} = \boldsymbol{b}$

where, in the formulas above:

A is a triangular matrix.
B is a general matrix.
b is a vector.
α is a scalar.

Notes:

1. The term ***X*** or ***x*** used in the systems of equations listed above represents the output solution matrix or vector, respectively. It is important to note that, in this subroutine, the solution matrix or vector is actually returned in the input-output argument *b*.
2. No data should be moved to form the matrix transpose; that is, the matrix should always be stored in its untransposed form.

If any of the assumed-shape arrays have a size of zero, no computation is performed, and the subroutine returns after doing some parameter checking.

See references [17], [30], [31], and [44].

Table 119. Data Types

α, ***A***, ***B***, ***b***	**Subprogram**
Long-precision real	TRSM

Syntax

HPF	**Solutions 1–4**	CALL TRSM (*alpha*, *a*, *b*, *uplo*, *side*) CALL TRSM (*alpha*, *a*, *b*, *uplo*, *side*, *transa*, *diag*)
HPF	**Solutions 5 and 6**	CALL TRSM (*a*, *b*, *uplo*) CALL TRSM (*a*, *b*, *uplo*, *transa*, *diag*)

On Entry

alpha
is the scalar α.

Type: **required** (solutions 1–4); **not present** (solutions 5 and 6)

Specified as: a number of the data type indicated in Table 119.

a

is the triangular matrix ***A*** used in the system of equations, where:

If *uplo* = `'U'`, the array contains the upper triangle of the triangular matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the triangular matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 119 on page 834, where size(*a*,1) = size(*a*,2).

b

is the general matrix ***B*** or the vector ***b***, containing the right-hand side(s) of the triangular system to be solved.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 119 on page 834.

uplo

indicates whether the upper or lower triangular part of the triangular matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

side

indicates whether ***A*** is located to the left or right of ***B*** in the system of equations, where:

If *side* = `'L'`, ***A*** is to the left of ***B***, resulting in solution 1 or 2.

If *side* = `'R'`, ***A*** is to the right of ***B***, resulting in solution 3 or 4.

Type: **required** (solutions 1–4); **not present** (solutions 5 and 6)

Specified as: a single character; *side* = `'L'` or `'R'`.

transa

indicates the form of matrix ***A*** used in the system of equations, where:

If *transa* = `'N'`, ***A*** is used in the system of equations, resulting in solution 1, 3, or 5.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the system of equations, resulting in solution 2, 4, or 6.

Type: **optional**

Default: *transa* = `'N'`

Specified as: a single character; *transa* = `'N'` or `'T'`.

diag

indicates the characteristics of the diagonal of matrix ***A***, where:

If *diag* = `'U'`, ***A*** is a unit triangular matrix.

If *diag* = `'N'`, ***A*** is not a unit triangular matrix.

Type: **optional**

Default: *diag* = `'N'`

Specified as: a single character; *diag* = `'U'` or `'N'`.

On Return

b

is the updated matrix ***B*** or vector ***b***, containing the solution vector(s).

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 119 on page 834.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For solutions 1 through 4:
     - If *side* = `'L'`, size(*b*,1) = size(*a*,1) = size(*a*,2)
     - If *side* = `'R'`, size(*b*,2) = size(*a*,1) = size(*a*,2)
   - For solutions 5 and 6: size(*b*) = size(*a*,1) = size(*a*,2)
2. For migration purposes, note that the *side* and *uplo* arguments appear in reverse order from the corresponding BLAS and PBLAS subroutines.
3. This subroutine accepts lowercase letters for the *side*, *uplo*, *transa*, and *diag* arguments.
4. If you specify `'C'` for *transa*, it is interpreted as though you specified `'T'`.
5. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
6. This subroutine assumes certain values in your array for parts of a triangular matrix. As a result, you do not have to set these values. For unit triangular matrices, the elements of the diagonal are assumed to be one. When using an upper or lower triangular matrix, the unreferenced elements in the strictly lower or upper triangular part, respectively, are assumed to be zero.
7. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
8. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
9. The restrictions given in “Notes and Coding Rules” on page 339 and “Notes and Coding Rules” on page 269 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 341 and “Error Conditions” on page 270 also apply to this subroutine.

Input-Argument Errors for Solutions 1–4

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *b*.
2. The process rank is not the same for *a* and *b*.
3. The process rank is not 1 or 2 for *a* or *b*.

Stage 2: The process grid is not the same for *a* and *b*.

Stage 3: The data distribution is inconsistent for *a* and *b*.

Stage 4

1. *side* is present, and *side* ≠ `'L'` or `'R'`.
2. *side* = `'L'` or `'R'`, and the shape of the assumed-shape arrays *a* and *b* is incompatible:
   a. *side* = `'L'` and:
      size(*b*,1) ≠ size(*a*,2) or
      size(*a*,1) ≠ size(*a*,2)
   b. *side* = `'R'` and:
      size(*b*,2) ≠ size(*a*,1) or
      size(*a*,1) ≠ size(*a*,2)
3. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a* or *b* is unsupported.

Input-Argument Errors for Solutions 5 and 6

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *b*.
2. The process rank is not the same for *a* and *b*.
3. The process rank is not 1 or 2 for *a* or *b*.

Stage 2: The process grid is not the same for *a* and *b*.

Stage 3: The data distribution is inconsistent for *a* and *b*.

Stage 4: The shape of the assumed-shape array for *a* is invalid:
size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a* or *b* is unsupported.

Stage 6: The shape of the assumed-shape arrays *a* and *b* is incompatible:
size(*a*,1) ≠ size(*b*)

Example 1: This example shows the solution $\boldsymbol{B} \leftarrow \alpha(\boldsymbol{A}^{-1})\boldsymbol{B}$. As in “Example 1” on page 343, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL TRSM(  1.0D0 , A , B , 'U' , 'L' )
-or-
CALL TRSM(  1.0D0 , A , B , 'U' , 'L' , TRANSA='N' , DIAG='N' )
```

Input: Triangular matrix ***A*** of order 5 is upper triangular:

$$\begin{bmatrix} 3.0 & -1.0 & 2.0 & 2.0 & 1.0 \\ . & -2.0 & 4.0 & -1.0 & 3.0 \\ . & . & -3.0 & 0.0 & 2.0 \\ . & . & . & 4.0 & -2.0 \\ . & . & . & . & 1.0 \end{bmatrix}$$

General 5 × 3 matrix ***B***:

$$\begin{bmatrix} 6.0 & 10.0 & -2.0 \\ -16.0 & -1.0 & 6.0 \\ -2.0 & 1.0 & -4.0 \\ 14.0 & 0.0 & -14.0 \\ -1.0 & 2.0 & 1.0 \end{bmatrix}$$

Output: General 5 × 3 matrix ***B***:

$$\begin{bmatrix} 2.0 & 3.0 & 1.0 \\ 5.0 & 5.0 & 4.0 \\ 0.0 & 1.0 & 2.0 \\ 3.0 & 1.0 & -3.0 \\ -1.0 & 2.0 & 1.0 \end{bmatrix}$$

Example 2: This example solves $\boldsymbol{b} \leftarrow \boldsymbol{A}^{-1}\boldsymbol{b}$, where ***A*** is a unit triangular matrix, and ***b*** is a row vector.

Array sections are specified for arguments *a* and *b*, resulting in the computation using a submatrix ***A*** starting at row 2 and column 2 in an array and a row vector ***b*** starting at element 2 in an array.

As in “Example 1” on page 271, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN B(:) WITH A(1,:)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL TRSM( A(2:13,2:13) , B(2:13) , 'L' , DIAG='U' )
-or-
CALL TRSM( A(2:13,2:13) , B(2:13) , 'L' , TRANSA='N' , DIAG='U' )
```

Input: Only a portion of the data structure is used—that is, submatrix ***A***. Following is the triangular submatrix ***A*** of order 12, starting at row 2 and column 2 in the array of order 13:

```
┌                                                           ┐
|  .    .    .    .    .    .    .    .    .    .    .    .    .  |
|  .   1.0   .    .    .    .    .    .    .    .    .    .    .  |
|  .   2.0  1.0   .    .    .    .    .    .    .    .    .    .  |
|  .   3.0  2.0  1.0   .    .    .    .    .    .    .    .    .  |
|  .   1.0  3.0  2.0  1.0   .    .    .    .    .    .    .    .  |
|  .   2.0  1.0  3.0  2.0  1.0   .    .    .    .    .    .    .  |
|  .   3.0  2.0  1.0  3.0  2.0  1.0   .    .    .    .    .    .  |
|  .   1.0  3.0  2.0  1.0  3.0  2.0  1.0   .    .    .    .    .  |
|  .   2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0   .    .    .    .  |
|  .   3.0  2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0   .    .    .  |
|  .   1.0  3.0  2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0   .    .  |
|  .   2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0   .  |
|  .   3.0  2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0  3.0  2.0  1.0 |
└                                                           ┘
```

Note: Because matrix ***A*** is unit triangular, the diagonal elements are not referenced. This subroutine assumes a value of 1.0 for the diagonal elements.

Only a portion of the data structure is used—that is, vector ***b***, which is a row vector. Following is the vector ***b*** of size 12, starting at element 2 in the array of size 13:

```
┌                                                                      ┐
|  .    2.0   7.0  13.0  15.0  17.0  26.0  28.0  27.0  39.0  41.0  37.0  52.0 |
└                                                                      ┘
```

Output: Only a portion of the data structure is used—that is, vector ***b***, which is a row vector. Following is the vector ***b*** of size 12, starting at element 2 in the array of size 13:

```
┌                                                           ┐
|  .   2.0  3.0  1.0  2.0  3.0  1.0  2.0  3.0  1.0  2.0  3.0  1.0 |
└                                                           ┘
```

SYRK—Rank-K Update of a Real Symmetric Matrix

This subroutine computes one of the following rank-k updates:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{A}^T + \beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^T\boldsymbol{A} + \beta \boldsymbol{C}$
3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{a}\boldsymbol{a}^T + \boldsymbol{C}$

where, in the formulas above:

A is a general matrix.
C is a symmetric matrix.
a is a vector.
α and β are scalars.

Note: No data should be moved to form the matrix transpose; that is, the matrix should always be stored in its untransposed form.

In the following cases, no computation is performed and the subroutine returns after doing some parameter checking:

- For equations 1 and 2:
  - Any of the assumed-shape arrays have a size of zero.
  - α is zero and β is one.
- For equation 3:
  - Any of the assumed-shape arrays have a size of zero.
  - α is zero.

See references [17], [30], [31], and [44].

Table 120. Data Types

α, β, ***A***, ***C***, ***a***	**Subprogram**
Long-precision real	SYRK

Syntax

HPF	**Equations 1 and 2**	CALL SYRK (*alpha*, *a*, *beta*, *c*, *uplo*) CALL SYRK (*alpha*, *a*, *beta*, *c*, *uplo*, *trans*)
HPF	**Equation 3**	CALL SYRK (*alpha*, *a*, *c*, *uplo*)

On Entry

alpha
is the scalar α.

Type: **required**

Specified as: a number of the data type indicated in Table 120.

a
is the general matrix ***A*** or the vector ***a***.

Type: **required**

Specified as: an assumed-shape array with shape (:,:) or (:), containing numbers of the data type indicated in Table 120.

beta
is the scalar β.

Type: **required** (equations 1 and 2); **not present** (equation 3)

Specified as: a number of the data type indicated in Table 120 on page 840.

c
is the symmetric matrix ***C***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***C*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***C*** in its lower triangle, and its strictly upper triangular part is not referenced.

For equations 1 and 2, when β is zero, ***C*** need not be set on input.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 120 on page 840, where size(*c*,1) = size(*c*,2).

uplo
indicates whether the upper or lower triangular part of the symmetric matrix ***C*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

trans
indicates which computation is performed, where:

If *trans* = `'N'`, the computation in equation 1 is performed.

If *trans* = `'T'`, the computation in equation 2 is performed.

Type: **optional** (equations 1 and 2); **not present** (equation 3)

Default: *trans* = `'N'`

Specified as: a single character; *trans* = `'N'` or `'T'`.

On Return

c
is the updated symmetric matrix ***C***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 120 on page 840.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For equations 1 and 2:
     - If *trans* = `'N'`, size(*c*,1) = size(*c*,2) = size(*a*,1)
     - If *trans* = `'T'`, size(*c*,1) = size(*c*,2) = size(*a*,2)
   - For equation 3: size(*c*,1) = size(*c*,2) = size(*a*)

2. This subroutine accepts lowercase letters for the *uplo* and *trans* arguments.
3. If you specify `'C'` for the *trans* argument, it is interpreted as though you specified `'T'`.
4. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
5. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
6. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
7. The restrictions given in “Notes and Coding Rules” on page 353 and “Notes and Coding Rules” on page 228 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 354 and “Error Conditions” on page 228 also apply to this subroutine.

Input-Argument Errors for Equations 1 and 2

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *c*.
2. The process rank is not the same for *a* and *c*.
3. The process rank is not 1 or 2 for *a* or *c*.

Stage 2: The process grid is not the same for *a* and *c*.

Stage 3: The data distribution is inconsistent for *a* and *c*.

Stage 4

1. *trans* is present, and *trans* ≠`'N'`, `'T'`, or `'C'`
2. *trans* = `'N'`, `'T'`, or `'C'`, and the shape of the assumed-shape arrays *a* and *c* is incompatible:
   a. *trans* = `'N'`:
      size(*c*,2) ≠ size(*a*,1) or
      size(*c*,1) ≠ size(*c*,2)
   b. *trans* = `'T'`:
      size(*c*,2) ≠ size(*a*,2) or
      size(*c*,1) ≠ size(*c*,2)
3. The shape of the assumed-shape array for *c* is invalid: size(*c*,1) ≠ size(*c*,2)

Stage 5: The data distribution for *a* or *c* is unsupported.

Input-Argument Errors for Equation 3

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *c*.
2. The process rank is not the same for *a* and *c*.

3. The process rank is not 1 or 2 for *a* or *c*.

Stage 2: The process grid is not the same for *a* and *c*.

Stage 3: The data distribution is unsupported for *c*.

Stage 4: The vector for *a* is replicated.

Stage 5: The data distribution for *a* is unsupported.

Stage 6: The shape of the assumed-shape array for *c* is invalid:
size(*c*,1) ≠ size(*c*,2)

Stage 7: The data distribution for *c* or *a* is unsupported.

Stage 8: The shape of the assumed-shape arrays *c* and *a* is incompatible:
size(*c*,1) ≠ size(*a*)

Example 1: This example computes $\boldsymbol{C} = \alpha \boldsymbol{A}\boldsymbol{A}^T + \beta \boldsymbol{C}$. As in "Example 1" on page 356, array data is block-cyclically distributed using a 2 × 3 process grid.

```
!HPF$ PROCESSORS PROC(2,3)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, C

CALL SYRK( 1.0D0 , A , 1.0D0 , C , UPLO='L' )
-or-
CALL SYRK( 1.0D0 , A , 1.0D0 , C , UPLO='L' , TRANS='N' )
```

Input: General 8 × 5 matrix ***A***:

```
┌                            ┐
| 0.0   8.0  16.0  24.0  32.0 |
| 1.0   9.0  17.0  25.0  33.0 |
| 2.0  10.0  18.0  26.0  34.0 |
| 3.0  11.0  19.0  27.0  35.0 |
| 4.0  12.0  20.0  28.0  36.0 |
| 5.0  13.0  21.0  29.0  37.0 |
| 6.0  14.0  22.0  30.0  38.0 |
| 7.0  15.0  23.0  31.0  39.0 |
└                            ┘
```

Symmetric matrix ***C*** of order 8:

```
┌                                              ┐
| 0.0    .     .     .     .     .     .     . |
| 1.0   8.0    .     .     .     .     .     . |
| 2.0   9.0  15.0    .     .     .     .     . |
| 3.0  10.0  16.0  21.0    .     .     .     . |
| 4.0  11.0  17.0  22.0  26.0    .     .     . |
| 5.0  12.0  18.0  23.0  27.0  30.0    .     . |
| 6.0  13.0  19.0  24.0  28.0  31.0  33.0    . |
| 7.0  14.0  20.0  25.0  29.0  32.0  34.0  35.0 |
└                                              ┘
```

Output: Symmetric matrix ***C*** of order 8:

```
┌                                                                        ┐
| 1920.0      .       .       .       .       .       .       .  |
| 2001.0  2093.0      .       .       .       .       .       .  |
| 2082.0  2179.0  2275.0      .       .       .       .       .  |
| 2163.0  2265.0  2366.0  2466.0      .       .       .       .  |
| 2244.0  2351.0  2457.0  2562.0  2666.0      .       .       .  |
| 2325.0  2437.0  2548.0  2658.0  2767.0  2875.0      .       .  |
| 2406.0  2523.0  2639.0  2754.0  2868.0  2981.0  3093.0      .  |
| 2487.0  2609.0  2730.0  2850.0  2969.0  3087.0  3204.0  3320.0 |
└                                                                        ┘
```

Example 2: This example computes $\boldsymbol{C} = \alpha \boldsymbol{a}\boldsymbol{a}^T + \boldsymbol{C}$. As in "Example 1" on page 230, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN A(:) WITH C(:,1)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL SYRK( 1.0D0 , A , C , UPLO='L' )
```

Input: Symmetric matrix ***C*** of order 9:

```
┌                                               ┐
| 1.0    .     .     .     .     .     .     .     .  |
| 2.0  12.0    .     .     .     .     .     .     .  |
| 3.0  13.0  23.0    .     .     .     .     .     .  |
| 4.0  14.0  24.0  34.0    .     .     .     .     .  |
| 5.0  15.0  25.0  35.0  45.0    .     .     .     .  |
| 6.0  16.0  26.0  36.0  46.0  56.0    .     .     .  |
| 7.0  17.0  27.0  37.0  47.0  57.0  67.0    .     .  |
| 8.0  18.0  28.0  38.0  48.0  58.0  68.0  78.0    .  |
| 9.0  19.0  29.0  39.0  49.0  59.0  69.0  79.0  89.0 |
└                                               ┘
```

Vector ***a*** of size 9:

```
┌     ┐
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
└     ┘
```

Output: Matrix ***C*** of order 9:

```
┌                                                ┐
|  2.0    .     .     .     .     .     .     .     .  |
|  3.0  13.0    .     .     .     .     .     .     .  |
|  4.0  14.0  24.0    .     .     .     .     .     .  |
|  5.0  15.0  25.0  35.0    .     .     .     .     .  |
|  6.0  16.0  26.0  36.0  46.0    .     .     .     .  |
|  7.0  17.0  27.0  37.0  47.0  57.0    .     .     .  |
|  8.0  18.0  28.0  38.0  48.0  58.0  68.0    .     .  |
|  9.0  19.0  29.0  39.0  49.0  59.0  69.0  79.0    .  |
| 10.0  20.0  30.0  40.0  50.0  60.0  70.0  80.0  90.0 |
└                                                ┘
```

SYR2K—Rank-2K Update of a Real Symmetric Matrix

This subroutine computes one of the following rank-2k updates:

1. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}\boldsymbol{B}^T+\alpha \boldsymbol{B}\boldsymbol{A}^T+\beta \boldsymbol{C}$
2. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{A}^T\boldsymbol{B}+\alpha \boldsymbol{B}^T\boldsymbol{A}+\beta \boldsymbol{C}$
3. $\boldsymbol{C} \leftarrow \alpha \boldsymbol{a}\boldsymbol{b}^T+\alpha \boldsymbol{b}\boldsymbol{a}^T+\boldsymbol{C}$

where, in the formulas above:

A and ***B*** are general matrices.
C is a symmetric matrix.
a and ***b*** are vectors.
α and β are scalars.

Note: No data should be moved to form the matrix transposes; that is, the matrices should always be stored in their untransposed forms.

In the following cases, no computation is performed and the subroutine returns after doing some parameter checking:

- For equations 1 and 2:
  - All of the assumed-shape arrays have a size of zero.
  - β is one, and (α is zero or the assumed-shape arrays for *a* and *b* have a size of zero).
- For equation 3:
  - Any of the assumed-shape arrays have a size of zero.
  - α is zero.

See references [17], [30], [31], and [44].

Table 121. Data Types

α, β, ***A***, ***B***, ***C***, ***a***, ***b***	**Subprogram**
Long-precision real	SYR2K

Syntax

HPF	**Equations 1 and 2**	CALL SYR2K (*alpha*, *a*, *b*, *beta*, *c*, *uplo*) CALL SYR2K (*alpha*, *a*, *b*, *beta*, *c*, *uplo*, *trans*)
HPF	**Equation 3**	CALL SYR2K (*alpha*, *a*, *b*, *c*, *uplo*)

On Entry

alpha
is the scalar α.

Type: **required**

Specified as: a number of the data type indicated in Table 121.

a
is the general matrix ***A*** or the vector ***a***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 121.

b

is the general matrix ***B*** or the vector ***b***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**) or (**:**), containing numbers of the data type indicated in Table 121 on page 846.

beta

is the scalar β.

Type: **required** (equations 1 and 2); **not present** (equation 3)

Specified as: a number of the data type indicated in Table 121 on page 846.

c

is the symmetric matrix ***C***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***C*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***C*** in its lower triangle, and its strictly upper triangular part is not referenced.

For equations 1 and 2, when β is zero, ***C*** need not be set on input.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 121 on page 846, where size(*c*,1) = size(*c*,2).

uplo

indicates whether the upper or lower triangular part of the symmetric matrix ***C*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

trans

indicates which computation is performed, where:

If *trans* = `'N'`, the computation in equation 1 is performed.

If *trans* = `'T'`, the computation in equation 2 is performed.

Type: **optional** (equations 1 and 2); **not present** (equation 3)

Default: *trans* = `'N'`

Specified as: a single character; *trans* = `'N'` or `'T'`.

On Return

c

is the updated symmetric matrix ***C***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 121 on page 846.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:

- For equations 1 and 2:
  - If *trans* = `'N'`:
    - size(*c*,1) = size(*c*,2) = size(*a*,1) = size(*b*,1)
    - size(*a*,2) = size(*b*,2)
  - If *trans* = `'T'`:
    - size(*c*,1) = size(*c*,2) = size(*a*,2) = size(*b*,2)
    - size(*a*,1) = size(*b*,1)
- For equation 3: size(*c*,1) = size(*c*,2) = size(*a*) = size(*b*)

2. This subroutine accepts lowercase letters for the *uplo* and *trans* arguments.
3. If you specify `'C'` for the *trans* argument, it is interpreted as though you specified `'T'`.
4. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
5. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
6. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
7. The restrictions given in “Notes and Coding Rules” on page 371 and “Notes and Coding Rules” on page 241 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Notes and Coding Rules” on page 371 and “Notes and Coding Rules” on page 241 also apply to this subroutine.

Input-Argument Errors for Equations 1 and 2

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: The process grid is not the same for *a*, *b*, and *c*.

Stage 3: The data distribution is inconsistent for *a*, *b*, and *c*.

Stage 4

1. *trans* is present, and *trans* ≠ `'N'`, `'T'`, or `'C'`
2. *trans* = `'N'`, `'T'`, or `'C'`, and the shape of the assumed-shape arrays for *a*, *b*, and *c* is incompatible:
   a. *trans* = `'N'`:
      size(*c*,1) ≠ size(*c*,2) or
      size(*c*,2) ≠ size(*a*,1) or
      size(*a*,1) ≠ size(*b*,1) or
      size(*a*,2) ≠ size(*b*,2)
   b. *trans* = `'T'`:

size(c,1) ≠ size(c,2) or
size(c,2) ≠ size(a,2) or
size(a,2) ≠ size(b,2) or
size(a,1) ≠ size(b,1)

3. The shape of the assumed-shape array for c is invalid: size(c,1) ≠ size(c,2)

Stage 5: The data distribution for *a*, *b*, or *c* is unsupported.

Input-Argument Errors for Equation 3

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *c*.
2. The process rank is not the same for *a*, *b*, and *c*.
3. The process rank is not 1 or 2 for *a*, *b*, or *c*.

Stage 2: The process grid is not the same for *a*, *b*, and *c*.

Stage 3: The data distribution is unsupported for *c*.

Stage 4: The vector for *a* or *b* is replicated.

Stage 5: The data distribution is unsupported for *a* or *b*.

Stage 6

1. The shape of the assumed-shape arrays for *a*, *b*, and *c* is incompatible:
   size(c,1) ≠ size(c,2) or
   size(c,1) ≠ size(a) or
   size(c,1) ≠ size(b)
2. The shape of the assumed-shape array for c is invalid: size(c,1) ≠ size(c,2)

Stage 7: The data distribution for *a*, *b*, or *c* is unsupported.

Example 1: This example computes $\boldsymbol{C} = \alpha \boldsymbol{A}^T\boldsymbol{B}+\alpha \boldsymbol{B}^T\boldsymbol{A}+\beta \boldsymbol{C}$. As in “Example 1” on page 376, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B, C

CALL SYR2K( 1.0D0 , A , B , 0.0D0 , C , 'U' , 'T' )
```

Input: General 8 × 9 matrix ***A***:

```
┌                                                   ┐
|  0.0  -1.0  -1.0   0.0   0.0   0.0   0.0   0.0   1.0 |
|  0.0   1.0   0.0   1.0   0.0   1.0   0.0   1.0   1.0 |
|  0.0   0.0  -1.0  -1.0   0.0   0.0   1.0   0.0   1.0 |
|  0.0   1.0   0.0  -1.0   1.0   1.0   0.0   1.0   1.0 |
|  1.0   0.0   0.0   0.0  -1.0   0.0   0.0   0.0   1.0 |
|  1.0   0.0   0.0   0.0   1.0   1.0   0.0   0.0   1.0 |
|  0.0   0.0  -1.0   0.0  -1.0   0.0   0.0   0.0   1.0 |
| -1.0   0.0   0.0   0.0   0.0   0.0  -1.0   0.0   1.0 |
└                                                   ┘
```

General 8 × 9 matrix ***B***:

```
┌                                                 ┐
│  0.0   1.0   1.0   0.0   0.0   0.0   0.0   0.0  -1.0 │
│  0.0  -1.0   0.0  -1.0   0.0  -1.0   0.0  -1.0  -1.0 │
│  0.0   0.0   1.0   1.0   0.0   0.0  -1.0   0.0  -1.0 │
│  0.0  -1.0   0.0   1.0  -1.0  -1.0   0.0  -1.0  -1.0 │
│ -1.0   0.0   0.0   0.0   1.0   0.0   0.0   0.0  -1.0 │
│ -1.0   0.0   0.0   0.0  -1.0  -1.0   0.0   0.0  -1.0 │
│  0.0   0.0   1.0   0.0   1.0   0.0   0.0   0.0  -1.0 │
│  1.0   0.0   0.0   0.0   0.0   0.0   1.0   0.0  -1.0 │
└                                                 ┘
```

Output: Symmetric matrix ***C*** of order 9:

```
┌                                                          ┐
│ -6.0    0.0    0.0    0.0    0.0   -2.0   -2.0    0.0   -2.0 │
│    .   -6.0   -2.0    0.0   -2.0   -4.0    0.0   -4.0   -2.0 │
│    .      .   -6.0   -2.0   -2.0    0.0    2.0    0.0    6.0 │
│    .      .      .   -6.0    2.0    0.0    2.0    0.0    2.0 │
│    .      .      .      .   -8.0   -4.0    0.0   -2.0    0.0 │
│    .      .      .      .      .   -6.0    0.0   -4.0   -6.0 │
│    .      .      .      .      .      .   -4.0    0.0    0.0 │
│    .      .      .      .      .      .      .   -4.0   -4.0 │
│    .      .      .      .      .      .      .      .  -16.0 │
└                                                          ┘
```

Example 2: This example computes ***C*** = α***ab***^{T}+α***ba***^{T}+***C***. As in “Example 1” on page 245, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN A(:) WITH C(:,1)
!HPF$ ALIGN B(:) WITH C(:,1)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: C

CALL SYR2K( 1.0D0 , A , B , C , 'L' )
```

Input: Symmetric matrix ***C*** of order 9:

```
┌                                                  ┐
│ 1.0     .     .     .     .     .     .     .     . │
│ 2.0  12.0     .     .     .     .     .     .     . │
│ 3.0  13.0  23.0     .     .     .     .     .     . │
│ 4.0  14.0  24.0  34.0     .     .     .     .     . │
│ 5.0  15.0  25.0  35.0  45.0     .     .     .     . │
│ 6.0  16.0  26.0  36.0  46.0  56.0     .     .     . │
│ 7.0  17.0  27.0  37.0  47.0  57.0  67.0     .     . │
│ 8.0  18.0  28.0  38.0  48.0  58.0  68.0  78.0     . │
│ 9.0  19.0  29.0  39.0  49.0  59.0  69.0  79.0  89.0 │
└                                                  ┘
```

Vector ***a*** of size 9:

```
┌     ┐
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
└     ┘
```

Vector ***b*** of size 9:

```
┌     ┐
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
└     ┘
```

Output: Matrix ***C*** of order 9:

```
┌                                                    ┐
|  5.0    .     .     .     .     .     .     .     . |
|  6.0  16.0    .     .     .     .     .     .     . |
|  7.0  17.0  27.0    .     .     .     .     .     . |
|  8.0  18.0  28.0  38.0    .     .     .     .     . |
|  9.0  19.0  29.0  39.0  49.0    .     .     .     . |
| 10.0  20.0  30.0  40.0  50.0  60.0    .     .     . |
| 11.0  21.0  31.0  41.0  51.0  61.0  71.0    .     . |
| 12.0  22.0  32.0  42.0  52.0  62.0  72.0  82.0    . |
| 13.0  23.0  33.0  43.0  53.0  63.0  73.0  83.0  93.0 |
└                                                    ┘
```

TRAN—Matrix Transpose for a General Matrix

This subroutine performs the following matrix computation:

$$\boldsymbol{C} \leftarrow \beta \boldsymbol{C} + \alpha \boldsymbol{A}^T$$

where, in the formula above:

A and ***C*** are general matrices.
α and β are scalars.

Note: No data should be moved to form the matrix transpose; that is, the matrix should always be stored in its untransposed form.

In the following two cases, no computation is performed and the subroutine returns after doing some parameter checking:

- All of the assumed-shape arrays have a size of zero.
- α is zero and β is one.

See references [17], [30], [31], and [44].

Table 122. Data Types

α, β, ***A***, ***C***	**Subprogram**
Long-precision real	TRAN

Syntax

HPF	CALL TRAN (*alpha*, *a*, *beta*, *c*)

On Entry

alpha
is the scalar α.

Type: **required**

Specified as: a number of the data type indicated in Table 122.

a
is the general matrix ***A***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 122.

beta
is the scalar β.

Type: **required**

Specified as: a number of the data type indicated in Table 122.

c
is the general matrix ***C***. When β is zero, ***C*** need not be set on input.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 117 on page 822.

On Return

c

is the updated general matrix ***C***, containing the results of the computation.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 122 on page 852.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - size(*c*,1) = size(*a*,2)
   - size(*c*,2) = size(*a*,1)
2. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
3. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
4. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
5. The restrictions given in “Notes and Coding Rules” on page 389 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 390 also apply to this subroutine.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *c*.
2. The process rank is not the same for *a* and *c*.
3. The process rank is not 1 or 2 for *a* or *c*.

Stage 2: The process grid is not the same for *a* and *c*.

Stage 3: The data distribution is inconsistent for *a* and *c*.

Stage 4: The shape of the assumed-shape arrays *a* and *c* is incompatible:

size(*c*,1) ≠ size(*a*,2) or
size(*c*,2) ≠ size(*a*,1)

Stage 5: The data distribution for *a* or *c* is unsupported.

Example: This example computes $\boldsymbol{C} = \beta\boldsymbol{C}+\alpha\boldsymbol{A}^T$. As in “Example 1” on page 392, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, C

CALL TRAN( 1.0D0 , A , 1.0D0 , C )
```

Input: General 8 × 9 matrix ***A***:

$$\begin{bmatrix}
0.0 & -1.0 & -1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
0.0 & 1.0 & 0.0 & 1.0 & 0.0 & 1.0 & 0.0 & 1.0 & 1.0 \\
0.0 & 0.0 & -1.0 & -1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 1.0 \\
0.0 & 1.0 & 0.0 & -1.0 & 1.0 & 1.0 & 0.0 & 1.0 & 1.0 \\
1.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
1.0 & 0.0 & 0.0 & 0.0 & 1.0 & 1.0 & 0.0 & 0.0 & 1.0 \\
0.0 & 0.0 & -1.0 & 0.0 & -1.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
-1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 1.0
\end{bmatrix}$$

General 9 × 8 matrix ***C***:

$$\begin{bmatrix}
0.0 & 1.0 & 1.0 & 5.0 & 6.0 & 7.0 & 8.0 & 9.0 \\
0.0 & -1.0 & 0.0 & -1.0 & 0.0 & -1.0 & 0.0 & 1.0 \\
0.0 & 0.0 & 1.0 & 1.0 & 0.0 & 0.0 & -1.0 & 0.0 \\
0.0 & -1.0 & 0.0 & 1.0 & -1.0 & -1.0 & 0.0 & 1.0 \\
-1.0 & 2.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
-1.0 & 3.0 & 0.0 & 0.0 & -1.0 & -1.0 & 0.0 & 0.0 \\
0.0 & 4.0 & 1.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 5.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\
1.0 & 2.0 & 3.0 & 4.0 & 1.0 & 1.0 & 1.0 & 1.0
\end{bmatrix}$$

Output: General 9 × 8 matrix ***C***:

$$\begin{bmatrix}
0.0 & 1.0 & 1.0 & 5.0 & 7.0 & 8.0 & 8.0 & 8.0 \\
-1.0 & 0.0 & 0.0 & 0.0 & 0.0 & -1.0 & 0.0 & 1.0 \\
-1.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & -2.0 & 0.0 \\
0.0 & 0.0 & -1.0 & 0.0 & -1.0 & -1.0 & 0.0 & 1.0 \\
-1.0 & 2.0 & 0.0 & 1.0 & 0.0 & 1.0 & -1.0 & 0.0 \\
-1.0 & 4.0 & 0.0 & 1.0 & -1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 4.0 & 2.0 & 0.0 & 1.0 & 0.0 & 0.0 & -1.0 \\
1.0 & 6.0 & 0.0 & 1.0 & 0.0 & 0.0 & 1.0 & 0.0 \\
2.0 & 3.0 & 4.0 & 5.0 & 2.0 & 2.0 & 2.0 & 2.0
\end{bmatrix}$$

Chapter 14. Linear Algebraic Equations (HPF)

This chapter describes the linear algebraic equation subroutines that can be called from an HPF program. These subroutines include a subset of the ScaLAPACK subroutines.

Note: These subroutines are designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Overview of the Dense Linear Algebraic Equation Subroutines

The dense linear algebraic equation subroutines provide solutions to linear systems of equations for real and complex general matrices and their transposes, and for positive definite real symmetric and complex Hermitian matrices.

Table 123. List of Dense Linear Algebraic Equation Subroutines (HPF)

Descriptive Name	Long-Precision Subroutine	Page
General Matrix Factorization	GETRF	858
General Matrix Solve	GETRS	863
Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization	POTRF	868
Positive Definite Real Symmetric or Complex Hermitian Matrix Solve	POTRS	873

Overview of the Banded Linear Algebraic Equation Subroutines

The banded linear algebraic equation subroutines provide solutions to linear systems of equations for real positive definite symmetric band matrices, real general tridiagonal matrices, diagonally-dominant real general tridiagonal matrices, and real positive definite symmetric tridiagonal matrices.

Table 124 (Page 1 of 2). List of Banded Linear Algebraic Equation Subroutines

Descriptive Name	Long-Precision Subroutine	Page
Positive Definite Symmetric Band Matrix Factorization and Solve	PBSV	879
Positive Definite Symmetric Band Matrix Factorization	PBTRF	884
Positive Definite Symmetric Band Matrix Solve	PBTRS	889
General Tridiagonal Matrix Factorization and Solve	GTSV	894
General Tridiagonal Matrix Factorization	GTTRF	899
General Tridiagonal Matrix Solve	GTTRS	908
Diagonally-Dominant General Tridiagonal Matrix Factorization and Solve	DTSV	894
Diagonally-Dominant General Tridiagonal Matrix Factorization	DTTRF	899

Table 124 (Page 2 of 2). List of Banded Linear Algebraic Equation Subroutines

Descriptive Name	**Long-Precision Subroutine**	**Page**
Diagonally-Dominant General Tridiagonal Matrix Solve	DTTRS	908
Positive Definite Symmetric Tridiagonal Matrix Factorization and Solve	PTSV	916
Positive Definite Symmetric Tridiagonal Matrix Factorization	PTTRF	921
Positive Definite Symmetric Tridiagonal Matrix Solve	PTTRS	927

Dense Linear Algebraic Equation Subroutines

This section contains the dense linear algebraic equation subroutine descriptions.

GETRF—General Matrix Factorization

This subroutine factors general matrix ***A*** using Gaussian elimination with partial pivoting, ***ipiv***, to compute the ***LU*** factorization of ***A***.

On output, the transformed matrix ***A*** contains ***U*** in the upper triangle (if size(*a*,1) ≥ size(*a*,2)) or upper trapezoid (if size(*a*,1) < size(*a*,2)) and ***L*** the strict lower triangle (if size(*a*,1) ≤ size(*a*,2)) or lower trapezoid (if size(*a*,1) > size(*a*,2)). ***ipiv*** contains the pivots representing permutation ***P***, such that ***A*** = ***PLU***.

To solve the system of equations with any number of right-hand sides, follow the call to this subroutine with one or more calls to GETRS.

If any of the assumed-shape arrays have a size of zero, no computation is performed, and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 125. Data Types

A	***ipiv***	**Subroutine**
Long-precision real	Integer	GETRF
Long-precision complex	Integer	GETRF

Syntax

HPF	CALL GETRF (*a*, *ipiv*)
	CALL GETRF (*a*, *ipiv*, *info*)

On Entry

a
is the general matrix ***A***, used in the system of equations.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 125.

ipiv
See On Return.

info
See On Return.

On Return

a
is the updated general matrix ***A***, containing the results of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 125.

ipiv
is the vector ***ipiv***, containing the pivot indices.

The elements of ***ipiv*** must be replicated across each element of the corresponding row of ***A***; that is, a copy of ***ipiv*** is aligned with every column of ***A***:

```
!HPF$ ALIGN IPIV(:) WITH A(:,*)
```

Type: **required**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 125 on page 858.

info

has the following meaning, when *info* is **present**:

If *info* = 0, matrix ***A*** is not singular, and the factorization completed normally.

If *info* > 0, matrix ***A*** is singular; that is, one or more columns of ***L*** and the corresponding diagonal of ***U*** contain all zeros. All columns of ***L*** are checked. *info* is set equal to *i*, the first column of ***L*** with a corresponding ***U*** = 0 diagonal element, encountered at position (*i*,*i*) in ***A***. The factorization is completed; however, if you call GETRS with these factors, results are unpredictable.

When *info* is **not present** and matrix ***A*** is singular, the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*a*,1) = size(*ipvt*).

   If you plan to call GETRS, then additionally size(*a*,1) = size(*a*,2).
2. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
3. The ***A*** and ***ipiv*** input to GETRS must be the same as for the corresponding output arguments for GETRF.
4. The way this subroutine handles singularity differs from ScaLAPACK. This subroutine uses the *info* argument to provide information about the singularity of ***A***, like ScaLAPACK, but also provides an error message.
5. On both input and output, matrix ***A*** conforms to ScaLAPACK format.
6. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
7. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vector and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
8. The restrictions given in “Notes and Coding Rules” on page 424 also apply to this subroutine.
9. For information about optimizing performance in this subroutine, see “Performance Considerations” on page 425.

Error Conditions: HPF-specific errors are listed below. All errors listed in "Error Conditions" on page 426 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *ipiv*.
2. The process rank is not the same for *a* and *ipiv*.
3. The process rank is not 1 or 2 for *a* or *ipiv*.

Stage 2: The process grid is not the same for *a* and *ipiv*.

Stage 3: The data distribution is unsupported for *a*.

Stage 4

1. The row block size for *a* and the block size for *ipiv* are incompatible.
2. The data distribution is unsupported for *ipiv*.

Stage 5: The shape of the assumed-shape arrays for *a* and *ipiv* is incompatible: size(*a*,1) $\neq$ size(*ipiv*)

Stage 6: The abstract process row indices for *a* and *ipiv* are incompatible.

Stage 7: The data distribution for *a* is unsupported.

Example 1: This example factors a 9 × 9 real general matrix. As in "Example 1" on page 427, array data is block-cyclically distributed using a 2 × 2 process grid, with ***ipiv*** being replicated across each element of the corresponding row of ***A***; that is, a copy of *ipiv* is aligned with every column of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN IPIV(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL GETRF( A , IPIV )
-or-
CALL GETRF( A , IPIV , INFO )
```

Input: General 9 × 9 matrix ***A***:

$$\begin{bmatrix}
1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 & 2.2 & 2.4 & 2.6 \\
1.2 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 & 2.2 & 2.4 \\
1.4 & 1.2 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 & 2.2 \\
1.6 & 1.4 & 1.2 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 & 2.0 \\
1.8 & 1.6 & 1.4 & 1.2 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
2.0 & 1.8 & 1.6 & 1.4 & 1.2 & 1.0 & 1.2 & 1.4 & 1.6 \\
2.2 & 2.0 & 1.8 & 1.6 & 1.4 & 1.2 & 1.0 & 1.2 & 1.4 \\
2.4 & 2.2 & 2.0 & 1.8 & 1.6 & 1.4 & 1.2 & 1.0 & 1.2 \\
2.6 & 2.4 & 2.2 & 2.0 & 1.8 & 1.6 & 1.4 & 1.2 & 1.0
\end{bmatrix}$$

Output: General 9 × 9 transformed matrix ***A***:

```
┌                                                        ┐
│ 2.6   2.4   2.2   2.0   1.8   1.6   1.4   1.2   1.0 │
│ 0.4   0.3   0.6   0.8   1.1   1.4   1.7   1.9   2.2 │
│ 0.5  -0.4   0.4   0.8   1.2   1.6   2.0   2.4   2.8 │
│ 0.5  -0.3   0.0   0.4   0.8   1.2   1.6   2.0   2.4 │
│ 0.6  -0.3   0.0   0.0   0.4   0.8   1.2   1.6   2.0 │
│ 0.7  -0.2   0.0   0.0   0.0   0.4   0.8   1.2   1.6 │
│ 0.8  -0.2   0.0   0.0   0.0   0.0   0.4   0.8   1.2 │
│ 0.8  -0.1   0.0   0.0   0.0   0.0   0.0   0.4   0.8 │
│ 0.9  -0.1   0.0   0.0   0.0   0.0   0.0   0.0   0.4 │
└                                                        ┘
```

Vector ***ipiv*** of size 9:

```
┌   ┐
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
└   ┘
```

info = 0 (if *info* is present)

Example 2: This example factors a 9 × 9 complex matrix.

As in "Example 2" on page 430, array data is block-cyclically distributed using a 2 × 2 process grid, with ***ipiv*** being replicated across each element of the corresponding row of ***A***; that is, a copy of *ipiv* is aligned with every column of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN IPIV(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL GETRF( A , IPIV )
-or-
CALL GETRF( A , IPIV , INFO )
```

Input: General 9 × 9 matrix ***A***:

```
┌                                                                                                                        ┐
│ (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0)  (5.2,-1.0) │
│ (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0)  (4.8,-1.0) │
│ (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0)  (4.4,-1.0) │
│ (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0)  (4.0,-1.0) │
│ (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0)  (3.6,-1.0) │
│ (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0)  (3.2,-1.0) │
│ (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0)  (2.8,-1.0) │
│ (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0)  (2.4,-1.0) │
│ (5.2, 1.0)  (4.8, 1.0)  (4.4, 1.0)  (4.0, 1.0)  (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) │
└                                                                                                                        ┘
```

Output: General 9 × 9 transformed matrix ***A***:

```
┌                                                                                                                           ┐
│ (5.2, 1.0)  (4.8, 1.0)   (4.4, 1.0)   (4.0, 1.0)   (3.6, 1.0)  (3.2, 1.0)  (2.8, 1.0)  (2.4, 1.0)  (2.0, 1.0) │
│ (0.4, 0.1)  (0.6,-2.0)   (1.1,-1.9)   (1.7,-1.9)   (2.3,-1.8)  (2.8,-1.8)  (3.4,-1.7)  (3.9,-1.7)  (4.5,-1.6) │
│ (0.5, 0.1)  (0.0,-0.1)   (0.6,-1.9)   (1.2,-1.8)   (1.8,-1.7)  (2.5,-1.6)  (3.1,-1.5)  (3.7,-1.4)  (4.3,-1.3) │
│ (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)   (0.7,-1.9)   (1.3,-1.7)  (2.0,-1.6)  (2.7,-1.5)  (3.4,-1.4)  (4.0,-1.2) │
│ (0.6, 0.1)  (0.0,-0.1)  (-0.1,-0.1)  (-0.1, 0.0)   (0.7,-1.9)  (1.5,-1.7)  (2.2,-1.6)  (2.9,-1.5)  (3.7,-1.3) │
│ (0.7, 0.1)  (0.0,-0.1)   (0.0, 0.0)  (-0.1, 0.0)  (-0.1, 0.0)  (0.8,-1.9)  (1.6,-1.8)  (2.4,-1.6)  (3.2,-1.5) │
│ (0.8, 0.0)  (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.8,-1.9)  (1.7,-1.8)  (2.5,-1.8) │
│ (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0)  (1.7,-1.9) │
│ (0.9, 0.0)  (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)   (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)  (0.0, 0.0)  (0.8,-2.0) │
└                                                                                                                           ┘
```

Vector ***ipiv*** of size 9:

```
┌   ┐
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
│ 9 │
└   ┘
```

info = 0 (if *info* is present)

GETRS—General Matrix Solve

GETRS solves one of the following systems of equations for multiple right-hand sides:

1. $\boldsymbol{AX} = \boldsymbol{B}$
2. $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$
3. $\boldsymbol{A}^{H}\boldsymbol{X} = \boldsymbol{B}$

In the formulas above:

A is the square general matrix containing the ***LU*** factorization.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to GETRF. For details on the factorization, see "GETRF—General Matrix Factorization" on page 858.

If any of the assumed-shape arrays have a size of zero, no computation is performed, and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 126. Data Types

A, ***B***	***ipiv***	**Subroutine**
Long-precision real	Integer	GETRS
Long-precision complex	Integer	GETRS

Syntax

HPF	CALL GETRS (*a*, *ipiv*, *b*)
	CALL GETRS (*a*, *ipiv*, *b*, *transa*, *info*)

On Entry

a

is the general matrix ***A***, containing the factorization of matrix ***A*** produced by a preceding call to GETRF.

Type: **required**

Specified as: an assumed-shape array with shape (`:`,`:`), containing numbers of the data type indicated in Table 126, where size(*a*,1) = size(*a*,2).

ipiv

is the vector ***ipiv***, containing the pivoting indices produced on a preceding call to GETRF.

The elements of ***ipiv*** must be replicated across each element of the corresponding row of ***A***; that is, a copy of ***ipiv*** is aligned with every column of ***A***:

```
!HPF$ ALIGN IPIV(:) WITH A(:,*)
```

Type: **required**

Specified as: an assumed-shape array with shape (`:`), containing numbers of the data type indicated in Table 126.

b

is the general matrix ***B***, containing the right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (`:,:`), containing numbers of the data type indicated in Table 126 on page 863.

transa

indicates the form of matrix ***A*** to use in the computation, where:

If *transa* = `'N'`, ***A*** is used in the computation, resulting in solution 1.

If *transa* = `'T'`, $\boldsymbol{A}^T$ is used in the computation, resulting in solution 2.

If *transa* = `'C'`, $\boldsymbol{A}^H$ is used in the computation, resulting in solution 3.

Type: **optional**

Default: *transa* = `'N'`

Specified as: a single character; *transa* = `'N'`, `'T'`, or `'C'`.

info

See On Return.

On Return

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (`:,:`), containing numbers of the data type indicated in Table 126 on page 863.

info

indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*a*,1) = size(*a*,2) = size(*b*,1) = size(*ipiv*).
2. This subroutine accepts lowercase letters for the *transa* argument.
3. When using real data, if you specify `'C'` for the *transa* argument, it is interpreted as though you specified `'T'`.
4. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
5. The ***A*** and ***ipiv*** input to GETRS must be the same as for the corresponding output arguments for GETRF.
6. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
7. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
8. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify

any data distribution for your vector and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.

9. The restrictions given in “Notes and Coding Rules” on page 437 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 438 also apply to this subroutine.

Computational Errors: None

Note: If the factorization performed by GETRF failed because of a singular matrix ***A***, the results returned by GETRS are unpredictable. For details, see the *info* output argument for GETRF.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *b*, or *ipiv*.
2. The process rank is not the same for *a*, *b*, and *ipiv*.
3. The process rank is not 1 or 2 for *a*, *b*, or *ipiv*.

Stage 2: The process grid is not the same for *a*, *b*, and *ipiv*.

Stage 3: The data distribution is unsupported for *a* and *b*.

Stage 4

1. The row block sizes for *a* and *b* and the block size for *ipiv* are incompatible.
2. The data distribution is unsupported for *ipiv*.

Stage 5

1. The shape of the assumed-shape arrays for *a*, *b*, and *ipiv* is incompatible:

   size(*a*,1) ≠ size(*a*,2) or
   size(*a*,1) ≠ size(*b*,1) or
   size(*a*,1) ≠ size(*ipiv*)

2. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 6: The abstract process row indices for *a*, *b*, and *ipiv* are incompatible.

Stage 7: The data distribution for *a* or *b* is unsupported.

Example 1: This example solves the real system ***AX*** = ***B*** with 5 right-hand sides. The input ***ipiv*** vector and transformed matrix ***A*** are the output from “Example 1” on page 860.

An array section is specified for argument *b*, resulting in the computation using a submatrix ***B*** starting at row 1 and column 2 in the array.

As in “Example 1” on page 440, array data is block-cyclically distributed using a 2 × 2 process grid, with ***ipiv*** being replicated across each element of the corresponding row of ***A***; that is, a copy of *ipiv* is aligned with every column of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN IPIV(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL GETRS( A , IPIV , B(1:9,2:6) )
-or-
CALL GETRS( A , IPIV , B(1:9,2:6) , 'N' , INFO )
```

Input: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                   ┐
|  .   93.0  186.0  279.0  372.0  465.0 |
|  .   84.4  168.8  253.2  337.6  422.0 |
|  .   76.6  153.2  229.8  306.4  383.0 |
|  .   70.0  140.0  210.0  280.0  350.0 |
|  .   65.0  130.0  195.0  260.0  325.0 |
|  .   62.0  124.0  186.0  248.0  310.0 |
|  .   61.4  122.8  184.2  245.6  307.0 |
|  .   63.6  127.2  190.8  254.4  318.0 |
|  .   69.0  138.0  207.0  276.0  345.0 |
└                                   ┘
```

Output: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                ┐
|  .    1.0   2.0   3.0   4.0   5.0 |
|  .    2.0   4.0   6.0   8.0  10.0 |
|  .    3.0   6.0   9.0  12.0  15.0 |
|  .    4.0   8.0  12.0  16.0  20.0 |
|  .    5.0  10.0  15.0  20.0  25.0 |
|  .    6.0  12.0  18.0  24.0  30.0 |
|  .    7.0  14.0  21.0  28.0  35.0 |
|  .    8.0  16.0  24.0  32.0  40.0 |
|  .    9.0  18.0  27.0  36.0  45.0 |
└                                ┘
```

info = 0 (if *info* is present)

Example 2: This example solves the complex system ***AX*** = ***B*** with 5 right-hand sides. The input ***ipiv*** vector and transformed matrix ***A*** are the output from “Example 2” on page 861.

An array section is specified for argument *b*, resulting in the computation using a submatrix ***B*** starting at row 1 and column 2 in the array.

As in “Example 2” on page 443, array data is block-cyclically distributed using a 2 × 2 process grid, with ***ipiv*** being replicated across each element of the corresponding row of ***A***; that is, a copy of *ipiv* is aligned with every column of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN IPIV(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL GETRS( A , IPIV , B(1:9,2:6) )
-or-
CALL GETRS( A , IPIV , B(1:9,2:6) , 'N' , INFO )
```

Input: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                                                                 ┐
|   .   (193.0,-10.6)  (200.0, 21.8)  (207.0, 54.2)  (214.0, 86.6)  (221.0,119.0) |
|   .   (173.8, -9.4)  (178.8, 20.2)  (183.8, 49.8)  (188.8, 79.4)  (193.8,109.0) |
|   .   (156.2, -5.4)  (159.2, 22.2)  (162.2, 49.8)  (165.2, 77.4)  (168.2,105.0) |
|   .   (141.0,  1.4)  (142.0, 27.8)  (143.0, 54.2)  (144.0, 80.6)  (145.0,107.0) |
|   .   (129.0, 11.0)  (128.0, 37.0)  (127.0, 63.0)  (126.0, 89.0)  (125.0,115.0) |
|   .   (121.0, 23.4)  (118.0, 49.8)  (115.0, 76.2)  (112.0,102.6)  (109.0,129.0) |
|   .   (117.8, 38.6)  (112.8, 66.2)  (107.8, 93.8)  (102.8,121.4)   (97.8,149.0) |
|   .   (120.2, 56.6)  (113.2, 86.2)  (106.2,115.8)   (99.2,145.4)   (92.2,175.0) |
|   .   (129.0, 77.4)  (120.0,109.8)  (111.0,142.2)  (102.0,174.6)   (93.0,207.0) |
└                                                                                 ┘
```

Output: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                                                  ┐
|   .   (1.0, 1.0)  (1.0, 2.0)  (1.0, 3.0)  (1.0, 4.0)  (1.0, 5.0) |
|   .   (2.0, 1.0)  (2.0, 2.0)  (2.0, 3.0)  (2.0, 4.0)  (2.0, 5.0) |
|   .   (3.0, 1.0)  (3.0, 2.0)  (3.0, 3.0)  (3.0, 4.0)  (3.0, 5.0) |
|   .   (4.0, 1.0)  (4.0, 2.0)  (4.0, 3.0)  (4.0, 4.0)  (4.0, 5.0) |
|   .   (5.0, 1.0)  (5.0, 2.0)  (5.0, 3.0)  (5.0, 4.0)  (5.0, 5.0) |
|   .   (6.0, 1.0)  (6.0, 2.0)  (6.0, 3.0)  (6.0, 4.0)  (6.0, 5.0) |
|   .   (7.0, 1.0)  (7.0, 2.0)  (7.0, 3.0)  (7.0, 4.0)  (7.0, 5.0) |
|   .   (8.0, 1.0)  (8.0, 2.0)  (8.0, 3.0)  (8.0, 4.0)  (8.0, 5.0) |
|   .   (9.0, 1.0)  (9.0, 2.0)  (9.0, 3.0)  (9.0, 4.0)  (9.0, 5.0) |
└                                                                  ┘
```

info = 0 (if *info* is present)

POTRF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization

This subroutine uses Cholesky factorization. It factors a positive definite real symmetric matrix ***A*** into one of the following forms:

$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^{\mathrm{T}}$ if ***A*** is lower triangular.
$\boldsymbol{A} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{U}$ if ***A*** is upper triangular.

It factors a positive definite complex Hermitian matrix ***A*** into one of the following forms:

$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^{\mathrm{H}}$ if ***A*** is lower triangular.
$\boldsymbol{A} = \boldsymbol{U}^{\mathrm{H}}\boldsymbol{U}$ if ***A*** is upper triangular.

In the formulas above:

A is the positive definite real symmetric or complex Hermitian matrix to be factored.
L is a lower triangular matrix.
U is an upper triangular matrix.

To solve the system of equations with any number of right-hand sides, follow the call to this subroutine with one or more calls to POTRS.

If the assumed-shape array has a size of zero, no computation is performed, and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 127. Data Types

A	**Subroutine**
Long-precision real	POTRF
Long-precision complex	POTRF

Syntax

HPF	CALL POTRF (*a*, *uplo*)
	CALL POTRF (*a*, *uplo*, *info*)

On Entry

a

is the real symmetric or complex Hermitian matrix ***A***, used in the system of equations, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (`:`,`:`), containing numbers of the data type indicated in Table 127, where size(*a*,1) = size(*a*,2).

uplo

indicates whether the upper or lower triangular part of the real symmetric or complex Hermitian submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

a

is the updated matrix ***A***, containing the results of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 127 on page 868.

info

has the following meaning, when *info* is **present**:

If *info* = 0, real symmetric or complex Hermitian matrix ***A*** is positive definite, and the factorization completed normally.

If *info* > 0, the leading minor of order *k* of the real symmetric or complex hermitian matrix ***A*** is not positive definite. *info* is set equal to *k*, where the leading minor was encountered at position (*k*,*k*) in ***A***. The factorization is not completed. ***A*** is overwritten with the partial factors.

When *info* is **not present** and matrix ***A*** is not positive definite, the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. The assumed-shape array must have the exact size required for the computation, that is: size(*a*,1) = size(*a*,2).
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. The imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values. On output, they are set to zero.
4. The ***A*** input to POTRS must be the same as for the corresponding output argument for POTRF.
5. The way this subroutine handles nonpositive definiteness differs from ScaLAPACK. This subroutine uses the *info* argument to provide information about the nonpositive definiteness of ***A***, like ScaLAPACK, but also provides an error message.
6. On both input and output, matrix ***A*** conforms to ScaLAPACK format.

7. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
8. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.
9. The restrictions given in “Notes and Coding Rules” on page 463 also apply to this subroutine.
10. For information about optimizing performance in this subroutine, see “Performance Considerations” on page 425.

Error Conditions: HPF-specific errors are listed below. All errors listed in “Error Conditions” on page 465 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*.
2. The process rank is not 1 or 2 for *a*.

Stage 2: The data distribution is inconsistent for *a*.

Stage 3: The shape of the assumed-shape array for *a* is invalid:
size(*a*,1) $\neq$ size(*a*,2)

Stage 4: The data distribution for *a* is unsupported.

Example 1: This example factors a 9 × 9 positive definite real symmetric matrix. As in “Example 1” on page 466, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL POTRF(  A , 'L' )
-or-
CALL POTRF(  A , 'L' , INFO )
```

Input: Real symmetric matrix ***A*** of order 9:

```
┌                                          ┐
| 1.0   .    .    .    .    .    .    .    .  |
| 1.0  2.0   .    .    .    .    .    .    .  |
| 1.0  2.0  3.0   .    .    .    .    .    .  |
| 1.0  2.0  3.0  4.0   .    .    .    .    .  |
| 1.0  2.0  3.0  4.0  5.0   .    .    .    .  |
| 1.0  2.0  3.0  4.0  5.0  6.0   .    .    .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0   .    .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0   .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  9.0 |
└                                          ┘
```

Output: Real symmetric matrix ***A*** of order 9:

```
┌                                             ┐
| 1.0   .    .    .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
| 1.0  1.0  1.0   .    .    .    .    .    .  |
| 1.0  1.0  1.0  1.0   .    .    .    .    .  |
| 1.0  1.0  1.0  1.0  1.0   .    .    .    .  |
| 1.0  1.0  1.0  1.0  1.0  1.0   .    .    .  |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0   .  |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
└                                             ┘
```

info = 0 (if *info* is present)

Example 2: This example factors a 9 × 9 positive definite complex Hermitian matrix.

As in “Example 2” on page 468, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL POTRF(  A , 'L' )
-or-
CALL POTRF(  A , 'L' , INFO )
```

Input: Complex Hermitian matrix ***A*** of order 9:

```
┌                                                                                                                  ┐
| (18.0,  . )      .           .           .           .            .            .            .           .       |
|  (1.0, 1.0)  (18.0,  . )     .           .           .            .            .            .           .       |
|  (1.0, 1.0)   (3.0, 1.0)  (18.0,  . )    .           .            .            .            .           .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)  (18.0,  . )   .            .            .            .           .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)   (7.0, 1.0)  (18.0,  . )   .            .            .           .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)   (7.0, 1.0)   (9.0, 1.0)  (18.0,  . )   .            .           .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)  (18.0,  . )   .           .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)  (13.0, 1.0)  (18.0,  . )  .       |
|  (1.0, 1.0)   (3.0, 1.0)   (5.0, 1.0)   (7.0, 1.0)   (9.0, 1.0)  (11.0, 1.0)  (13.0, 1.0)  (15.0, 1.0)  (18.0, . ) |
└                                                                                                                  ┘
```

Note: On input, the imaginary parts of the diagonal elements of the complex Hermitian matrix ***A*** are assumed to be zero, so you do not have to set these values.

Output: Complex Hermitian matrix ***A*** of order 9:

```
┌                                                                                                                        ┐
|   (4.2, 0.0)       .            .           .            .            .            .            .            .        |
| (0.24, 0.24)    (4.2, 0.0)      .           .            .            .            .            .            .        |
| (0.24, 0.24)  (0.68, 0.24)   (4.2, 0.0)     .            .            .            .            .            .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24)  (4.0, 0.0)      .            .            .            .            .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24) (1.3, 0.25)   (3.8, 0.0)      .            .            .            .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24) (1.3, 0.25)  (1.4, 0.26)   (3.5, 0.0)      .            .            .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24) (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)   (3.2, 0.0)      .            .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24) (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)  (1.6, 0.32)   (2.7, 0.0)      .        |
| (0.24, 0.24)  (0.68, 0.24)  (1.1, 0.24) (1.3, 0.25)  (1.4, 0.26)  (1.5, 0.28)  (1.6, 0.32)  (1.6, 0.37)   (2.2, 0.0)  |
└                                                                                                                        ┘
```

Note: On output, the imaginary parts of the diagonal elements of the matrix are set to zero.

info = 0 (if *info* is present)

POTRS—Positive Definite Real Symmetric or Complex Hermitian Matrix Solve

This subroutine solves the following systems of equations for multiple right-hand sides:

AX = ***B***

where, in the formula above:

A is the positive definite real symmetric or complex Hermitian matrix factored by Cholesky factorization.
B is the general matrix ***B***, containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to POTRF. For details on the factorization, see “POTRF—Positive Definite Real Symmetric or Complex Hermitian Matrix Factorization” on page 868.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking. See references [16], [18], [22], [36], and [37].

Table 128. Data Types

A, ***B***	**Subroutine**
Long-precision real	POTRS
Long-precision complex	POTRS

Syntax

HPF	CALL POTRS (*a*, *b*, *uplo*) CALL POTRS (*a*, *b*, *uplo*, *info*)

On Entry

a

is the real symmetric or complex Hermitian matrix ***A***, containing the factorization of matrix ***A*** produced by a preceding call to POTRF, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 128.

b

is the general matrix ***B***, containing the right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 128.

uplo

indicates whether the upper or lower triangular part of the real symmetric or complex Hermitian submatrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 128 on page 873.

info

indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*a*,1) = size(*a*,2) = size(*b*,1).
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
4. The ***A*** input to POTRS must be the same as for the corresponding output argument for POTRF.
5. On both input and output, matrices ***A*** and ***B*** conform to ScaLAPACK format.
6. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
7. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 9 on page 108.
8. The restrictions given in "Notes and Coding Rules" on page 474 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in "Error Conditions" on page 475 also apply to this subroutine.

Computational Errors: None

Note: If the factorization performed by POTRF failed because of a nonpositive definite matrix ***A***, the results returned by POTRS are unpredictable. For details, see the *info* output argument for POTRF.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *b*.
2. The process rank is not the same for *a* and *b*.
3. The process rank is not 1 or 2 for *a* or *b*.

Stage 2: The process grid is not the same for *a* and *b*.

Stage 3: The data distribution is inconsistent for *a* and *b*.

Stage 4

1. The shape of the assumed-shape arrays for *a* and *b* is incompatible:
   a. size(*a*,1) ≠ size(*a*,2) or
   b. size(*a*,1) ≠ size(*b*,1)
2. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 5: The data distribution for *a* or *b* is unsupported.

Example 1: This example solves the positive definite real symmetric system ***AX*** = ***B*** with 5 right-hand sides. The transformed matrix ***A*** is the output from “Example 1” on page 870.

An array section is specified for argument *b*, resulting in the computation using a submatrix ***B*** starting at row 1 and column 2 in the array.

As in “Example 1” on page 477, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL POTRS( A , B(1:9,2:6) , 'L' )
-or-
CALL POTRS( A , B(1:9,2:6) , 'L' , INFO )
```

Input: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                      ┐
|  .   18.0   27.0   36.0   45.0    9.0 |
|  .   34.0   51.0   68.0   85.0   17.0 |
|  .   48.0   72.0   96.0  120.0   24.0 |
|  .   60.0   90.0  120.0  150.0   30.0 |
|  .   70.0  105.0  140.0  175.0   35.0 |
|  .   78.0  117.0  156.0  195.0   39.0 |
|  .   84.0  126.0  168.0  210.0   42.0 |
|  .   88.0  132.0  176.0  220.0   44.0 |
|  .   90.0  135.0  180.0  225.0   45.0 |
└                                      ┘
```

Output: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                              ┐
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
|  .   2.0  3.0  4.0  5.0  1.0 |
└                              ┘
```

info = 0 (if *info* is present)

Example 2: This example solves the positive definite complex Hermitian system ***AX*** = ***B*** with 5 right-hand sides. The transformed matrix ***A*** is the output from "Example 2" on page 871.

An array section is specified for argument *b*, resulting in the computation using a submatrix ***B*** starting at row 1 and column 2 in the array.

As in "Example 2" on page 479, array data is block-cyclically distributed using a 2 × 2 process grid.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, B

CALL POTRS( A , B(1:9,2:6) , 'L' )
-or-
CALL POTRS( A , B(1:9,2:6) , 'L' , INFO )
```

Input: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                                                                      ┐
|   .    (60.0, 10.0)   (86.0,  2.0)  (112.0,  -6.0)  (138.0,-14.0)   (34.0, 18.0) |
|   .    (86.0, 28.0)  (126.0, 22.0)  (166.0,  16.0)  (206.0, 10.0)   (46.0, 34.0) |
|   .   (108.0, 44.0)  (160.0, 40.0)  (212.0,  36.0)  (264.0, 32.0)   (56.0, 48.0) |
|   .   (126.0, 58.0)  (188.0, 56.0)  (250.0,  54.0)  (312.0, 52.0)   (64.0, 60.0) |
|   .   (140.0, 70.0)  (210.0, 70.0)  (280.0,  70.0)  (350.0, 70.0)   (70.0, 70.0) |
|   .   (150.0, 80.0)  (226.0, 82.0)  (302.0,  84.0)  (378.0, 86.0)   (74.0, 78.0) |
|   .   (156.0, 88.0)  (236.0, 92.0)  (316.0,  96.0)  (396.0, 100.0)  (76.0, 84.0) |
|   .   (158.0, 94.0)  (240.0,100.0)  (322.0, 106.0)  (404.0, 112.0)  (76.0, 88.0) |
|   .   (156.0, 98.0)  (238.0,106.0)  (320.0, 114.0)  (402.0, 122.0)  (74.0, 90.0) |
└                                                                                      ┘
```

Output: Only a portion of the data structure is used—that is, submatrix ***B***. Following is the 9 × 5 submatrix ***B***, starting at row 1 and column 2 in the 9 × 6 array:

```
┌                                                                    ┐
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
|   .   (2.0, 1.0)  (3.0, 1.0)  (4.0, 1.0)  (5.0, 1.0)  (1.0, 1.0)  |
└                                                                    ┘
```

info = 0 (if *info* is present)

Banded Linear Algebraic Equation Subroutines

This section contains the banded linear algebraic equation subroutine descriptions.

PBSV—Positive Definite Symmetric Band Matrix Factorization and Solve

This subroutine solves the following system of equations for multiple right-hand sides:

AX = ***B***

where, in the formula above:

A is the positive definite symmetric band matrix, factored by Cholesky factorization.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [23], [2], [16], [18], [22], [36], and [37].

Table 129. Data Types

A, ***B***	**Subroutine**
Long-precision real	PBSV

Syntax

HPF	CALL PBSV (*a*, *b*, *uplo*) CALL PBSV (*a*, *b*, *uplo*, *info*)

On Entry

a

is the positive definite symmetric band matrix ***A*** with half bandwidth *k*, where *k*=size(*a*,1)−1, to be factored. Matrix ***A*** is stored in upper- or lower-band-packed storage mode, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric band matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric band matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 129.

On output array `A` is overwritten; that is, the original input is not preserved.

b

is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 129.

uplo

indicates whether the upper or lower triangular part of the matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

a

is overwritten; that is, the original input is not preserved. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 129 on page 879.

info

has the following meaning, when *info* is **present**:

If *info* = 0, matrix ***A*** is positive definite, and the factorization completed normally.

If *info* > 0, the leading minor of order *i* of the matrix ***A*** is not positive definite. *info* is set equal to *i*, where the first leading minor was encountered at position (*i*,*i*) in ***A***. The results contained in matrix ***A*** are not defined.

When *info* is **not present** and matrix ***A*** is not positive definite, the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*a*,2) = size(*b*,1). Also, in this subroutine, the half bandwidth *k*=size(*a*,1)−1.
2. For performances reasons, it is suggested that you specify *uplo* = `'L'`. For information on how bandwidth affects performance, see [2].
3. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
4. This subroutine accepts lowercase letters for the *uplo* argument.
5. The band matrix ***A*** must be positive definite. If ***A*** is not positive definite, this subroutine uses the *info* argument to provide information about ***A*** and issues

an error message. This differs from ScaLAPACK, which only uses the *info* argument to provide information about ***A***.

6. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
7. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. For details, see the section on symmetric matrices in "Matrices" on page 40.

   Matrix ***A*** must be distributed over a one-dimensional process grid, using block-column data distribution. For more information on using block-column data distribution, see "Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations" on page 26.

   Matrix ***B*** must be distributed over a one-dimensional process grid, using block-row data distribution. For more information on using block-row data distribution, see the section on block distributing a general matrix containing the right-hand sides in "Matrices" on page 40.

   Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 10 on page 109.
8. The restrictions given in "Notes and Coding Rules" on page 489 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in "Error Conditions" on page 490 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *a* or *b*.
2. The process rank is not 1 for *a* or *b*.

Stage 2

1. The process grid is not the same for *a* and *b*.
2. *a* is not distributed (*,BLOCK).
3. *b* is not distributed (BLOCK,*).

Stage 3

1. The shape of the assumed-shape arrays for *a* and *b* is incompatible: size(*a*,2) ≠ size(*b*,1)
2. The column block size for *a* and the row block size for *b* are not equal.
3. The abstract process indices for *a* and *b* are not equal.
4. The data distribution for *a* or *b* is unsupported.

Example: This example shows a factorization of the positive definite symmetric band matrix ***A*** of order 9 with a half bandwidth of 7:

```
┌                                              ┐
│ 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0 │
│ 1.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  1.0 │
│ 1.0  2.0  3.0  3.0  3.0  3.0  3.0  3.0  2.0 │
│ 1.0  2.0  3.0  4.0  4.0  4.0  4.0  4.0  3.0 │
│ 1.0  2.0  3.0  4.0  5.0  5.0  5.0  5.0  4.0 │
│ 1.0  2.0  3.0  4.0  5.0  6.0  6.0  6.0  5.0 │
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0  6.0 │
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  7.0 │
│ 0.0  1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0 │
└                                              ┘
```

Matrix ***A*** is stored in lower-band-packed storage mode:

```
┌                                              ┐
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  8.0 │
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0   .  │
│ 1.0  2.0  3.0  4.0  5.0  6.0  6.0   .    .  │
│ 1.0  2.0  3.0  4.0  5.0  5.0   .    .    .  │
│ 1.0  2.0  3.0  4.0  4.0   .    .    .    .  │
│ 1.0  2.0  3.0  3.0   .    .    .    .    .  │
│ 1.0  2.0  2.0   .    .    .    .    .    .  │
│ 1.0  1.0   .    .    .    .    .    .    .  │
└                                              ┘
```

where "." means you do not have to store a value in that position in the local array. However, these storage positions are required and are overwritten during the computation.

As in "Example" on page 492, array data is block distributed over 3 processes.

Note: On output, the matrix ***A*** is overwritten by this subroutine.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (*,BLOCK) ONTO PROC :: A
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL PBSV( A , B , 'L' )
-or-
CALL PBSV( A , B , 'L' , INFO=INFO )
```

Input: Matrix ***A***, stored in an 8 × 9 array:

```
┌                                              ┐
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  8.0 │
│ 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0   .  │
│ 1.0  2.0  3.0  4.0  5.0  6.0  6.0   .    .  │
│ 1.0  2.0  3.0  4.0  5.0  5.0   .    .    .  │
│ 1.0  2.0  3.0  4.0  4.0   .    .    .    .  │
│ 1.0  2.0  3.0  3.0   .    .    .    .    .  │
│ 1.0  2.0  2.0   .    .    .    .    .    .  │
│ 1.0  1.0   .    .    .    .    .    .    .  │
└                                              ┘
```

Rectangular 9 × 3 matrix ***B***:

```
┌                    ┐
│  8.0   36.0   44.0 │
│ 16.0   80.0   80.0 │
│ 23.0  122.0  108.0 │
│ 29.0  161.0  129.0 │
│ 34.0  196.0  144.0 │
│ 38.0  226.0  154.0 │
│ 41.0  250.0  160.0 │
│ 43.0  267.0  163.0 │
│ 36.0  240.0  120.0 │
└                    ┘
```

Output: Rectangular 9 × 3 matrix ***B***:

```
┌               ┐
│ 1.0  1.0  9.0 │
│ 1.0  2.0  8.0 │
│ 1.0  3.0  7.0 │
│ 1.0  4.0  6.0 │
│ 1.0  5.0  5.0 │
│ 1.0  6.0  4.0 │
│ 1.0  7.0  3.0 │
│ 1.0  8.0  2.0 │
│ 1.0  9.0  1.0 │
└               ┘
```

info = 0 (if *info* is present)

PBTRF—Positive Definite Symmetric Band Matrix Factorization

This subroutine uses Cholesky factorization to factor a positive definite symmetric band matrix ***A***, stored in upper- or lower-band-packed storage mode, into one of the following forms:

$\boldsymbol{A} = \boldsymbol{U}^T\boldsymbol{U}$ if ***A*** is upper triangular.
$\boldsymbol{A} = \boldsymbol{L}\boldsymbol{L}^T$ if ***A*** is lower triangular.

where, in the formulas above:

A is the positive definite symmetric band matrix to be factored.
U is an upper triangular matrix.
L is a lower triangular matrix.

To solve the system of equations with multiple right-hand sides, follow the call to this subroutine with one of more calls to PBTRS. The output from this factorization subroutine should be used only as input to PBTRS.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [23], [2], [16], [18], [22], [36], and [37].

Table 130. Data Types

A, *af*	**Subroutine**
Long-precision real	PBTRF

Syntax

HPF	CALL PBTRF (*a*, *af*, *uplo*) CALL PBTRF (*a*, *af*, *uplo*, *info*)

On Entry

a

is the positive definite symmetric band matrix ***A*** with half bandwidth *k*, where *k*=size(*a*,1)−1, to be factored. Matrix ***A*** is stored in upper- or lower-band-packed storage mode, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric band matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric band matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (`:`,`:`), containing numbers of the data type indicated in Table 130.

On output array `A` is overwritten; that is, the original input is not preserved.

af

is a reserved output area.

Type: **required**

Specified as: for migration purposes, you should specify a one-dimensional long-precision assumed-shape array with shape (**:**), where:

size(*af*) ≥
number_of_processors() [(ceiling{size(*a*,2) / number_of_processors()} + (2)(*k*)) (*k*)]

uplo

indicates whether the upper or lower triangular part of the matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

a

is the updated matrix ***A***, containing the results of the factorization, where:

If *uplo* = `'U'`, the array contains the results of the factorization of the symmetric band matrix ***A*** in its upper triangle. The remaining elements stored in the array are overwritten by this subroutine.

If *uplo* = `'L'`, the array contains the results of the factorization of the symmetric band matrix ***A*** in its lower triangle. The remaining elements stored in the array are overwritten by this subroutine.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 130 on page 884.

af

is a reserved output area.

info

has the following meaning, when *info* is **present**:

If *info* = 0, matrix ***A*** is positive definite, and the factorization completed normally.

If *info* > 0, the leading minor of order *i* of the matrix ***A*** is not positive definite. *info* is set equal to *i*, where the first leading minor was encountered at position (*i*,*i*) in ***A***. The results contained in matrix ***A*** are not defined.

When *info* is **not present** and matrix ***A*** is not positive definite, the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. In this subroutine, the half bandwidth *k*=size(*a*,1)−1.
2. For performances reasons, it is suggested that you specify *uplo* = `'L'`. For information on how bandwidth affects performance, see [2].
3. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
4. This subroutine accepts lowercase letters for the *uplo* argument.
5. The output from this factorization subroutine should be used only as input to the solve subroutine PBTRS.

   The data specified for input argument *uplo* must be the same for both PBTRF and PBTRS.

   The matrix ***A*** and *af* input to PBTRS must be the same as the corresponding output arguments for PBTRF.
6. The matrix ***A*** must remain unchanged between calls to PBTRF and PBTRS. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.
7. The band matrix ***A*** must be positive definite. If ***A*** is not positive definite, this subroutine uses the *info* argument to provide information about ***A*** and issues an error message. This differs from ScaLAPACK, which only uses the *info* argument to provide information about ***A***.
8. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
9. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. For details, see the section on symmetric matrices in “Matrices” on page 40.

   Matrix ***A*** must be distributed over a one-dimensional process grid, using block-column data distribution. For more information on using block-column data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.

   Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vector and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 10 on page 109.
10. The restrictions given in “Notes and Coding Rules” on page 501 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in “Error Conditions” on page 502 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *a* or *af*.
2. The process rank is not 1 for *a* or *af*.

Stage 2

1. The process grid is not the same for *a* and *af*.
2. *a* is not distributed (*,BLOCK).
3. *af* is not distributed (BLOCK).

Stage 3: The data distribution for *a* is unsupported.

Example: This example shows a factorization of the positive definite symmetric band matrix ***A*** of order 9 with a half bandwidth of 7:

```
┌                                             ┐
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  0.0 |
| 1.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  1.0 |
| 1.0  2.0  3.0  3.0  3.0  3.0  3.0  3.0  2.0 |
| 1.0  2.0  3.0  4.0  4.0  4.0  4.0  4.0  3.0 |
| 1.0  2.0  3.0  4.0  5.0  5.0  5.0  5.0  4.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  6.0  6.0  5.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0  6.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  7.0 |
| 0.0  1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0 |
└                                             ┘
```

Matrix ***A*** is stored in lower-band-packed storage mode:

```
┌                                             ┐
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  8.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0   .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  6.0   .    .  |
| 1.0  2.0  3.0  4.0  5.0  5.0   .    .    .  |
| 1.0  2.0  3.0  4.0  4.0   .    .    .    .  |
| 1.0  2.0  3.0  3.0   .    .    .    .    .  |
| 1.0  2.0  2.0   .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
└                                             ┘
```

where "." means you do not have to store a value in that position in the local array. However, these storage positions are required and are overwritten during the computation.

As in "Example" on page 503, array data is block distributed over 3 processes.

Notes:

1. Matrix ***A***, output from PBTRF, must be passed, unchanged, to the solve subroutine PBTRS.
2. The *af* argument is reserved and not shown in this example.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (*,BLOCK) ONTO PROC :: A
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: AF

CALL PBTRF( A , AF , 'L' )
-or-
CALL PBTRF( A , AF , 'L' , INFO=INFO )
```

Input: Matrix ***A***, stored in an 8 × 9 array:

```
┌                                              ┐
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  8.0  8.0 |
| 1.0  2.0  3.0  4.0  5.0  6.0  7.0  7.0   .  |
| 1.0  2.0  3.0  4.0  5.0  6.0  6.0   .    .  |
| 1.0  2.0  3.0  4.0  5.0  5.0   .    .    .  |
| 1.0  2.0  3.0  4.0  4.0   .    .    .    .  |
| 1.0  2.0  3.0  3.0   .    .    .    .    .  |
| 1.0  2.0  2.0   .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
└                                              ┘
```

Output: Matrix ***A***, stored in an 8 × 9 array:

```
┌                                              ┐
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0   .  |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
| 1.0  1.0  1.0  1.0  1.0  1.0   .    .    .  |
| 1.0  1.0  1.0  1.0  1.0   .    .    .    .  |
| 1.0  1.0  1.0  1.0   .    .    .    .    .  |
| 1.0  1.0  1.0   .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
└                                              ┘
```

info = 0 (if *info* is present)

PBTRS—Positive Definite Symmetric Band Matrix Solve

This subroutine solves the following system of equations for multiple right-hand sides:

$\boldsymbol{AX} = \boldsymbol{B}$

where, in the formula above:

A is the positive definite symmetric band matrix, factored by Cholesky factorization.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to PBTRF. The output from PBTRF should be used only as input to this solve subroutine.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [23], [2], [16], [18], [22], [36], and [37].

Table 131. Data Types

A, ***B***, *af*	**Subroutine**
Long-precision real	PBTRS

Syntax

HPF	CALL PBTRS (*a*, *b*, *af*, *uplo*) CALL PBTRS (*a*, *b*, *af*, *uplo*, *info*)

On Entry

a

is the positive definite symmetric band matrix ***A*** with half bandwidth *k*, where *k*=size(*a*,1)−1, containing the factorization of matrix ***A*** produced by a preceding call to PBTRF. Matrix ***A*** is stored in upper- or lower-band-packed storage mode, where:

If *uplo* = `'U'`, the array contains the results of the factorization of the symmetric band matrix ***A*** in its upper triangle.

If *uplo* = `'L'`, the array contains the results of the factorization of the symmetric band matrix ***A*** in its lower triangle.

Type: **required**

Specified as: an assumed-shape array with shape (`:`,`:`), containing numbers of the data type indicated in Table 131.

b

is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 131.

af

is a reserved area.

Type: **required**

Specified as: for migration purposes, you should specify a one-dimensional long-precision assumed-shape array with shape (**:**), where:

size(*af*) ≥
number_of_processors() [(ceiling{size(*a*,2) / number_of_processors()} + (2)(*k*)) (*k*)]

uplo

indicates whether the upper or lower triangular part of the matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 131 on page 889.

info

indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*a*,2) = size(*b*,1). Also, in this subroutine, the half bandwidth *k*=size(*a*,1)–1.
2. For performances reasons, it is suggested that you specify *uplo* = `'L'`. For information on how bandwidth affects performance, see [2].
3. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
4. This subroutine accepts lowercase letters for the *uplo* argument.
5. The output from the factorization subroutine PBTRF should be used only as input to this solve subroutine.

   The data specified for input argument *uplo* must be the same for both PBTRF and PBTRS.

   The matrix ***A*** and *af* input to PBTRS must be the same as the corresponding output arguments for PBTRF.

6. The matrix ***A*** must remain unchanged between calls to PBTRF and PBTRS. This subroutine overwrites data in positions that do not contain the positive definite symmetric band matrix ***A*** stored in upper- or lower-band-packed storage mode.
7. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
8. The global positive definite symmetric band matrix ***A*** must be stored in upper- or lower-band-packed storage mode. For details, see the section on symmetric matrices in “Matrices” on page 40.

   Matrix ***A*** must be distributed over a one-dimensional process grid, using block-column data distribution. For more information on using block-column data distribution, see “Specifying Block-Cyclically-Distributed Matrices for the Banded Linear Algebraic Equations” on page 26.

   Matrix ***B*** must be distributed over a one-dimensional process grid, using block-row data distribution. For more information on using block-row data distribution, see the section on block distributing a general matrix containing the right-hand sides in “Matrices” on page 40.

   Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vector and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 10 on page 109.
9. The restrictions given in “Notes and Coding Rules” on page 512 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 514 also apply to this subroutine.

Computational Errors: None

Note: If the factorization performed by PBTRF failed because of a nonpositive definite matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PBTRF.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *a*, *b*, or *af*.
2. The process rank is not 1 for *a*, *b*, or *af*.

Stage 2

1. The process grid is not the same for *a*, *b*, and *af*.
2. *a* is not distributed (*,BLOCK).
3. *b* is not distributed (BLOCK,*).
4. *af* is not distributed (BLOCK).

Stage 3

1. The shape of the assumed-shape arrays for *a* and *b* is incompatible: size(*a*,2) ≠ size(*b*,1)

2. The column block size for *a* and the row block size for *b* are not equal.
3. The abstract process indices for *a* and *b* are not equal.
4. The data distribution for *a* or *b* is unsupported.

Example: This example solves the system ***AX***=***B***, where matrix ***A*** is the same matrix factored in “Example” on page 887 for PBTRF.

As in “Example” on page 515, array data is block distributed over 3 processes.

Notes:

1. Matrix ***A***, output from PBTRF, must be passed, unchanged, to the solve subroutine PBTRS.
2. The *af* argument, output from PBTRF, must be passed, unchanged, to the solve subroutine PBTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (*,BLOCK) ONTO PROC :: A
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: AF

CALL PBTRS( A , B , AF , 'L' )
-or-
CALL PBTRS( A , B , AF , 'L' , INFO=INFO )
```

Input: Matrix ***A***, stored in an 8 × 9 array:

```
┌                                             ┐
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0 |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.0   .  |
| 1.0  1.0  1.0  1.0  1.0  1.0  1.0   .    .  |
| 1.0  1.0  1.0  1.0  1.0  1.0   .    .    .  |
| 1.0  1.0  1.0  1.0  1.0   .    .    .    .  |
| 1.0  1.0  1.0  1.0   .    .    .    .    .  |
| 1.0  1.0  1.0   .    .    .    .    .    .  |
| 1.0  1.0   .    .    .    .    .    .    .  |
└                                             ┘
```

Rectangular 9 × 3 matrix ***B***:

```
┌                     ┐
|  8.0   36.0    44.0 |
| 16.0   80.0    80.0 |
| 23.0  122.0   108.0 |
| 29.0  161.0   129.0 |
| 34.0  196.0   144.0 |
| 38.0  226.0   154.0 |
| 41.0  250.0   160.0 |
| 43.0  267.0   163.0 |
| 36.0  240.0   120.0 |
└                     ┘
```

Output: Rectangular 9 × 3 matrix ***B***:

```
┌                ┐
| 1.0   1.0   9.0 |
| 1.0   2.0   8.0 |
| 1.0   3.0   7.0 |
| 1.0   4.0   6.0 |
| 1.0   5.0   5.0 |
| 1.0   6.0   4.0 |
| 1.0   7.0   3.0 |
| 1.0   8.0   2.0 |
| 1.0   9.0   1.0 |
└                ┘
```

info = 0 (if *info* is present)

GTSV and DTSV—General Tridiagonal Matrix Factorization and Solve

GTSV solves the tridiagonal systems of linear equations, ***AX*** = ***B***, using Gaussian elimination with partial pivoting for the general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

DTSV solves the tridiagonal systems of linear equations, ***AX*** = ***B***, using Gaussian elimination for the diagonally dominant general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

In these subroutines:

A is the square general tridiagonal matrix.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 132. Data Types

dl, ***d***, ***du***, ***B***	**Subroutine**
Long-precision real	GTSV and DTSV

Syntax

HPF	CALL GTSV | DTSV (*dl, d, du, b*) CALL GTSV | DTSV (*dl, d, du, b, info*)

On Entry

dl

is the vector ***dl***, containing the subdiagonal of the general tridiagonal matrix ***A*** in elements 2 through size(*dl*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 132.

On output, `DL` is overwritten; that is, the original input is not preserved.

d

is the vector ***d***, containing the main diagonal of the general tridiagonal matrix ***A*** in elements 1 through size(*d*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 132.

On output, `D` is overwritten; that is, the original input is not preserved.

du

is the vector ***du***, containing the superdiagonal of the general tridiagonal matrix ***A*** in elements 1 through size(*du*)−1.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 132 on page 894.

On output, `DU` is overwritten; that is, the original input is not preserved.

b
: is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 132 on page 894.

info
: See On Return.

On Return

dl
: is overwritten; that is, the original input is not preserved.

d
: is overwritten; that is, the original input is not preserved.

du
: is overwritten; that is, the original input is not preserved.

b
: is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 132 on page 894.

info
: is the vector ***info*** of length equal to the number_of_processors(), where, if you are running on the *j*-th process, then $info_j$ has the following meaning, when *info* is **present**:

If $info_j = 0$ for all *j*, the factorization completed normally.

Note: For DTSV, if the input matrix ***A*** is not diagonally dominant, the subroutine may still complete the factorization; however, results are unpredictable.

If $1 \le info_j \le$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j-1$ and factored locally, is singular or reducible (for GTSV), or not diagonally dominant (for DTSV). The magnitude of a pivot element was zero or too small.

If $info_j >$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j-$number_of_processors()-1 representing interactions with other processes, is singular or reducible (for GTSV), or not diagonally dominant (for DTSV). The magnitude of a pivot element was zero or too small.

If $info_j > 0$ for any *j*, the factorization is completed; however, the results are unpredictable.

All elements of *info* will have the same value.

When *info* is **not present** or size(*info*)=0, and matrix ***A*** is singular or reducible (for GTSV) or not diagonally dominant (for DTSV), then the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: an assumed-shape array with shape (**:**), containing fullword integers, where $info_j \ge 0$ for $j = 1$...number_of_processors().

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*dl*) = size(*d*) = size(*du*) = size(*b*,1).
2. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
3. For GTSV, the general tridiagonal matrix ***A*** must be non-singular and irreducible. For DTSV, the general tridiagonal matrix ***A*** must be diagonally dominant to ensure numerical accuracy because no pivoting is performed. These subroutines use the *info* argument to provide information about ***A***, like ScaLAPACK. However, these subroutines also issue an error message, which differs from ScaLAPACK.
4. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
5. The general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode. For details, see the section on tridiagonal matrices in "Matrices" on page 40.

   Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 10 on page 109.
6. The restrictions given in "Notes and Coding Rules" on page 526 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in "Error Conditions" on page 528 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument or if size(*info*)=0, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *dl*, *d*, *du*, *b*, or *info* (if *info* is present).
2. The process rank is not 1 for *dl*, *d*, *du*, *b*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *dl*, *d*, and *du*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *dl*, *d*, and *du*.
4. *b* is not distributed (BLOCK,*).
5. *dl*, *d*, or *du* is not distributed (BLOCK).

6. The vector for *dl*, *d*, or *du* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *dl*, *d*, *du*, and *b* is incompatible:
   a. size(*dl*) ≠ size(*b*,1) or
   b. size(*d*) ≠ size(*b*,1) or
   c. size(*du*) ≠ size(*b*,1)
2. The block sizes for *dl*, *d*, *du*, and *b* are incompatible.
3. The abstract process indices for *dl*, *d*, *du*, and *b* are incompatible.
4. The data distribution for *dl*, *d*, or *du* is unsupported.

Example 1: This example shows a factorization of the general tridiagonal matrix ***A***, of order 12, where matrix ***A*** is stored in tridiagonal storage mode:

$$\begin{bmatrix}
2.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 3.0
\end{bmatrix}$$

As in “Example” on page 530, array data is block distributed over 3 processes.

Note: On output, vectors ***dl***, ***d***, and ***du*** are overwritten by this subroutine.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL GTSV( DL , D , DU , B )   -or-   CALL DTSV( DL , D , DU , B )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL GTSV( DL , D , DU , B , INFO )   -or-   CALL DTSV( DL , D , DU , B , INFO )
```

Input: Vector ***dl*** of size 12:

$$\begin{bmatrix} . & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

Vector ***d*** of size 12:

$$\begin{bmatrix} 2.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 & 3.0 \end{bmatrix}$$

Vector ***du*** of size 12:

$$\begin{bmatrix} 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & . \end{bmatrix}$$

Rectangular 12 × 3 matrix ***B***:

$$\begin{bmatrix} 46.0 & 6.0 & 4.0 \\ 65.0 & 13.0 & 6.0 \\ 59.0 & 19.0 & 6.0 \\ 53.0 & 25.0 & 6.0 \\ 47.0 & 31.0 & 6.0 \\ 41.0 & 37.0 & 6.0 \\ 35.0 & 43.0 & 6.0 \\ 29.0 & 49.0 & 6.0 \\ 23.0 & 55.0 & 6.0 \\ 17.0 & 61.0 & 6.0 \\ 11.0 & 67.0 & 6.0 \\ 5.0 & 47.0 & 4.0 \end{bmatrix}$$

Output: Rectangular 12 × 3 matrix ***B***:

$$\begin{bmatrix} 12.0 & 1.0 & 1.0 \\ 11.0 & 2.0 & 1.0 \\ 10.0 & 3.0 & 1.0 \\ 9.0 & 4.0 & 1.0 \\ 8.0 & 5.0 & 1.0 \\ 7.0 & 6.0 & 1.0 \\ 6.0 & 7.0 & 1.0 \\ 5.0 & 8.0 & 1.0 \\ 4.0 & 9.0 & 1.0 \\ 3.0 & 10.0 & 1.0 \\ 2.0 & 11.0 & 1.0 \\ 1.0 & 12.0 & 1.0 \end{bmatrix}$$

Vector ***info*** of size 3: (if *info* is present)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

GTTRF and DTTRF—General Tridiagonal Matrix Factorization

GTTRF factors the general tridiagonal matrix ***A***, stored in tridiagonal storage mode, using Gaussian elimination with partial pivoting.

DTTRF factors the diagonally dominant general tridiagonal matrix ***A***, stored in tridiagonal storage mode, using Gaussian elimination.

In these subroutines, ***A*** is a square general tridiagonal matrix.

To solve a tridiagonal system of linear equations with multiple right-hand sides, follow the call to GTTRF or DTTRF with one or more calls to GTTRS or DTTRS, respectively. The output from these factorization subroutines should be used only as input to the solve subroutines GTTRS and DTTRS, respectively.

If the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 133. Data Types

dl, ***d***, ***du***, ***du2***, *af*	***ipiv***	**Subroutine**
Long-precision real	Integer	GTTRF and DTTRF

Syntax

HPF	CALL GTTRF (*dl, d, du, du2, ipiv, af*) CALL GTTRF (*dl, d, du, du2, ipiv, af, info*)
HPF	CALL DTTRF (*dl, d, du, af*) CALL DTTRF (*dl, d, du, af, info*)

On Entry

dl

is the vector ***dl***, containing the subdiagonal of the general tridiagonal matrix ***A*** in elements 2 through size(*dl*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133.

On output, DL is overwritten; that is, the original input is not preserved.

d

is the vector ***d***, containing the main diagonal of the general tridiagonal matrix ***A*** in elements 1 through size(*d*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133.

On output, D is overwritten; that is, the original input is not preserved.

du

is the vector ***du***, containing the superdiagonal of the general tridiagonal matrix ***A*** in elements 1 through size(*du*)−1.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

On output, `DU` is overwritten; that is, the original input is not preserved.

du2
: See On Return.

ipiv
: See On Return.

af
: See On Return.

info
: See On Return.

On Return

dl
: is the updated vector ***dl***, containing part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

On output, `DL` is overwritten; that is, the original input is not preserved.

d
: is the updated vector ***d***, containing part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

On output, `D` is overwritten; that is, the original input is not preserved.

du
: is the updated vector ***du***, containing part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

On output, `DU` is overwritten; that is, the original input is not preserved.

du2
: is the vector ***du2***, containing part of the factorization.

Type: **required** (GTTRF); **not present** (DTTRF)

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

ipiv
: is the vector ***ipiv***, containing the pivot information needed by GTTRS.

Type: **required** (GTTRF); **not present** (DTTRF)

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899.

af

is a work area used by these subroutines and contains part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 133 on page 899, where:

For GTTRF:
  size(*af*) ≥
    number_of_processors() { (12)(number_of_processors())
    + 3 (ceiling(size(*dl*) / number_of_processors())) }

For DTTRF:
  size(*af*) ≥
    number_of_processors() { (12)(number_of_processors())
    + 2 (ceiling(size(*dl*) / number_of_processors())) }

info

is the vector ***info*** of length equal to the number_of_processors(), where, if you are running on the *j*-th process, then $info_j$ has the following meaning, when *info* is **present**:

If $info_j = 0$ for all *j*, the factorization completed normally.

Note: For DTTRF, if the input matrix ***A*** is not diagonally dominant, the subroutine may still complete the factorization; however, results are unpredictable.

If $1 \le info_j \le$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j$–1 and factored locally, is singular or reducible (for GTTRF), or not diagonally dominant (for DTTRF). The magnitude of a pivot element was zero or too small.

If $info_j >$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j$–number_of_processors()–1 representing interactions with other processes, is singular or reducible (for GTTRF), or not diagonally dominant (for DTTRF). The magnitude of a pivot element was zero or too small.

If $info_j > 0$ for any *j*, the factorization is completed; however, if you call GTTRS/DTTRS with these factors, the results are unpredictable.

All elements of *info* will have the same value.

When *info* is **not present** or size(*info*)=0, and matrix ***A*** is singular or reducible (for GTTRF) or not diagonally dominant (for DTTRF), then the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: an assumed-shape array with shape (**:**), containing fullword integers, where $info_j \ge 0$ for $j = 1...$number_of_processors().

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:

   - For GTTRF: size(*dl*) = size(*d*) = size(*du*) = size(*du2*) = size(*ipiv*)
   - For DTTRF: size(*dl*) = size(*d*) = size(*du*)

2. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
3. The output from the factorization subroutines should be used only as input to the solve subroutines GTTRS and DTTRS, respectively.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   | The format of the output from DTTRF has changed. Therefore, the factorization
   | and solve must be performed using Parallel ESSL Version 2 Release 1.2.

   The vectors for ***dl***, ***d***, ***du***, ***du2***, ***ipiv***, and *af* input to GTTRS must be the same as the corresponding output arguments for GTTRF.

   The vectors for ***dl***, ***d***, ***du***, and *af* input to DTTRS must be the same as the corresponding output arguments for DTTRF.
4. For GTTRF, the general tridiagonal matrix ***A*** must be non-singular and irreducible. For DTTRF, the general tridiagonal matrix ***A*** must be diagonally dominant to ensure numerical accuracy because no pivoting is performed. These subroutines use the *info* argument to provide information about ***A***, like ScaLAPACK. However, these subroutines also issue an error message, which differs from ScaLAPACK.
5. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
6. The general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode. For details, see the section on general tridiagonal matrices in "Matrices" on page 40.

   Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 10 on page 109.
7. The restrictions given in "Notes and Coding Rules" on page 542 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in "Error Conditions" on page 543 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument or if size(*info*)=0, your program terminates as a result of the computational error.

Input-Argument Errors for GTTRF

Stage 1

1. The rank of the ultimate align target is not 1 for *dl*, *d*, *du*, *du2*, *ipiv*, or *info* (if *info* is present).
2. The process rank is not 1 for *dl*, *d*, *du*, *du2*, *ipiv*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *dl*, *d*, *du*, *du2*, and *ipiv*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.

   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *dl*, *d*, *du*, *du2*, and *ipiv*.
4. *dl*, *d*, *du*, *du2*, or *ipiv* is not distributed (BLOCK).
5. The vector for *dl*, *d*, *du*, or *du2* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *dl*, *d*, *du*, *du2*, and *ipiv* is incompatible:
   a. size(*dl*) ≠ size(*ipiv*) or
   b. size(*d*) ≠ size(*ipiv*) or
   c. size(*du*) ≠ size(*ipiv*) or
   d. size(*du2*) ≠ size(*ipiv*)
2. The block sizes for *dl*, *d*, *du*, *du2*, and *ipiv* are incompatible.
3. The abstract process indices for *dl*, *d*, *du*, *du2*, and *ipiv* are incompatible.
4. The data distribution for *dl*, *d*, *du*, *du2*, or *ipiv* is unsupported.

Input-Argument Errors for DTTRF

Stage 1

1. The rank of the ultimate align target is not 1 for *dl*, *d*, *du*, or *info* (if *info* is present).
2. The process rank is not 1 for *dl*, *d*, *du*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *dl*, *d*, and *du*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *dl*, *d*, and *du*.
4. *dl*, *d*, or *du* is not distributed (BLOCK).
5. The vector for *dl*, *d*, or *du* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *dl*, *d*, and *du* is incompatible:
   a. size(*dl*) ≠ size(*du*) or
   b. size(*d*) ≠ size(*du*)
2. The block sizes for *dl*, *d*, and *du* are incompatible.
3. The abstract process indices for *dl*, *d*, and *du* are incompatible.
4. The data distribution for *dl*, *d*, or *du* is unsupported.

Example 1: This example shows a factorization of the general tridiagonal matrix ***A***, of order 12, where matrix ***A*** is stored in tridiagonal storage mode:

```
┌                                                                       ┐
| 2.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 1.0  3.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  1.0  3.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  1.0  3.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  1.0  3.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  1.0  3.0  2.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  1.0  3.0  2.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  1.0  3.0  2.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  3.0  2.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  3.0  2.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  3.0  2.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  3.0 |
└                                                                       ┘
```

As in “Example 1” on page 545, array data is block distributed over 3 processes.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from GTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine GTTRS.
2. The contents of vectors *du2* and *af*, output from GTTRF, is not shown. These vectors are passed, unchanged, to the solve subroutine GTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, DU2, IPIV, AF

CALL GTTRF( DL , D , DU , DU2 , IPIV , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, DU2, IPIV, AF
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL GTTRF( DL , D , DU , DU2 , IPIV , AF , INFO )
```

Input: Vector ***dl*** of size 12:

```
┌     ┐
|  .  |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
| 1.0 |
└     ┘
```

Vector ***d*** of size 12:

```
┌     ┐
| 2.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
| 3.0 |
└     ┘
```

Vector ***du*** of size 12:

```
┌     ┐
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
|  .  |
└     ┘
```

Output: Vector ***dl*** of size 12:

```
┌      ┐
|  .   |
| 0.5  |
| 0.5  |
| 0.5  |
| 1.0  |
| 0.33 |
| 0.43 |
| 0.47 |
| 1.0  |
| 1.0  |
| 1.0  |
| 1.0  |
└      ┘
```

Vector ***d*** of size 12:

```
┌      ┐
│ 0.5  │
│ 0.5  │
│ 0.5  │
│ 2.0  │
│ 0.33 │
│ 0.43 │
│ 0.47 │
│ 2.07 │
│ 2.07 │
│ 0.47 │
│ 0.43 │
│ 0.33 │
└      ┘
```

Vector ***du*** of size 12:

```
┌      ┐
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 2.0  │
│ 0.93 │
│ 0.86 │
│ 0.67 │
│  .   │
└      ┘
```

Vector ***ipiv*** of size 12:

```
┌   ┐
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
│ 0 │
└   ┘
```

Vector ***info*** of size 3: (if *info* is present)

```
┌   ┐
│ 0 │
│ 0 │
│ 0 │
└   ┘
```

Example 2: This example shows a factorization of the diagonally dominant general tridiagonal matrix ***A***, of order 12, where matrix ***A*** is stored in tridiagonal storage mode.

Matrix ***A*** and the input and/or output values for *dl*, *d*, *du*, and *info* in this example are the same as shown for “Example 1” on page 903.

As in “Example 2” on page 552, array data is block distributed over 3 processes.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from DTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine DTTRS.
2. The contents of vector *af*, output from DTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine DTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, AF

CALL DTTRF( DL , D , DU , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, AF
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL DTTRF( DL , D , DU , AF , INFO )
```

GTTRS and DTTRS—General Tridiagonal Matrix Solve

GTTRS solves the tridiagonal systems of linear equations, using Gaussian elimination with partial pivoting for the general tridiagonal matrix ***A*** stored in tridiagonal storage mode.

1. $\boldsymbol{AX} = \boldsymbol{B}$

| DTTRS solves one of the following tridiagonal systems of linear equations, using
| Gaussian elimination for the diagonally dominant general tridiagonal matrix ***A*** stored
| in tridiagonal storage mode.

| 1. $\boldsymbol{AX} = \boldsymbol{B}$
| 2. $\boldsymbol{A}^{T}\boldsymbol{X} = \boldsymbol{B}$

In these subroutines:

A is the factored square general tridiagonal matrix.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

These subroutines use the results of the factorization of matrix ***A***, produced by a preceding call to GTTRF or DTTRF, respectively. The output from these factorization subroutines should be used only as input to the solve subroutines GTTRS and DTTRS, respectively.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 134. Data Types

dl, ***d***, ***du***, ***du2***, ***B***, *af*	***ipiv***	**Subroutine**
Long-precision real	Integer	GTTRS and DTTRS

Syntax

HPF	CALL GTTRS (*dl, d, du, du2, ipiv, b, af*) CALL GTTRS (*dl, d, du, du2, ipiv, b, af, transa, info*)
HPF	CALL DTTRS (*dl, d, du, b, af*) CALL DTTRS (*dl, d, du, b, af, transa, info*)

On Entry

dl

is the updated vector ***dl***, containing part of the factorization, produced by a preceding call to GTTRF or DTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134.

d

is the updated vector ***d***, containing part of the factorization, produced by a preceding call to GTTRF or DTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134 on page 908.

du

is the updated vector ***du***, containing part of the factorization, produced by a preceding call to GTTRF or DTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134 on page 908.

du2

is the vector ***du2***, containing part of the factorization, produced by a preceding call to GTTRF.

Type: **required** (GTTRS); **not present** (DTTRS)

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134 on page 908.

ipiv

is the vector ***ipiv***, containing the pivot information produced by a preceding call to GTTRF.

Type: **required** (GTTRS); **not present** (DTTRS)

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134 on page 908.

b

is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 134 on page 908.

af

is a work area used by these subroutines and contains part of the factorization, produced by a preceding call to GTTRF or DTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 134 on page 908, where:

For GTTRS:
size(*af*) ≥
number_of_processors() { (12)(number_of_processors())
+ 3 (ceiling(size(*dl*) / number_of_processors())) }

For DTTRS:
size(*af*) ≥
number_of_processors() { (12)(number_of_processors())
+ 2 (ceiling(size(*dl*) / number_of_processors())) }

transa

indicates matrix ***A*** is used in the computation, resulting in solution 1.

Type: **optional**

Default: *transa* = `'N'`

Specified as: a single character, where:

| - For GTTRS, it must be `'N'`.
| - For DTTRS, it must be `'N'`, `'T'`, or `'C'`.

info
See On Return.

On Return

b
is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (`:`,`:`), containing numbers of the data type indicated in Table 134 on page 908.

info
is the vector ***info*** of length equal to the number_of_processors(). ***info*** is set to zero, indicating that a successful computation occurred on each process.

Type: **optional**

Returned as: an assumed-shape array with shape (`:`), containing fullword integers.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - For GTTRS: size(*dl*) = size(*d*) = size(*du*) = size(*du2*) = size(*ipiv*) = size(*b*,1)
   - For DTTRS: size(*dl*) = size(*d*) = size(*du*) = size(*b*,1)
2. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
3. The subroutine accepts lowercase letters for the *transa* argument.
4. The output from the factorization subroutines GTTRF and DTTRF should be used only as input to the solve subroutines GTTRS and DTTRS, respectively.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   | The format of the output from DTTRF has changed. Therefore, the factorization
   | and solve must be performed using Parallel ESSL Version 2 Release 1.2.

   The vectors for ***dl***, ***d***, ***du***, ***du2***, ***ipiv***, and *af* input to GTTRS must be the same as the corresponding output arguments for GTTRF.

   The vectors for ***dl***, ***d***, ***du***, and *af* input to DTTRS must be the same as the corresponding output arguments for DTTRF.
5. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
6. The general tridiagonal matrix ***A*** must be stored in tridiagonal storage mode. For details, see the section on general tridiagonal matrices in “Matrices” on page 40.

Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 10 on page 109.

7. The restrictions given in "Notes and Coding Rules" on page 561 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in "Error Conditions" on page 562 also apply to this subroutine.

Computational Errors: None

Note: If the factorization performed by GTTRF or DTTRF failed because of a singular or reducible matrix ***A*** (for GTTRF) or not diagonally dominant matrix ***A*** (for DTTRF), the results returned by this subroutine are unpredictable. For details, see the *info* output argument for GTTRF or DTTRF.

Input-Argument Errors for GTTRS

Stage 1

1. The rank of the ultimate align target is not 1 for *dl*, *d*, *du*, *du2*, *ipiv*, *b*, or *info* (if *info* is present).
2. The process rank is not 1 for *dl*, *d*, *du*, *du2*, *ipiv*, *b*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *dl*, *d*, *du*, *du2*, and *ipiv*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *dl*, *d*, *du*, *du2*, and *ipiv*.
4. *b* is not distributed (BLOCK,*).
5. *dl*, *d*, *du*, *du2*, or *ipiv* is not distributed (BLOCK).
6. The vector for *dl*, *d*, *du*, or *du2* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *dl*, *d*, *du*, *du2*, *ipiv*, and *b* is incompatible:
   a. size(*dl*) ≠ size(*b*,1) or
   b. size(*d*) ≠ size(*b*,1) or
   c. size(*du*) ≠ size(*b*,1) or
   d. size(*du2*) ≠ size(*b*,1)
   e. size(*ipiv*) ≠ size(*b*,1)
2. The block sizes for *dl*, *d*, *du*, *du2*, *ipiv*, and *b* are incompatible.
3. The abstract process indices for *dl*, *d*, *du*, *du2*, *ipiv*, and *b* are incompatible.
4. The data distribution for *dl*, *d*, *du*, *du2*, or *ipiv* is unsupported.

Input-Argument Errors for DTTRS

Stage 1

1. The rank of the ultimate align target is not 1 for *dl*, *d*, *du*, *b*, or *info* (if *info* is present).
2. The process rank is not 1 for *dl*, *d*, *du*, *b*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *dl*, *d*, and *du*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *dl*, *d*, and *du*.
4. *b* is not distributed (BLOCK,*).
5. *dl*, *d*, or *du* is not distributed (BLOCK).
6. The vector for *dl*, *d*, or *du* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *dl*, *d*, *du*, and *b* is incompatible:
   a. size(*dl*) ≠ size(*b*,1) or
   b. size(*d*) ≠ size(*b*,1) or
   c. size(*du*) ≠ size(*b*,1)
2. The block sizes for *dl*, *d*, *du*, and *b* are incompatible.
3. The abstract process indices for *dl*, *d*, *du*, and *b* are incompatible.
4. The data distribution for *dl*, *d*, or *du* is unsupported.

Example 1: This example shows how to solve the system ***AX*** = ***B***, where matrix ***A*** is the same matrix factored in "Example 1" on page 903 for GTTRF.

As in "Example 1" on page 565, array data is block distributed over 3 processes.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from GTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine GTTRS.
2. The contents of vectors *du2* and *af*, output from GTTRF, is not shown. These vectors are passed, unchanged, to the solve subroutine GTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, DU2, IPIV, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL GTTRS( DL , D , DU , DU2 , IPIV , B , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, DU2, IPIV, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL GTTRS( DL , D , DU , DU2 , IPIV , B , AF , TRANSA='N' , INFO=INFO )
```

Input: Vector ***dl*** of size 12:

```
┌      ┐
|  .   |
| 0.5  |
| 0.5  |
| 0.5  |
| 1.0  |
| 0.33 |
| 0.43 |
| 0.47 |
| 1.0  |
| 1.0  |
| 1.0  |
| 1.0  |
└      ┘
```

Vector ***d*** of size 12:

```
┌      ┐
| 0.5  |
| 0.5  |
| 0.5  |
| 2.0  |
| 0.33 |
| 0.43 |
| 0.47 |
| 2.07 |
| 2.07 |
| 0.47 |
| 0.43 |
| 0.33 |
└      ┘
```

Vector ***du*** of size 12:

```
┌      ┐
| 2.0  |
| 2.0  |
| 2.0  |
| 2.0  |
| 2.0  |
| 2.0  |
| 2.0  |
| 2.0  |
| 0.93 |
| 0.86 |
| 0.67 |
|  .   |
└      ┘
```

Vector ***ipiv*** of size 12:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Rectangular 12 × 3 matrix ***B***:

$$\begin{bmatrix}
46.0 & 6.0 & 4.0 \\
65.0 & 13.0 & 6.0 \\
59.0 & 19.0 & 6.0 \\
53.0 & 25.0 & 6.0 \\
47.0 & 31.0 & 6.0 \\
41.0 & 37.0 & 6.0 \\
35.0 & 43.0 & 6.0 \\
29.0 & 49.0 & 6.0 \\
23.0 & 55.0 & 6.0 \\
17.0 & 61.0 & 6.0 \\
11.0 & 67.0 & 6.0 \\
5.0 & 47.0 & 4.0
\end{bmatrix}$$

Output: Rectangular 12 × 3 matrix ***B***:

$$\begin{bmatrix}
12.0 & 1.0 & 1.0 \\
11.0 & 2.0 & 1.0 \\
10.0 & 3.0 & 1.0 \\
9.0 & 4.0 & 1.0 \\
8.0 & 5.0 & 1.0 \\
7.0 & 6.0 & 1.0 \\
6.0 & 7.0 & 1.0 \\
5.0 & 8.0 & 1.0 \\
4.0 & 9.0 & 1.0 \\
3.0 & 10.0 & 1.0 \\
2.0 & 11.0 & 1.0 \\
1.0 & 12.0 & 1.0
\end{bmatrix}$$

Vector ***info*** of size 3: (if *info* is present)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Example 2: This example shows how to solve the system ***AX*** = ***B***, where matrix ***A*** is the same matrix factored in “Example 2” on page 907 for DTTRF.

The input and/or output values for *dl*, *d*, *du*, *b*, *transa*, and *info* in this example are the same as shown for “Example 1” on page 912.

As in “Example 2” on page 571, array data is block distributed over 3 processes.

Notes:

1. The vectors ***dl***, ***d***, and ***du***, output from DTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine DTTRS.
2. The contents of vector *af*, output from DTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine DTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL DTTRS( DL , D , DU , B , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: DL, D, DU, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL DTTRS( DL , D , DU , B , AF , TRANSA='N' , INFO=INFO )
```

PTSV—Positive Definite Symmetric Tridiagonal Matrix Factorization and Solve

This subroutine solves the tridiagonal systems of linear equations, ***AX*** = ***B***, where the positive definite symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode. In this description:

A is the positive definite symmetric tridiagonal matrix.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 135. Data Types

d, ***e***, ***B***	**Subroutine**
Long-precision real	PTSV

Syntax

HPF	CALL PTSV (*d, e, b*) CALL PTSV (*d, e, b, info*)

On Entry

d
is the vector ***d***, containing the main diagonal of the positive definite symmetric tridiagonal matrix ***A*** in elements 1 through size(*d*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 135.

On output, `D` is overwritten; that is, the original input is not preserved.

e
is the vector ***e***, containing the off-diagonal of the positive definite symmetric tridiagonal matrix ***A*** in elements 1 through size(*e*)–1.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 135.

On output, `E` is overwritten; that is, the original input is not preserved.

b
is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 135.

info
See On Return.

On Return

d

is overwritten; that is, the original input is not preserved.

e

is overwritten; that is, the original input is not preserved.

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 135 on page 916.

info

is the vector ***info*** of length equal to the number_of_processors(), where, if you are running on the *j*-th process, then $info_j$ has the following meaning, when *info* is **present**:

If $info_j = 0$ for all *j*, matrix ***A*** is positive definite, and the factorization completed normally.

If $1 \leq info_j \leq$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j-1$ and factored locally, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info_j >$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j-$number_of_processors()-1 representing interactions with other processes, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info_j > 0$ for any *j*, the results of the computation are unpredictable.

All elements of *info* will have the same value.

When *info* is **not present** or size(*info*)=0, and matrix ***A*** is not positive definite, then the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: an assumed-shape array with shape (**:**), containing fullword integers, where $info_j \geq 0$ for $j = 1$...number_of_processors().

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is: size(*d*) = size(*e*) = size(*b*,1).
2. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
3. The symmetric tridiagonal matrix ***A*** must be positive definite. This subroutine uses the *info* argument to provide information about ***A***, like ScaLAPACK. However, this subroutine also issues an error message, which differs from ScaLAPACK.
4. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105

5. The positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode. For details, see the section on tridiagonal matrices in “Matrices” on page 40.

   Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 10 on page 109.
6. The restrictions given in “Notes and Coding Rules” on page 580 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in “Error Conditions” on page 581 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument or if size(*info*)=0, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *d*, *e*, *b*, or *info* (if *info* is present).
2. The process rank is not 1 for *d*, *e*, *b*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *d* and *e*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *d* and *e*.
4. *b* is not distributed (BLOCK,*).
5. *d* or *e* is not distributed (BLOCK).
6. The vector for *d* or *e* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *d*, *e*, and *b* is incompatible:
   a. size(d) $\neq$ size(b,1) or
   b. size(e) $\neq$ size(b,1)
2. The block sizes for *d*, *e*, and *b* are incompatible.
3. The abstract process indices for *d*, *e*, and *b* are incompatible.
4. The data distribution for *d* or *e* is unsupported.

Example 1: This example shows a factorization of the positive definite symmetric tridiagonal matrix ***A***, of order 12, where matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode:

$$\begin{bmatrix}
4.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0 & 2.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 2.0 & 5.0
\end{bmatrix}$$

As in “Example” on page 584, array data is block distributed over 3 processes.

Note: On output, vectors ***d*** and ***e*** are overwritten by this subroutine.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL PTSV( D , E , B )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL PTSV( D , E , B , INFO )
```

Input: Vector ***d*** of size 12:

$$\begin{bmatrix} 4.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 & 5.0 \end{bmatrix}$$

Vector ***e*** of size 12:

$$\begin{bmatrix} 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & 2.0 & . \end{bmatrix}$$

Rectangular 12 × 3 matrix ***B***:

$$\begin{bmatrix} 70.0 & 8.0 & 6.0 \\ 99.0 & 18.0 & 9.0 \\ 90.0 & 27.0 & 9.0 \\ 81.0 & 36.0 & 9.0 \\ 72.0 & 45.0 & 9.0 \\ 63.0 & 54.0 & 9.0 \\ 54.0 & 63.0 & 9.0 \\ 45.0 & 72.0 & 9.0 \\ 36.0 & 81.0 & 9.0 \\ 27.0 & 90.0 & 9.0 \\ 18.0 & 99.0 & 9.0 \\ 9.0 & 82.0 & 7.0 \end{bmatrix}$$

Output: Rectangular 12 $\times$ 3 matrix ***B***:

$$\begin{bmatrix} 12.0 & 1.0 & 1.0 \\ 11.0 & 2.0 & 1.0 \\ 10.0 & 3.0 & 1.0 \\ 9.0 & 4.0 & 1.0 \\ 8.0 & 5.0 & 1.0 \\ 7.0 & 6.0 & 1.0 \\ 6.0 & 7.0 & 1.0 \\ 5.0 & 8.0 & 1.0 \\ 4.0 & 9.0 & 1.0 \\ 3.0 & 10.0 & 1.0 \\ 2.0 & 11.0 & 1.0 \\ 1.0 & 12.0 & 1.0 \end{bmatrix}$$

Vector ***info*** of size 3: (if *info* is present)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

PTTRF—Positive Definite Symmetric Tridiagonal Matrix Factorization

This subroutine factors the positive definite symmetric tridiagonal matrix ***A***, stored in parallel-symmetric-tridiagonal storage mode.

To solve a tridiagonal system of linear equations with multiple right-hand sides, follow the call to PTTRF with one or more calls to PTTRS, respectively. The output from these factorization subroutines should be used only as input to the solve subroutines PTTRS, respectively.

If the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 136. Data Types

d, ***e***, *af*	**Subroutine**
Long-precision real	PTTRF

Syntax

HPF	CALL PTTRF (*d, e, af*)
	CALL PTTRF (*d, e, af, info*)

On Entry

d

is the vector ***d***, containing the main diagonal of the tridiagonal matrix ***A*** in elements 1 through size(*d*).

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 136.

On output, D is overwritten; that is, the original input is not preserved.

e

is the vector ***e***, containing the off-diagonal of the positive definite symmetric tridiagonal matrix ***A*** in elements 1 through size(*e*)−1.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 136.

On output, E is overwritten; that is, the original input is not preserved.

af

See On Return.

info

See On Return.

On Return

d

is the updated vector ***d***, containing part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 136.

On output, D is overwritten; that is, the original input is not preserved.

e

is the updated vector ***e***, containing part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 136 on page 921.

On output, E is overwritten; that is, the original input is not preserved.

af

is a work area used by these subroutines and contains part of the factorization.

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 136 on page 921, where:

size(*af*) ≥
number_of_processors() { (12)(number_of_processors())
+ 3 (ceiling(size(*e*) / number_of_processors())) }

info

is the vector ***info*** of length equal to the number_of_processors(), where, if you are running on the *j*-th process, then $info_j$ has the following meaning, when *info* is **present**:

If $info_j = 0$ for all *j*, matrix ***A*** is positive definite, and the factorization completed normally.

If $1 \leq info_j \leq$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j-1$ and factored locally, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info_j >$ number_of_processors() for any *j*, the portion of global submatrix ***A*** stored on process $info_j$–number_of_processors()–1 representing interactions with other processes, is not positive definite. A pivot element whose value is less than or equal to a small positive number was detected.

If $info_j > 0$ for any *j*, the factorization is completed; however, if you call PTTRS with these factors, the results are unpredictable.

All elements of *info* will have the same value.

When *info* is not present or size(*info*) = 0 and matrix ***A*** is not positive definite,then the information for the above computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: an assumed-shape array with shape (**:**), containing fullword integers, where $info_j \geq 0$ for $j = 1$...number_of_processors().

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:

   size(*d*) = size(*e*)

2. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
3. The output from this factorization subroutine should be used only as input to the solve subroutine PTTRS.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   The vectors for ***d***, ***e***, and *af* input to PTTRS must be the same as the corresponding output arguments for PTTRF.
4. The symmetric tridiagonal matrix ***A*** must be positive definite. This subroutine uses the *info* argument to provide information about ***A***, like ScaLAPACK. However, these subroutines also issue an error message, which differs from ScaLAPACK.
5. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
6. The positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode. For details, see the section on tridiagonal matrices in “Matrices” on page 40.

   Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 10 on page 109.
7. The restrictions given in “Notes and Coding Rules” on page 593 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. All errors listed in “Error Conditions” on page 595 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument or if size(*info*)=0, your program terminates as a result of the computational error.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *d*, *e*, or *info* (if *info* is present).
2. The process rank is not 1 for *d*, *e*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *d* and *e*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *d* and *e*.
4. *d* or *e* is not distributed (BLOCK).
5. The vector for *d* or *e* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *d* and *e* is incompatible:
   size(d) ≠ size(e)
2. The block sizes for *d* and *e* are incompatible.
3. The abstract process indices for *d* and *e* are incompatible.
4. The data distribution for *d* or *e* is unsupported.

Example 1: This example shows a factorization of the positive definite symmetric tridiagonal matrix ***A***, of order 12, where matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode:

```
┌                                                               ┐
| 4.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 2.0  5.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  2.0  5.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  2.0  5.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  2.0  5.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  2.0  5.0  2.0  0.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  2.0  5.0  2.0  0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  2.0  5.0  2.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  5.0  2.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  5.0  2.0  0.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  5.0  2.0 |
| 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  5.0 |
└                                                               ┘
```

As in “Example” on page 596, array data is block distributed over 3 processes.

Notes:

1. The vectors, ***d*** and ***e***, output from PTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PTTRS.
2. The contents of vector *af*, output from PTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine PTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E, AF

CALL PTTRF( D , E , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E, AF
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL PTTRF( D , E , AF , INFO )
```

Input: Vector ***d*** of size 12:

```
┌     ┐
| 4.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
| 5.0 |
└     ┘
```

Vector ***e*** of size 12:

```
┌     ┐
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
| 2.0 |
|  .  |
└     ┘
```

Output: Vector ***d*** of size 12:

```
┌       ┐
|   .25 |
|   .25 |
|   .25 |
| 4.0   |
|   .20 |
|   .24 |
|   .25 |
| 4.01  |
| 4.01  |
|   .25 |
|   .24 |
|   .20 |
└       ┘
```

Vector ***e*** of size 12:

```
┌        ┐
| 2.0    |
| 2.0    |
| 2.0    |
| 2.0    |
| 2.0    |
| 2.0    |
| 2.0    |
| 2.0    |
|   .49  |
|   .48  |
|   .40  |
|   .    |
└        ┘
```

Vector ***info*** of size 3: (if *info* is present)

```
┌   ┐
| 0 |
| 0 |
| 0 |
└   ┘
```

PTTRS—Positive Definite Symmetric Tridiagonal Matrix Solve

This subroutine solves the following systems of equations for multiple right-hand sides:

1. ***AX*** = ***B***

PTTRS solves the tridiagonal systems of linear equations, where the positive definite symmetric tridiagonal matrix ***A*** is stored in parallel-symmetric-tridiagonal storage mode, where:

A is the factored positive definite symmetric tridiagonal matrix.
B is the general matrix containing the right-hand sides in its columns.
X represents the general matrix ***B***, containing the output solution vectors in its columns.

This subroutine uses the results of the factorization of matrix ***A***, produced by a preceding call to PTTRF. The output from these factorization subroutines should be used only as input to the solve subroutines PTTRS.

If any of the assumed-shape arrays have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [51], [16], [18], [22], [36], and [37].

Table 137. Data Types

d, ***e***, ***B***, *af*	**Subroutine**
Long-precision real	PTTRS

Syntax

HPF	CALL PTTRS (*d, e, b, af*) CALL PTTRS (*d, e, b, af, info*)

On Entry

d
is the updated vector ***d***, containing part of the factorization, produced by a preceding call to PTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 137.

e
is the updated vector ***e***, containing part of the factorization, produced by a preceding call to PTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 137.

b
is the general matrix ***B***, containing the multiple right-hand sides of the system.

Type: **required**

Specified as: an assumed-shape array with shape (:,:), containing numbers of the data type indicated in Table 137.

af

is a work area used by these subroutines and contains part of the factorization, produced by a preceding call to PTTRF.

Type: **required**

Specified as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 137 on page 927, where:

size(*af*) ≥
number_of_processors() { (12)(number_of_processors())
+ 3 (ceiling(size(*e*) / number_of_processors())) }

info

See On Return.

On Return

b

is the updated matrix ***B***, containing the solution vectors.

Type: **required**

Returned as: an assumed-shape array with shape (:,:), containing numbers of the data type indicated in Table 137 on page 927.

info

is the vector ***info*** of length equal to the number_of_processors(). ***info*** is set to zero, indicating that a successful computation occurred on each process.

Type: **optional**

Returned as: an assumed-shape array with shape (:), containing fullword integers.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:

   size(*d*) = size(*e*) = size(*b*,1)

2. The assumed-shape arrays must have no common elements; otherwise results are unpredictable.
3. The output from the factorization subroutine PTTRF should be used only as input to the solve subroutine PTTRS.

   The factored matrix ***A*** is stored in an internal format that depends on the number of processes.

   The vectors for ***d***, ***e***, and *af* input to PTTRS must be the same as the corresponding output arguments for PTTRF.
4. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
5. The positive definite symmetric tridiagonal matrix ***A*** must be stored in parallel-symmetric-tridiagonal storage mode. For details, see the section on tridiagonal matrices in "Matrices" on page 40.

   Block data distribution is required for all array data, except the array for *info*, which requires cyclic data distribution. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 10 on page 109.

6. The restrictions given in “Notes and Coding Rules” on page 609 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 610 also apply to this subroutine.

Computational Errors: None

Note: If the factorization performed by PTTRF failed because of a nonpositive definite matrix ***A***, the results returned by this subroutine are unpredictable. For details, see the *info* output argument for PTTRF.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is not 1 for *d*, *e*, *b*, or *info* (if *info* is present).
2. The process rank is not 1 for *d*, *e*, *b*, or *info* (if *info* is present).

Stage 2

1. The data distribution is inconsistent for *d* and *e*.
2. *info* is present and:
   a. The data distribution is unsupported for *info*.
   b. *info* is not distributed (CYCLIC).
   c. The vector for *info* is replicated.
3. The process grid is not the same for *d* and *e*.
4. *b* is not distributed (BLOCK,*).
5. *d* or *e* is not distributed (BLOCK).
6. The vector for *d* or *e* is replicated.

Stage 3

1. The shape of the assumed-shape arrays for *d*, *e*, and *b* is incompatible:
   a. size(*d*) ≠ size(*b*,1) or
   b. size(*e*) ≠ size(*b*,1)
2. The block sizes for *d*, *e*, and *b* are incompatible.
3. The abstract process indices for *d*, *e*, and *b* are incompatible.
4. The data distribution for *d* or *e* is unsupported.

Example 1: This example shows how to solve the system ***AX*** = ***B***, where matrix ***A*** is the same matrix factored in “Example 1” on page 924 for PTTRF.

As in “Example” on page 613, array data is block distributed over 3 processes.

Notes:

1. The vectors ***d*** and ***e*** output from PTTRF, are stored in an internal format that depends on the number of processes. These vectors are passed, unchanged, to the solve subroutine PTTRS.
2. The contents of vector *af* output from PTTRF, is not shown. This vector is passed, unchanged, to the solve subroutine PTTRS.

```
!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B

CALL PTTRS( D , E , B , AF )

      -or-

!HPF$ PROCESSORS PROC(3)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: D, E, AF
!HPF$ DISTRIBUTE (BLOCK,*) ONTO PROC :: B
!HPF$ DISTRIBUTE (CYCLIC) ONTO PROC :: INFO

CALL PTTRS( D , E , B , AF , INFO=INFO )
```

Input: Vector ***d*** of size 12:

```
 ┌       ┐
 |  .25  |
 |  .25  |
 |  .25  |
 | 4.0   |
 |  .20  |
 |  .24  |
 |  .25  |
 | 4.01  |
 | 4.01  |
 |  .25  |
 |  .24  |
 |  .20  |
 └       ┘
```

Vector ***e*** of size 12:

```
 ┌       ┐
 | 2.0   |
 | 2.0   |
 | 2.0   |
 | 2.0   |
 | 2.0   |
 | 2.0   |
 | 2.0   |
 | 2.0   |
 |  .49  |
 |  .48  |
 |  .40  |
 |   .   |
 └       ┘
```

Rectangular 12 × 3 matrix ***B***:

```
┌                   ┐
│ 70.0    8.0   6.0 │
│ 99.0   18.0   9.0 │
│ 90.0   27.0   9.0 │
│ 81.0   36.0   9.0 │
│ 72.0   45.0   9.0 │
│ 63.0   54.0   9.0 │
│ 54.0   63.0   9.0 │
│ 45.0   72.0   9.0 │
│ 36.0   81.0   9.0 │
│ 27.0   90.0   9.0 │
│ 18.0   99.0   9.0 │
│  5.0   82.0   7.0 │
└                   ┘
```

Output: Rectangular 12 × 3 matrix ***B***:

```
┌                   ┐
│ 12.0    1.0   1.0 │
│ 11.0    2.0   1.0 │
│ 10.0    3.0   1.0 │
│  9.0    4.0   1.0 │
│  8.0    5.0   1.0 │
│  7.0    6.0   1.0 │
│  6.0    7.0   1.0 │
│  5.0    8.0   1.0 │
│  4.0    9.0   1.0 │
│  3.0   10.0   1.0 │
│  2.0   11.0   1.0 │
│  1.0   12.0   1.0 │
└                   ┘
```

Vector ***info*** of size 3: (if *info* is present)

```
┌   ┐
│ 0 │
│ 0 │
│ 0 │
└   ┘
```

Chapter 15. Eigensystem Analysis and Singular Value Analysis (HPF)

This chapter describes the eigensystem analysis and singular value analysis subroutines that can be called from an HPF program.

Overview of the Eigensystem Analysis and Singular Value Analysis Subroutines

The eigensystem analysis and singular value analysis subroutines include a subset of the ScaLAPACK subroutines.

Note: These subroutines are designed to be consistent with the proposals for the Fortran 90 BLAS and the Fortran 90 LAPACK. (See references [30] and [31].) If these subroutines do not comply with any eventual proposal for HPF interfaces to the PBLAS and ScaLAPACK, IBM will consider updating them to do so. If IBM updates these subroutines, the update could require modifications of the calling application program.

Table 138. List of Eigensystem Analysis and Singular Value Analysis Subroutines (HPF)

Descriptive Name	Long-Precision Subroutine	Page
Selected Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix	SYEVX	935
Reduce a Real Symmetric Matrix to Tridiagonal Form	SYTRD	946
Reduce a General Matrix to Upper Hessenberg Form	GEHRD	952
Reduce a General Matrix to Bidiagonal Form	GEBRD	957

Eigensystem Analysis and Singular Value Analysis Subroutines

This section contains the eigensystem analysis subroutine descriptions.

SYEVX—Selected Eigenvalues and, Optionally, the Eigenvectors of a Real Symmetric Matrix

This subroutine computes selected eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A***. Eigenvalues and eigenvectors can be selected by specifying a range of values or a range of indices for the eigenvalues.

The eigenvalue computation performed by this subroutine depends on which of the *vl*, *vu*, *il*, and *iu* arguments you specify:

Case 1:	If *vl*, *vu*, *il*, *iu* are all not present, all eigenvalues are found.
Case 2:	If *vl* and *vu* are present and *il* and *iu* not present, then all eigenvalues in the interval [*vl*,*vu*] are found.
Case 3:	If *il* or *iu*, or both, are present and *vl* and *vu* are not present, then the *il*-th through *iu*-th eigenvalues are found.

Any other combination of *vl*, *vu*, *il*, *iu* being present or not present is considered an input-argument error.

If the assumed-shape arrays for ***A*** and ***w*** have a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13], [21], [31], [41], [24], [25], and [26].

Table 139. Data Types

A, *vl*, *vu*, *abstol*, *orfac*, ***Z***, ***w***, ***gap***	***ifail***, ***iclustr***	**Subroutine**
Long-precision real	Integer	SYEVX

Syntax

HPF	**Case 1**	CALL SYEVX (*a*, *w*, *uplo*) CALL SYEVX (*a*, *w*, *uplo*, *abstol=*, *m=*, *nz=*, *orfac=*, *z=*, *ifail=*, *iclustr=*, *gap=*, *iclustrsz=*, *info=*)
HPF	**Case 2**	CALL SYEVX (*a*, *w*, *uplo*, *vl=*, *vu=*) CALL SYEVX (*a*, *w*, *uplo*, *vl=*, *vu=*, *abstol=*, *m=*, *nz=*, *orfac=*, *z=*, *ifail=*, *iclustr=*, *gap=*, *iclustrsz=*, *info=*)
HPF	**Case 3**	CALL SYEVX (*a*, *w*, *uplo*, *il=*, *iu=*) CALL SYEVX (*a*, *w*, *uplo*, *il=*, *iu=*, *abstol=*, *m=*, *nz=*, *orfac=*, *z=*, *ifail=*, *iclustr=*, *gap=*, *iclustrsz=*, *info=*)
Note: Specify the indicated arguments as keywords only.		

On Entry

a

is the symmetric matrix ***A***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (`:,:`), containing numbers of the data type indicated in Table 139 on page 935, where size(*a*,1) = size(*a*,2).

w

See On Return.

uplo

indicates whether the upper or lower triangular part of the symmetric matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

vl

has the following meaning:

If *vl* and *vu* are present and *il* and *iu* are not present, then *vl* is the lower bound of the interval to be searched for eigenvalues.

Type: **optional**

Default: none

Specified as: a number of the data type indicated in Table 139 on page 935; $vl < vu$.

vu

has the following meaning:

If *vl* and *vu* are present and *il* and *iu* are not present, then *vu* is the upper bound of the interval to be searched for eigenvalues.

Type: **optional**

Default: none

Specified as: a number of the data type indicated in Table 139 on page 935; $vl < vu$.

il

has the following meaning:

If *il* is present and *vl* and *vu* are not present, then *il* is the index (from smallest to largest) of the smallest eigenvalue to be returned.

Type: **optional**

Default: $il = 1$

Specified as: a fullword integer; $il \geq 1$.

iu

has the following meaning:

If *iu* is present and *vl* and *vu* are not present, then *iu* is the index (from smallest to largest) of the largest eigenvalue to be returned.

Type: **optional**

Default: *iu* = size(*a*,1)

Specified as: a fullword integer; min(*il*, size(*a*,1)) ≤ *iu* ≤ size(*a*,1).

abstol

is the absolute tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [*a*,*b*] of width less than or equal to:

abstol+ε(max(|*a*|, |*b*|))

where ε is the machine precision. If *abstol* is less than or equal to zero, then ε(norm(***T***)) is used in its place, where norm(***T***) is the 1-norm of the tridiagonal matrix obtained by reducing ***A*** to tridiagonal form. For most problems, this is the appropriate level of accuracy to request.

For certain strongly graded matrices, greater accuracy can be obtained in very small eigenvalues by setting *abstol* to a very small positive number. However, if *abstol* is less than:

$$\sqrt{unfl}$$

where *unfl* is the underflow threshold, then:

$$\sqrt{unfl}$$

is used in its place.

Eigenvalues are computed most accurately when *abstol* is set to twice the underflow threshold—that is, (2)(*unfl*).

If *z* is present, then setting *abstol* to *unfl*, the underflow threshold, yields the most orthogonal eigenvectors.

Note:

- ε is approximately $0.222044604925031308\text{E}-15$
- $unfl$ is approximately $0.222507385850720138\text{E}-307$
- $\sqrt{unfl}$ is approximately $0.149166814624004135\text{E}-153$

Type: **optional**

Default: *abstol* = ε(norm(***T***))

Specified as: a number of the data type indicated in Table 139 on page 935.

m

See On Return.

nz

See On Return.

orfac

specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within:

ortol = (*orfac*)(norm(***A***))

of each other (where norm(***A***) is the 1-norm of ***A***) are to be reorthogonalized.

However, if the workspace is insufficient (see *iclustrsz*), *ortol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process.

If *orfac* is zero, no reorthogonalization is done.

If *orfac* is less than zero, a default value of 10^{-3} is used.

Type: **optional**

Default: *orfac* = 10^{-3}

Specified as: a number of the data type indicated in Table 139 on page 935.

z
See On Return.

ifail
See On Return.

iclustr
See On Return.

gap
See On Return.

iclustrsz
is the variable used to calculate how much additional workspace to use for the eigenvector computation.

The computed eigenvectors may not be orthogonal if the minimum workspace is used and *ortol* is too small; therefore, if you want to guarantee orthogonality (at the cost of potentially compromising performance), set *iclustrsz*, then the following **additional** workspace is used to compute the eigenvectors:

$(iclustrsz-1)(\text{size}(a,1))$

where *iclustrsz* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:

$\{w_k, ..., w_{k+iclustrsz-1} \mid w_{j+1} \leq w_j + orfac(2)(\text{norm}(\boldsymbol{A}))\}$

If the workspace is too small to guarantee orthogonality, this subroutine attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.

Relationship between workspace, orthogonality, and performance:

- If *iclustrsz* is:

$$iclustrsz \geq \text{size}(a,1) / \sqrt{number\_of\_processors()}$$

  then providing enough space to compute all the eigenvectors orthogonally causes serious degradation in performance. In the limit ($iclustrsz = \text{size}(a,1)-1$), performance may be no better than using one process.

- If *iclustrsz* is:

$$iclustrsz = \text{size}(a,1) / \sqrt{number\_of\_processors()}$$

  then reorthogonalizing all eigenvectors increases the total execution time by a factor of 2 or more.

- If *iclustrsz* is:

$$iclustrsz > \text{size}(a,1) / \sqrt{number\_of\_processors()}$$

  then execution time grows as the square of the cluster size, assuming all other factors remain equal and there is enough workspace. Less workspace means less reorthogonalization, but faster execution.

Type: **optional**

Default: *iclustrsz* = 1

Specified as: a fullword integer; *iclustrsz* > 0.

info

See On Return.

On Return

a

is the symmetric matrix ***A***, where:

If *uplo* = `'U'`, the upper triangle and diagonal of the symmetric matrix ***A*** are overwritten; that is, the original input is not preserved.

If *uplo* = `'L'`, the lower triangle and diagonal of the symmetric matrix ***A*** are overwritten; that is, the original input is not preserved.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 139 on page 935.

w

On normal exit (see *info*), it is the vector ***w***, containing the selected eigenvalues in ascending order in the first *m* elements of ***w***.

A copy of ***w*** is aligned with every element of ***A***:

```
!HPF$ ALIGN W(*) WITH A(*,*)
```

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 139 on page 935, where size(*w*) = size(*a*,1).

m

is the number of eigenvalues found.

Type: **optional**

Returned as: a fullword integer; $0 \le m \le$ size(*a*,1).

nz

has the following meaning:

If *z* is not present, then *nz* is ignored.

If *z* is present, then *nz* is the number of eigenvectors computed—that is, the number of columns of ***Z*** used in the computation. On output, *nz* = *m*, unless there is insufficient space and this subroutine is not able to detect it before starting the computation.

Note: This subroutine is able to detect insufficient space without computation, unless *vl* and *vu* are present and eigenvalues are being selected by specifying a range of values.

To get all the eigenvectors requested, you must supply sufficient space to hold the eigenvectors in ***Z*** and sufficient workspace to compute them must be available.

Type: **optional**

Returned as: a fullword integer; $0 \le nz \le m$.

z

is the general matrix ***Z***. On normal exit (see *info*), the first *m* columns of the matrix ***Z*** contain the orthonormal eigenvectors of the symmetric matrix ***A***, corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of the general matrix ***Z*** contains the last approximation to the eigenvector, and the index of the eigenvector is returned in ***ifail***, if *ifail* is present.

Type: **optional**

Returned as: an assumed-shape array with shape (:,:), containing numbers of the data type indicated in Table 139 on page 935, where size(*z*,1) = size(*a*,1) and size(*z*,2) = size(*a*,2).

ifail

has the following meaning:

If *z* is not present, then *ifail* is ignored.

If *z* is present, it is vector ***ifail***, where:

- If there is a normal exit (see *info*), the first *m* elements of ***ifail*** are zero.
- If there is an error exit (where one or more eigenvectors failed to converge—see *info*), ***ifail*** contains the indices of the eigenvectors that failed to converge.

A copy of ***ifail*** is aligned with every element of ***A***:

```
!HPF$ ALIGN IFAIL(*) WITH A(*,*)
```

Type: **optional**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 139 on page 935; $0 \le ifail_i \le$ size(*a*,1), where size(*ifail*) = size(*a*,1).

iclustr

has the following meaning:

If *z* is not present, then *iclustr* is ignored.

If *z* is present, it is vector ***iclustr***, containing the indices of the eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace. Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr_{2i-1}$ to $iclustr_{2i}$ could not be reorthogonalized due to lack of workspace. **Hence, the eigenvectors corresponding to these clusters may not be orthogonal.**

iclustr is a zero-terminated vector; that is, the last element of ***iclustr*** is set to zero. Assuming that *k* is the number of clusters, then:

$iclustr_{2k} \ne 0$ and $iclustr_{2k+1} = 0$

A copy of ***iclustr*** is aligned with every element of ***A***:

```
!HPF$ ALIGN ICLUSTR(*) WITH A(*,*)
```

Type: **optional**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 139 on page 935; $0 \leq iclustr_i \leq$ size(a,1), where size(*iclustr*) = (2)(number_of_processors()).

gap

has the following meaning:

If *z* is not present, then *gap* is ignored.

If *z* is present, it is vector ***gap***, containing the gap between the eigenvalues whose eigenvectors could not be reorthogonalized. The values in this vector correspond to the clusters indicated by ***iclustr***. As a result, the dot product between the eigenvectors corresponding to the *i*-th cluster may be as high as ((C)(size(a,1)))/gap_i, where C is a small constant.

A copy of ***gap*** is aligned with every element of ***A***:

```
!HPF$ ALIGN GAP(*) WITH A(*,*)
```

Type: **optional**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 139 on page 935, where size(*gap*) = (number_of_processors()).

info

has the following meaning, when *info* is **present**:

If *info* = 0, then no input-argument errors or computational errors occurred. This indicates a normal exit.

Note: One use of *info* in ScaLAPACK is to identify whether input-argument errors occurred. Because Parallel ESSL terminates the application if input-argument errors occur, the setting of *info* is irrelevant for these errors.

If *info* > 0, then one or more of the following computational errors occurred and the appropriate error messages were issued, indicating an error exit, where:

- If mod(*info*, 2) ≠ 0, then one or more eigenvectors failed to converge. Their indices are stored in ***ifail***, if *ifail* is present. (Ensure that *abstol* = (2)(*unfl*).)
- If mod(*info*/2, 2) ≠ 0, then the eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in ***iclustr***, if *iclustr* is present.
- If mod(*info*/4, 2) ≠ 0, then all the eigenvectors between *vl* and *vu* could not be computed because of insufficient space. The number of eigenvectors computed is returned in *nz*, if *nz* is present.
- If mod(*info*/8, 2) ≠ 0, then one of more eigenvalues were not computed. (Ensure that *abstol* = (2)(*unfl*).)

When *info* is **not present** and a computational error occurs, the information for the computational error is issued in an error message, and your program is terminated.

Type: **optional**

Returned as: a fullword integer; *info* ≥ 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - size(*a*,1) = size(*a*,2) = size(*w*)
   - If *z* is present:
     – size(*z*,1) = size(*a*,1)
     – size(*z*,2) = size(*a*,2)
   - If *ifail* and *z* are present, size(*ifail*) = size(*a*,1)
   - If *iclustr* and *z* are present, size(*iclustr*) = (2)(number_of_processors())
   - If *gap* and *z* are present, size(*gap*) = (number_of_processors())
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
4. Eigenvectors associated with tightly clustered eigenvalues may not be orthogonal.
5. Eigenvectors that are on different processes are not reorthogonalized. For details, see the argument description for *iclustrsz*.
6. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
7. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrices, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 9 on page 108.
8. The restrictions given in "Notes and Coding Rules" on page 703 also apply to this subroutine.
9. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See "Program Main (HPF)" on page 1036.

Function: This subroutine computes selected eigenvalues and, optionally, the eigenvectors of a real symmetric matrix ***A***. Eigenvalues and eigenvectors can be selected by specifying a range of values or a range of indices for the eigenvalues. The computation involves the following steps:

1. Reduce the real symmetric matrix ***A*** to real symmetric tridiagonal form.
2. Compute the requested eigenvalues of the real symmetric tridiagonal matrix using bisection.
3. If requested, compute the eigenvectors of the real symmetric tridiagonal matrix using inverse iteration, and then back transform the eigenvectors to obtain the eigenvectors of the real symmetric matrix ***A***.

Error Conditions: HPF-specific errors are listed below. All errors listed in “Error Conditions” on page 705 also apply to this subroutine; however, for computational errors, if you do **not** specify the optional *info* argument, your program terminates as a result of the computational error.

Note: If a computational error occurs, information is stored in *ifail*, *iclustr*, and *nz* only if these arguments are present.

Input-Argument Errors

Stage 1

1. It is not possible to determine the type of eigenvalue computation to perform—that is, one of the following valid combinations of *vl*, *vu*, *il*, and *iu* did **not** occur:
   - *vl*, *vu*, *il*, *iu* are all not present (all eigenvalues are found).
   - *vl* and *vu* are present and *il* and *iu* not present (all eigenvalues in the interval [*vl*,*vu*] are found).
   - *il* or *iu*, or both, are present and *vl* and *vu* are not present (the *il*-th through *iu*-th eigenvalues are found).
2. *iclustrsz* is present and *iclustrsz* < 1.

Stage 2

1. The rank of the ultimate align target is greater than 2 for *a*.
2. The process rank is not 1 or 2 for *a*.
3. *z* is present and:
   a. The rank of the ultimate align target is greater than 2 for *z*.
   b. The process rank is not 1 or 2 for *z*.
   c. The process rank is not the same for *a* and *z*.

Stage 3: *z* is present, and the process grid is not the same for *a* and *z*.

Stage 4

1. *z* is present, and the data distribution is inconsistent for *a* and *z*.
2. *z* is not present, and the data distribution is unsupported for *a*.

Stage 5: *w* is not replicated and collapsed.

Stage 6

1. *z* is present, and the shape of the assumed-shape arrays *a*, *z*, and *w* is incompatible:
   a. size(*a*,1) ≠ size(*a*,2) or
   b. size(*a*,1) ≠ size(*w*) or
   c. size(*a*,1) ≠ size(*z*,1) or
   d. size(*z*,2) ≠ size(*a*,2)
2. *z* is not present, and the shape of the assumed-shape arrays *a* and *w* is incompatible:
   a. size(*a*,1) ≠ size(*a*,2) or
   b. size(*a*,1) ≠ size(*w*)
3. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)

Stage 7: *z* and the optional arrays, indicated below, are present, and:

1. *ifail* is not replicated and collapsed.
2. The shape of the assumed-shape arrays for *a* and *ifail* is incompatible: size(*a*,1) ≠ size(*ifail*).
3. *iclustr* is not replicated and collapsed.
4. The shape of the assumed-shape array for *iclustr* is invalid: size(*iclustr*,1) ≠ (2)(number_of_processors()).
5. *gap* is not replicated and collapsed.
6. The shape of the assumed-shape array for *gap* is invalid: size(*gap*,1) ≠ (number_of_processors()).

Stage 8

1. The data distribution for *a* is unsupported.
2. *z* is present, and the data distribution for *z* is unsupported.

Example: This example shows how to find all the eigenvalues and eigenvectors of a real symmetric matrix ***A*** of order 4. As in "Example" on page 708, array data for ***A*** and ***Z*** is block-cyclically distributed using a 2 × 2 process grid, with a copy of vectors ***w***, ***ifail***, ***iclustr***, and ***gap*** each being aligned with every element of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN W(*) WITH A(*,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, Z

CALL SYEVX( A , W , 'U' , Z=Z )

      -or-

!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN W(*) WITH A(*,*)
!HPF$ ALIGN IFAIL(*) WITH A(*,*)
!HPF$ ALIGN ICLUSTR(*) WITH A(*,*)
!HPF$ ALIGN GAP(*) WITH A(*,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A, Z

 CALL SYEVX( A, W, 'U', ABSTOL=-1.0D0, M=M, NZ=NZ, ORFAC=-1.0D0, Z=Z,
           IFAIL=IFAIL, ICLUSTR=ICLUSTR, GAP=GAP, ICLUSTRSZ=1, INFO=INFO)
```

Input: Symmetric matrix ***A*** of order 4:

$$\begin{bmatrix} 5.0 & 4.0 & 1.0 & 1.0 \\ . & 5.0 & 1.0 & 1.0 \\ . & . & 4.0 & 2.0 \\ . & . & . & 4.0 \end{bmatrix}$$

Output: The upper triangle, including the diagonal, of the symmetric matrix ***A*** is overwritten; that is, the original input is not preserved.

m = 4 (if *m* is present)

nz = 4 (if *nz* is present)

Vector ***w*** of length 4:

w = (1.00, 2.00, 5.00, 10.00)

General matrix ***Z*** of order 4:

$$\begin{bmatrix} 0.7071 & 0.0000 & -0.3162 & -0.6325 \\ -0.7071 & 0.0000 & -0.3162 & -0.6325 \\ 0.0000 & -0.7071 & 0.6325 & -0.3162 \\ 0.0000 & 0.7071 & 0.6325 & -0.3162 \end{bmatrix}$$

Vector ***ifail*** of length 4: (if *ifail* is present)

ifail = (0, 0, 0, 0)

Vector ***iclustr*** of length 8 (= (2)(number_of_processors())): (if *iclustr* is present)

iclustr = (0, 0, 0, 0, 0, 0, 0, 0)

Vector ***gap*** of length 4 (= number_of_processors()): (if *gap* is present)

gap = (–1.0, –1.0, –1.0, –1.0)

info = 0 (if *info* is present)

SYTRD—Reduce a Real Symmetric Matrix to Tridiagonal Form

This subroutine reduces a real symmetric matrix ***A*** to symmetric tridiagonal form ***T*** by an orthogonal similarity transformation:

$$\boldsymbol{T} = \boldsymbol{Q}^{T}\boldsymbol{A}\boldsymbol{Q}$$

If the assumed-shape array for ***A*** has a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13], [21], [31], and [41].

Table 140. Data Types

A, ***d***, ***e***, τ	**Subroutine**
Long-precision real	SYTRD

Syntax

HPF	CALL SYTRD (*a*, *d*, *e*, *tau*, *uplo*) CALL SYTRD (*a*, *d*, *e*, *tau*, *uplo*, *info*)

On Entry

a

is the symmetric matrix ***A***, where:

If *uplo* = `'U'`, the array contains the upper triangle of the symmetric matrix ***A*** in its upper triangle, and its strictly lower triangular part is not referenced.

If *uplo* = `'L'`, the array contains the lower triangle of the symmetric matrix ***A*** in its lower triangle, and its strictly upper triangular part is not referenced.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 140, where size(*a*,1) = size(*a*,2).

d

See On Return.

e

See On Return.

tau

See On Return.

uplo

indicates whether the upper or lower triangular part of the symmetric matrix ***A*** is referenced, where:

If *uplo* = `'U'`, the upper triangular part is referenced.

If *uplo* = `'L'`, the lower triangular part is referenced.

Type: **required**

Specified as: a single character; *uplo* = `'U'` or `'L'`.

info

See On Return.

On Return

a

is the updated symmetric matrix ***A***, containing the results of the computation, where:

If *uplo* = `'U'`, the diagonal and first superdiagonal of ***A*** are overwritten by the corresponding elements of the tridiagonal matrix ***T***. The elements above the first superdiagonal, with τ, represent the orthogonal matrix ***Q*** as a product of elementary reflectors.

If *uplo* = `'L'`, the diagonal and first subdiagonal of ***A*** are overwritten by the corresponding elements of the tridiagonal matrix ***T***. The elements below the first subdiagonal, with τ, represent the orthogonal matrix ***Q*** as a product of elementary reflectors.

See “Function” on page 948, for more information.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 140 on page 946.

d

is the updated vector ***d***, containing the diagonal elements of the tridiagonal matrix ***T***.

The elements of ***d*** must be replicated across each element of the corresponding column of ***A***; that is, a copy of ***d*** is aligned with every row of ***A***:

```
!HPF$ ALIGN D(:) WITH A(*,:)
```

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 140 on page 946, where size(*d*) = size(*a*,1).

e

is the updated vector ***e***, containing the off-diagonal elements of the tridiagonal matrix ***T***, where:

If *uplo* = `'U'`, then $\boldsymbol{e}_1 = 0$ and $\boldsymbol{e}_{2:\text{size}(e)}$ contains the superdiagonal elements of the tridiagonal matrix ***T***.

If *uplo* = `'L'`, then $\boldsymbol{e}_{1:\text{size}(e)-1}$ contains the subdiagonal elements of the tridiagonal matrix ***T***, and $\boldsymbol{e}_{\text{size}(e)} = 0$.

The elements of ***e*** must be replicated across each element of the corresponding column of ***A***; that is, a copy of ***e*** is aligned with every row of ***A***:

```
!HPF$ ALIGN E(:) WITH A(*,:)
```

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 140 on page 946, where size(*e*) = size(*a*,1).

tau

is the updated vector τ, containing the scalar factors of the elementary reflectors, where:

If *uplo* = `'U'`, then $\tau_1 = 0$ and $\tau_{2:\text{size}(tau)}$ contains the scalar factors of the elementary reflectors.

If *uplo* = `'L'`, then $\tau_{1:\text{size}(tau)-1}$ contains the scalar factors of the elementary reflectors and $\tau_{\text{size}(tau)} = 0$

The elements of τ must be replicated across each element of the corresponding column of ***A***; that is, a copy of τ is aligned with every row of ***A***:

```
    !HPF$ ALIGN TAU(:) WITH A(*,:)
```

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 140 on page 946, where size(*tau*) = size(*a*,1).

info
indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:

   size(*a*,1) = size(*a*,2) = size(*d*) = size(*e*) = size(*tau*)
2. This subroutine accepts lowercase letters for the *uplo* argument.
3. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
4. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
5. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vectors and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 9 on page 108.
6. The restrictions given in "Notes and Coding Rules" on page 715 also apply to this subroutine.

Function: This subroutine reduces a real symmetric matrix $\boldsymbol{A}$ to symmetric tridiagonal form $\boldsymbol{T}$ by an orthogonal similarity transformation:

$$\boldsymbol{T} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{AQ}$$

where:

- $\boldsymbol{A}$ is a symmetric matrix, where n = size(*a*,1) = size(*a*,2)
- Matrix $\boldsymbol{Q}$ represents the following:
  - For *uplo* = `'U'`, the matrix $\boldsymbol{Q}$ is the product of elementary reflectors: $\boldsymbol{Q} = \boldsymbol{H}_{n-1} \ldots \boldsymbol{H}_2\ \boldsymbol{H}_1$,

    where:

    For each i: $\boldsymbol{H}_i = \boldsymbol{I} - \tau \boldsymbol{v}\boldsymbol{v}^{\mathrm{T}}$
    τ is a real scalar
    $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{i+1:n} = 0$ and $\boldsymbol{v}_i = 1$
    $\boldsymbol{v}_{1:i-1}$ is stored on return in submatrix $\boldsymbol{A}_{1:i-1,\ i+1}$
    τ is stored on return in τ_i
    $\boldsymbol{I}$ is the identity matrix

If *uplo* = `'U'`, then the following example shows the contents of ***A*** on return with *n* = 5:

$$\begin{bmatrix} d & e & v_2 & v_3 & v_4 \\ . & d & e & v_3 & v_4 \\ . & . & d & e & v_4 \\ . & . & . & d & e \\ . & . & . & . & d \end{bmatrix}$$

where:

d represents the diagonal elements of ***T***
e represents the superdiagonal elements of ***T***
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.

– For *uplo* = `'L'`, the matrix ***Q*** is the product of elementary reflectors: $\boldsymbol{Q} = \boldsymbol{H}_1\boldsymbol{H}_2 \ldots \boldsymbol{H}_{n-1}$,

where:

For each *i*: $\boldsymbol{H}_i = \boldsymbol{I} - \tau \boldsymbol{v}\boldsymbol{v}^T$
τ is a real scalar
$\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{1:i} = 0$ and $\boldsymbol{v}_{i+1} = 1$.
$\boldsymbol{v}_{i+2:n}$ is stored on return in submatrix $\boldsymbol{A}_{i+2:n,\ i}$
τ is stored on return in τ_i
I is the identity matrix.

If *uplo* = `'L'`, then the following example shows the contents of ***A*** on return with *n* = 5:

$$\begin{bmatrix} d & . & . & . & . \\ e & d & . & . & . \\ v_1 & e & d & . & . \\ v_1 & v_2 & e & d & . \\ v_1 & v_2 & v_3 & e & d \end{bmatrix}$$

where:

d represents the diagonal elements of ***T***
e represents the subdiagonal elements of ***T***
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 717 also apply to this subroutine.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *d*, *e*, or *tau*.
2. The process rank is not the same for *a*, *d*, *e*, and *tau*.
3. The process rank is not 1 or 2 for *a*, *d*, *e*, or *tau*.

Stage 2: The process grid is not the same for *a*, *d*, *e*, and *tau*.

Stage 3: The data distribution is unsupported for *a*.

Stage 4: The data distribution is unsupported for *d*, *e*, or *tau*.

Stage 5

1. The shape of the assumed-shape arrays *a*, *d*, *e*, and *tau* is incompatible:
   a. size(*a*,1) ≠ size(*a*,2) or
   b. size(*a*,2) ≠ size(*d*) or
   c. size(*d*) ≠ size(*e*) or
   d. size(*e*) ≠ size(*tau*)
2. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)
3. The column block size for *a* and the block sizes for *d*, *e*, and *tau* are incompatible.

Stage 6

1. The abstract process column indices for *a*, *d*, *e*, and *tau* are incompatible.
2. The data distribution for *a* is unsupported.

Example: This example shows the reduction of a symmetric matrix of order 4 to symmetric tridiagonal form. As in “Example” on page 718, array data for ***A*** is block-cyclically distributed using a 2 × 2 process grid, with ***d***, ***e***, and τ replicated across each element of the corresponding column of ***A***; that is, a copy of ***d***, *e*, and *tau* is aligned with every row of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN D(:) WITH A(*,:)
!HPF$ ALIGN E(:) WITH A(*,:)
!HPF$ ALIGN TAU(:) WITH A(*,:)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL SYTRD( A , D , E , TAU , 'U' )
-or-
CALL SYTRD( A , D , E , TAU , 'U' , INFO )
```

Input: Symmetric matrix ***A*** of order 4:

```
┌                     ┐
| 5.0  4.0  1.0  1.0 |
|  .   5.0  1.0  1.0 |
|  .    .   4.0  2.0 |
|  .    .    .   4.0 |
└                     ┘
```

Output: Symmetric matrix ***A*** of order 4:

```
┌                                ┐
| 1.00   0.00   0.41    0.22 |
|  .     6.00   2.83    0.22 |
|  .      .     7.00   -2.45 |
|  .      .      .      4.00 |
└                                ┘
```

Vector ***d*** of length 4:

```
┌                        ┐
| 1.00  6.00  7.00  4.00 |
└                        ┘
```

Vector ***e*** of length 4:

```
┌                           ┐
| 0.00   0.00   2.83  -2.45 |
└                           ┘
```

Vector τ of length 4:

```
┌                        ┐
| 0.00  0.00  1.71  1.82 |
└                        ┘
```

info = 0 (if *info* is present)

GEHRD—Reduce a General Matrix to Upper Hessenberg Form

This subroutine reduces a real general matrix ***A*** to upper Hessenberg form ***H*** by an orthogonal similarity transformation:

$$\boldsymbol{H} = \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{Q}$$

where ***A*** represents the general submatrix $\boldsymbol{A}_{ilo:ihi,\ ilo:ihi}$. $\tau_{1:ilo-1}$ and

If the assumed-shape array ***A*** has a size of zero, no computation is performed and the subroutine returns after doing some error checking. Then, if *ihi* = *ilo*, the subroutine returns after doing some parameter checking and setting $\tau_{ihi:\mathrm{size}(tau)}$ to zero.

See references [13], [21], [31], and [41].

Table 141. Data Types

A, τ	**Subroutine**
Long-precision real	GEHRD

Syntax

HPF	CALL GEHRD (*a*, *tau*) CALL GEHRD (*a*, *tau*, *ilo*, *ihi*, *info*)

On Entry

a
is the general matrix ***A***.

Type: **required**

Specified as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 141, where size(*a*,1) = size(*a*,2).

tau
See On Return.

ilo
lower range of the rows or columns in the general submatrix ***A*** used in the computation.

Type: **optional**

Default: *ilo* = 1

Specified as: a fullword integer; 1 ≤ *ilo* ≤ max(1,size(*a*,1)).

ihi
upper range of the rows or columns in the general submatrix ***A*** used in the computation.

Type: **optional**

Default: *ihi* = size(*a*,1)

Specified as: a fullword integer; min(*ilo*, size(*a*,1)) ≤ *ihi* ≤ size(*a*,1).

info
See On Return.

On Return

a

is the general matrix ***A***, containing the results of the computation.

The upper triangle and the first subdiagonal of ***A*** are overwritten by the corresponding elements of the upper Hessenberg matrix ***H***. The elements below the first subdiagonal, with τ, represent the orthogonal matrix ***Q*** as a product of elementary reflectors.

See "Function" on page 954, for more information.

Type: **required**

Returned as: an assumed-shape array with shape (**:**,**:**), containing numbers of the data type indicated in Table 141 on page 952.

tau

is the updated vector τ, where:

- $\tau_{ilo:ihi-1}$ contains the scalar factors of the elementary reflectors.
- $\tau_{1:ilo-1}$ are set to zero.
- $\tau_{ihi:\text{size}(tau)}$ are set to zero.

The elements of τ must be replicated across each element of the corresponding column of ***A***; that is, a copy of τ is aligned with every row of ***A***:

```
!HPF$ ALIGN TAU(:) WITH A(*,:)
```

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 141 on page 952, where size(*tau*) = max(size(*a*,1)−1,0).

info

indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - size(*a*,1) = size(*a*,2)
   - size(*tau*) = max(size(*a*,1)−1,0)
2. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
3. On entry, ***A*** must already be:
   - Upper triangular in rows (1:*ilo*−1) and (*ihi*+1:size(*a*,1))
   - Upper triangular in columns (1:*ilo*−1) and (*ihi*+1:size(*a*,2))

   If this is not the case, do one of the following:
   - Do not specify the *ilo* and *ihi* arguments.
   - Set *ilo* = 1 and *ihi* = size(*a*,1).

   If you specify the *ilo* and *ihi* arguments, then:
   - If size(*a*,1) = 0, set *ilo* = 1 and *ihi* = 0.

- If size(a,1) > 0, set $1 \le ilo \le ihi \le$ size(a,1).

4. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
5. Block-cyclic data distribution is required for your array data. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vector and matrix, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79. For a sample program including directives, see Figure 9 on page 108.
6. The restrictions given in "Notes and Coding Rules" on page 725 also apply to this subroutine.

Function: This subroutine reduces a real general matrix ***A*** to upper Hessenberg form ***H*** by an orthogonal similarity transformation:

$$\boldsymbol{H} = \boldsymbol{Q}^T\boldsymbol{A}\boldsymbol{Q}$$

where:

- ***A*** represents the general submatrix $\boldsymbol{A}_{ilo:ihi,\ ilo:ihi}$, where n = size(a,1) = size(a,2)
- Matrix ***Q*** is represented as a product of ($ihi - ilo$) elementary reflectors:

  $\boldsymbol{Q} = \boldsymbol{H}_{ilo}\ \boldsymbol{H}_{ilo+1}\ \dots\ \boldsymbol{H}_{ihi-1}$

  where:

  For each i: $\boldsymbol{H}_i = \boldsymbol{I} - \tau \boldsymbol{v}\boldsymbol{v}^T$
  τ is a real scalar
  $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{1:i} = 0$, $\boldsymbol{v}_{i+1} = 1$, and $\boldsymbol{v}_{ihi+1:n} = 0$
  $\boldsymbol{v}_{i+2:ihi}$ is stored on return in $\boldsymbol{A}_{i+ilo+1:ihi,\ ilo+i-1}$
  τ is stored on return in $\tau_{i+ilo-1}$
  $\boldsymbol{I}$ is the identity matrix

The following example shows the contents of the general submatrix ***A***, within a matrix of order 7, on entry with $ilo = 2$ and $ihi = 6$:

$$\begin{bmatrix} a & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & a & a & a & a & a & a \\ . & . & . & . & . & . & a \end{bmatrix}$$

Following is the general submatrix ***A*** on return:

$$\begin{bmatrix} a & a & h & h & h & h & a \\ . & a & h & h & h & h & a \\ . & h & h & h & h & h & h \\ . & v_2 & h & h & h & h & h \\ . & v_2 & v_3 & h & h & h & h \\ . & v_2 & v_3 & v_4 & h & h & h \\ . & . & . & . & . & . & a \end{bmatrix}$$

where:

a represents an element of the original submatrix ***A***.
h represents a updated element of the upper Hessenberg matrix ***H***.
v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_{ilo+i-1}$.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in "Error Conditions" on page 727 also apply to this subroutine.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a* or *tau*.
2. The process rank is not the same for *a* and *tau*.
3. The process rank is not 1 or 2 for *a* or *tau*.

Stage 2: The process grid is not the same for *a* and *tau*.

Stage 3: The data distribution is unsupported for *a*.

Stage 4: The data distribution is unsupported for *tau*.

Stage 5

1. The shape of the assumed-shape arrays *a* and *tau* is incompatible:
   a. size(*a*,1) ≠ size(*a*,2) or
   b. size(*tau*) ≠ max (size(*a*,1)−1,0)
2. The shape of the assumed-shape array for *a* is invalid: size(*a*,1) ≠ size(*a*,2)
3. size(*tau*) ≠ 0, and the column block size for *a* and the block size for *tau* are incompatible.

Stage 6

1. *ilo* is present, and *ilo* < 1 or *ilo* > max(1,size(*a*,1)).
2. *ihi* is present, and *ihi* < min(*ilo*, size(*a*,1)) or *ihi* > size(*a*,1)

Stage 7

1. size(*tau*) ≠ 0, and the abstract process column indices for *a* and *tau* are incompatible.
2. The data distribution for *a* is unsupported.

Example: This example shows the reduction of a general matrix of order 3 to upper Hessenberg form. As in "Example" on page 728, array data for ***A*** is block-cyclically distributed using a 2 × 2 process grid, with τ replicated across each element of the corresponding column of ***A***; that is, a copy of *tau* is aligned with every row of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN TAU(:) WITH A(*,:)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL GEHRD( A , TAU )
-or-
CALL GEHRD( A , TAU , 1 , 3 , INFO )
```

Input: General matrix ***A*** of order 3:

```
┌                     ┐
│  33.0   16.0   72.0 │
│ -24.0  -10.0  -57.0 │
│  -8.0   -4.0  -17.0 │
└                     ┘
```

Output: General matrix ***A*** of order 3:

```
┌                        ┐
│ 33.00  -37.95   63.25 │
│ 25.30  -29.00   53.00 │
│  0.16    0.00    2.00 │
└                        ┘
```

Vector τ of length 2:

```
┌            ┐
│ 1.95  0.00 │
└            ┘
```

info = 0 (if *info* is present)

GEBRD—Reduce a General Matrix to Bidiagonal Form

This subroutine reduces a real general matrix ***A*** to upper or lower bidiagonal form ***B*** by an orthogonal transformation:

$$\boldsymbol{B} = \boldsymbol{Q}^{T}\boldsymbol{AP}$$

where:

- If size(*a*,1) ≥ size(*a*,2), then ***B*** is upper bidiagonal.
- If size(*a*,1) < size(*a*,2), then ***B*** is lower bidiagonal.

If the assumed-shape array for ***A*** has a size of zero, no computation is performed and the subroutine returns after doing some parameter checking.

See references [13], [21], [31], and [41].

Table 142. Data Types

A, d, e, τ_q, τ_p	**Subroutine**
Long-precision real	GEBRD

Syntax

HPF	CALL GEBRD (*a*, *d*, *e*, *tauq*, *taup*) CALL GEBRD (*a*, *d*, *e*, *tauq*, *taup*, *info*)

On Entry

a
is the general matrix ***A***.

Type: **required**

Specified as: an assumed-shape array with shape (`:,:`), containing numbers of the data type indicated in Table 142.

d
See On Return.

e
See On Return.

tauq
See On Return.

taup
See On Return.

info
See On Return.

On Return

a
is the updated general matrix ***A***, containing the results of the computation, where:

- If size(*a*,1) ≥ size(*a*,2), the diagonal and first superdiagonal of ***A*** are overwritten by the corresponding elements of the upper bidiagonal matrix ***B***. The elements below the diagonal, with τ_q, represent the orthogonal matrix ***Q*** as a product of elementary reflectors. The elements above the first superdiagonal, with τ_p, represent the orthogonal matrix ***P*** as a product of elementary reflectors.

- If size(*a*,1) < size(*a*,2), the diagonal and first subdiagonal of ***A*** are overwritten by the corresponding elements of the lower bidiagonal matrix ***B***. The elements below the first subdiagonal, with τ_q, represent the orthogonal matrix ***Q*** as a product of elementary reflectors. The elements above the diagonal, with τ_p, represent the orthogonal matrix ***P*** as a product of elementary reflectors.

See "Function" on page 960, for more information.

Type: **required**

Returned as: an assumed-shape array with shape (:,:), containing numbers of the data type indicated in Table 142 on page 957.

d

is the updated vector ***d***, where:

- If size(*a*,1) ≥ size(*a*,2), then ***d*** contains the diagonal elements of the bidiagonal matrix ***B***.

  The elements of ***d*** must be replicated across each element of the corresponding column of ***A***; that is, a copy of ***d*** is aligned with every row of ***A***:

  ```
  !HPF$ ALIGN D(:) WITH A(*,:)
  ```

- If size(*a*,1) < size(*a*,2), then ***d*** contains the diagonal elements of the bidiagonal matrix ***B***.

  The elements of ***d*** must be replicated across each element of the corresponding row of ***A***; that is, a copy of ***d*** is aligned with every column of ***A***:

  ```
  !HPF$ ALIGN D(:) WITH A(:,*)
  ```

Type: **required**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 142 on page 957, where size(*d*) = min(size(*a*,1), size(*a*,2)).

e

is the updated vector ***e***, where:

- If size(*a*,1) ≥ size(*a*,2), then ***e*** contains the superdiagonal elements of the bidiagonal matrix ***B***.

  The elements of ***e*** must be replicated across each element of the corresponding row of ***A***; that is, a copy of ***e*** is aligned with every column of ***A***:

  ```
  !HPF$ ALIGN E(:) WITH A(:,*)
  ```

- If size(*a*,1) < size(*a*,2), then ***e*** contains the subdiagonal elements of the bidiagonal matrix ***B***.

  The elements of ***e*** must be replicated across each element of the corresponding column of ***A***; that is, a copy of ***e*** is aligned with every row of ***A***:

  ```
  !HPF$ ALIGN E(:) WITH A(*,:)
  ```

Type: **required**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 142 on page 957, where size(*e*) = max(min(size(*a*,1), size(*a*,2))−1, 0).

tauq

is the updated matrix τ_q, containing the scalar factors of the elementary reflectors which represent the orthogonal matrix ***Q***. See “Function” on page 960 for more details.

The elements of τ_q must be replicated across each element of the corresponding column of ***A***; that is, a copy of τ_q is aligned with every row of ***A***:

```
!HPF$ ALIGN TAUQ(:) WITH A(*,:)
```

Type: **required**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 142 on page 957, where size(*tauq*) = min(size(*a*,1), size(*a*,2)).

taup

is the updated matrix τ_p, containing the scalar factors of the elementary reflectors which represent the orthogonal matrix ***P***. See “Function” on page 960 for more details.

The elements of τ_p must be replicated across each element of the corresponding row of ***A***; that is, a copy of τ_p is aligned with every column of ***A***:

```
!HPF$ ALIGN TAUP(:) WITH A(:,*)
```

Type: **required**

Returned as: an assumed-shape array with shape (:), containing numbers of the data type indicated in Table 142 on page 957, where size(*taup*) = min(size(*a*,1), size(*a*,2)).

info

indicates that a successful computation occurred.

Type: **optional**

Returned as: a fullword integer; *info* = 0.

Notes and Coding Rules

1. The assumed-shape arrays must have the exact size required for the computation, that is:
   - size(*d*) = min(size(*a*,1), size(*a*,2))
   - size(*e*) = max(min(size(*a*,1), size(*a*,2))−1, 0)
   - size(*tauq*) = min(size(*a*,1), size(*a*,2))
   - size(*taup*) = min(size(*a*,1), size(*a*,2))
2. Data directives for this subroutine cannot be included in the interface module PESSL_HPF, because the alignment requirements for *d* and *e* depend on the size of the matrix ***A***; therefore, you must code the data directives for all the assumed-shape arrays explicitly in your program according to the rules described above.

   Block-cyclic data distribution is required for your array data. For how to code your HPF directives, see “Distributing Data in an HPF Program” on page 79. For a sample program including directives, see Figure 9 on page 108.

3. The assumed-shape arrays must have no common elements; otherwise, results are unpredictable.
4. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
5. The restrictions given in “Notes and Coding Rules” on page 737 also apply to this subroutine.

Function: This subroutine reduces a real general matrix ***A*** to upper or lower bidiagonal form ***B*** by an orthogonal transformation:

$\mathbf{B} = \mathbf{Q}^T\mathbf{AP}$

where:

- ***A*** is a general matrix, where m = size(a,1) and n = size(a,2).
- If $m \geq n$, then ***B*** is upper bidiagonal, and matrices ***Q*** and ***P*** are represented as the product of elementary reflectors:

  $\mathbf{Q} = \mathbf{H}_1 \mathbf{H}_2 \ldots \mathbf{H}_n$
  $\mathbf{P} = \mathbf{G}_1 \mathbf{G}_2 \ldots \mathbf{G}_{n-1}$

  where:

  For each i: $\mathbf{H}_i = \mathbf{I} - \tau_q \mathbf{v}\mathbf{v}^T$
  For each i: $\mathbf{G}_i = \mathbf{I} - \tau_p \mathbf{u}\mathbf{u}^T$
  τ_q and τ_p are real scalars, where:

  $$\tau_q \text{ is stored on return in } \tau_{q_i}$$

  $$\tau_p \text{ is stored on return in } \tau_{p_i}$$

  $\mathbf{v}$ is a real vector with $\mathbf{v}_{1:i-1} = 0$ and $\mathbf{v}_i = 1$
  $\mathbf{v}_{i+1:m}$ is stored on return in submatrix $\mathbf{A}_{i+1:m,\, i}$
  $\mathbf{u}$ is a real vector with $\mathbf{u}_{1:i} = 0$ and $\mathbf{u}_{i+1} = 1$
  $\mathbf{u}_{i+2:n}$ is stored on return in submatrix $\mathbf{A}_{i,\, i+2:n}$
  I is the identity matrix

  The following example shows the contents of ***A*** on return with m = 6 and n = 5:

  $$\begin{bmatrix} d & e & u_1 & u_1 & u_1 \\ v_1 & d & e & u_2 & u_2 \\ v_1 & v_2 & d & e & u_3 \\ v_1 & v_2 & v_3 & d & e \\ v_1 & v_2 & v_3 & v_4 & d \\ v_1 & v_2 & v_3 & v_4 & v_5 \end{bmatrix}$$

  where:

  d represents the diagonal elements of ***B***
  e represents the off-diagonal elements of ***B***
  v_i represents the corresponding elements of the vector defining $\mathbf{H}_i$.
  u_i represents the corresponding elements of the vector defining $\mathbf{G}_i$.

- If $m < n$, then $\boldsymbol{B}$ is lower bidiagonal, and matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$ are represented as the product of elementary reflectors:

  $\boldsymbol{Q} = \boldsymbol{H}_1 \boldsymbol{H}_2 \ldots \boldsymbol{H}_{m-1}$
  $\boldsymbol{P} = \boldsymbol{G}_1 \boldsymbol{G}_2 \ldots \boldsymbol{G}_m$

  where:

  For each i: $\boldsymbol{H}_i = \boldsymbol{I} - \tau_q \boldsymbol{v}\boldsymbol{v}^T$
  For each i: $\boldsymbol{G}_i = \boldsymbol{I} - \tau_p \boldsymbol{u}\boldsymbol{u}^T$
  τ_q and τ_p are real scalars, where:

$$\tau_q \text{ is stored on return in } \tau_{q_i}$$

$$\tau_p \text{ is stored on return in } \tau_{p_i}$$

  $\boldsymbol{v}$ is a real vector with $\boldsymbol{v}_{1:i} = 0$ and $\boldsymbol{v}_{i+1} = 1$
  $\boldsymbol{v}_{i+2:m}$ is stored on return in submatrix $\boldsymbol{A}_{i+2:m,i}$
  $\boldsymbol{u}$ is a real vector with $\boldsymbol{u}_{1:i-1} = 0$ and $\boldsymbol{u}_i = 1$
  $\boldsymbol{u}_{i+1:n}$ is stored on return in submatrix $\boldsymbol{A}_{i,i+1:n}$
  $\boldsymbol{I}$ is the identity matrix

  The following example shows the contents of $\boldsymbol{A}$ on return with $m = 5$ and $n = 6$:

$$\begin{bmatrix} d & u_1 & u_1 & u_1 & u_1 & u_1 \\ e & d & u_2 & u_2 & u_2 & u_2 \\ v_1 & e & d & u_3 & u_3 & u_3 \\ v_1 & v_2 & e & d & u_4 & u_4 \\ v_1 & v_2 & v_3 & e & d & u_5 \end{bmatrix}$$

  where:

  d represents the diagonal elements of $\boldsymbol{B}$
  e represents the off-diagonal elements of $\boldsymbol{B}$
  v_i represents the corresponding elements of the vector defining $\boldsymbol{H}_i$.
  u_i represents the corresponding elements of the vector defining $\boldsymbol{G}_i$.

Error Conditions: HPF-specific errors are listed below. The input-argument errors and computational errors listed in "Error Conditions" on page 740 also apply to this subroutine.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 2 for *a*, *d*, *e*, *tauq*, or *taup*.
2. The process rank is not the same for *a*, *d*, *e*, *tauq*, and *taup*.
3. The process rank is not 1 or 2 for *a*, *d*, *e*, *tauq*, or *taup*.

Stage 2: The process grid is not the same for *a*, *d*, *e*, *tauq*, and *taup*.

Stage 3: The data distribution is unsupported for *a*.

Stage 4: The data distribution is unsupported for *d*, *e*, *tauq*, or *taup*.

Stage 5

1. The shape of the assumed-shape arrays *a*, *d*, *e*, *tauq*, and *taup* is incompatible:
   a. size(*d*) ≠ min(size(*a*,1), size(*a*,2)) or
   b. size(*e*) ≠ max(min(size(*a*,1), size(*a*,2))−1, 0) or
   c. size(*tauq*) ≠ min(size(*a*,1), size(*a*,2)) or
   d. size(*taup*) ≠ min(size(*a*,1), size(*a*,2))
2. The block sizes for *d*, *e* (if size(*e*) ≠ 0), *tauq*, and *taup* are incompatible with the corresponding row or column block size of *a*.

Stage 6

1. The abstract process indices for *a*, *d*, *e* (if size(*e*) ≠ 0), *tauq*, and *taup* are incompatible.
2. The data distribution for *a* is unsupported.

Example: This example shows the reduction of a general matrix of order 4 by 3 to bidiagonal form. As in "Example" on page 741, array data for ***A*** is block-cyclically distributed using a 2 × 2 process grid, with:

- τ_q and ***d*** replicated across each element of the corresponding column of ***A***; that is, a copy of τ_q and ***d*** is aligned with every row of ***A***.
- τ_p and ***e*** replicated across each element of the corresponding row of ***A***; that is, a copy of τ_p and ***e*** is aligned with every column of ***A***.

```
!HPF$ PROCESSORS PROC(2,2)
!HPF$ ALIGN D(:) WITH A(*,:)
!HPF$ ALIGN E(:) WITH A(:,*)
!HPF$ ALIGN TAUQ(:) WITH A(*,:)
!HPF$ ALIGN TAUP(:) WITH A(:,*)
!HPF$ DISTRIBUTE (CYCLIC, CYCLIC) ONTO PROC :: A

CALL GEBRD( A , D , E , TAUQ , TAUP )
-or-
CALL GEBRD( A , D , E , TAUQ , TAUP , INFO )
```

Input: General matrix ***A*** of order 4 × 3:

```
┌                  ┐
│ 10.0   5.0   9.0 │
│  2.0  16.0  10.0 │
│  3.0   7.0  21.0 │
│  4.0   8.0  12.0 │
└                  ┘
```

Output: General matrix ***A*** of order 4 × 3:

```
┌                      ┐
│ -11.36  22.80   0.56 │
│   0.09  23.32   1.67 │
│   0.14   0.46  -9.68 │
│   0.19   0.22   0.08 │
└                      ┘
```

Vector ***d*** of length 3:

```
┌                        ┐
| -11.36  23.32  -9.68 |
└                        ┘
```

Vector ***e*** of length 2:

```
┌       ┐
| 22.80 |
|  1.67 |
└       ┘
```

Vector τ_q of length 3:

```
┌                  ┐
| 1.88  1.59  1.99 |
└                  ┘
```

Vector τ_p of length 3:

```
┌      ┐
| 1.52 |
| 0.00 |
| 0.00 |
└      ┘
```

info = 0 (if *info* is present)

Chapter 16. Fourier Transforms (HPF)

This chapter describes the Fourier Transforms subroutines that can be called from an HPF program.

Overview of the Fourier Transforms Subroutines

The Fourier transform subroutines perform mixed-radix transforms of in two and three dimensions. These subroutines are available in short- and long-precision versions. See references [1] and [3].

Table 143. List of Fourier Transform Subroutines (HPF)

Descriptive Name	**Short- and Long-Precision Subroutine**	**Page**
Fourier Transforms in Two Dimensions	FFT	968
Fourier Transforms in Three Dimensions	FFT	976

Acceptable Lengths for the Transforms

Use the following formula to determine acceptable transform lengths:

$n = (2^h)\ (3^i)\ (5^j)\ (7^k)\ (11^m) \quad \text{for } n \leq 37748736$

where:

$h = 1, 2, ..., 25$
$i = 0, 1, 2$
$j, k, m = 0, 1$

Figure 13 on page 966 lists all the acceptable values for transform lengths in the Fourier transform subroutines.

2	4	6	8	10	12	14	16	18
20	22	24	28	30	32	36	40	42
44	48	56	60	64	66	70	72	80
84	88	90	96	110	112	120	126	128
132	140	144	154	160	168	176	180	192
198	210	220	224	240	252	256	264	280
288	308	320	330	336	352	360	384	396
420	440	448	462	480	504	512	528	560
576	616	630	640	660	672	704	720	768
770	792	840	880	896	924	960	990	1008
1024	1056	1120	1152	1232	1260	1280	1320	1344
1386	1408	1440	1536	1540	1584	1680	1760	1792
1848	1920	1980	2016	2048	2112	2240	2304	2310
2464	2520	2560	2640	2688	2772	2816	2880	3072
3080	3168	3360	3520	3584	3696	3840	3960	4032
4096	4224	4480	4608	4620	4928	5040	5120	5280
5376	5544	5632	5760	6144	6160	6336	6720	6930
7040	7168	7392	7680	7920	8064	8192	8448	8960
9216	9240	9856	10080	10240	10560	10752	11088	11264
11520	12288	12320	12672	13440	13860	14080	14336	14784
15360	15840	16128	16384	16896	17920	18432	18480	19712
20160	20480	21120	21504	22176	22528	23040	24576	24640
25344	26880	27720	28160	28672	29568	30720	31680	32256
32768	33792	35840	36864	36960	39424	40320	40960	42240
43008	44352	45056	46080	49152	49280	50688	53760	55440
56320	57344	59136	61440	63360	64512	65536	67584	71680
73728	73920	78848	80640	81920	84480	86016	88704	90112
92160	98304	98560	101376	107520	110880	112640	114688	118272
122880	126720	129024	131072	135168	143360	147456	147840	157696
161280	163840	168960	172032	177408	180224	184320	196608	197120
202752	215040	221760	225280	229376	236544	245760	253440	258048
262144	270336	286720	294912	295680	315392	322560	327680	337920
344064	354816	360448	368640	393216	394240	405504	430080	443520
450560	458752	473088	491520	506880	516096	524288	540672	573440
589824	591360	630784	645120	655360	675840	688128	709632	720896
737280	786432	788480	811008	860160	887040	901120	917504	946176
983040	1013760	1032192	1048576	1081344	1146880	1179648	1182720	1261568
1290240	1310720	1351680	1376256	1419264	1441792	1474560	1572864	1576960
1622016	1720320	1774080	1802240	1835008	1892352	1966080	2027520	2064384
2097152	2162688	2293760	2359296	2365440	2523136	2580480	2621440	2703360
2752512	2838528	2883584	2949120	3145728	3153920	3244032	3440640	3548160
3604480	3670016	3784704	3932160	4055040	4128768	4194304	4325376	4587520
4718592	4730880	5046272	5160960	5242880	5406720	5505024	5677056	5767168
5898240	6291456	6307840	6488064	6881280	7096320	7208960	7340032	7569408
7864320	8110080	8257536	8388608	8650752	9175040	9437184	9461760	10092544
10321920	10485760	10813440	11010048	11354112	11534336	11796480	12582912	12615680
12976128	13762560	14192640	14417920	14680064	15138816	15728640	16220160	16515072
16777216	17301504	18350080	18874368	18923520	20185088	20643840	20971520	21626880
22020096	22708224	23068672	23592960	25165824	25231360	25952256	27525120	28385280
28835840	29360128	30277632	31457280	32440320	33030144	33554432	34603008	36700160
37748736								

Figure 13. Table of Acceptable Lengths for the Transforms

Fourier Transforms Subroutines

This section contains the Fourier transform subroutine descriptions.

FFT—Fourier Transforms in Two Dimensions

These subroutines compute the mixed-radix two-dimensional discrete Fourier transform of short- or long-precision data:

$$y_{k1,k2} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} x_{j1,j2} \; W_{n1}^{(Isign)j1k1} \; W_{n2}^{(Isign)j2k2}$$

for:

$k1$ = 0, 1, ..., $n1$–1
$k2$ = 0, 1, ..., $n2$–1

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$

and where:

$x_{j1,j2}$ are elements of array X.
$y_{k1,k2}$ are elements of array Y.
Isign is + or – (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/(($n1$)($n2$)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 144 (Page 1 of 2). Data Types and Sizes for X and Y

Short- and Long-Precision Data Types	**Size of Assumed-Shaped Array X**	**Size of Assumed-Shaped Array Y**
Case 1: Complex-to-Complex	*n1* × *n2*	*n2* × *n1* (output is stored in transposed format) *n1* × *n2* (output is stored in normal format) If *transpose* is not specified, see the *transpose* argument and its default description to find out how the output is stored.
Case 2: Real-to-Complex	*n1* × *n2*	*n2* × *n1*/2†
Case 3: Complex-to-Real	*n2* × *n1*/2†	*n1* × *n2*

Table 144 (Page 2 of 2). Data Types and Sizes for X and Y		
Short- and Long-Precision Data Types	**Size of Assumed-Shaped Array X**	**Size of Assumed-Shaped Array Y**

† This array is stored in FFT-packed storage mode.

Notes for All Cases:

1. *n1* and *n2* must be fullword integers that are less than or equal to 37748736, and must be one of the values listed in Figure 13 on page 966.
2. For Cases 1 and 2:

   n1 = size(*x*,1)
   n2 = size(*x*,2)

   For Case 3:

   n1 = 2(size(*x*,2))
   n2 = size(*x*,1)
3. *scale* is a short- or long-precision real number that must be of the same precision as both *x* and *y*.

Syntax

HPF	**Case 1**	CALL FFT (*x*) CALL FFT (*x*, *y*, *transpose*, *isign*, *scale*)
HPF	**Cases 2 and 3**	CALL FFT (*x, y*) CALL FFT (*x*, *y*, *isign*, *scale*)

On Entry

x
is the array X containing the two-dimensional data to be transformed.

Type: **required**

Specified as: an assumed-shape array with shape (:,:), containing numbers of the data type indicated in Table 144 on page 968. This array must be aligned on a doubleword boundary.

y
See On Return.

transpose
has the following meaning:

- If *y* is specified, array Y contains the results of the computation, where:

  If *transpose* = 'T', *y* is returned in transposed form; that is, *y* has dimensions *n2* × *n1*.

  If *transpose* = 'N', *y* is returned in normal form; that is, *y* has dimensions *n1* × *n2*.

- For case 1, if *y* is not specified, array X contains the result of the computation, where:

  If *transpose* = 'T', *x* is returned in transposed form; that is, *x* has dimensions *n2* × *n1*.

  If *transpose* = 'N', *x* is returned in normal form; that is, *x* has dimensions *n1* × *n2*.

Type: **optional** (case 1); **not present** (cases 2 and 3)

Default:

For case 1, if *transpose* is not present, the subroutine returns the output in transposed form if the output array has a shape which is the transpose of the shape of the input array X. Otherwise, the output is returned in normal form, as follows:

- $n1 = n2$, the output array is stored in transposed form and has dimensions $n2 \times n1$
- $n1 \neq n2$ and the output array has dimensions $n2 \times n1$, the output array is stored in transposed form.
- $n1 \neq n2$ and the output array has dimensions $n1 \times n2$, the output array is stored in normal form.

Specified as: a single character; *transpose* = `'T'` or `'N'`.

isign

controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Type: **optional**

Default: *isign* is a positive value.

Specified as: a fullword integer; where $isign > 0$ or $isign < 0$.

scale

is the scaling constant *scale*.

Type: **optional**

Default: *scale* has the following value, where:

- If *isign* is a positive value or is not specified, $scale = 1$.
- If *isign* is negative, $scale = 1/((n1)(n2))$.

Specified as: a number of the data type indicated in Table 144 on page 968, where $scale > 0.0$ or $scale < 0.0$.

On Return

x

can only be returned in the complex-to-complex subroutines (case 1) and has the following meaning:

- If *y* is specified, array Y contains the results of the computation and *x* is not returned.
- If *y* is not specified, array X contains the result of the computation, where:

  If *transpose* = `'T'`, *x* is returned in transposed form; that is, *x* has dimensions $n2 \times n1$. In this case, *n1* must equal *n2*.

  If *transpose* = `'N'`, *x* is returned in normal form; that is *x* has dimensions $n1 \times n2$.

  In this case, output overwrites input.

Returned as: an assumed-shape array with shape (:,:), containing the numbers of the data type indicated in Table 144 on page 968. This array must be aligned on a doubleword boundary.

y

has the following meaning:

- If *y* is specified, array `Y` contains the results of the computation, where:

  If *transpose* = `'T'`, *y* is returned in transposed form; that is, *y* has dimensions $n2 \times n1$.

  If *transpose* = `'N'`, *y* is returned in normal form; that is *y* has dimensions $n1 \times n2$

  In this case, `X` and `Y` must have no common elements; otherwise, results are unpredictable.

- For case 1, if *y* is not specified, array `X` contains the result of the computation.

Type: **optional** (case 1); **required** (cases 2 and 3)

Default: Array `X` contains the result of the computation.

Returned as: an assumed-shape array with shape (:,:), containing the numbers of the data type indicated in Table 144 on page 968. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. All subroutines accept lowercase letters for the *transpose* argument.
2. The *x*, *y*, and *scale* arguments must be of the same precision.
3. The assumed shape arrays must have no common elements; otherwise, results are unpredictable.
4. If the output array does not have the correct shape for storing the transformed data, an error message is issued and the application program is terminated. However, this error check occurs before the problem size is verified to consist of only valid transform lengths. This means that if you correct the shape and rerun the program, you may still receive an additional error message if the values for the problem size are not valid transform lengths.
5. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See subroutine fourier in “Module Fourier (HPF)” on page 1046.
6. For details on how to set up and code your HPF program using Parallel ESSL, see “Coding Your HPF Program” on page 105
7. Block-column data distribution is required for your array data. In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

   Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your sequences, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Sequences (Fourier Transform)” on page 81. For a sample program including directives, see Figure 10 on page 109.

8. The restrictions given in “Notes and Coding Rules” on page 751, “Notes and Coding Rules” on page 757, and “Notes and Coding Rules” on page 763 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 751, “Error Conditions” on page 758, and “Error Conditions” on page 764 also apply to these subroutines.

Input-Argument Errors for Case 1

Stage 1

1. The rank of the ultimate align target is greater than 1 for *x* or *y*.
2. The process rank is not the same for *x* and *y*.
3. The process rank is not 1 for *x* or *y*.

Stage 2

1. The process grid is not the same for *x* and *y*.
2. The data distribution is unsupported for *x* or *y*.

Stage 3

1. *transpose* is present and *transpose* ≠ `'N'` or `'T'`

Stage 4

1. If *transpose* = `'T'` and *y* is present and the shape of *y* is not $n2 \times n1$.
2. If *transpose* = `'N'` and *y* is present and the shape of *y* is not $n1 \times n2$.
3. If *y* is not present (which means *x* contains the result of the computation) and *transpose* = `'T'` and $n1 \neq n2$.

Input-Argument Errors for Cases 2 and 3

Stage 1

1. The rank of the ultimate align target is greater than 1 for *x* or *y*.
2. The process rank is not the same for *x* and *y*.
3. The process rank is not 1 for *x* or *y*.

Stage 2

1. The process grid is not the same for *x* and *y*.
2. The data distribution is unsupported for *x* or *y*.

Stage 3

1. The shape of the assumed-shape arrays *x* and *y* are incompatible, as follows:

   For case 2:

   $2(\text{size}(y,2)) \neq n1$ or
   $\text{size}(y,1) \neq n2$

   For case 3:

   $\text{size}(y,2) \neq n2$ or
   $\text{size}(y,1) \neq n1$

Example 1: This example shows how to compute a complex-to-complex, two-dimensional transform. As in “Example 1” on page 751, the array data is block-column distributed using a one-dimensional process grid with two processes. Array Y is returned in transposed form. The arrays are declared as follows:

```
COMPLEX*16 X(0:7,0:5), Y(0:5,0:7)
REAL*8 SCALE
```

Input

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, BLOCK) ONTO PROC :: X, Y

CALL FFT( X , Y , ISIGN=-1 )
-or-
CALL FFT( X , Y , TRANSPOSE='T' , ISIGN=-1 , SCALE=1.0D0/48.0D0 )
```

Matrix ***X*** of order 8 × 6:

```
┌                                                                          ┐
| (48.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
|  (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0)   (0.0,0.0) |
└                                                                          ┘
```

Output: Matrix for ***Y***:

```
┌                                                                                          ┐
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
└                                                                                          ┘
```

Example 2: This example shows how to compute a real-to-complex, two-dimensional transform. As in “Example” on page 758, the array data is block-column distributed using a one-dimensional process grid with two processes. The arrays are declared as follows:

```
REAL*8 X(0:11,0:3)
COMPLEX*16 Y(0:6,0:3)
REAL*8 SCALE
```

Input

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, BLOCK) ONTO PROC :: X, Y

CALL FFT( X(0:7,0:3) , Y(0:3,0:3))
-or-
CALL FFT( X(0:7,0:3) , Y(0:3,0:3) , ISIGN=1 , SCALE=1.0D0 )
```

Matrix ***X*** of order 8 × 4:

$$\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

Output: The following matrix ***Y*** is returned in transposed form and stored in FFT-packed storage mode:

$$\begin{bmatrix} (1.0,1.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,1.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

Example 3: This example shows how to compute a complex-to-real, two-dimensional transform. As in “Example” on page 764, the array data is block-column distributed using a one-dimensional process grid with two processes. The arrays are declared as follows:

```
COMPLEX*16 X(0:6,0:3)
REAL*8 Y(0:11,0:3)
REAL*8 SCALE
```

Input

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, BLOCK) ONTO PROC :: X, Y

CALL FFT( X(0:3,0:3) , Y(0:7,0:3) ISIGN=-1 )
-or-
CALL FFT( X(0:3,0:3) , Y(0:7,0:3) , ISIGN=-1 , SCALE=1.0D0/32.0D0 )
```

The following matrix ***X*** is stored in FFT-packed storage mode:

$$\begin{bmatrix} (1.0,1.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,1.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ (1.0,0.0) & (1.0,0.0) & (1.0,0.0) & (1.0,0.0) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix}$$

Output: Matrix ***Y***:

```
┌                     ┐
│ 1.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
└                     ┘
```

FFT—Fourier Transforms in Three Dimensions

These subroutines compute the mixed-radix three-dimensional discrete Fourier transform of short- or long-precision data:

$$y_{k1,k2,k3} = scale \sum_{j1=0}^{n1-1} \sum_{j2=0}^{n2-1} \sum_{j3=0}^{n3-1} x_{j1,j2,j3}\ W_{n1}^{(Isign)j1k1}\ W_{n2}^{(Isign)j2k2}\ W_{n3}^{(Isign)j3k3}$$

for:

$k1 = 0, 1, ..., n1-1$
$k2 = 0, 1, ..., n2-1$
$k3 = 0, 1, ..., n3-1$

where:

$$W_{n1} = e^{-2\pi\left(\sqrt{-1}\right)/n1}$$
$$W_{n2} = e^{-2\pi\left(\sqrt{-1}\right)/n2}$$
$$W_{n3} = e^{-2\pi\left(\sqrt{-1}\right)/n3}$$

and where:

$x_{j1,j2,j3}$ are elements of array `X`.
$y_{k1,k2,k3}$ are elements of array `Y`.
Isign is + or − (determined by argument *isign*).
scale is a scalar value.

For *scale* = 1 and *isign* being positive, you obtain the discrete Fourier transform. For *scale* = 1/((*n1*)(*n2*)(*n3*)) and *isign* being negative, you obtain the inverse Fourier transform.

See references [1] and [3].

Table 145 (Page 1 of 2). Data Types and Sizes for X and Y

Short- and Long-Precision Data Types	**Size of Assumed-Shaped Array X**	**Size of Assumed-Shaped Array Y**
Case 1: Complex-to-Complex	*n1* × *n2* × *n3*	*n3* × *n2* × *n1* (output is stored in transposed format) *n1* × *n2* × *n3* (output is stored in normal format) If *transpose* is not specified, see the *transpose* argument and its default description to find out how the output is stored.
Case 2: Real-to-Complex	*n1* × *n2* × *n3*	*n3* × *n2* × *n1*/2†
Case 3: Complex-to-Real	*n3* × *n2* × *n1*/2†	*n1* × *n2* × *n3*

Table 145 (Page 2 of 2). Data Types and Sizes for X and Y

Short- and Long-Precision Data Types	**Size of Assumed-Shaped Array X**	**Size of Assumed-Shaped Array Y**

† This array is stored in FFT-packed storage mode.

Notes for All Cases:

1. *n1*, *n2*, *n3* must be fullword integers that are less than or equal to 37748736, and must be one of the values listed in Figure 13 on page 966.
2. For cases 1 and 2:

   n1 = size(*x*,1)
   n2 = size(*x*,2)
   n3 = size(*x*,3)

   For case 3:

   n1 = 2(size(*x*,3))
   n2 = size(*x*,2)
   n3 = size(*x*,1)
3. *scale* is a short- or long-precision real number that must be of the same precision as both *x* and *y*.

Syntax

HPF	**Case 1**	CALL FFT (*x*) CALL FFT (*x*, *y*, *transpose*, *isign*, *scale*,)
HPF	**Cases 2 and 3**	CALL FFT (*x, y*) CALL FFT (*x*, *y*, *isign*, *scale*)

On Entry

x
is the array X containing the three-dimensional data to be transformed.

Type: **required**

Specified as: an assumed-shape array with shape (:,:,:), containing numbers of the data type indicated in Table 145 on page 976. This array must be aligned on a doubleword boundary.

y
See On Return.

transpose
has the following meaning:

- If *y* is specified, array Y contains the result of the computation, where:

  If *transpose* = 'T', *y* is returned in transposed form; that is, *y* has dimensions *n3* × *n2* × *n1*

  If *transpose* = 'N', *y* is returned in normal form; that is, *y* has dimensions *n1* × *n2* × *n3*.

- For case 1, if *y* is not specified, array X contains the result of the computation, where:

  If *transpose* = 'T', *x* is returned in transposed form; that is, *x* has dimensions *n3* × *n2* × *n1*.

If *transpose* = `'N'`, *x* is returned in normal form; that is, *x* has dimensions *n1* × *n2* × *n3*.

Type: **optional** (case 1); **not present** (cases 2 and 3)

Default:

For case 1, if *transpose* is not present, the subroutine returns the output in transposed form if the output array has a shape which is the transpose of the shape of the input array `X`. Otherwise, the output is returned in normal form, as follows:

- *n1* = *n3*, the output array is stored in transposed form and has dimensions *n3* × *n2* × *n1*
- *n1* ≠ *n3* and the output array has dimensions *n3* × *n2* × *n1*, the output array is stored in transposed form.
- *n1* ≠ *n3* and the output array has dimensions *n1* × *n2* × *n3*, the output array is stored in normal form.

Specified as: a single character; *transpose* = `'T'` or `'N'`.

isign

controls the direction of the transform, determining the sign, *isign*, of the exponent of W_n, where:

If *isign* = positive value, *Isign* = + (transforming time to frequency).

If *isign* = negative value, *Isign* = − (transforming frequency to time).

Type: **optional**

Default: *isign* is a positive value.

Specified as: a fullword integer; where *isign* > 0 or *isign* < 0.

scale

is the scaling constant *scale*.

Type: **optional**

Default: *scale* has the following value, where:

- If *isign* is a positive value or is not specified, *scale* = 1.
- If *isign* is negative, *scale* = 1/((*n1*)(*n2*)(*n3*)).

Specified as: a number of the data type indicated in Table 145 on page 976, where *scale* > 0.0 or *scale* < 0.0.

On Return

x

can only be returned in the complex-to-complex subroutines (case 1) and has the following meaning:

- If *y* is specified, array `Y` contains the result of the computation and *x* is not returned.
- If *y* is not specified, array `X` contains the result of the computation, where:

  If *transpose* = `'T'`, *x* is returned in transposed form; that is, *x* has dimensions *n3* × *n2* × *n1*. In this case, *n1* must equal *n3*.

If *transpose* = `'N'`, *x* is returned in normal form; that is, *x* has dimensions *n1* × *n2* × *n3*.

In this case, output overwrites input.

Returned as: an assumed-shape array with shape (**:**,**:**,**:**), containing numbers of the data type indicated in Table 145 on page 976. This array must be aligned on a doubleword boundary.

y

has the following meaning:

- If *y* is specified, array `Y` contains the results of the computation, where:

  If *transpose* = `'T'`, *y* is returned in transposed form; that is, *y* has dimensions *n3* × *n2* × *n1*

  If *transpose* = `'N'`, *y* is returned in normal form; that is, *y* has dimensions *n1* × *n2* × *n3*

  In this case, `X` and `Y` must have no common elements; otherwise, results are unpredictable.

- For case 1, if *y* is not specified, array `X` contains the result of the computation.

Type: **optional** (case 1); **required** (cases 2 and 3)

Default: Array `X` contains the result of the computation. This array must be aligned on a doubleword boundary.

Returned as: an assumed-shape array with shape (**:**,**:**,**:**), containing the numbers of the data type indicated in Table 145 on page 976. This array must be aligned on a doubleword boundary.

Notes and Coding Rules

1. All subroutines accept lowercase letters for the *transpose* argument.
2. The *x*, *y*, and *scale* arguments must be of the same precision.
3. The assumed shape arrays must have no common elements; otherwise, results are unpredictable.
4. If the output array does not have the correct shape for storing the transformed data, an error message is issued and the application program is terminated. However, this error check occurs before the problem size is verified to consist of only valid transform lengths. This means that if you correct the shape and rerun the program, you may still receive an additional error message if the values for the problem size are not valid transform lengths.
5. An example of the use of this subroutine in a thermal diffusion application program is shown in Appendix B. See subroutine fourier in "Module Fourier (HPF)" on page 1046.
6. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
7. Block-column data distribution is required for your array data. In general, distributing your data evenly provides the best work load balance among the processes and allows the use of the most efficient collective communication. However, for your specific problem size and number of processes available, experimentation is necessary to achieve optimal performance.

Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your sequences, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see “Sequences (Fourier Transform)” on page 81. For a sample program including directives, see Figure 10 on page 109.

8. The restrictions given in “Notes and Coding Rules” on page 770, “Notes and Coding Rules” on page 779, and “Notes and Coding Rules” on page 786 also apply to this subroutine.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in “Error Conditions” on page 770, “Error Conditions” on page 779, and “Error Conditions” on page 786 also apply to these subroutines.

Input-Argument Errors for Case 1

Stage 1

1. The rank of the ultimate align target is greater than 1 for *x* or *y*.
2. The process rank is not the same for *x* and *y*.
3. The process rank is not 1 for *x* or *y*.

Stage 2

1. The process grid is not the same for *x* and *y*.
2. The data distribution is unsupported for *x* or *y*.

Stage 3

1. *transpose* is present and *transpose* ≠ `'N'` or `'T'`

Stage 4

1. If *transpose* = `'T'` and *y* is present and the shape of *y* is not *n3* × *n2* × *n1*.
2. If *transpose* = `'N'` and *y* is present and the shape of *y* is not *n1* × *n2* × *n3*.
3. If *y* is not present (which means *x* contains the result of the computation) and *transpose* = `'T'` and *n1* ≠ *n3*.

Input-Argument Errors for Cases 2 and 3

Stage 1

1. The rank of the ultimate align target is greater than 1 for *x* or *y*.
2. The process rank is not the same for *x* and *y*.
3. The process rank is not 1 for *x* or *y*.

Stage 2

1. The process grid is not the same for *x* and *y*.
2. The data distribution is unsupported for *x* or *y*.

Stage 3

1. The shape of the assumed-shape arrays *x* and *y* are incompatible, as follows:

   For case 2:

   2(size(*y*,3)) ≠ *n1* or
   size(*y*,2) ≠ *n2* or
   size(*y*,1) ≠ *n3*

For case 3:

size(y,3) $\neq$ $n3$ or
size(y,2) $\neq$ $n2$ or
size(y,1)) $\neq$ $n1$

Example 1: This example shows how to compute a complex-to-complex, three-dimensional transform. As in “Example” on page 764, the array data is block-plane distributed using a one-dimensional process grid with two processes. The arrays are declared as follows:

```
COMPLEX*16 X(0:3,0:3,0:1)
COMPLEX*16 Y(0:3,0:3,0:1)
REAL*8 SCALE
```

Input

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, *, BLOCK) ONTO PROC :: X, Y

CALL FFT( X , Y )
-or-
CALL FFT( X , Y , TRANSPOSE='N', ISIGN=1 , SCALE=1.0D0 )
```

Following is matrix ***X*** in two planes:

Plane 0:

```
┌                                              ┐
| (1.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
└                                              ┘
```

Plane 1:

```
┌                                              ┐
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
| (0.0,0.0)  (0.0,0.0)  (0.0,0.0)  (0.0,0.0) |
└                                              ┘
```

Output: Following is matrix ***Y***:

Plane 0:

```
┌                                              ┐
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
└                                              ┘
```

Plane 1:

```
┌                                              ┐
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
└                                              ┘
```

Example 2: This example shows how to compute a real-to-complex, three-dimensional transform. As in “Example” on page 780, the array data is block-plane distributed using a one-dimensional process grid with two processes. The arrays are declared as follows:

```
REAL*8 X(0:8,0:3,0:3)
COMPLEX*16 Y(0:4,0:3,0:1)
REAL*8 SCALE
```

Input

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, *, BLOCK) ONTO PROC :: X, Y

CALL FFT( X(0:3,0:3,0:3) , Y(0:3,0:3,0:1) )
-or-
CALL FFT( X(0:3,0:3,0:3) , Y(0:3,0:3,0:1) , ISIGN=1 , SCALE=1.0D0 )
```

Following is matrix ***X*** in four planes:

Plane 0:

```
┌                    ┐
| 1.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Plane 1:

```
┌                    ┐
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Plane 2:

```
┌                     ┐
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
└                     ┘
```

Plane 3:

```
┌                     ┐
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│ 0.0  0.0  0.0  0.0 │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
│  .    .    .    .  │
└                     ┘
```

Output: The following matrix ***Y*** is returned in transposed form and stored in FFT-packed storage mode:

Plane 0:

```
┌                                                ┐
│ (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) │
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│ (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) │
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│     .          .          .          .     │
└                                                ┘
```

Plane 1:

```
┌                                                ┐
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│ (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) │
│     .          .          .          .     │
└                                                ┘
```

Example 3: This example shows how to compute a complex-to-real, three-dimensional transform. As in “Example” on page 787, the array data is block-plane distributed using a one-dimensional process grid with two processes. Array `Y` is returned in transposed form. The arrays are declared as follows:

```
COMPLEX*16 X(0:4,0:3,0:1)
REAL*8 Y(0:8,0:3,0:3)
REAL*8 SCALE
```

Input:

```
!HPF$ PROCESSORS PROC(2)
!HPF$ DISTRIBUTE (*, *, BLOCK) ONTO PROC :: X, Y

CALL FFT( X(0:3,0:3,0:1) , Y(0:3,0:3,0:3) , ISIGN=-1 )
-or-
CALL FFT( X(0:3,0:3:0:1) , Y(0:3,0:3,0:3) , ISIGN=-1 , SCALE=1.0D0/64.0D0 )
```

The following matrix ***X*** is stored in FFT-packed storage mode:

Plane 0:

```
┌                                              ┐
| (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,1.0)  (1.0,0.0)  (1.0,1.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
|     .          .          .          .     |
└                                              ┘
```

Plane 1:

```
┌                                              ┐
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
| (1.0,0.0)  (1.0,0.0)  (1.0,0.0)  (1.0,0.0) |
|     .          .          .          .     |
└                                              ┘
```

Output: The following matrix ***Y*** is returned:

Plane 0:

```
┌                    ┐
| 1.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Plane 1:

```
┌                    ┐
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Plane 2:

```
┌                    ┐
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Plane 3:

```
┌                    ┐
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
| 0.0  0.0  0.0  0.0 |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
|  .    .    .    .  |
└                    ┘
```

Chapter 17. Random Number Generation (HPF)

This chapter describes the random number generation subroutine that can be called from an HPF program.

Overview of the Random Number Generation Subroutine

The random number generation subroutine generates uniformly distributed random numbers.

Table 146. List of Random Number Generation Subroutine (HPF)

Descriptive Name	**Long-Precision Subroutine**	**Page**
Uniform Random Number Generator	URNG	989

Random Number Generation Subroutine

This section contains the random number generation subroutine.

URNG—Uniform Random Number Generator

This subroutine generates a vector ***x*** of uniform pseudo-random numbers in the ranges (0,1) or (–1,1), depending on the *iopt* argument. The random numbers are generated using the multiplicative congruential method with a user-specified seed, as follows:

$s_i = (a(s_{i-1})) \bmod (m) = (a^i s_0) \bmod (m)$
$x_i = s_i/m$ if $iopt = 0$
$x_i = (2s_i/m)-1$ if $iopt = 1$
for $i = 1, 2, ..., n$

where:

s_0 is the initial seed provided by the caller.
s_i for $i = 1, n$ is a random sequence.
x_i for $i = 1, n$ are the random numbers.
$a = 44485709377909.0$
$m = 2.0^{48}$
n = size(x) = number of random numbers to be generated.

If the assumed-shape array has a size of zero, no computation is performed, the subroutine returns, and the initial seed is unchanged.

The process rank must be one for the array data in this subroutine; that is, given *np* processes, you must use a $1 \times np$ or $np \times 1$ process grid. In addition, the array data must be **evenly distributed** across all processes, using **block distribution**.

Note: This differs from the message-passing version of this subroutine, PDURNG, which supports block-cyclic data distribution.

See references [8], [41], [42], [47], [48], [49], and [50].

Table 147. Data Types

x, *seed*	**Subroutine**
Long-precision real	URNG

Syntax

HPF	CALL URNG (*seed*, *x*) CALL URNG (*seed*, *x*, *iopt*)

On Entry

seed
is the initial value s_0 used to generate the random numbers.

Type: **required**

Specified as: a number of the data type indicated in Table 147. You should specify *seed* to be an **odd**, **whole** number; otherwise, this subroutine sets it to an odd, whole number and continues with the computation. The value of *seed* must be $1.0 \leq seed < 2.0^{48}$.

x
See On Return.

iopt
indicates the range of uniform random numbers to generate, where:

If *iopt* = 0, the range is (0,1).

If *iopt* = 1, the range is (−1,1).

Type: **optional**

Default: *iopt* = 0

Specified as: a fullword integer; *iopt* = 0 or 1.

On Return

seed
is the new seed that is to be used to generate additional random numbers in subsequent invocations of this subroutine, having a value of *seed* = $(a^n s_0)$ mod (m).

Type: **required**

Returned as: a number of the data type indicated in Table 147 on page 989. It is an **odd**, **whole** number, where $1.0 \le seed < 2.0^{48}$.

x
is the vector ***x***, containing the uniform pseudo-random numbers, where:

If *iopt* = 0, they are in the range (0,1).

If *iopt* = 1, they are in the range (−1,1).

Type: **required**

Returned as: an assumed-shape array with shape (**:**), containing numbers of the data type indicated in Table 147 on page 989, where size(*x*)mod(*np*) = 0.

Notes and Coding Rules

1. To generate more than $(2^{31}-1)$ random numbers, you should make multiple calls to this subroutine.
2. For details on how to set up and code your HPF program using Parallel ESSL, see "Coding Your HPF Program" on page 105
3. Block data distribution is required for your array data, where the array data is evenly distributed across all processes. Because data directives are included in the interface module PESSL_HPF, you can specify any data distribution for your vector, and the XL HPF compiler will, if necessary, redistribute the data prior to calling this subroutine. For how to code your HPF directives, see "Distributing Data in an HPF Program" on page 79.

Error Conditions: HPF-specific errors are listed below. Resource and input-argument errors listed in "Error Conditions" on page 795 also apply to this subroutine.

Input-Argument Errors

Stage 1

1. The rank of the ultimate align target is greater than 1 for *x*.
2. The process rank is not 1 for *x*.

Stage 2: The vector for *x* is replicated.

Stage 3: The size of the assumed-shape array for *x* is invalid—that is, it is not a multiple of the product of the blocksize and the number of processes.

Example: This example generates 30 random numbers in vector ***x***. The array data is block distributed, evenly, over 5 processes, unlike the array data in “Example” on page 795 which is block-cyclically distributed.

```
!HPF$ PROCESSORS PROC(5)
!HPF$ DISTRIBUTE (BLOCK) ONTO PROC :: X

CALL URNG( SEED , X )
-or-
CALL URNG( SEED , X , IOPT=0 )
```

Input: `SEED` = 31415926535897.0

Output: `SEED` = $(a^{30}s_0) \bmod (m)$ = 6316434292705.0

Vector ***x***:

$$\begin{bmatrix} 0.683821516135299845 \\ 0.058874407800946215 \\ 0.391855250856924187 \\ 0.755994653022330709 \\ 0.557764301423606668 \\ 0.001333801764989317 \\ 0.056855932753212101 \\ 0.331063036202269956 \\ 0.347339794409027292 \\ 0.649429020370863697 \\ 0.386144876217390021 \\ 0.457224855098420591 \\ 0.892518134165118937 \\ 0.074548748224632532 \\ 0.912379366805073033 \\ 0.112809499110515077 \\ 0.857547605095465570 \\ 0.756480901897081282 \\ 0.046993364463578046 \\ 0.889457684002341153 \\ 0.167775766106718294 \\ 0.504952722600595649 \\ 0.999725924546471134 \\ 0.696269487398215148 \\ 0.671896598019703362 \\ 0.271472156040264423 \\ 0.566418406688985243 \\ 0.464684865759100063 \\ 0.982442539763031419 \\ 0.022440482512937620 \end{bmatrix}$$

Part 4. Appendixes

Appendix A. BLACS Quick Reference Guide

This quick reference guide shows the Fortran calling sequences for the BLACS subroutines, which are included with Parallel ESSL.

If you are coding your program in C or C++, you must pass the BLACS arguments by reference to the Parallel ESSL subroutine. See “Initializing the BLACS” on page 87 for sample C and C++ calling sequences that show arguments passed by reference.

To receive a complete copy of the *BLACS User's Guide* send email to `netlib@ornl.gov` and in the mail message type: `send blacs_ug.ps from blacs`.

See reference [52].

Notes:

1. In the calling sequences, an underlined argument indicates it is an output argument.
2. In the subroutine names, GE indicates general rectangular matrix and TR indicates trapezoidal matrix.

BLACS Initialization Subroutines

CALL BLACS_PINFO (*mypnum*, *nprocs*)
CALL BLACS_SETUP (*mypnum*, *nprocs*)
CALL BLACS_GET (*icontxt*, *what*, *val*)
CALL BLACS_SET (*icontxt*, *what*, *val*)
CALL BLACS_GRIDINIT (*icontxt*, *order*, *nprow*, *npcol*)
CALL BLACS_GRIDMAP (*icontxt*, *usermap*, *ldumap*, *nprow*, *npcol*)

BLACS Deallocating Resources Subroutines

CALL BLACS_FREEBUFF (*icontxt*, *wait*)
CALL BLACS_GRIDEXIT (*icontxt*)
CALL BLACS_ABORT (*icontxt*, *errornum*)
CALL BLACS_EXIT (*doneflag*)

BLACS Sending Subroutines

CALL SGESD2D | DGESD2D | CGESD2D | ZGESD2D | IGESD2D (*icontxt*, *m*, *n*, *a*, *lda*, *rdest*, *cdest*)
CALL SGEBS2D | DGEBS2D | CGEBS2D | ZGEBS2D | IGEBS2D (*icontxt*, *scope*, *top*, *m*, *n*, *a*, *lda*)
CALL STRSD2D | DTRSD2D | CTRSD2D | ZTRSD2D | ITRSD2D (*icontxt*, *uplo*, *diag*, *m*, *n*, *a*, *lda rdest*, *cdest*)
CALL STRBS2D | DTRBS2D | CTRBS2D | ZTRBS2D | ITRBS2D (*icontxt*, *scope*, *top*, *uplo*, *diag*, *m*, *n*, *a*, *lda*)

BLACS Receiving Subroutines

CALL SGERV2D | DGERV2D | CGERV2D | ZGERV2D | IGERV2D (*icontxt*, *m*, *n*, <u>*a*</u>, *lda*, *rsrc*, *csrc*)
CALL SGEBR2D | DGEBR2D | CGEBR2D | ZGEBR2D | IGEBR2D (*icontxt*, *scope*, *top*, *m*, *n*, <u>*a*</u>, *lda*, *rsrc*, *csrc*)
CALL STRRV2D | DTRRV2D | CTRRV2D | ZTRRV2D | ITRRV2D (*icontxt*, *uplo*, *diag*, *m*, *n*, <u>*a*</u>, *lda rsrc*, *csrc*)
CALL STRBR2D | DTRBR2D | CTRBR2D | ZTRBR2D | ITRBR2D (*icontxt*, *scope*, *top*, *uplo*, *diag*, *m*, *n*, <u>*a*</u>, *lda*, *rsrc*, *csrc*)

BLACS Global Operation Subroutines

Absolute Maximum Value of a General Matrix	CALL SGAMX2D | DGAMX2D | CGAMX2D | ZGAMX2D | IGAMX2D (*icontxt*, *scope*, *top*, *m*, *n*, <u>*a*</u>, *lda*, <u>*ra*</u>, <u>*ca*</u>, *rcflag*, *rdest*, *cdest*)
Absolute Minimum Value of a General Matrix	CALL SGAMN2D | DGAMN2D | CGAMN2D | ZGAMN2D | IGAMN2D (*icontxt*, *scope*, *top*, *m*, *n*, <u>*a*</u>, *lda*, <u>*ra*</u>, <u>*ca*</u>, *rcflag*, *rdest*, *cdest*)
Sum of the Elements of a General Matrix	CALL SGSUM2D | DGSUM2D | CGSUM2D | ZGSUM2D | IGSUM2D (*icontxt*, *scope*, *top*, *m*, *n*, <u>*a*</u>, *lda*, *rdest*, *cdest*)

BLACS Informational and Miscellaneous Subroutines

Note: BLACS_PNUM returns an integer.

CALL BLACS_GRIDINFO (*icontxt*, <u>*nprow*</u>, <u>*npcol*</u>, <u>*myprow*</u>, <u>*mypcol*</u>)
ipnum = BLACS_PNUM (*icontxt*, *prow*, *pcol*)
CALL BLACS_PCOORD (*icontxt*, *pnum*, <u>*prow*</u>, <u>*pcol*</u>)
CALL BLACS_BARRIER (*icontxt*, *scope*)

Data Types

This section shows the type of data for each BLACS argument.

Data Type	**Argument**
Character	*diag*, *order*, *scope*, *top*, and *uplo*
Integer	`A(LDA,*)`, BLACS_PNUM, `CA(*)`, *cdest*, *csrc*, *doneflag*, *errornum*, *icontxt*, KBRID, KBSID, KRECVID, KSENDID, *lda*, *ldumap*, *m*, *mypcol*, *mypnum*, *myprow*, *n*, *npcol*, *nprocs*, *nprow*, *ntasks*, *pcol*, *pnum*, *prow*, `RA(*)`, *rcflag*, *rdest*, *rsrc*, `TIDS(*)`, `USERMAP(LDUMAP,NPCOL)`, `VAL(*)`, *wait*, *what*
Short-precision real	`A(LDA,*)`
Long-precision real	`A(LDA,*)`, DCPUTIME00, DWALLTIME00
Short-precision complex	`A(LDA,*)`
Long-precision complex	`A(LDA,*)`

Argument Options

1. *order* = `'Row-major'` or `'Column-major'`

   The default is row-major natural ordering.

2. *uplo* = `'Upper triangular'` or `'Lower triangular'`
3. *diag* = `'Nonunit triangular'` or `'Unit triangular'`
4. *scope* = `'All'`, `'Row'`, or `'Column'`
5. For broadcast topologies, *top* = `''`, `'I'`, `'D'`, `'H'`, `'S'`, `'F'`, `'M'`, `'T'`, `'1'`, `'2'`, `'3'`, `'4'`, `'5'`, `'6'` `'7'`, `'8'`, or `'9'`

   For global topologies, *top* = `''`, `'H'`, `'F'`, `'T'`, `'1'`, `'2'`, `'3'`, `'4'`, `'5'`, `'6'` `'7'`, `'8'`, or `'9'`

In the MPI implementation of the BLACS for Parallel ESSL for AIX, the broadcast and global topology arguments are ignored. In all cases, the equivalent MPI collective communication subroutine is used.

Appendix B. Sample Programs

This appendix contains information about sample programs provided with the Parallel ESSL product.

Sample Programs and Utilities Provided with Parallel ESSL

A variety of sample programs are shipped with the Parallel ESSL product in the following directories:

- `/usr/lpp/pessl.rte.common/example/c`
- `/usr/lpp/pessl.rte.common/example/fortran`
- `/usr/lpp/pessl.rte.common/example/hpf`

For file names and installation procedures, see the *Parallel ESSL Installation Memo*.

The sample programs include:

- A diffusion calculation example program, which is discussed at length later in this chapter. Three different versions of this sample program, in C, Fortran, and HPF, are provided.
- Three sample message passing programs, image.f, pdgexmp.f, and simple.f, plus utilities in a utility library. These are provided in the Fortran directory, along with a makefile. More information regarding these programs and utilities is provided in the file `/usr/lpp/pessl.rte.common/example/fortran/examples.readme`. Following is a description of their functions:
  - The image program demonstrates the use of the complex to real Fourier transform subroutines and calculates a correlation between two images.
  - The pdgexmp program is a simple performance program, allowing you to vary the processor size and shape, as well as the block size, to gauge their effect on the performance of solutions to linear equations.
  - The final program, simple.f, is an example program showing how to set up and use the sample utilities.
  - The sample utility library consists of a set of Fortran 90 routines implementing commonly-used message passing functions. This example subroutine library provides you with a simplified interface to some very commonly-used message passing routines, including:
    - Library initialization:

initutils	BLACS initialization and initialization of library variables. This must be the first routine called in order to use the remainder of the library.
exitutils	Performs a BLACS grid exit and marks the library as uninitialized.

    - Scatter operation:

scatter	Scatter a matrix on a single node to a distributed matrix.

    - Gather operation:

gather	Gather a matrix onto a single node from a distributed matrix.

- Nearest neighbor communication:

sendnorthborder	Sends top row of each block to north processor.
sendwestborder	Sends first column of each block to west processor.
sendsouthborder	Sends last row of each block to south processor.
sendeastborder	Sends last column of each block to east processor.
rcvnorthborder	Receives last row of each block from north processor.
rcvwestborder	Receives last column of each block from west processor.
rcvsouthborder	Receives top row of each block from south processor.
rcveastborder	Receives first column of each block from east processor.

- Array creation and descriptor vector initialization:

create_array	Allocates memory for array and initializes array descriptor.

- Global-to-local mapping routines:

g2l	Creates global to local index arrays.
l2g	Creates local to global index arrays.
number_row_blocks	Returns number of local row blocks in an array.
number_col_blocks	Returns number of local column blocks in an array.
last_row_block_size	Returns size of last local row block in an array.
last_col_block_size	Returns size of last local column block an in array.

- Three sample message passing programs using the sparse linear algebraic equations subroutines plus some utility programs. These programs are discussed in "Sample Sparse Linear Algebraic Equations Programs" on page 1055. They are provided in the Fortran directory along with a makefile. More information regarding these programs and utilities is provided in the file `/usr/lpp/pessl.rte.common/example/fortran/examples.readme`.

Sample Thermal Diffusion Programs

Both a message passing Fortran 90 sample program and an HPF sample program are presented in this section, along with the commands for processing them in a parallel processing environment. The sample programs, solving a thermal diffusion problem, use vectors and matrices distributed across a one-dimensional process grid and call Level 2 PBLAS, Eigensystem analysis, and Fourier transform subroutines.

A copy of these sample programs, plus an equivalent C program, are provided with the Parallel ESSL product. For file names and installation procedures, see the *Parallel ESSL Installation Memo*.

Following is a table of contents for this section, along with a description of each section for both the message passing Fortran 90 sample program and the HPF sample program.

Table 148 (Page 1 of 2). Table of Contents for the Sample Thermal Diffusion Programs

Contents	Page (Message Passing)	Page (HPF)
Thermal Diffusion Discussion Paper: A technical description of the problem to be solved.	1002	1002
Program main: Finds the cooling rate for a specified set of points in an anisotropic rectangular beam, immersed in a constant heat bath with a temperature of zero.	1006	1036
Module parameters: Defines system wide parameters and the index structure used to help map global indices to local indices.	1012	1042
Module diffusion: Assigns problem parameters and initial data. **Subroutine init_diffusion:** Initializes the problem size, the number of output points, the functional form of the diffusion constant, and the initial temperature distribution. **Subroutine init_temp:** Returns the initial temperature of the bar at a particular point. **Subroutine diff_coef:** Returns the value of the thermal diffusion coefficient at an arbitrary point.	1013	1042
Module fourier: Represents both the diffusion operator and the temperature profile in a sine function basis. **Subroutine get_diffusion_matrix:** Obtains the matrix representation of the diffusion operator in a sine function basis. **Subroutine expand_temp_profile:** Obtains the expansion coefficients of the initial temperature profile in a sine function expansion. **Subroutine factor_nodes:** Obtains the powers of prime factorization of the number of nodes, failing if the factorization is not compatible with FFT supported transform lengths. **Subroutine min_power2:** Obtains the smallest number which is a power of 2 and greater than or equal to the input argument.	1017	1046
Module scale: Initializes the communications and provides a few communication utility routines. **Subroutine initialize_scale:** Initializes BLACS and calculates a block size. **Subroutine initialize_rarray:** Allocates space for a real array and creates the associated descriptor array and index array. **Subroutine initialize_carray:** Allocates space for a complex array and creates the associated descriptor array and index array. **Subroutine clocal_to_rglobal:** Gathers the real parts of the local portions of the block-cyclically-distributed complex array to generate the corresponding global matrix. **Subroutine rlocal_to_rglobal:** Gathers the local portions of the block-cyclically-distributed real array to generate the corresponding global matrix.	1025	—
Input Data: Sample input data in namelist format, used by subroutine init_diffusion in module diffusion.	1052	1052
Output Data: Sample output data, based on the sample input data, issued at the end of program main.	1052	1052
Makefile: The makefile used to build the thermal diffusion program.	1097	1101

Table 148 (Page 2 of 2). Table of Contents for the Sample Thermal Diffusion Programs

Contents	Page (Message Passing)	Page (HPF)
Run Script: The script file used to execute the thermal diffusion program.	1101	1101

Thermal Diffusion Discussion Paper

The objective of the diffusion program is to solve for the temperature of a beam at any point and at any time, given an initial temperature distribution. The following assumptions are made concerning the beam and its properties:

- The beam has the dimensions l_x, l_y, and l_z, where $l_z >> l_x$ and $l_z >> l_y$.
- The thermal diffusion coefficient is a function of only:

  $$\bar{x} \text{ and } \bar{y}$$

  That is, the physical properties of the beam do not depend on the direction:

  $$\bar{z}$$

  Note: The bar over a variable indicates it has dimension; whereas, the unbarred form is dimensionless.
- The beam is immersed in a zero degree heat bath.

The general diffusion equation is given by:

$$\frac{\partial}{\partial \bar{t}} T(\bar{x},\bar{y},\bar{z},\bar{t}) = \nabla \cdot \overline{D} \nabla T(\bar{x},\bar{y},\bar{z},\bar{t}) \qquad (1)$$

where:

$$\overline{D}$$

is the position-dependent diffusion coefficient. Equation 1 may be rewritten, ignoring the *z* dimension, using the following dimensionless variables:

$$x = \frac{\pi \bar{x}}{l_x}$$

$$y = \frac{\pi \bar{y}}{l_y}$$

$$D = \frac{\bar{D}}{D_r}$$

$$t = \frac{l_x^2 \bar{t}}{\pi^2 D_r}$$

as follows:

$$\frac{\partial}{\partial t} T(x,y,t) = \left(\frac{\partial}{\partial x} D \frac{\partial}{\partial x} + \frac{\partial}{\partial y} l_r^2 D \frac{\partial}{\partial y} \right) T(x,y,t) \qquad (2)$$

where D_r is a reference diffusion constant, and l_r is the beam dimension ratio l_x/l_y. This equation is subject to the initial and boundary conditions given by:

$T(x, y, 0) = T_0(x, y)$
$T(0, y, t) = T(x, 0, t) = T(\pi, y, t) = T(x, \pi, t) = 0$

In the program, the initial condition $T_0(x, y)$, the diffusion coefficient $D(x, y)$, and the ratio l_r are determined in the initialization subroutine **init_diffusion**. n_x and n_y, defined later, are also initialized here. Equation 2 is solved by representing the operators in a sine function basis and solving the resulting matrix equations. We begin by expanding T in sine functions, as follows:

$$T(x,y,t) = \frac{2}{\pi} \sum_{k'=1}^{\infty} \sum_{j'=1}^{\infty} a_{k'j'}(t) \sin(k'x) \sin(j'y) \qquad (3)$$

The initial set of expansion coefficients $a_{kj}(0)$ are determined from the initial temperature profile by:

$$a_{kj}(0) = \frac{2}{\pi} \int_0^{\pi} \int_0^{\pi} \sin(kx) \sin(jy) T_0(x,y)\, dx\, dy \qquad (4)$$

where the orthogonality of the sine functions has been used. If we extend the range of T from π to 2π, as an odd function, equation 4 can be written in terms of a discrete Fourier transform:

$$a_{kj}(0) = \frac{-1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} e^{-ikx-ijy} T_0(x,y)\, dx\, dy \qquad (5)$$

$a_{kj}(0)$ is calculated in the program in the subroutine **expand_temp_profile**, with the actual temperatures calculated in the function **init_temp**. The subroutine call to DCFT2 performs the Fourier transform, and the results are stored in the array `A`. Substitute equation 3 into equation 2, multiply by $(2/\pi)\sin(kx)\sin(jy)$ and integrate over x and y to obtain:

$$\frac{\partial a_{kj}(t)}{\partial t} = -\sum_{k'=1}^{\infty}\sum_{j'=1}^{\infty} F_{kj;k'j'} a_{k'j'}(t) \qquad (6)$$

where:

$$F_{kj;k'j'} = \frac{-4}{\pi^2}\int_0^{\pi}\int_0^{\pi} \sin(kx)\sin(jy) D_{op} \sin(k'x)\sin(j'y)\, dx\, dy \qquad (7)$$

and:

$$D_{op} = \left(\frac{\partial}{\partial x} D \frac{\partial}{\partial x} + \frac{\partial}{\partial y} l_r^2 D \frac{\partial}{\partial y}\right) \qquad (8)$$

Note that $\sin(kx)\sin(k'x)$ and $\sin(jy)\sin(j'y)$ are even functions about π. Therefore, if we define $D(2\pi-x, y) = D(x, y)$ and $D(x, 2\pi-y) = D(x, y)$, where $0 \le x \le \pi$ and $0 \le y \le \pi$, the limits on the integral in equation 7 may be extended to 2π:

$$F_{kj;k'j'} = \frac{1}{\pi^2}\int_0^{2\pi}\int_0^{2\pi} \Big(kk' D\cos(kx)\cos(k'x)\sin(jy)\sin(j'y) + l_r^2 jj' D\cos(jy)\cos(j'y)\sin(kx)\sin(k'x)\Big)\, dx\, dy \qquad (9)$$

Equation 9 may be further reduced to sums of Fourier transforms using the identities:

$\cos(kx)\cos(k'x) = (1/2)(\cos((k-k')x)+\cos((k+k')x))$
$\sin(kx)\sin(k'x) = (1/2)(\cos((k-k')x)-\cos((k+k')x))$
$\sin(kx) = (i/2)(e^{ikx}-e^{-ikx})$
$\cos(kx) = (1/2)(e^{ikx}+e^{-ikx})$

Substituting these identities into equation 9, we have:

$$F_{kj;k'j'} = \left(kk' + l_r^2 jj'\right)\left(\tilde{D}_{k^-j^-} - \tilde{D}_{k^+j^+}\right) + \left(kk' - l_r^2 jj'\right)\left(\tilde{D}_{k^+j^-} - \tilde{D}_{k^-j^+}\right) \qquad (10)$$

where:

$k^+ = k+k'$
$k^- = |k-k'|$
$j^+ = j+j'$
$j^- = |j-j'|$

and where:

$$\widetilde{D}_{kj} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} D e^{-ikx} e^{-ijy} dx\ dy$$

Equation 10 is the simplest form for the matrix elements of the diffusion operator, and these elements are calculated in the subroutine **get_diffusion_matrix**. The diffusion coefficient is evaluated with the function **diff_coef**. The Parallel ESSL Fourier transform subroutine PDCFT2, called in subroutine **get_diffusion_matrix**, is used to determine the following elements:

$$\widetilde{D}_{kj}$$

which are stored in the array DF. Because DF is an array which is block cyclically distributed among the nodes and each node requires elements of DF not locally available, this array is collected to the global array DFG on each node. The array DFG is subsequently used to calculate the matrix *F*. Now that we have determined the matrix elements of equation 6, we must truncate it in order to solve it. This may be done by truncating the *k* summation at n_x and the *j* summation at n_y. The dual indices may be collapsed into a single index 1 by:

$l = (j-1)n_x + k$

The *j* and *k* indices can similarly be recovered from the 1 index by:

$j = ((l-1)/n_x)+1$
$k = \text{mod}(l-1, n_x)+1$

Rewriting the truncated equation 6, we have:

$$\frac{\partial a_t(t)}{\partial t} = -\sum_{l'=1}^{n_x n_y} F_{l;l'} a_{l'}(t) \qquad (11)$$

Equation 11 has the general solution:

$$a_l(t) = \sum_{l'=1}^{n_x n_y} e^{-\lambda_{l'} t} B_{ll'} \left[\sum_{m=1} B_{ml'} a_m(0) \right] \qquad (12)$$

where λ is the eigenvalue vector and *B* is the matrix of eigenvectors of the matrix *F*. The eigenvectors *B* and eigenvalues λ correspond to the arrays B and LAMBDA in the program. These eigenvalues and eigenvectors are determined with the Parallel ESSL subroutine PDSYEVX. The inner sum:

$$\sum_{m=1} B_{ml'} a_m(0)$$

corresponds to the array AB, with ABT containing the extra factor of:

$$e^{-\lambda_l t}$$

Also, $a_l(t)$ is represented by the array `X`, which is reused for each solution time.

Each of these matrix multiplications are done using the Parallel ESSL subroutine PDGEMV. The final solution is obtained by summing over the expansion coefficients:

$$T(x,y,t) = \sum_{k=1}^{n_x}\sum_{j=1}^{n_y} \sin(kx)\sin(jy)a_{kj}(t) \qquad (13)$$

The final temperature array is `TEMP` in the program, with the indices into the array corresponding to specific values of *x*, *y*, and *t*.

Program Main (Message Passing)

```
        program main
!
!   Purpose, to  find the cooling rate for a specified set of
!   points in an anisotropic rectangular beam, immersed in a constant
!   heat bath with a temperature of 0.
!
!   Routines called:
!     expand_temp_profile
!     get_diffusion_matrix
!     igamx2d
!     init_diffusion
!     initialize_rarray
!     initialize_scale
!     pdgemv
!     pdsyevx
!     rlocal_to_rglobal
!
        use parameters
        use scale
        use diffusion
        use fourier
        implicit none

        integer :: n, ix, jx, iy, jy, k, i, j, stat, it, ib, ig
        integer :: num_errors, lwork, ilwork
        integer, allocatable :: iwork(:)
        real(long), allocatable :: work(:)
!
!       a contains the sine expansion coefficients of the initial
!                temperature profile.
!       b contains the eigenvectors of the diffusion operator in the
!                sine function basis.
!       ab contains the initial temperature profile expanded in the
!                eigenvectors of the diffusion operator.
!       f contains the matrix elements of the diffusion operation in
!                sine function basis.
!       lambda contains the eigenvalues of the diffusion operator.
```

```
!
!       df contains the Fourier transform of the diffusion coefficient function.
!
        real(long), allocatable ::  lambda(:), xg(:,:)
        real(long), allocatable ::  gap(:)
        real(long) :: dum

        real(long), pointer :: f(:,:), b(:,:), a(:,:)
        type (g_index), pointer :: f_i, b_i, a_i
        integer :: f_d(DESC_DIM), b_d(DESC_DIM), a_d(DESC_DIM)

        real(long), pointer :: x(:,:), ab(:,:), abt(:,:)
        type (g_index), pointer :: x_i, ab_i, abt_i
        integer :: x_d(DESC_DIM), ab_d(DESC_DIM), abt_d(DESC_DIM)

        real(long), allocatable :: xsines(:,:), ysines(:,:), temp(:,:)
        integer, allocatable :: ifail(:), iclustr(:)
        integer :: biga_index, num_eigvalues, num_vectors, info


!
!   Read in the problem size, initialize the problem dimensions,
!   choose functional form for the spatially dependent heat diffusion
!   coefficients, choose functional form of initial temperature distribution
!   and choose the number of points, in both space and time, of the solution
!   to print out.
!
        call init_diffusion
        num_errors = 0
!
!       Read in how many sine functions to include in both the
!            x and y directions.
!
!       Because we need to get the Fourier coefficients of the sums
!       and differences of the indices, we need to include twice as
!       many Fourier coefficients as the number of sine expansion coefficients.
!       Also, because the top and bottom halves of the Fourier
!       transform are identical,
!       an artifact of artificially extending the diffusion coefficient
!       function and the initial temperature distribution, we need to
!       double the number of Fourier coefficients again. Finally, the
!       extra sum of one comes from the fact that the sine function
!       expansion starts a 1 and the Fourier expansion starts at 0.
!

!  n is the order of the diffusion operator equation.
        n = dif_nx * dif_ny
!
!  Initialize BLACS and calculate default block sizes.
!
        call initialize_scale(n, biga_index)



!
!
!   Allocate room for the eigenvalues of diffusion operator.
!
        allocate( lambda(n), stat=stat)
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for sines of x and y coordinates.
!
        allocate( xsines(dif_npts, dif_nx) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( ysines(dif_npts, dif_ny) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for temperature history at selected points.
!
        allocate( temp(dif_npts, dif_ntemps) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for global temperature profile expansion vector at time t.
!
        allocate( xg(1, n) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        call igamx2d(sc_icontext,'A',' ',1,1,num_errors,1,-1,-1,-1,            &
     &              -1,-1)
        if( num_errors .gt. 0 ) then
           if( sc_iam .eq. 0 ) then
               write(*,*) 'Error in allocating arrays in main'
               call blacs_abort(sc_icontext, 1)
           endif
        endif


!
!      A call to expand_temp_profile returns the sine expansion
!      coefficients of the initial temperature profile.
!
!
!  Get matrices.
!
!  Diffusion operator matrix.
        call initialize_rarray(f, f_d, f_i, n, n, biga_index)

!  Eigenvectors of diffusion operator matrix.
        call initialize_rarray(b, b_d, b_i, n, n, biga_index)

!  Initial temperature profile, in row vector.
        call initialize_rarray(a, a_d, a_i, 1, n, biga_index)

!  Initial temperature profile, in eigenfunction basis, in row vector.
        call initialize_rarray(ab, ab_d, ab_i, 1, n, biga_index)

!  Temperature profile, at time t, in eigenfunction basis, in row vector.
        call initialize_rarray(abt, abt_d, abt_i, 1, n, biga_index)

!  Temperarure profile in at time t in sine expansion basis, in row vector.
        call initialize_rarray(x, x_d, x_i, 1, n, biga_index)
```

```
!
!  Represent initial temperature in sine function expansion.
!
      call expand_temp_profile(a,a_i,a_d)
!
!  Here, we are calculating the initial set of coefficients
!  in the sine function expansion as given in equations 3 and 4 of
!  the discussion paper
!

!
!     The call to get_diffusion_matrix returns the diffusion
!      operator in the sine function basis.
!
      call get_diffusion_matrix(f,f_i,f_d)
!
!   This last call determines the matrix elements defined by equation 10
!   in the discussion paper.
!



!
!    Here we precalculate the sine functions, sin(kx) and sin(jy) used
!    in equation 13 of the discussion paper.
!    These sine functions are only evaluated at the points x and y for
!    which the solution is evaluated.
!

      do i = 1, dif_nx
        do j = 1, dif_npts
          xsines(j,i) = sqrt(2.d0/pi) * sin( i * dif_x(j))
        enddo
      enddo

      do i = 1, dif_ny
        do j = 1, dif_npts
          ysines(j,i) = sqrt(2.d0/pi) * sin( i * dif_y(j))
        enddo
      enddo



!
!  Allocate arrays for eigenvalue decomposition.
!
      allocate( ifail(n), stat=stat)
      if( stat .ne. 0) num_errors = num_errors + 1
      allocate( iclustr(n), stat=stat)
      if( stat .ne. 0) num_errors = num_errors + 1
      allocate( gap(sc_nnodes), stat=stat)
      if( stat .ne. 0) num_errors = num_errors + 1


!
!  Allocate scratch space for the symetric eigenvector solver.
!
      lwork = 20*n  + max( 5*n, n*(n+ sc_nnodes-1)/sc_nnodes )            &
   &         + n*( (n+ sc_nnodes-1)/sc_nnodes) + 2*f_d(MB_) * f_d(MB_)

      ilwork = 2*n + max((3*n+1+sc_nnodes),max(4*n,14))
```

```
          allocate( work(lwork) , stat=stat )
          if( stat .ne. 0) num_errors = num_errors + 1
          allocate( iwork(ilwork) , stat=stat )
          if( stat .ne. 0) num_errors = num_errors + 1

!
!   Test to see if we had any allocation errors.
!

          call igamx2d(sc_icontext,'A',' ',1,1,num_errors,1,-1,-1,-1,            &
     &             -1,-1)
          if( num_errors .gt. 0 ) then
             if( sc_iam .eq. 0 ) then
                write(*,*) 'Error in allocating arrays for pdsyevx in ',        &
     &                   'main'
                call blacs_abort(sc_icontext, 1)
             endif
          endif

          do i = 1, sc_nnodes
             gap(i) = 0.d0
          enddo
          do i = 1, n
             ifail(i) = 0
             iclustr(i) = 0
          enddo
!
! The call to pdsyevx will find both the eigenvalues and eigenvectors
! of the diffusion matrix operator f. The eigenvalues will be stored in
! the vector lambda and the corresponding eigenvectors will be stored in
! the matrix b.  The f and b matrices in the program correspond to the
! F and B matrices in equations 11 and 12 in the
! discussion paper.
!
!
          call pdsyevx('V','A','U',n,f,1,1,f_d,-1.d30,1.d30,0,n,                &
     &       0.d0,num_eigvalues,num_vectors,lambda,1.d-5,b,1,1,b_d,             &
     &       work, lwork, iwork, ilwork, ifail, iclustr, gap, info)

          if( sc_iam .eq. 0) then
            if( info .ne. 0 ) then
               write(*,*) ' info is ', info
               call blacs_abort(sc_icontext, 1)
            endif
          endif
!
!         Multiply the transpose of the eigenvector matrix, b, with the sine
!         expansion of the initial temperature profile, a, to obtain the
!         initial temperature profile in terms of the eigenfuctions of the
!         diffusion operator.
!
          call pdgemv('T', n, n, 1.d0, b, 1, 1, b_d, a, 1, 1, a_d, 1,            &
     &            0.d0, ab, 1, 1, ab_d, 1)
!
!  This first matrix multiplication, yielding the matrix ab,
!  corresponds to the inner summation in equation 10
!  of the discussion paper.
!

!
!    Calculate temperature profile for each time step.
!
        do it = 1, dif_ntemps
          i = 0
          do ib = 1, ab_i%num_col_blks
            do ig = ab_i%scb(ib), ab_i%ecb(ib)
              i = i + 1
              abt(1,i) = exp( -lambda(ig) * it * dif_delta_t) * ab(1,i)
            enddo
          enddo
!
!   abt now has the expansion of the temperature profile in terms of the
!   eigenvectors of the diffusion operator.
!

!
!   Multiply the eigenvector matrix with abt to give the sine function
!   expansion of the temperature profile at time t, x.
!
          call pdgemv('N', n, n, 1.d0, b, 1, 1, b_d, abt, 1, 1, abt_d,         &
     &             1, 0.d0, x, 1, 1, x_d, 1)
!  This last sum corresponds to the outer sum of equation 12, where the
!  time dependent expansion coefficients a{sub l}(t) are stored in the
!  temporary array x in the program.
!

!
!   Gather all of the local pieces of the array x to the array xg.
!
          call rlocal_to_rglobal(x, x_d, xg )

          do k = 1, dif_npts
            temp(k, it) = 0.d0
          enddo

          do iy = 1, dif_ny
            do ix = 1, dif_nx
              i = (iy -1) * dif_nx + ix
              do k = 1, dif_npts
                temp(k,it) = temp(k,it) + xsines(k,ix) * ysines(k,iy)         &
     &                       * xg(1,i)
              enddo
            enddo
          enddo
!
!  This last do loop corresponds to the double summation in equation
!  13 of the discussion paper.
!

        enddo  ! end of time loop


!
!   Here, we are just writing out the temperatures at the selected times
!   and points.
!
        if( sc_iam .eq. 0 ) then    ! if I am node 0
           write(*,*) '   point #      X         Y'
           do i = 1, dif_npts
             write(*,'(2x, i6, 2x, 2f11.4)') i, dif_x(i), dif_y(i)
           enddo
           write(*,*)
           do k = 1, dif_npts, 6
             write(*,*)
             write(*,'(30X,'Points'')')
             write(*,'(''   TIME   '',6(5x,''#'', i4))') (i, i=k, k+5)
             do i = 1, dif_ntemps
                write(*,'(7f10.5)') i*dif_delta_t,                             &
     &                          (temp(j,i),j=k,min(k+5,dif_npts))
             enddo
          enddo
        endif

        deallocate(xg)
        deallocate(xsines)
        deallocate(ysines)
        deallocate(lambda)
        deallocate(temp)
        deallocate( ifail)
        deallocate( iclustr)
        deallocate( gap)
        stop
        end
```

Module Parameters (Message Passing)

```
      module parameters
!
!   Purpose: Define system wide parameters and index structure
!         used to help map global indices to local indices.
!
        implicit none
        public
        integer, parameter :: long=8, short=4
        real(long), parameter :: pi = 3.141592653589793d0
        real(long), parameter :: twopi = 2.d0*pi

        type g_index
         integer :: num_row_blks, num_col_blks
         integer, pointer :: srb(:), scb(:), erb(:), ecb(:)
        end type g_index
!
!  The g_index type was created for convenience
!     components:
!        num_row_blks: number of block repetitions over matrix rows.
!        num_col_blks: number of block repetitions over matrix columns.
!        srb: global row index at start of a block
!             corresponding local index is ( block # -1) * mb
!             where mb is the number of rows in the block.
!
!        scb: global column index at start of a block
!            corresponding local index is ( block # -1) * nb
!            where nb is the number of columns in the block.
!
!       erb: last global row index in the block.
!       ecb: last global column index in the block.
      public g_index

      end module parameters
```

Module Diffusion (Message Passing)

```
      module diffusion
!
!   Purpose: Assign problem parameters and initial data.
!
!   Routines called:
!       none
!
      use parameters
      use scale
      implicit none
      private
!
! Make all entities private by default.
! Have all public entities have the prefix dif_.
!
!  The following are the publicly available routines.
!
      public init_diffusion, init_temp, diff_coef

!
!     The following are publicly available variables.
!

      real, public :: dif_ly_ratio
      integer, public :: dif_nx, dif_ny, dif_npts, dif_ntemps
      real(long), public :: dif_delta_t
      real(long), public, allocatable :: dif_x(:), dif_y(:)

!
!     dif_ly_ratio  is the ratio of the x and y lengths of the beam.
!     dif_nx        is the number of sine expansion coefficients to use
!                   in the x direction.
!     dif_ny        is the number of sine expansion coefficients to use
!                   in the y direction.
!     dif_delta_t   is the size of the time step to be display on output.
!     dif_ntemps    is the total number of temperatures to display per point.
!     dif_npts      is the total number of points to output.
!     dif_x         is the x coordinates of the points.
!     dif_y         is the y coordinates of the points.
!
!


!
!     Private variables
!
      integer :: init_f=1, diff_f=1
!   init_f chooses the functional form of initial distribution of temperature.
!   diff_f chooses the functional form for spatially dependent head diffusion
!          coefficient.

      contains

!**********************************************************************!
!*                                                                    *!
!*   Module routine init_diffusion                                    *!
!*                                                                    *!
!*   Purpose: Initialize problem size, number of output point and     *!
!*            functional form of diffusion constant and initial temperature *!
!*            distribution                                            *!
!*                                                                    *!
!**********************************************************************!
        subroutine init_diffusion
        namelist /input/ ly_ratio, delta_t, numx, numy, nx, ny, numt,        &
     &                   init_f, diff_f
        integer :: numx=5, numy=5, nx=7, ny=7, numt=20
        real(long) :: ly_ratio=1.d0, delta_t=0.1
        real(long) :: delx, dely
        integer :: i, j, ij
        logical :: ex
!======================================================================!
!          Start of executable code                                    !

        inquire ( file='diffus.naml', exist=ex)
        if( ex ) then
          open( 10, file='diffus.naml', action='read')
          read( 10, input)
          close(10)
        endif

        dif_ly_ratio = ly_ratio
        dif_npts = numx*numy
        dif_delta_t = delta_t
        dif_ntemps  = numt
        dif_nx = nx
        dif_ny = ny
        allocate( dif_x(numx*numy) )
        allocate( dif_y(numy*numx) )
!
! Assign a simple linear array of points.
!
        delx = PI/ ( numx + 1.d0)
        dely = PI/ ( numy + 1.d0)
        do i = 1, numx
          do j = 1, numy
            ij = numx*(j-1)  + i
            dif_x(ij) = delx* i
            dif_y(ij) = dely * j
          enddo
        enddo
        return
        end subroutine init_diffusion

!**********************************************************************!
!*                                                                    *!
!*   Module routine init_temp                                         *!
!*                                                                             *!
!*   Purpose: Return the initial temperature of the bar at a particular       *!
!*            point                                                            *!
!*                                                                             *!
!******************************************************************************!
        function init_temp(x, y)
!
!     Arguments:
!       x: real*8 (in), x coordinate
!       y: real*8 (in), y coordinate
!     Function return:
!     init_temp: real*8 (out), initial temperature at (x,y)
!
        real(long), intent(in) :: x, y
        real(long) :: init_temp

!
!   The problem has been scaled to go from 0 to pi in both the x
!   and y directions. To calculate the expansion coefficients, we
!   define the function to be odd about pi and use the range 0 < x < 2*pi
!
!  Local variables.
        integer :: isign
        real(long) :: x1, y1
!
        isign = 1
        x1 = x
        if( x .gt. pi ) then
          isign = -isign
          x1 = twopi - x
        endif
        y1 = y
        if( y .gt. pi ) then
          isign = -isign
          y1 = twopi - y
        endif

!
! Choose very simple temperature profile cases.
!
        select case (init_f)
          case (1)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)
          case (2)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*y1
          case (3)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*x1
          case (4)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*x1*y1
          case (5)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)
          case (6)
             init_temp = isign*(x1*(pi-x1))**2 *y1*(pi-y1)
          case (7)
             init_temp = isign*(x1*(pi-x1))*(y1*(pi-y1))**2
          case default
             init_temp = isign*sin(x1)*sin(y1)
        end select
        return
        end function init_temp



!****************************************************************************!
!*                                                                          *!
!*   Module routine diff_coef                                               *!
!*                                                                          *!
!*   Purpose: Return the value of the thermal diffusion coefficient at      *!
!*            an arbitrary point                                            *!
!*                                                                          *!
!****************************************************************************!
        function diff_coef(x, y)
!     Arguments:
!       x: real*8 (in), x coordinate
!       y: real*8 (in), y coordinate
!     Function return:
!       diff_coef: real*8 (out), diffusion coefficient at (x,y)
!
         real(long), intent(in) :: x, y
         real(long) :: diff_coef

!
!   The problem has been scaled to go from 0 to pi in both the x
!   and y directions. To simplify the matrix element calculations,
!   we define the function to be even about pi.
!

!  Local variables.
        real(long) :: x1, y1

!==============================================================================!
!         Start of executable code.                                            !
        x1 = x
        if( x .gt. pi )  x1 = twopi - x
        y1 = y
        if( y .gt. pi ) y1 = twopi - y

!
! Choose very simple diffusion coefficient cases.
!
        select case (diff_f)
           case (1)
             diff_coef = .5d0 + (x1 + y1) / (2 * twopi)
           case (2)
             diff_coef = ((1.d0 + x1)*(pi - x1 + 1.d0)*(y1 + pi))/ 3*pi
           case (3)
             diff_coef = (y1 + pi) * pi/((pi + x1) * (2* pi - x1))
           case default
             diff_coef = 1.d0
           end select
        return
        end function diff_coef


      end module diffusion
```

Module Fourier (Message Passing)

```
      module fourier
!
!   Purpose: To represent both the diffusion operator and
!            the temperature profile in a sine function basis.
!
!   Routines called:
!     blacs_abort
!     clocal_to_rglobal
!     dcft2
!     igamx2d
!     initialize_carray
!     initialize_scale
!     pdcft2
!
      use parameters
      use scale
      use diffusion
      implicit none
      private
!
!     all entities private by default
!
      external pdcft2
!
!   publicly available routines
!
      public expand_temp_profile, get_diffusion_matrix
      public g_index
!
!   publicly available variables
!


!
!   private variables
!
      integer :: pn_fac(5) = 5*0  ! prime factors of number of nodes
!     nnodes = 2**pn_fac(1) * 3**pn_fac(2) * 5**pn_fac(3) *
!              7**pn_fac(4) * 11**pn_fac(5)


!
!   private routines
!
      private minpower2, factor_nodes
      contains

!*****************************************************************************!
!*                                                                           *!
!*   Module routine get_diffusion_matrix                                     *!
!*                                                                           *!
!*   Purpose: To obtain the matrix representation of the diffusion          *!
!*            operator in a sine function basis                              *!
!*                                                                           *!
!*****************************************************************************!
        subroutine get_diffusion_matrix(f,f_i, f_d)
!
!   Arguments:
```

```
!     f: real*8,dimension(:,:),(out), local part of the global matrix
!               containing the diffusion operator in sine function basis.
!     f_d: integer*4, dimension(:),(in), array descriptor for f.
!     f_i: g_type, (in), g_type structure for f, see parameter.f.
!
        real(long), intent(out) :: f(:,:)
        integer, intent(in) :: f_d(DESC_DIM)
        type (g_index), intent(in) :: f_i

! Local variables
!
!  df contains the diffusion coefficient before the call to pdcft2.
!  df contains the Fourier transpose of diffusion coefficients after the call.
!  dfg contains the entire Fourier transpose of df on each node.
!
        complex(long), pointer :: df(:,:)
        type (g_index), pointer :: df_i
        integer :: df_d(DESC_DIM)

        real(long), allocatable :: dfg(:,:)
!
!  ixi and iyi are arrays which, given a global index,
!  return the x and y offsets.  Recall that the large arrays
!  are 4 dimensional arrays collapsed into 2 dimensions,
!  where i = (ix-1)*dif_ny + iy.
!
        integer, allocatable :: ixi(:), iyi(:)
        real(long) :: scale_f
        integer :: nx, ny, ix, iy, ixp, iyp, istat, nerrs
        integer :: ixdiff, iydiff, num_errors
        integer :: naux1, naux2, i, j, factor1, factor2, idum
        integer :: ib, jb, il, jl
!
!  ip is a support array for pdcft2.
!
        integer :: ip(40)
        integer :: blk_index
!
!   Fourier transform of diffusion coefficient function
        nerrs=0

        call factor_nodes()
        factor1 = 3**pn_fac(2) * 5**pn_fac(3) * 7**pn_fac(4) *                 &
     &                           11**pn_fac(5)
!
!  Here we are trying to find the smallest number which is evenly
!  divisible by the number of processes and is larger than 4*(n+1).
!
        factor2 = (4*(dif_nx+1) + factor1 -1)/factor1
        nx = minpower2( factor2,idum) * factor1

        factor2 = (4*(dif_ny+1) + factor1 -1)/factor1
        ny = minpower2( factor2,idum) * factor1

        scale_f = 1.d0/ real(nx*ny, long)
```

```
!
!  Get storage for diffusion array.
!
        call  initialize_scale(ny, blk_index)
        call  initialize_carray(df, df_d, df_i, nx, ny,                        &
     &                          blk_index)
!
!    Here, we initialize the local part of the global array df, which
!    contains the value of the diffusion coefficient function, evenly
!    evaluated between (0, 2*pi).  We do a two dimensional Fourier
!    transform on the data.  Because the size of this array is so small,
!    nx by ny, and ultimately we have to transfer the whole array to
!    each node, it would probably be more efficient to do the calculation
!    locally on each node.
!

!    Get the value of the diffusion coefficient function at
!    the necessary points.
!


!
!   This loop can be simplified considerably. Because blocks of the
!   array are column-distributed with the block size equal to the number
!   of columns divided by the number of processes, there is only a single
!   column block.  Also, because the processes are distributed in a 1 x np
!   arrangement, the local row index will equal the global row index.
!   However, the loop is perfectly general for other process arrangements
!   and is correct for this particular case.
!
        jl = 0
        do jb = 1, df_i%num_col_blks   ! loop over the number of column blocks
          do j = df_i%scb(jb), df_i%ecb(jb)
                         ! loop over columns in block
                         ! j is a global index
            jl = jl + 1    ! jl is local array column index
            il = 0
            do ib = 1, df_i%num_row_blks   ! loop over the number of row blocks
              do i = df_i%srb(ib),  df_i%erb(ib)
                         ! loop over rows in block
                         ! i is a global index
                il = il + 1    ! il is local array row index
                df(il,jl) = diff_coef((twopi*(i-1))/nx,                        &
     &                                  (twopi*(j-1))/ny)
              enddo
            enddo
          enddo
        enddo
!
!  This last loop just determined the diffusion coefficient at evenly
!  spaced points along the x and y coordinates.
!


!
!
!  Do the Fourier transform.
!
        do i= 1, 40
          ip(i) = 0
```

```
         enddo
!  Store the array in normal mode overwriting the original array.
         ip(1) = 1
         ip(2) = 1

!
! Because the size of the 2d Fourier transform is nx by ny, which is much
! smaller than the size of the eigenvalue problem, this could probably
! be done serially on each node more quickly.
!
         call pdcft2(df, df, nx, ny, 1,scale_f, sc_icontext, ip)
!
!
!   df now has the Fourier coefficients for the diffusion coefficient
!      function, which correspond to the D(tilde)(sub ij) given in the
!      discussion paper.
!
!   Because each process will need most of the Fourier transformed diffusion
!   coefficients, it is useful to broadcast all parts of this matrix
!   to each process.
!
!  First allocate the index arrays.
!
         num_errors=0
         allocate(ixi(dif_nx*dif_ny), stat=istat)
         if( istat .ne. 0 ) num_errors = num_errors + 1
         allocate(iyi(dif_nx*dif_ny), stat=istat)
         if( istat .ne. 0 ) num_errors = num_errors + 1
!  Allocate array for holding global Fourier transform.
         allocate(dfg(nx,ny), stat = istat)
         if( istat .ne. 0 ) num_errors = num_errors + 1

         call igamx2d(sc_icontext,'A',' ',1,1,num_errors,1,-1,-1,-1,            &
     &               -1,-1)
         if( num_errors .gt. 0 ) then
            if( sc_iam .eq. 0 ) then
                write(*,*) 'Error in allocating scratch arrays in ',            &
     &                      'get_diffusion_matrix'
                call blacs_abort(sc_icontext, 1)
            endif
         endif


         call clocal_to_rglobal( df, df_d, dfg )

!   Here df contains only local portions of the global array, while
!   dfg contains the entire global array.
!


!
!  Now load up the diffusion operator
!    f(ix,iy;ix',iy').
!
!    Here we transform the 4d matrix into the 2d matrix where
!       i = (iy-1)* dif_nx + ix + 1
!    and
!       j = (iy'-1)* dif_nx + ix' + 1.
!
```

```
!  First calculate the index arrays.
!
        do ix = 1, dif_nx
          do iy = 1, dif_ny
            i = (iy-1)* dif_nx + ix
            ixi(i) = ix
            iyi(i) = iy
          enddo
        enddo

!
! This final loop loads the matrix elements up for F as given in
! equation 10.
!
        jl = 0
        do jb = 1, f_i%num_col_blks   ! loop over the number of column blocks
          do j = f_i%scb(jb), f_i%ecb(jb)
                        ! loop over columns in block
                        ! j is a global index
            jl = jl + 1    ! jl is local array column index
            iyp = iyi(j)
            ixp = ixi(j)
            il = 0
            do ib = 1, f_i%num_row_blks   ! loop over the number of row blocks
              do i = f_i%srb(ib), f_i%erb(ib)
                        ! loop over rows in block
                        ! i is a global index
                il = il + 1    ! il is local array row index
                iy = iyi(i)
                ix = ixi(i)
                ixdiff = iabs(ix-ixp) + 1
                iydiff = iabs(iy-iyp) + 1
                f(il,jl) = ( ( ix*ixp + iy*iyp*dif_ly_ratio ) *              &
     &              (dfg(ixdiff, iydiff)    - dfg(ix+ixp+1,iy+iyp+1))        &
     &              +     ( ix*ixp - iy*iyp*dif_ly_ratio ) *                 &
     &              (dfg(ix+ixp+1,iydiff )  - dfg(ixdiff,iy+iyp+1)))
              enddo
            enddo
          enddo
        enddo
        deallocate(dfg)
        deallocate(ixi)
        deallocate(iyi)
!
!   We should add routines to free df.
!
        return
        end subroutine get_diffusion_matrix



!***************************************************************************!
!*                                                                         *!
!*   Module routine expand_temp_profile                                    *!
!*                                                                         *!
!*   Purpose: To obtain the expansion coefficients of the initial          *!
!*            temperature profile in a sine function expansion             *!
!*                                                                        *!
!**************************************************************************!
       subroutine expand_temp_profile(a,a_i,a_d)
!
!   Arguments:
!     a: real*8,dimension(:,:),(out), local part of the global matrix,
!                 containing the sine coefficients for initial
!                 temperature distribution.
!     a_d: integer*4, dimension(:),(in), array descriptor for a.
!     a_i: g_type, (in), g_type structure for a, see parameter.f.
!
       real(long), intent(out) :: a(:,:)
       integer, intent(in) :: a_d(DESC_DIM)
       type (g_index), intent(in) :: a_i

! Local variables
       complex(long), allocatable :: atmp(:,:)
       real(long), allocatable :: aux1(:), aux2(:)
       integer :: i,j, nx, ny, istat, naux1, naux2, nerrs, jl
       integer :: idum, jb, jx, jy
       real(long) :: x, y, scale_f

!
!      Calculate the minimum power of 2 to hold twice the number of
!      Fourier coefficients as sine coefficients. The top half of the
!      Fourier coefficients will equal minus the bottom half because
!      we are forcing the temperature profile to be odd.
!
       nx = minpower2( 2*(dif_nx+1), idum)
       ny = minpower2( 2*(dif_ny+1), idum)
       scale_f = -twopi / real( nx*ny,long)

       nerrs = 0
!
!       Temperature profile allocation.
       allocate(atmp(nx,ny), stat=istat )
       if( istat .ne. 0 ) nerrs = nerrs + 1
!
!
       naux1 = 40000 + 2.28*( nx + ny)
       naux2 = 20000 + 66*( 256 + 2*max(nx , ny))
!
!  Allocate work storage.
       allocate(aux1(naux1), stat=istat)
       if( istat .ne. 0 ) nerrs = nerrs + 1
       allocate(aux2(naux2), stat=istat)
       if( istat .ne. 0 ) nerrs = nerrs + 1

!
!   Check for allocation errors.
!
       call igamx2d(sc_icontext,'A',' ',1,1,nerrs,1,-1,-1,-1,-1,-1)
       if( nerrs .gt. 0 ) then
          if( sc_iam .eq. 0 ) then
              write(*,*) 'Error in allocating scratch arrays in ',           &
     &                    'expand_temp_profile'
              call blacs_abort(sc_icontext, 1)
          endif
```

```
      endif

!
!
!     Fill atmp with the initial temperatures.
!
!  atmp contains the initial temperature profile T(sub 0)(x,y) as used
!  in equation 5 in the discussion paper.
!
      do i = 1, nx
         do j = 1, ny
           atmp(i,j) = init_temp((twopi*(i-1))/nx, (twopi*(j-1))/ny)
         enddo
      enddo

!
!  Do the 2d Fourier transform of atmp.
!
!  First initialize.
!
!  The 2d Fourier transform can be done in parallel, however it
!  is such a small part of the problem, it is probably faster to do
!  it serially on each node.
!
!  Note that we could have used DSINF to obtain these expansion coefficients
!  as well.
!
      call dcft2(1,atmp,1,nx,atmp,1,nx,nx,ny,1,scale_f ,aux1,naux1,          &
     &              aux2,naux2 )
      call dcft2(0,atmp,1,nx,atmp,1,nx,nx,ny,1,scale_f ,aux1,naux1,          &
     &              aux2,naux2 )
!
! The calls to dcft2 calculated the dual Fourier transform as
! defined by equation 5 in the discussion paper.
!


!
!  This final loop is to load only those portions of the global array
!  corresponding to the local portion of that array for this process.
!
      jl = 0
      do jb =  1, a_i%num_col_blks    ! loop over all column blocks
        do j = a_i%scb(jb),  a_i%ecb(jb)  ! j is global index
          jx = mod(j-1, dif_nx) + 2
          jy = (j-1) / dif_nx + 2
          jl = jl + 1
          a(1,jl) = real(atmp(jx,jy),long)
        enddo
      enddo

      deallocate(atmp)
      deallocate(aux1)
      deallocate(aux2)
      return
      end subroutine expand_temp_profile
```

```
!************************************************************************!
!*                                                                      *!
!*   Module routine factor_nodes                                        *!
!*                                                                      *!
!*   Purpose: To obtain the powers of prime factorization of the number *!
!*            nodes, failing if the factorization is not compatible with *!
!*            FFT supported transform lengths                           *!
!*                                                                      *!
!************************************************************************!
        subroutine factor_nodes()
!  Arguments: None
!
!  Local variables
        integer n1, n2, l2
!
!   Determine the prime factorization of nnodes, which must
!       be of the form 2**n * 3**m * 5**i * 7**j * 11**k
!       where m cannot be greater than 2 and i, j, and k cannot
!       be greater than 1
!
        n2 = sc_nnodes
        n1 = n2/11
        if( n1*11 .eq. n2) then
            pn_fac(5) = 1
            n2 = n1
        endif

        n1 = n2/7
        if( n1*7 .eq. n2) then
            pn_fac(4) = 1
            n2 = n1
        endif

        n1 = n2/5
        if( n1*5 .eq. n2) then
            pn_fac(3) = 1
            n2 = n1
        endif

        n1 = n2/3
        if( n1*3 .eq. n2) then
          if ( (n1/3)*3 .eq. n1 ) then
            pn_fac(2) = 2
            n2 = n1/3
          else
            pn_fac(2) = 1
            n2 = n1
          endif
        endif

        n1 = minpower2(n2,l2)
        pn_fac(1) = l2

        if( n1 .ne. n2) then
          if( sc_iam .eq. 0) then
            write(*,*) 'Invalid number of nodes, it must have the form:'
            write(*,*) '2**n * 3**m * 5**i * 7**j * 11**k, where '
            write(*,*) ' n >= 0, 0<=m<=2 and 0<= i,j,k <=1 '
```

```
              write(*,*) ' choose a power of 2 for best performance'
              call blacs_abort(sc_icontext,1)
            endif
          endif
          end subroutine factor_nodes


!****************************************************************************!
!*                                                                          *!
!*   Module function minpower2                                              *!
!*                                                                          *!
!*   Purpose: To obtain the smallest number which is a power of 2 and       *!
!*            greater than or equal to the input argument                   *!
!*                                                                          *!
!****************************************************************************!
        function minpower2( n, log2n )
!
!   Arguments:
!      n: integer*4, (in), input number
!      log2n: integer*4, (out), log base 2 of the function result
!   Function return:
!      minpower2: integer*4 (out), smallest number, which is a power of 2
!                 and greater than or equal to n.
!
        integer n, minpower2, log2n
!
!   Local variables.
        integer m, i
        m=n

        if( n < 0 ) write(*,*) 'n cannot be negative'
        powerloop: do i= 1, bit_size(n)
          m = m / 2
          if( m .eq. 0 ) exit powerloop
        enddo powerloop
        if ( 2**(i-1) .ne. n ) then
           if( 2**i < 0) write(*,*) 'n too large'
           log2n = i
           minpower2 = 2**i
        else
           log2n = i-1
           minpower2 = n
        endif
        return
        end function minpower2

      end module fourier
```

Module Scale (Message Passing)

```
      module scale
!
!  Purpose: To initialize the communications and provide a few
!           communication utility routines.
!
!  Routines called:
!    blacs_abort
!    blacs_get
!    blacs_gridinfo
!    blacs_gridinit
!    blacs_pinfo
!    dgebr2d
!    dgebs2d
!    numroc
!
        use parameters
        implicit none
        external numroc
!
!  All variables private by default.
!
        private
!
!   Publicly accessible routines follow.
!
        public initialize_scale
        public initialize_rarray, initialize_carray
        public clocal_to_rglobal, rlocal_to_rglobal
        public g_index

!
!   Public variables follow.
!
        integer, public :: sc_icontext, sc_iam, sc_nnodes

!
!   Private variables follow.
!
!   MAXBLOCK is the maximum block size. Here it is set to a very
!           large number to force an equal noncyclic block distribution
!           for the FFTs.
!
!
        integer, public, parameter :: MAXBLOCK=50000
        integer, public, parameter :: MAX_SC_INDEX=10
        integer, public, parameter :: DESC_DIM=9
        integer, public, parameter :: DTYPE_=1
        integer, public, parameter :: CTXT_=2
        integer, public, parameter :: M_=3
        integer, public, parameter :: N_=4
        integer, public, parameter :: MB_=5
        integer, public, parameter :: NB_=6
        integer, public, parameter :: RSRC_=7
        integer, public, parameter :: CSRC_=8
        integer, public, parameter :: LLD_=9
        integer :: numroc
        integer :: nprow, npcol, myrow, mycol
!
```

```
!  The block sizes for a given array size are indexed in the
!  nblock, mblock and nsize arrays so that, for a given array size,
!  the block size calculation is done only once and exactly the same
!  block sizes are returned for any particular array size.
!
       integer :: nblock(MAX_SC_INDEX), mblock(MAX_SC_INDEX)
       integer :: nsize(MAX_SC_INDEX)
!
!  The number of different array sizes is limited by the fixed
!  dimension of the arrays nblock, mblock and nsize.
!
       integer :: icontext, iam, nnodes
       integer, save :: sc_indx=0
       integer, parameter :: rsrc=0, csrc=0
       logical, save :: initialized = .false.
!
!  All of the manipulation of the PESSL and SP variables will be
!  done in this module and held privately.
!
     contains

!****************************************************************************!
!*                                                                          *!
!*   Module routine initialize_scale                                        *!
!*                                                                          *!
!*   Purpose: Initialize blacs and calculate a block size                   *!
!*                                                                          *!
!****************************************************************************!
       subroutine initialize_scale (n, index)
!
!     Arguments:
!       n: integer*4 (in), matrix or vector size.
!       index: integer*4 (out), index into an array of block sizes.
!              Provides a mechanism for creating similar descriptor arrays.
!


!
!  This routine assigns the block size based on a given
!  n and returns an index, so that multiple arrays can be created with
!  compatible block sizes.
!
       integer n, index
       integer  npc, npr, i

       if ( .not. initialized ) then
         call blacs_pinfo( iam, nnodes )
         sc_iam = iam
         sc_nnodes = nnodes
!
!      Get the number of nodes.
!


!
!   The Fourier transform routines require that the processors
!   be laid out in a 1 by nnodes arrangement.
!
         nprow = 1
         npcol = nnodes
```

```
          call blacs_get(0,0,icontext)
          sc_icontext = icontext
          call blacs_gridinit( icontext, 'R', nprow, npcol)
          sc_icontext = icontext
          call blacs_gridinfo( icontext, npr, npc, myrow, mycol)
!
!      Check that the system is gridded as expected.
!
          if( npr .ne. nprow .or. npc .ne. npcol) then
            if( iam .eq. 0) then
              write(*,*) 'number of processor rows and columns'
              write(*,*) 'incorrect ', nprow, npr, npcol, npc
              call blacs_abort(icontext,1)
            endif
          endif
          initialized = .true.
        endif

!
!  If we have already calculated a block size based on estimated
!  array size, return the index for that block size.
!
        do i = 1, sc_indx
           if( n .eq. nsize(i)) then
              index = i
              return
           endif
        enddo

!
!     Compute a block size.
!
        sc_indx = sc_indx + 1
        if ( sc_indx .GT. MAX_SC_INDEX ) then
          if( iam .eq. 0 ) then
            write(*,*) 'Used more than the maximum number of '
            write(*,*) 'indices in initialize_scale, Maximum is ',             &
     &                  MAX_SC_INDEX
            call blacs_abort(icontext,1)
          endif
        endif

        index = sc_indx
        nsize(index) = n

!
!  Always choose a square block with a maximum dimension of MAXBLOCK.
!
        if( ( n ) / nnodes .gt. MAXBLOCK ) then
            mblock(index) = MAXBLOCK
        else
            mblock(index) = (n ) / nnodes
        endif
        if( mblock(index) .lt. 1 ) then
            if( iam .eq. 0 ) then
              write(*,*) 'problem size too small for number of nodes'
              write(*,*) 'try increasing the nx and ny'
```

```
               write(*,*) n, nnodes
               call blacs_abort(sc_icontext, 1)
             endif
         endif
         nblock(index) = mblock(index)

         return
         end subroutine initialize_scale


!
!   This routine is provided to allocate dynamically the space
!   needed for the local part of a global array and initialize the
!   associated array descriptor.  It also returns array-useful
!   indices to do local to global index conversion.
!
!  initialize_rarray => real array initialization.
!  initialize_carray => complex array initialization.
!

!***********************************************************************!
!*                                                                     *!
!*   Module routine initialize_rarray                                  *!
!*                                                                     *!
!*   Purpose: Allocate space for a real array and create associated    *!
!*            descriptor array and index array                         *!
!*                                                                     *!
!***********************************************************************!
        subroutine initialize_rarray( array, desc_array,                  &
     &                              index_array, m, n, blk_index)
!
!    Arguments:
!       array: pointer to real*8 (out), pointer to real array, initialized.
!       desc_array: integer*4 (out), empty descriptor array, initialized.
!       index_array: g_index (out), pointer to g_index structure,
!                   see parameter.f, initialized.
!       m: integer*4 (out), scalar number of rows in global array.
!       n: integer*4 (out), scalar number of columns in global array.
!       blk_index: integer*4 (in), index into array of block sizes to use
!                   for initializing the descriptor array.
!
        integer, intent(out) :: desc_array(DESC_DIM)
        integer, intent(in) :: m, n, blk_index
        type (g_index), pointer :: index_array
        real(long), pointer :: array(:,:)

! Local variables
        integer :: irows, icols, istat, i, j
        integer :: start_row_block, start_col_block
        integer :: mb, nb, num_mb, num_nb

!
!   Check to see if the block sizes were already calculated.
!
        if ( blk_index .lt.1 .or. blk_index .gt. sc_indx ) then
           if( iam .eq. 0 ) then
              write(*,*) 'No initialization done for index ', blk_index
              call blacs_abort(icontext, 1)
           endif
        endif

        mb = mblock(blk_index)
        nb = nblock(blk_index)

        irows = numroc( m, mb, myrow, rsrc, nprow )
        icols = numroc( n, nb, mycol, csrc, npcol )

        allocate(array(max(irows,1),max(icols,1)), stat=istat)
        if ( istat .ne. 0) then
           write(*,*) 'allocate failed in initialize_array'
           call blacs_abort(icontext,1)
        endif

!
!   Calculate the number of row and column blocks.
!
        num_mb = ( (irows + mb -1)/ mb )
        num_nb = ( (icols + nb -1)/ nb )

        allocate (index_array)
        index_array%num_row_blks = num_mb
        index_array%num_col_blks = num_nb

        allocate(index_array%srb(num_mb))
        allocate(index_array%erb(num_mb))
        allocate(index_array%scb(num_nb))
        allocate(index_array%ecb(num_nb))

        desc_array(DTYPE_) = 1
        desc_array(M_) = m
        desc_array(N_) = n
        desc_array(MB_) = mb
        desc_array(NB_) = nb
        desc_array(RSRC_) = rsrc
        desc_array(CSRC_) = csrc
        desc_array(CTXT_) = icontext
        desc_array(LLD_) = max(irows,1)

!
        start_row_block = mod( nprow + myrow - rsrc , nprow )
        start_col_block = mod( npcol + mycol - csrc , npcol )

        do i = 1, index_array%num_row_blks
          index_array%srb(i) = (start_row_block + (i - 1)*nprow) *             &
     &                     mb+1
          index_array%erb(i) = index_array%srb(i) + mb - 1
        enddo
        index_array%erb(num_mb) = mod(irows-1,mb) +                             &
     &                            index_array%srb(num_mb)

        do i = 1, index_array%num_col_blks
          index_array%scb(i) = (start_col_block + (i - 1)*npcol) *             &
     &                        nb + 1
          index_array%ecb(i) = index_array%scb(i) + nb - 1
        enddo
        index_array%ecb(num_nb) = mod(icols-1,nb) +                             &
```

```
     &                              index_array%scb(num_nb)


        end subroutine initialize_rarray

! Complex array initialization.
!
!***************************************************************************!
!*                                                                         *!
!*   Module routine initialize_carray                                      *!
!*                                                                         *!
!*   Purpose: Allocate space for a complex array and create associated     *!
!*            descriptor array and index array                             *!
!*                                                                         *!
!***************************************************************************!
        subroutine initialize_carray( array, desc_array,                      &
     &                               index_array, m, n, blk_index)
!
!    Arguments:
!       array: pointer to complex (out), pointer to real array, initialized.
!       desc_array: integer*4 (out), empty descriptor array, initialized.
!       index_array: g_index (out), pointer to g_index structure,
!                   see parameter.f, initialized.
!       m: integer*4 (out), scalar number of rows in global array.
!       n: integer*4 (out), scalar number of columns in global array.
!       blk_index: integer*4 (in), index into array of block sizes to use
!                   for initializing the descriptor array.
!
        integer desc_array(DESC_DIM), m, n, blk_index
        type (g_index),pointer :: index_array
        complex(long), pointer :: array(:,:)


! Local variables
        integer :: irows, icols, istat, i, j
        integer :: start_row_block, start_col_block
        integer :: mb, nb, num_mb, num_nb

!
!   Check to see if the block sizes were already calculated.
!
        if ( blk_index .lt.1 .or. blk_index .gt. sc_indx ) then
           if( iam .eq. 0 ) then
              write(*,*) 'No initialization done for index ', blk_index
              call blacs_abort(icontext, 1)
           endif
        endif

        mb = mblock(blk_index)
        nb = nblock(blk_index)

        irows = numroc( m, mb, myrow, rsrc, nprow )
        icols = numroc( n, nb, mycol, csrc, npcol )

        allocate(array(max(irows,1),max(icols,1)), stat=istat)
        if ( istat .ne. 0) then
           write(*,*) 'allocate failed in initialize_array'
           call blacs_abort(icontext,1)
        endif


!
!   Calculate the number of row and column blocks.
!
        num_mb = ( (irows + mb -1)/ mb )
        num_nb = ( (icols + nb -1)/ nb )

        allocate(index_array, stat=istat)
        if ( istat .ne. 0) then
           write(*,*) 'allocate failed in initialize_array'
           call blacs_abort(icontext,1)
        endif

        index_array%num_row_blks = num_mb
        index_array%num_col_blks = num_nb

        allocate(index_array%srb(num_mb))
        allocate(index_array%erb(num_mb))
        allocate(index_array%scb(num_nb))
        allocate(index_array%ecb(num_nb))



        desc_array(DTYPE_) = 1
        desc_array(M_) = m
        desc_array(N_) = n
        desc_array(MB_) = mb
        desc_array(NB_) = nb
        desc_array(RSRC_) = rsrc
        desc_array(CSRC_) = csrc
        desc_array(CTXT_) = icontext
        desc_array(LLD_) = max(irows,1)


!
        start_row_block = mod( nprow + myrow - rsrc , nprow )
        start_col_block = mod( npcol + mycol - csrc , npcol )


        do i = 1, index_array%num_row_blks
          index_array%srb(i) = (start_row_block + (i - 1)*nprow) *             &
     &                       mb+1
          index_array%erb(i) = index_array%srb(i) + mb - 1
        enddo
        index_array%erb(num_mb) = mod(irows-1,mb) +                            &
     &                              index_array%srb(num_mb)

        do i = 1, index_array%num_col_blks
          index_array%scb(i) = (start_col_block + (i - 1)*npcol) *             &
     &                          nb + 1
          index_array%ecb(i) = index_array%scb(i) + nb - 1
        enddo
        index_array%ecb(num_nb) = mod(icols-1,nb) +                            &
     &                              index_array%scb(num_nb)

        end subroutine initialize_carray
```

```
!
!***************************************************************************!
!*                                                                         *!
!*   Module routine clocal_to_rglobal                                      *!
!*                                                                         *!
!*   Purpose: Take the real parts of the local portions of a complex matrix *!
!*            and distribute them globally to each node                    *!
!*                                                                         *!
!***************************************************************************!
        subroutine clocal_to_rglobal(a,a_d,a_glob)
!
!    Arguments:
!     a: complex*16, dimension(:,:), is the local part of a
!                                    global complex array.
!     a_d: integer*4, array descriptor for a.
!     a_glob: real*8, dimension(:,:), entire matrix A on each node.
!

        complex(long), intent(in) :: a(:,:)
        integer, intent(in) :: a_d(DESC_DIM)
        real(long), intent(out) :: a_glob(:,:)
!
!    Local variables.
!
        integer :: nrow_blks, ncol_blks, ib, jb, ibl, jbl, i, j
        integer :: m, n, nb, mb, ig, jg, lda, il, jl, prow, pcol
        integer :: iarow, iacol, ni, nj
!
!    m is number of rows in global matrix.
!    n is number of columns in global matrix.
!    mb and nb are rows and columns in each block.
!    prow and  pcol are the processor row and column containing a block.
!    ib, jb, ibl, jbl are global and local  block indices.
!    il, jl, ig, jg are local and global matrix indices.
!

!
!
!     Start of executable code.
!
!==================================================================
!

! Determine the total number of row and column blocks.
        m = a_d(M_)
        n = a_d(N_)
        mb = a_d(MB_)
        nb = a_d(NB_)
        iarow = a_d(RSRC_)
        iacol = a_d(CSRC_)
        nrow_blks =  ( m + mb -1 )/ mb
        ncol_blks = ( n + nb - 1 )/ nb
!
! Determine leading dimension of global array.
        lda = size(a_glob, dim=1)
```

```
!
!  Loop over all of the blocks.
!
        do jb = 0, ncol_blks - 1
!
!  Loop over column blocks, determining both local and global j indices.
          jg = jb * nb + 1
          nj = nb
          if (jb .eq. ncol_blks - 1) nj = mod( n - 1, nb) + 1
          jbl = ( jb + iacol ) / npcol  - (mycol + iacol) / npcol
          jl = jbl * nb + 1
          pcol = mod((jb + iacol), npcol )

          do ib = 0, nrow_blks - 1
!
!  Loop over row blocks, determining both local and global i indices.
            ig = ib * mb + 1
            ni = mb
            if (ib .eq. nrow_blks - 1) ni = mod( m - 1, mb) + 1
            ibl = ( ib + iarow ) / nprow - (myrow + iarow) /nprow
            il = ibl * mb  + 1
            prow = mod((ib + iarow), nprow )
!
!   Determine if this block is on my processor.
!
            if( prow .eq. myrow .and. pcol .eq. mycol) then
!
!   Block is on my processor.
!
!   using Fortran 90 array language this is
!       a_glob(ig:ig+ni-1,jg:jg+nj-1) = a(il:il+ni-1:jl+nj-1)
!
              do j = 1, nj
                 do i  = 1, ni
                    a_glob( ig+i-1, jg+j-1 ) = a( il+i-1, jl+j-1)
                 enddo
              enddo
              call dgebs2d(icontext,'ALL',' ',ni,nj,a_glob(ig,jg),lda)
            else
!
!   Block is on somebody elses processor.
!
              call dgebr2d(icontext,'ALL',' ',ni,nj,a_glob(ig,jg),            &
     &                    lda, prow, pcol)
            endif

          enddo
        enddo

        return
        end subroutine clocal_to_rglobal


!**********************************************************************!
!*                                                                    *!
!*   Module routine rlocal_to_rglobal                                 *!
!*                                                                    *!
!*   Purpose: Take the local portions of a real matrix                *!
!*           and distribute them globally to each node                    *!
!*                                                                        *!
!*************************************************************************!
        subroutine rlocal_to_rglobal(a,a_d,a_glob)
!
!    Arguments:
!     a: real*8, dimension(:,:), is the local part of a global real array.
!     a_d: integer*4, array descriptor for a.
!     a_glob: real*8, dimension(:,:), entire matrix A on each node.
!
!   Input arguments.
!
        real(long), intent(in) :: a(:,:)
        integer, intent(in) :: a_d(DESC_DIM)
        real(long), intent(out) :: a_glob(:,:)

!
!    Local variables.
!
        integer :: nrow_blks, ncol_blks, ib, jb, ibl, jbl, i, j
        integer :: m, n, nb, mb, ig, jg, lda, il, jl, prow, pcol
        integer :: iarow, iacol, ni, nj
!
!    m is number of rows in global matrix.
!    n is number of columns in global matrix.
!    mb and nb are rows and columns in each block.
!    prow and  pcol are the processor row and column containing a block.
!    ib, jb, ibl, jbl are global and local  block indices.
!    il, jl, ig, jg are local and global matrix indices.
!


!
!
!     Start of executable code.
!
!=================================================================
!

! Determine the total number of row and column blocks.
        m = a_d(M_)
        n = a_d(N_)
        mb = a_d(MB_)
        nb = a_d(NB_)
        iarow = a_d(RSRC_)
        iacol = a_d(CSRC_)
        nrow_blks =  ( m + mb -1 )/ mb
        ncol_blks = ( n + nb - 1 )/ nb
!
! Determine leading dimension of global array.
        lda = size(a_glob, dim=1)

!
!  Loop over all of the blocks.
!
        do jb = 0, ncol_blks - 1
!
!  Loop over column blocks, determining both local and global j indices
          jg = jb * nb + 1
```

```
              nj = nb
              if (jb .eq. ncol_blks - 1) nj = mod( n - 1, nb) + 1
              jbl = ( jb + iacol ) / npcol  - (mycol + iacol) / npcol
              jl = jbl * nb + 1
              pcol = mod((jb + iacol), npcol )

              do ib = 0, nrow_blks - 1
!
!  Loop over row blocks, determining both local and global i indices.
                ig = ib * mb + 1
                ni = mb
                if (ib .eq. nrow_blks - 1) ni = mod( m - 1, mb) + 1
                ibl = ( ib + iarow ) / nprow - (myrow + iarow) /nprow
                il = ibl * mb  + 1
                prow = mod((ib + iarow), nprow )
!
!   Determine if this block is on my processor.
!
                if( prow .eq. myrow .and. pcol .eq. mycol) then
!
!   Block is on my processor.
!
                  do j = 1, nj
                     do i  = 1, ni
                        a_glob( ig+i-1, jg+j-1 ) = a( il+i-1, jl+j-1)
                     enddo
                  enddo
                  call dgebs2d(icontext,'ALL',' ',ni,nj,a_glob(ig,jg),lda)
                else
!
!   Block is on somebody elses processor.
!
                  call dgebr2d(icontext,'ALL',' ',ni,nj,a_glob(ig,jg),             &
     &                     lda, prow, pcol)
                endif

              enddo
            enddo

            return
            end subroutine rlocal_to_rglobal

          end module scale
```

Program Main (HPF)

```
            program main
!
!   Purpose, to  find the cooling rate for a specified set of
!   points in an anisotropic rectangular beam, immersed in a constant
!   heat bath with a temperature of 0.
!
!   Routines called:
!     expand_temp_profile
!     get_diffusion_matrix
!
            use parameters
```

```
        use diffusion
        use fourier
        use pessl_hpf
        implicit none

        integer :: n, ix, jx, iy, jy, k, i, j, stat, it, ij
        integer :: num_errors
!
!       a contains the sine expansion coefficients of the initial
!                temperature profile.
!       b contains the eigenvectors of the diffusion operator in the
!                sine function basis.
!       ab contains the initial temperature profile expanded in the
!                eigenvectors of the diffusion operator.
!       f contains the matrix elements of the diffusion operation in
!                sine function basis.
!       lambda contains the eigenvalues of the diffusion operator.
!
!       df contains the Fourier transform of the diffusion coefficient function.
!
        real(long)              ::  t1, t2, delx, dely
        integer                 ::  j1, j2
        real(long), allocatable ::  dif_x(:), dif_y(:)
        real(long), allocatable ::  lambda(:)
        real(long), allocatable ::  gap(:)
        real(long), allocatable :: f(:,:), b(:,:), a(:), a_rep(:)
        real(long), allocatable :: x(:,:), ab(:), abt(:,:)
        real(long), allocatable :: xsines(:,:), ysines(:,:), temp(:,:)

        integer :: biga_index, num_eigvalues, num_vectors

!   HPF directives, we will only use descriptive mapping except in main
!HPF$  DISTRIBUTE (*,BLOCK) :: f, b, x, abt, temp
!HPF$  DISTRIBUTE (BLOCK)   :: a, ab

! lambda is a replicated array.

!
!   Read in the problem size, initialize the problem dimensions,
!   choose functional form for the spatially dependent heat diffusion
!   coefficients, choose funtional form of initial temperature distribution
!   and choose the number of points, in both space and time, of the solution
!   to print out.
!
        call init_diffusion
        allocate( dif_x(dif_numx*dif_numy) )
        allocate( dif_y(dif_numy*dif_numx) )
!
! Assign a simple linear array of points.
!
        delx = PI/ ( dif_numx + 1.d0)
        dely = PI/ ( dif_numy + 1.d0)
        do i = 1, dif_numx
          do j = 1, dif_numy
            ij = dif_numx*(j-1)  + i
            dif_x(ij) = delx* i
            dif_y(ij) = dely * j
          enddo
        enddo

        num_errors = 0
!
!       Read in how many sine functions to include in both the
!            x and y directions.
!
!       Because we need to get the Fourier coefficients of the sums
!       and differences of the indices, we need to include twice as
!       many Fourier coefficients as the number of sine expansion coefficients.
!       Also, because the top and bottom halves of the Fourier
!       transform are identical,
!       an artifact of artificially extending the diffusion coefficient
!       function and the initial temperature distribution, we need to
!       double the number of Fourier coefficients again. Finally, the
!       extra sum of one comes from the fact that the sine function
!       expansion starts a 1 and the Fourier expansion starts at 0.
!

!  n is the order of the diffusion operator equation.
        n = dif_nx * dif_ny
!
!
!   Allocate room for the eigenvalues of diffusion operator.
!
        allocate( lambda(n), stat=stat)
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( a(n), stat=stat)
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( ab(n), stat=stat)
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( a_rep(n), stat=stat)
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for sines of x and y coordinates.
!
        allocate( xsines(dif_npts, dif_nx) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( ysines(dif_npts, dif_ny) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for temperature history at selected points.
!
        allocate( temp(dif_npts, dif_ntemps) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( f(n,n) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( b(n,n) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1
        allocate( abt(n,dif_ntemps) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

        allocate( x(n,dif_ntemps) , stat=stat )
        if( stat .ne. 0) num_errors = num_errors + 1

!
!   Allocate room for global temperature profile expansion vector at time t.
!

        if( num_errors .gt. 0 ) then
          write(*,*) 'Error in allocating arrays in main'
          stop
        endif


!
!      A call to expand_temp_profile returns the sine expansion
!      coefficients of the initial temperature profile.
!
!


!
!  Represent initial temperature in sine function expansion.
!
        call expand_temp_profile(a)
        a_rep = a
!
!  Here, we are calculating the initial set of coefficients
!  in the sine function expansion as given in equations 3 and 4 of
!  the discussion paper
!


!
!       The call to get_diffusion_matrix returns the diffusion
!        operator in the sine function basis.
!
        call get_diffusion_matrix(f)
!
!   This last call determines the matrix elements defined by equation 10
!   in the discussion paper.
!



!
!    Here we precalculate the sine functions, sin(kx) and sin(jy) used
!    in equation 13 of the discussion paper.
!    These sine functions are only evaluated at the points x and y for
!    which the solution is evaluated.
!

        do i = 1, dif_nx
          do j = 1, dif_npts
            t1 = i*dif_x(j)
            xsines(j,i) = sqrt(2.d0/pi) * dsin(t1)
          enddo
        enddo
        do i = 1, dif_ny
          do j = 1, dif_npts
            t2 = i*dif_y(j)
            ysines(j,i) = sqrt(2.d0/pi) * dsin( t2)
          enddo
        enddo

!
!   Test to see if we had any allocation errors.
!

        if( num_errors .gt. 0 ) then
          write(*,*) 'Error in allocating arrays for syevx in main'
          stop
        endif

!
! The call to syevx will find both the eigenvalues and eigenvectors
! of the diffusion matrix operator f. The eigenvalues will be stored in
! the vector lambda and the corresponding eigenvectors will be stored in
! the matrix b.  The f and b matrices in the program correspond to the
! F and B matrices in equations 11 and 12 in the
! discussion paper.
!
!
        call syevx(f,lambda,'u', z=b)

!
!       Multiply the transpose of the eigenvector matrix, b, with the sine
!       expansion of the initial temperature profile, a, to obtain the
!       initial temperature profile in terms of the eigenfunctions of the
!       diffusion operator.
!
        ab(:) = 0.d0
        do i = 1, n
         do j = 1, n
          ab(i) = b(j,i)*a_rep(j) + ab(i)
         enddo
        enddo

!
!  This first matrix multiplication, yielding the matrix ab,
!  corresponds to the inner summation in equation 10
!  of the discussion paper.
!

!
!    Calculate temperature profile for each time step.
!
        do it = 1, dif_ntemps
          i = 0
          do i = 1, n
            abt(i,it) = exp( -lambda(i) *it*dif_delta_t) * ab(i)
          enddo
        enddo
!
!   abt now has the expansion of the temperature profile in terms of the
!   eigenvectors of the diffusion operator.
!


!
!   Multiply the eigenvector matrix with abt to give the sine function
!   expansion of the temperature profile at time t, x.
!

          call gemm(1.d0,b,abt,0.d0,x,'N','N')

!  This last sum corresponds to the outer sum of equation 12, where the
!  time dependent expansion coefficients a{sub l}(t) are stored in the
!  temporary array x in the program.
!



         do it = 1, dif_ntemps
          do k = 1, dif_npts
            temp(k, it) = 0.d0
          enddo
         enddo

         do it = 1, dif_ntemps
          do i = 1, dif_nx*dif_ny
            iy = (i-1)/dif_nx+1
            ix = mod(i-1,dif_nx)+1
            do k = 1, dif_npts
              temp(k,it) = temp(k,it) + xsines(k,ix) * ysines(k,iy)            &
     &                        * x(i,it)
            enddo
           enddo
          enddo



!
!  This last do loop corresponds to the double summation in equation
!  13 of the discussion paper.
!




!
!   Here, we are just writing out the temperatures at the selected times
!   and points.
!
         write(*,*) '   point #      X         Y'
         do i = 1, dif_npts
           write(*,'(2x, i6, 2x, 2f11.4)') i, dif_x(i), dif_y(i)
         enddo
         write(*,*)
         do k = 1, dif_npts, 6
           write(*,*)
           write(*,'(30X,''Points'')')
           write(*,'(''   TIME   '',6(5x,''#'', i4))') (i, i=k, k+5)
           do i = 1, dif_ntemps
              write(*,'(7f10.5)') i*dif_delta_t,                             &
     &                          (temp(j,i),j=k,min(k+5,dif_npts))
           enddo
        enddo

        deallocate(xsines)
        deallocate(ysines)
        deallocate(lambda)
        deallocate(temp)
        deallocate(a)
        deallocate(abt)
        deallocate(ab)
        deallocate(f)
        deallocate(dif_x)
        deallocate(dif_y)
        stop
        end

!
```

Module Parameters (HPF)

```
        module parameters
!
!   Purpose: Define system wide parameters and pessl structure.
!
          implicit none
          public
          integer, parameter :: long=8, short=4
          real(long), parameter :: pi = 3.141592653589793d0
          real(long), parameter :: twopi = 2.d0*pi

        end module parameters
```

Module Diffusion (HPF)

```
        module diffusion
!
!   Purpose: Assign problem parameters and initial data.
!
!   Routines called:
!       none
!
        use parameters
        implicit none
        private
!
! Make all entities private by default.
! Have all public entities have the prefix dif_.
!
!  The following are the publicly available routines.
!
        public init_diffusion, init_temp, diff_coef

!
!     The following are publicly available variables.
!

        real, public :: dif_ly_ratio
        integer, public :: dif_nx, dif_ny, dif_npts, dif_ntemps
        integer, public :: dif_numx, dif_numy
```

```
      real(long), public :: dif_delta_t

!
!     dif_ly_ratio  is the ratio of the x and y lengths of the beam.
!     dif_nx        is the number of sine expansion coefficients to use
!                   in the x direction.
!     dif_ny        is the number of sine expansion coefficients to use
!                   in the y direction.
!     dif_delta_t   is the size of the time step to be display on output.
!     dif_ntemps    is the total number of temperatures to display per point.
!     dif_npts      is the total number of points to output.
!
!


!
!     Private variables
!
      integer :: init_f=1, diff_f=1
!   init_f chooses the functional form of initial distribution of temperature.
!   diff_f chooses the functional form for spatially dependent head diffusion
!          coefficient.

      contains

!****************************************************************************!
!*                                                                          *!
!*   Module routine init_diffusion                                          *!
!*                                                                          *!
!*   Purpose: Initialize problem size, number of output point and           *!
!*            functional form of diffusion constant and initial temperature *!
!*            distribution                                                  *!
!*                                                                          *!
!****************************************************************************!
        subroutine init_diffusion
        namelist /input/ ly_ratio, delta_t, numx, numy, nx, ny, numt,         &
     &                  init_f, diff_f
        integer :: numx=5, numy=5, nx=7, ny=7, numt=20
        real(long) :: ly_ratio=1.d0, delta_t=0.1
        real(long) :: delx, dely
        integer :: i, j, ij
        logical :: ex
!==============================================================================!
!          Start of executable code                                            !

        inquire ( file='diffus.naml', exist=ex)
        if( ex ) then
          open( 10, file='diffus.naml', action='read')
          read( 10, input)
          close(10)
        endif

        dif_ly_ratio = ly_ratio
        dif_npts = numx*numy
        dif_delta_t = delta_t
        dif_ntemps  = numt
        dif_nx = nx
        dif_ny = ny
        dif_numx = numx
        dif_numy = numy
        return
        end subroutine init_diffusion

!*****************************************************************************!
!*                                                                           *!
!*   Module routine init_temp                                                *!
!*                                                                           *!
!*   Purpose: Return the initial temperature of the bar at a particular     *!
!*            point                                                          *!
!*                                                                           *!
!*****************************************************************************!
        function init_temp(x, y)
!
!     Arguments:
!       x: real*8 (in), x coordinate
!       y: real*8 (in), y coordinate
!     Function return:
!     init_temp: real*8 (out), initial temperature at (x,y)
!
        real(long), intent(in) :: x, y
        real(long) :: init_temp

!
!   The problem has been scaled to go from 0 to pi in both the x
!   and y directions. To calculate the expansion coefficients, we
!   define the function to be odd about pi and use the range 0 < x < 2*pi
!
!  Local variables.
        integer :: isign
        real(long) :: x1, y1
!
        isign = 1
        x1 = x
        if( x .gt. pi ) then
          isign = -isign
          x1 = twopi - x
        endif
        y1 = y
        if( y .gt. pi ) then
          isign = -isign
          y1 = twopi - y
        endif

!
! Choose very simple temperature profile cases.
!
        select case (init_f)
          case (1)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)
          case (2)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*y1
          case (3)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*x1
          case (4)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)*x1*y1
          case (5)
             init_temp = isign*(x1*(pi-x1))*y1*(pi-y1)
          case (6)
             init_temp = isign*(x1*(pi-x1))**2 *y1*(pi-y1)
          case (7)
             init_temp = isign*(x1*(pi-x1))*(y1*(pi-y1))**2
          case default
             init_temp = isign*sin(x1)*sin(y1)
        end select
        return
        end function init_temp



!*****************************************************************************!
!*                                                                           *!
!*   Module routine diff_coef                                                *!
!*                                                                           *!
!*   Purpose: Return the value of the thermal diffusion coefficient at      *!
!*            an arbitrary point                                             *!
!*                                                                           *!
!*****************************************************************************!
        function diff_coef(x, y)
!     Arguments:
!       x: real*8 (in), x coordinate
!       y: real*8 (in), y coordinate
!     Function return:
!       diff_coef: real*8 (out), diffusion coefficient at (x,y)
!
         real(long), intent(in) :: x, y
         real(long) :: diff_coef

!
!   The problem has been scaled to go from 0 to pi in both the x
!   and y directions. To simplify the matrix element calculations,
!   we define the function to be even about pi.
!

!  Local variables.
        real(long) :: x1, y1

!=============================================================================!
!          Start of executable code.                                          !
        x1 = x
        if( x .gt. pi )  x1 = twopi - x
        y1 = y
        if( y .gt. pi ) y1 = twopi - y

!
! Choose very simple diffusion coefficient cases.
!
        select case (diff_f)
           case (1)
             diff_coef = .5d0 + (x1 + y1) / (2 * twopi)
           case (2)
             diff_coef = ((1.d0 + x1)*(pi - x1 + 1.d0)*(y1 + pi))/ 3*pi
           case (3)
             diff_coef = (y1 + pi) * pi/((pi + x1) * (2* pi - x1))
           case default
             diff_coef = 1.d0
```

```
                 end select
              return
              end function diff_coef


            end module diffusion
```

Module Fourier (HPF)

```
            module fourier
!
!   Purpose: To represent both the diffusion operator and
!            the temperature profile in a sine function basis.
!
!   Routines called:
!     fft
!
            use parameters
            use diffusion
            use pessl_hpf
            use hpf_library
            implicit none
            private
!
!     all entities private by default
!
!
!   publicly available routines
!
            public expand_temp_profile, get_diffusion_matrix
!
!   publicly available variables
!


!
!   private variables
!
!     integer :: pn_fac(5) = 5*0  ! prime factors of number of nodes
!     nnodes = 2**pn_fac(1) * 3**pn_fac(2) * 5**pn_fac(3) *
!              7**pn_fac(4) * 11**pn_fac(5)


!
!   private routines
!
            private minpower2, factor_nodes
            contains

!*****************************************************************************!
!*                                                                           *!
!*   Module routine get_diffusion_matrix                                     *!
!*                                                                           *!
!*   Purpose: To obtain the matrix representation of the diffusion           *!
!*            operator in a sine function basis                              *!
!*                                                                           *!
!*****************************************************************************!
              subroutine get_diffusion_matrix(f)
!
!   Arguments:
```

```
!     f: real*8,dimension(:,:),(out), local part of the global matrix
!               containing the diffusion operator in sine function basis.
!
        real(long), intent(out) :: f(:,:)
!HPF$ DISTRIBUTE *(*,BLOCK) :: f

! Local variables
!
!  df contains the diffusion coefficient before the call to fft.
!  df contains the Fourier transpose of diffusion coefficients after the call.
!
        complex(long), allocatable :: df(:,:)
!HPF$ DISTRIBUTE (*,BLOCK) :: df
!
!  ixi and iyi are arrays which, given a global index,
!  return the x and y offsets.  Recall that the large arrays
!  are 4 dimensional arrays collapsed into 2 dimensions,
!  where i = (ix-1)*dif_ny + iy.
!
        integer, allocatable :: ixi(:), iyi(:)
        real(long) :: scale_f
        integer :: nx, ny, ix, iy, ixp, iyp, istat
        integer :: ixdiff, iydiff, num_errors
        integer :: naux1, naux2, i, j, factor1, factor2, idum
        integer :: pn_fac(5)   ! prime factors of number of nodes
!
!   Fourier transform of diffusion coefficient function

        pn_fac = 0
        call factor_nodes(pn_fac)
        factor1 = 3**pn_fac(2) * 5**pn_fac(3) * 7**pn_fac(4) *                &
     &                          11**pn_fac(5)
!
!  Here we are trying to find the smallest number which is evenly
!  divisible by the number of processes and is larger than 4*(n+1).
!
        factor2 = (4*(dif_nx+1) + factor1 -1)/factor1
        nx = minpower2( factor2,idum) * factor1

        factor2 = (4*(dif_ny+1) + factor1 -1)/factor1
        ny = minpower2( factor2,idum) * factor1

        scale_f = 1.d0/ real(nx*ny, long)


!
!  Get storage for diffusion array.
!
        allocate(df(nx,ny))
!
!    Here, we initialize the local part of the global array df, which
!    contains the value of the diffusion coefficient function, evenly
!    evaluated between (0, 2*pi).  We do a two dimensional Fourier
!    transform on the data.  Because the size of this array is so small,
!    nx by ny, and ultimately we have to transfer the whole array to
!    each node, it would probably be more efficient to do the calculation
!    locally on each node.
!
```

```
!    Get the value of the diffusion coefficient function at
!    the necessary points.
!
        do j = 1, ny
          do i =1,  nx
            df(i,j) = diff_coef((twopi*(i-1))/nx,(twopi*(j-1))/ny)
          enddo
        enddo
!
!  This last loop just determined the diffusion coefficient at evenly
!  spaced points along the x and y coordinates.
!

!
!
!  Do the Fourier transform.
!
        call fft(df,scale=scale_f,transpose='N')
!
!
!   df now has the Fourier coefficients for the diffusion coefficient
!      function, which correspond to the D(tilde)(sub ij) given in the
!      discussion paper.
!
!
!  First allocate the index arrays.
!
        num_errors=0
        allocate(ixi(dif_nx*dif_ny), stat=istat)
        if( istat .ne. 0 ) num_errors = num_errors + 1
        allocate(iyi(dif_nx*dif_ny), stat=istat)
        if( istat .ne. 0 ) num_errors = num_errors + 1

!
!  Now load up the diffusion operator
!    f(ix,iy;ix',iy').
!
!    Here we transform the 4d matrix into the 2d matrix where
!       i = (iy-1)* dif_nx + ix + 1
!    and
!       j = (iy'-1)* dif_nx + ix' + 1.
!
!  First calculate the index arrays.
!
        do ix = 1, dif_nx
          do iy = 1, dif_ny
            i = (iy-1)* dif_nx + ix
            ixi(i) = ix
            iyi(i) = iy
          enddo
        enddo

!
! This final loop loads the matrix elements up for F as given in
! equation 10.
!
        do j = 1, dif_nx*dif_ny
          iyp = iyi(j)
          ixp = ixi(j)
          do i = 1, dif_nx*dif_ny
            iy = iyi(i)
            ix = ixi(i)
            ixdiff = iabs(ix-ixp) + 1
            iydiff = iabs(iy-iyp) + 1
            f(i,j) = ( ( ix*ixp + iy*iyp*dif_ly_ratio ) *                   &
     &              real(df(ixdiff, iydiff)    - df(ix+ixp+1,iy+iyp+1))        &
     &              +     ( ix*ixp - iy*iyp*dif_ly_ratio ) *                &
     &              real(df(ix+ixp+1,iydiff )  - df(ixdiff,iy+iyp+1)))
          enddo
        enddo
        deallocate(df)
        deallocate(ixi)
        deallocate(iyi)
        return
        end subroutine get_diffusion_matrix



!****************************************************************************!
!*                                                                          *!
!*   Module routine expand_temp_profile                                     *!
!*                                                                          *!
!*   Purpose: To obtain the expansion coefficients of the initial          *!
!*            temperature profile in a sine function expansion             *!
!*                                                                          *!
!****************************************************************************!
        subroutine expand_temp_profile(a)
!
!   Arguments:
!     a: real*8,dimension(:),(out), local part of the global matrix,
!                 containing the sine coefficients for initial
!                 temperature distribution.
!
        real(long), intent(out) :: a(:)
!HPF$ DISTRIBUTE *(BLOCK) :: a

! Local variables
        complex(long), allocatable :: atmp(:,:)
!HPF$ DISTRIBUTE (*,BLOCK) :: atmp
        integer :: i,j, nx, ny, istat, naux1, naux2
        integer :: idum, jb, jx, jy
        real(long) :: x, y, scale_f

!
!      Calculate the minimum power of 2 to hold twice the number of
!      Fourier coefficients as sine coefficients. The top half of the
!      Fourier coefficients will equal minus the bottom half because
!      we are forcing the temperature profile to be odd.
!
        nx = minpower2( 2*(dif_nx+1), idum)
        ny = minpower2( 2*(dif_ny+1), idum)
        scale_f = -twopi / real( nx*ny,long)

!
!       Temperature profile allocation.
        allocate(atmp(nx,ny), stat=istat )
        if( istat .ne. 0 )  then
          write(*,*) 'allocation failure in expand_temp_profile'
          stop
        endif

!
!
        do i = 1, nx
          do j = 1, ny
            atmp(i,j) = init_temp((twopi*(i-1))/nx, (twopi*(j-1))/ny)
          enddo
        enddo

!
!  Do the 2d Fourier transform of atmp.
!
        call fft(atmp,scale=scale_f,transpose='N')
!
! The calls to fft calculated the dual Fourier transform as
! defined by equation 5 in the discussion paper.
!

!
!  This final loop is to load only those portions of the global array
!  corresponding to the local portion of that array for this processor.
!
        do j =  1, dif_nx * dif_ny
          jx = mod(j-1, dif_nx) + 2
          jy = (j-1) / dif_nx + 2
          a(j) = real(atmp(jx,jy),long)
        enddo

        deallocate(atmp)
        return
        end subroutine expand_temp_profile


!***************************************************************************!
!*                                                                         *!
!*   Module routine factor_nodes                                           *!
!*                                                                         *!
!*   Purpose: To obtain the powers of prime factorization of the number    *!
!*            nodes, failing if the factorization is not compatible with   *!
!*            FFT supported transform lengths                              *!
!*                                                                         *!
!***************************************************************************!
        subroutine factor_nodes(pn_fac)
!  Arguments:
        integer    :: pn_fac(:)
!
!  Local variables
        integer n1, n2, l2
!
!   Determine the prime factorization of nnodes, which must
!       be of the form 2**n * 3**m * 5**i * 7**j * 11**k
!       where 0 <= m <=2, 0 <= i <= 1, 0 <= j <= 1, 0 <= k <= 1,
!       and 0> n >= 25.
!
```

```
        n2 = number_of_processors()
        n1 = n2/11
        if( n1*11 .eq. n2) then
            pn_fac(5) = 1
            n2 = n1
        endif

        n1 = n2/7
        if( n1*7 .eq. n2) then
            pn_fac(4) = 1
            n2 = n1
        endif

        n1 = n2/5
        if( n1*5 .eq. n2) then
            pn_fac(3) = 1
            n2 = n1
        endif

        n1 = n2/3
        if( n1*3 .eq. n2) then
          if ( (n1/3)*3 .eq. n1 ) then
            pn_fac(2) = 2
            n2 = n1/3
          else
            pn_fac(2) = 1
            n2 = n1
          endif
        endif

        n1 = minpower2(n2,l2)
        pn_fac(1) = l2

        if( n1 .ne. n2) then
          write(*,*) 'Invalid number of nodes, it must have the form:'
          write(*,*) '2**n * 3**m * 5**i * 7**j * 11**k, where '
          write(*,*) ' n >= 0, 0<=m<=2 and 0<= i,j,k <=1 '
          write(*,*) ' choose a power of 2 for best performance'
          stop
        endif
        end subroutine factor_nodes


!***************************************************************************!
!*                                                                         *!
!*   Module function minpower2                                             *!
!*                                                                         *!
!*   Purpose: To obtain the smallest number which is a power of 2 and      *!
!*            greater than or equal to the input argument                  *!
!*                                                                         *!
!***************************************************************************!
        function minpower2( n, log2n )
!
!   Arguments:
!      n: integer*4, (in), input number
!      log2n: integer*4, (out), log base 2 of the function result
!   Function return:
!      minpower2: integer*4 (out), smallest number, which is a power of 2
!                  and greater than or equal to n.
!
        integer n, minpower2, log2n
!
!   Local variables.
        integer m, i
        m=n

        if( n < 0 ) write(*,*) 'n cannot be negative'
        powerloop: do i= 1, bit_size(n)
          m = m / 2
          if( m .eq. 0 ) exit powerloop
        enddo powerloop
        if ( 2**(i-1) .ne. n ) then
           if( 2**i < 0) write(*,*) 'n too large'
           log2n = i
           minpower2 = 2**i
        else
           log2n = i-1
           minpower2 = n
        endif
        return
        end function minpower2

      end module fourier
```

Input Data

```
&input
ly_ratio = 1.0, delta_t = .1, nx =15, ny=15, numx = 5, numy =5,
numt=20, init_f=1, diff_f =3
/
```

Output Data

```
 0:    point #      X         Y
 0:       1      0.5236     0.5236
 0:       2      1.0472     0.5236
 0:       3      1.5708     0.5236
 0:       4      2.0944     0.5236
 0:       5      2.6180     0.5236
 0:       6      0.5236     1.0472
 0:       7      1.0472     1.0472
 0:       8      1.5708     1.0472
 0:       9      2.0944     1.0472
 0:      10      2.6180     1.0472
 0:      11      0.5236     1.5708
 0:      12      1.0472     1.5708
 0:      13      1.5708     1.5708
 0:      14      2.0944     1.5708
 0:      15      2.6180     1.5708
 0:      16      0.5236     2.0944
 0:      17      1.0472     2.0944
 0:      18      1.5708     2.0944
 0:      19      2.0944     2.0944
 0:      20      2.6180     2.0944
 0:      21      0.5236     2.6180
 0:      22      1.0472     2.6180
```

```
0:     23       1.5708     2.6180
0:     24       2.0944     2.6180
0:     25       2.6180     2.6180
0:
0:
0:                                 Points
0:   TIME        #   1     #   2     #   3     #   4     #   5     #   6
0:   0.10000   1.62046   2.69937   3.06225   2.69937   1.62046   2.58264
0:   0.20000   1.41067   2.41366   2.76175   2.41366   1.41067   2.24055
0:   0.30000   1.23798   2.15271   2.48036   2.15271   1.23798   1.95718
0:   0.40000   1.09075   1.91572   2.21798   1.91572   1.09075   1.71506
0:   0.50000   0.96250   1.70103   1.97560   1.70103   0.96250   1.50484
0:   0.60000   0.84945   1.50710   1.75384   1.50710   0.84945   1.32086
0:   0.70000   0.74925   1.33257   1.55266   1.33257   0.74925   1.15925
0:   0.80000   0.66027   1.17611   1.37141   1.17611   0.66027   1.01707
0:   0.90000   0.58127   1.03638   1.20907   1.03638   0.58127   0.89197
0:   1.00000   0.51121   0.91203   1.06432   0.91203   0.51121   0.78192
0:   1.10000   0.44919   0.80169   0.93574   0.80169   0.44919   0.68518
0:   1.20000   0.39438   0.70404   0.82187   0.70404   0.39438   0.60020
0:   1.30000   0.34601   0.61781   0.72126   0.61781   0.34601   0.52560
0:   1.40000   0.30340   0.54179   0.63255   0.54179   0.30340   0.46016
0:   1.50000   0.26591   0.47487   0.55444   0.47487   0.26591   0.40277
0:   1.60000   0.23295   0.41604   0.48576   0.41604   0.23295   0.35247
0:   1.70000   0.20401   0.36436   0.42543   0.36436   0.20401   0.30841
0:   1.80000   0.17861   0.31901   0.37249   0.31901   0.17861   0.26982
0:   1.90000   0.15634   0.27924   0.32605   0.27924   0.15634   0.23604
0:   2.00000   0.13682   0.24438   0.28535   0.24438   0.13682   0.20647
0:
0:                                 Points
0:   TIME        #   7     #   8     #   9     #  10     #  11     #  12
0:   0.10000   4.31793   4.90301   4.31793   2.58264   2.85249   4.78806
0:   0.20000   3.84769   4.40918   3.84769   2.24055   2.43863   4.20461
0:   0.30000   3.41303   3.93800   3.41303   1.95718   2.10266   3.67816
0:   0.40000   3.01815   3.49799   3.01815   1.71506   1.82146   3.21228
0:   0.50000   2.66276   3.09466   2.66276   1.50484   1.58234   2.80362
0:   0.60000   2.34497   2.72994   2.34497   1.32086   1.37711   2.44657
0:   0.70000   2.06219   2.40319   2.06219   1.15925   1.19994   2.13515
0:   0.80000   1.81149   2.11234   1.81149   1.01707   1.04644   1.86371
0:   0.90000   1.58985   1.85460   1.58985   0.89197   0.91312   1.62712
0:   1.00000   1.39435   1.62693   1.39435   0.78192   0.79712   1.42087
0:   1.10000   1.22220   1.42626   1.22220   0.68518   0.69609   1.24100
0:   1.20000   1.07081   1.24971   1.07081   0.60020   0.60802   1.08410
0:   1.30000   0.93783   1.09457   0.93783   0.52560   0.53119   0.94718
0:   1.40000   0.82111   0.95838   0.82111   0.46016   0.46415   0.82766
0:   1.50000   0.71874   0.83892   0.71874   0.40277   0.40561   0.72330
0:   1.60000   0.62901   0.73420   0.62901   0.35247   0.35450   0.63215
0:   1.70000   0.55039   0.64244   0.55039   0.30841   0.30985   0.55254
0:   1.80000   0.48154   0.56208   0.48154   0.26982   0.27084   0.48298
0:   1.90000   0.42125   0.49171   0.42125   0.23604   0.23676   0.42220
0:   2.00000   0.36848   0.43011   0.36848   0.20647   0.20697   0.36908
0:
0:                                 Points
0:   TIME        #  13     #  14     #  15     #  16     #  17     #  18
0:   0.10000   5.44297   4.78806   2.85249   2.45333   4.13480   4.70628
0:   0.20000   4.82635   4.20461   2.43863   2.04155   3.53366   4.06306
0:   0.30000   4.25037   3.67816   2.10266   1.72459   3.02543   3.50098
0:   0.40000   3.72711   3.21228   1.82146   1.47101   2.59910   3.01855
0:   0.50000   3.26069   2.80362   1.58234   1.26277   2.23988   2.60657
```

```
0:   0.60000   2.84936   2.44657   1.37711   1.08877   1.93538   2.25472
0:   0.70000   2.48867   2.13515   1.19994   0.94166   1.67587   1.95359
0:   0.80000   2.17330   1.86371   1.04644   0.81630   1.45370   1.69519
0:   0.90000   1.89793   1.62712   0.91312   0.70885   1.26279   1.47284
0:   1.00000   1.65761   1.42087   0.79712   0.61636   1.09824   1.28104
0:   1.10000   1.44792   1.24100   0.69609   0.53650   0.95604   1.11524
0:   1.20000   1.26492   1.08410   0.60802   0.46739   0.83291   0.97163
0:   1.30000   1.10520   0.94718   0.53119   0.40745   0.72611   0.84705
0:   1.40000   0.96575   0.82766   0.46415   0.35539   0.63334   0.73883
0:   1.50000   0.84399   0.72330   0.40561   0.31012   0.55266   0.64471
0:   1.60000   0.73764   0.63215   0.35450   0.27071   0.48243   0.56279
0:   1.70000   0.64474   0.55254   0.30985   0.23638   0.42125   0.49142
0:   1.80000   0.56358   0.48298   0.27084   0.20646   0.36792   0.42920
0:   1.90000   0.49265   0.42220   0.23676   0.18036   0.32140   0.37493
0:   2.00000   0.43067   0.36908   0.20697   0.15758   0.28081   0.32758
0:
0:                                 Points
0:   TIME         #  19     #  20     #  21     #  22     #  23     #  24
0:   0.10000   4.13480   2.45333   1.42982   2.41948   2.75758   2.41948
0:   0.20000   3.53366   2.04155   1.14813   1.99356   2.29549   1.99356
0:   0.30000   3.02543   1.72459   0.95015   1.67033   1.93488   1.67033
0:   0.40000   2.59910   1.47101   0.79960   1.41458   1.64393   1.41458
0:   0.50000   2.23988   1.26277   0.67989   1.20679   1.40486   1.20679
0:   0.60000   1.93538   1.08877   0.58206   1.03495   1.20593   1.03495
0:   0.70000   1.67587   0.94166   0.50068   0.89108   1.03879   0.89108
0:   0.80000   1.45370   0.81630   0.43217   0.76952   0.89732   0.76952
0:   0.90000   1.26279   0.70885   0.37401   0.66612   0.77685   0.66612
0:   1.00000   1.09824   0.61636   0.32432   0.57769   0.67376   0.57769
0:   1.10000   0.95604   0.53650   0.28168   0.50176   0.58522   0.50176
0:   1.20000   0.83291   0.46739   0.24495   0.43633   0.50892   0.43633
0:   1.30000   0.72611   0.40745   0.21322   0.37981   0.44300   0.37981
0:   1.40000   0.63334   0.35539   0.18576   0.33088   0.38592   0.33088
0:   1.50000   0.55266   0.31012   0.16193   0.28844   0.33642   0.28844
0:   1.60000   0.48243   0.27071   0.14124   0.25158   0.29343   0.25158
0:   1.70000   0.42125   0.23638   0.12325   0.21953   0.25604   0.21953
0:   1.80000   0.36792   0.20646   0.10759   0.19163   0.22350   0.19163
0:   1.90000   0.32140   0.18036   0.09395   0.16733   0.19516   0.16733
0:   2.00000   0.28081   0.15758   0.08205   0.14614   0.17045   0.14614
0:
0:                                 Points
0:   TIME         #  25     #  26     #  27     #  28     #  29     #  30
0:   0.10000   1.42982
0:   0.20000   1.14813
0:   0.30000   0.95015
0:   0.40000   0.79960
0:   0.50000   0.67989
0:   0.60000   0.58206
0:   0.70000   0.50068
0:   0.80000   0.43217
0:   0.90000   0.37401
0:   1.00000   0.32432
0:   1.10000   0.28168
0:   1.20000   0.24495
0:   1.30000   0.21322
0:   1.40000   0.18576
0:   1.50000   0.16193
0:   1.60000   0.14124
0:   1.70000   0.12325
```

```
0:   1.80000   0.10759
0:   1.90000   0.09395
0:   2.00000   0.08205
```

Sample Sparse Linear Algebraic Equations Programs

This section contains the message passing sample programs and utilities that use the Fortran 90 and Fortran 77 sparse linear algebraic equations subroutines. You can use the makefile shown in “Makefile (Message Passing)” on page 1097 to build these sample programs. A copy of these programs and the makefile are provided with the Parallel ESSL product. For file names and installation procedures, see the *Parallel ESSL Installation Memo*.

The following list describes these sample programs and their utilities:

- The Fortran 90 and Fortran 77 sample programs shown in “Fortran 90 Sample Sparse Program” on page 1056 and “Fortran 77 Sample Sparse Program” on page 1066, respectively, solve a sparse linear system based on the discretization of the same elliptic partial differential equation:

$$-b_1 \frac{\partial^2(u)}{\partial x^2} - b_2 \frac{\partial^2(u)}{\partial y^2} - b_3 \frac{\partial^2(u)}{\partial z^2} - a_1 \frac{\partial(u)}{\partial x} - a_2 \frac{\partial(u)}{\partial y} - a_3 \frac{\partial(u)}{\partial z} + a_4 u = 0$$

  with the Dirichlet boundary conditions on the unit cube, that is, $0 \le x,y,z \le 1$. The equation is discretized with finite differences and uniform stepsize, where the resulting discrete equation is:

$$u(x,y,z)\,(2b_1+2b_2+2b_3+a_1+a_2+a_3) + u(x-1,y,z)\,(-b_1-a_1) + u(x,y-1,z)\,(-b_2-a_2)$$
$$+\, u(x,y,z-1)\,(-b_3-a_3) - u(x+1,y,z)b_1 - u(x,y+1,z)b_2 - u(x,y,z+1)\,(b_3)\left({}^{1}\!/_{h^2}\right)$$

  This elliptic partial differential equation is similar to an example problem discussed in [43].

  In these sample programs the index space of the discretized computational domain is first numbered sequentially in a standard way. Then, the corresponding vector is distributed using block data distribution, which is implemented using the subroutine shown in “PART_BLOCK (Block Data Distribution)” on page 1084.

  Boundary conditions are set in a very simple way, by adding equations of the form: *u*(*x,y,z*) = *rhs*(*x,y,z*)

  From the command line, you can specify the number of gridpoints along the edges of the unit cube, the iterative solution method, the preconditioner and the stopping criterion to be used.

- The Fortran 90 sample program shown in “Fortran 90 Sample Sparse Program (using the Harwell-Boeing exchange format)” on page 1077 shows how to build and solve a sparse linear system with the coefficient matrices read from external storage, using the Harwell-Boeing exchange format. Details on the Harwell-Boeing exchange format and sample matrices are available from `http://math.nist.gov/MatrixMarket/`.

From the command line, you can specify a file containing the input matrix, an iterative method and preconditioner, and a data distribution to be used.

This sample program uses the following subroutines:

– One of the following data distribution subroutines:
  - PART_BLOCK, which implements a block data distribution
  - PARTBCYC, which implements a block-cyclic data distribution
  - PARTRAND, which implements a random data distribution

  (These subroutines are documented in "Sample PARTS Subroutine" on page 1084.)
– READ_MAT, a serial module, reads a matrix in Harwell-Boeing format from a file (see "The READ_MAT Subroutine" on page 1088)
– MAT_DIST, a utility module, scatters a sparse matrix across a process grid according to a user-specified data distribution (see "The MAT_DIST Subroutine" on page 1092)
– DESYM, a utility subroutine, takes a matrix stored in symmetric format (that is, stores only the upper or lower triangle) and converts it into full storage format, assuming storage-by-rows representation. (see "The DESYM Subroutine" on page 1096)

Note: The performance of the iterative methods depends heavily on the choice of data distribution. The random data distribution is usually not a good choice. It is provided to serve as a template to help you implement a graph partitioning data distribution, which you can do by substituting the call to the random number generator in the PARTRAND initialization routine with a call to a graph partitioning package. The data distributions based on graph partitioning and/or physical considerations usually give the best performance; in general, experimentation is required to determine the best data distribution for your particular application.

Fortran 90 Sample Sparse Program

```
@process free(f90) init(f90ptr) nosave
!
! This sample program shows how to build and solve a sparse linear
! system using the subroutines in the  sparse section of Parallel
! ESSL. The matrix and RHS are generated
! in  parallel, so that there is no serial bottleneck.
!
! The program  solves a linear system based on the partial differential
! equation
!
!
!   b1 dd(u)  b2 dd(u)    b3 dd(u)    a1 d(u)   a2 d(u)  a3 d(u)
! -   ------ -  ------ -  ------ -  -----  -  ------  -  ------ + a4 u
!      dxdx     dydy       dzdz        dx       dy         dz
!
!  = 0
!
! with  Dirichlet boundary conditions on the unit cube
!
!    0<=x,y,z<=1
!
```

```
! The equation is discretized with finite differences and uniform stepsize;
! the resulting  discrete  equation is
!
! ( u(x,y,z)(2b1+2b2+2b3+a1+a2+a3)+u(x-1,y,z)(-b1-a1)+u(x,y-1,z)(-b2-a2)+
!  + u(x,y,z-1)(-b3-a3)-u(x+1,y,z)b1-u(x,y+1,z)b2-u(x,y,z+1)b3)*(1/h**2)
!
!
! In this sample program the index space of the discretized
! computational domain is first numbered sequentially in a standard way,
! then the corresponding vector is distributed according to an HPF BLOCK
! distribution directive.
!
! Boundary conditions are set in a very simple way, by adding
! equations of the form
!
!   u(x,y,z) = rhs(x,y,z)
!
Program PDE90
  USE F90SPARSE
  Implicit none

  INTERFACE PART_BLOCK
     !   .....user defined subroutine.....
     SUBROUTINE PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)
       IMPLICIT NONE
       INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
       INTEGER, INTENT(OUT) :: NV
       INTEGER, INTENT(OUT) :: PV(*)
     END SUBROUTINE PART_BLOCK
  END INTERFACE

  ! Input parameters
  Character*10 :: CMETHD, PREC
  Integer      :: IDIM, IRET

  ! Miscellaneous
  Integer, Parameter   :: IZERO=0, IONE=1
  Character, PARAMETER :: ORDER='R'
  INTEGER              :: IARGC
  REAL(KIND(1.D0)), PARAMETER :: DZERO = 0.D0, ONE = 1.D0
  REAL(KIND(1.D0)) :: TIMEF, T1, T2, TPREC, TSOLVE, T3, T4
  EXTERNAL  TIMEF

  ! Sparse Matrix and preconditioner
  TYPE(D_SPMAT) :: A
  TYPE(D_PRECN) :: APRC
  ! Descriptor
  TYPE(DESC_TYPE)    :: DESC_A
  ! Dense Matrices
  REAL(KIND(1.d0)), POINTER :: B(:), X(:)

  ! BLACS parameters
  INTEGER            :: nprow, npcol, icontxt, iam, np, myprow, mypcol

  ! Solver parameters
  INTEGER            :: ITER, ITMAX,IERR,ITRACE, METHD,IPREC, ISTOPC,&
      & IPARM(20)
  REAL(KIND(1.D0))   :: ERR, EPS, RPARM(20)
```

```
! Other variables
INTEGER            :: I,INFO
INTEGER            :: INTERNAL, M,II

! Initialize BLACS
CALL BLACS_PINFO(IAM, NP)
CALL BLACS_GET(IZERO, IZERO, ICONTXT)

! Rectangular Grid,  P x 1

CALL BLACS_GRIDINIT(ICONTXT, ORDER, NP, IONE)
CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

!
!  Get parameters
!
CALL GET_PARMS(ICONTXT,CMETHD,PREC,IDIM,ISTOPC,ITMAX,ITRACE)

!
!  Allocate and fill in the coefficient matrix, RHS and initial guess
!

CALL BLACS_BARRIER(ICONTXT,'All')
T1 = TIMEF()
CALL CREATE_MATRIX(PART_BLOCK,IDIM,A,B,X,DESC_A,ICONTXT)
T2 = TIMEF() - T1

CALL DGAMX2D(ICONTXT,'A',' ',IONE, IONE,T2,IONE,T1,T1,-1,-1,-1)
IF (IAM.EQ.0) Write(6,*) 'Matrix creation Time : ',T2/1.D3

!
!  Prepare the preconditioner.
!
SELECT CASE (PREC)
CASE ('ILU')
   IPREC = 2
CASE ('DIAGSC')
   IPREC = 1
CASE ('NONE')
   IPREC = 0
CASE DEFAULT
   WRITE(0,*) 'Unknown preconditioner'
   CALL BLACS_ABORT(ICONTXT,-1)
END SELECT
CALL BLACS_BARRIER(ICONTXT,'All')
T1 = TIMEF()
CALL PSPGPR(IPREC,A,APRC,DESC_A,INFO=IRET)
TPREC = TIMEF()-T1

CALL DGAMX2D(icontxt,'A',' ',IONE, IONE,TPREC,IONE,t1,t1,-1,-1,-1)

IF (IAM.EQ.0) WRITE(6,*) 'Preconditioner Time : ',TPREC/1.D3

IF (IRET.NE.0) THEN
   WRITE(0,*) 'Error on preconditioner',IRET
   CALL BLACS_ABORT(ICONTXT,-1)
   STOP
```

```
      END IF

      !
      ! Iterative method parameters
      !
      IF (CMETHD(1:6).EQ.'CGSTAB') Then
         METHD = 1
      ELSE IF (CMETHD(1:3).EQ.'CGS') Then
         METHD = 2
      ELSE IF (CMETHD(1:5).EQ.'TFQMR') THEN
         METHD = 3
      ELSE
         WRITE(0,*) 'Unknown method '
         CALL BLACS_ABORT(ICONTXT,-1)
      END IF
      EPS   = 1.D-9
      IPARM = 0
      RPARM = 0.D0
      IPARM(1) = METHD
      IPARM(2) = ISTOPC
      IPARM(3) = ITMAX
      IPARM(4) = ITRACE
      RPARM(1) = EPS
      CALL BLACS_BARRIER(ICONTXT,'All')
      T1 = TIMEF()
      CALL PSPGIS(A,B,X,APRC,DESC_A,&
           & IPARM=IPARM,RPARM=RPARM,INFO=IERR)
      CALL BLACS_BARRIER(ICONTXT,'All')
      T2 = TIMEF() - T1
      ITER = IPARM(5)
      ERR  = RPARM(2)
      CALL DGAMX2D(ICONTXT,'A',' ',IONE, IONE,T2,IONE,T1,T1,-1,-1,-1)

      IF (IAM.EQ.0) THEN
         WRITE(6,*) 'Time to Solve Matrix : ',T2/1.D3
         WRITE(6,*) 'Time per iteration : ',T2/(ITER*1.D3)
         WRITE(6,*) 'Number of iterations : ',ITER
         WRITE(6,*) 'Error on exit : ',ERR
         WRITE(6,*) 'INFO  on exit : ',IERR
      END IF

      !
      !  Cleanup storage and exit
      !
      CALL PGEFREE(B,DESC_A)
      CALL PGEFREE(X,DESC_A)

      CALL PSPFREE(APRC,DESC_A)
      CALL PSPFREE(A,DESC_A)

      CALL PADFREE(DESC_A)

      CALL BLACS_GRIDEXIT(ICONTXT)
      CALL BLACS_EXIT(0)

      STOP

    CONTAINS
```

```
  !
  !  Subroutine to allocate and fill in the coefficient matrix and
  !  the RHS.
  !
  SUBROUTINE CREATE_MATRIX(PARTS,IDIM,A,B,T,DESC_A,ICONTXT)
!
!   Discretize the partial diferential equation
!
!   b1 dd(u)  b2 dd(u)    b3 dd(u)    a1 d(u)   a2 d(u)  a3 d(u)
! -   ------ -  ------ -  ------ -  -----  -  ------  -  ------ + a4 u
!      dxdx     dydy       dzdz        dx       dy         dz
!
!  = 0
!
! boundary condition: Dirichlet
!    0< x,y,z<1
!
!  u(x,y,z)(2b1+2b2+2b3+a1+a2+a3)+u(x-1,y,z)(-b1-a1)+u(x,y-1,z)(-b2-a2)+
!  + u(x,y,z-1)(-b3-a3)-u(x+1,y,z)b1-u(x,y+1,z)b2-u(x,y,z+1)b3
    USE F90SPARSE
    Implicit None
    INTEGER                  :: IDIM
    INTERFACE PART_BLOCK
       SUBROUTINE PARTS(GLOBAL_INDX,N,P,PV,NV)
         IMPLICIT NONE
         INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, P
         INTEGER, INTENT(OUT) :: NV
         INTEGER, INTENT(OUT) :: PV(*)

       END SUBROUTINE PARTS
    END INTERFACE
    Real(Kind(1.D0)),Pointer :: B(:),T(:)
    Type (DESC_TYPE)         :: DESC_A
    Integer                  :: ICONTXT
    Type(D_SPMAT)            :: A
    Real(Kind(1.d0))         :: ZT(10),GLOB_X,GLOB_Y,GLOB_Z
    Integer                  :: M,N,NNZ,GLOB_ROW,J
    Type (D_SPMAT)           :: ROW_MAT
    Integer                  :: X,Y,Z,COUNTER,IA,I,INDX_OWNER
    INTEGER                  :: NPROW,NPCOL,MYPROW,MYPCOL
    Integer                  :: ELEMENT
    INTEGER                  :: INFO, NV, INV
    INTEGER, ALLOCATABLE     :: PRV(:)
    ! deltah dimension of each grid cell
    ! deltat discretization time
    Real(Kind(1.D0))         :: DELTAH
    Real(Kind(1.d0)),Parameter   :: RHS=0.d0,ONE=1.d0,ZERO=0.d0
    Real(Kind(1.d0))   :: TIMEF, T1, T2, TINS
    external            timef
    ! common area


    CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

    DELTAH = 1.D0/(IDIM-1)

    ! Initialize array descriptor and sparse matrix storage. Provide an
    ! estimate of the number of non zeroes
```

```
M   = IDIM*IDIM*IDIM
N   = M
NNZ = (N*7)/(NPROW*NPCOL)
Call PADALL(N,PARTS,DESC_A,ICONTXT)
Call PSPALL(A,DESC_A,NNZ=NNZ)
! Define  RHS from boundary conditions; also build initial guess
Call PGEALL(B,DESC_A)
Call PGEALL(T,DESC_A)

! We build an auxiliary matrix consisting of one row at a
! time
ROW_MAT%DESCRA(1:1) = 'G'
ROW_MAT%FIDA        = 'CSR'
ALLOCATE(ROW_MAT%AS(20))
ALLOCATE(ROW_MAT%IA1(20))
ALLOCATE(ROW_MAT%IA2(20))
ALLOCATE(PRV(NPROW))
ROW_MAT%IA2(1)=1

TINS = 0.D0
CALL BLACS_BARRIER(ICONTXT,'ALL')
T1 = TIMEF()

! Loop over rows belonging to current process in a BLOCK
! distribution.

DO GLOB_ROW = 1, N
   CALL PARTS(GLOB_ROW,N,NPROW,PRV,NV)
   DO INV = 1, NV
      INDX_OWNER = PRV(INV)
      IF (INDX_OWNER == MYPROW) THEN
         ! Local matrix pointer
         ELEMENT=1
         ! Compute gridpoint Coordinates
         IF (MOD(GLOB_ROW,(IDIM*IDIM)).EQ.0) THEN
            X = GLOB_ROW/(IDIM*IDIM)
         ELSE
            X = GLOB_ROW/(IDIM*IDIM)+1
         ENDIF
         IF (MOD((GLOB_ROW-(X-1)*IDIM*IDIM),IDIM).EQ.0) THEN
            Y = (GLOB_ROW-(X-1)*IDIM*IDIM)/IDIM
         ELSE
            Y = (GLOB_ROW-(X-1)*IDIM*IDIM)/IDIM+1
         ENDIF
         Z = GLOB_ROW-(X-1)*IDIM*IDIM-(Y-1)*IDIM
      ! GLOB_X, GLOB_Y, GLOB_X coordinates
      GLOB_X=X*DELTAH
      GLOB_Y=Y*DELTAH
      GLOB_Z=Z*DELTAH

         ! Check on boundary points
         IF (X.EQ.1) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE IF (Y.EQ.1) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE IF (Z.EQ.1) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE IF (X.EQ.IDIM) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE IF (Y.EQ.IDIM) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE IF (Z.EQ.IDIM) THEN
            ROW_MAT%AS(ELEMENT)=ONE
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ELSE
            ! Internal point: build discretization
            !
            !  Term depending on   (x-1,y,z)
            !
            ROW_MAT%AS(ELEMENT)=-B1(GLOB_X,GLOB_Y,GLOB_Z)&
                 & -A1(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X-2)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x,y-1,z)
            ROW_MAT%AS(ELEMENT)=-B2(GLOB_X,GLOB_Y,GLOB_Z)&
                 & -A2(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-2)*IDIM+(Z)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x,y,z-1)
            ROW_MAT%AS(ELEMENT)=-B3(GLOB_X,GLOB_Y,GLOB_Z)&
                 & -A3(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z-1)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x,y,z)
            ROW_MAT%AS(ELEMENT)=2*B1(GLOB_X,GLOB_Y,GLOB_Z)&
                 & +2*B2(GLOB_X,GLOB_Y,GLOB_Z)&
                 & +2*B3(GLOB_X,GLOB_Y,GLOB_Z)&
                 & +A1(GLOB_X,GLOB_Y,GLOB_Z)&
                 & +A2(GLOB_X,GLOB_Y,GLOB_Z)&
                 & +A3(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x,y,z+1)
            ROW_MAT%AS(ELEMENT)=-B1(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
```

```
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z+1)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x,y+1,z)
            ROW_MAT%AS(ELEMENT)=-B2(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y)*IDIM+(Z)
            ELEMENT=ELEMENT+1
            !  Term depending on     (x+1,y,z)
            ROW_MAT%AS(ELEMENT)=-B3(GLOB_X,GLOB_Y,GLOB_Z)
            ROW_MAT%AS(ELEMENT) = ROW_MAT%AS(ELEMENT)/(DELTAH*&
                 & DELTAH)
            ROW_MAT%IA1(ELEMENT)=(X)*IDIM*IDIM+(Y-1)*IDIM+(Z)
            ELEMENT=ELEMENT+1
         ENDIF
         ROW_MAT%M=1
         ROW_MAT%N=N
         ROW_MAT%IA2(2)=ELEMENT
         ! IA== GLOBAL ROW INDEX
         IA=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
         T3 = TIMEF()
         CALL PSPINS(A,IA,1,ROW_MAT,DESC_A)
         TINS = TINS + (TIMEF()-T3)
      ! Build RHS
         IF (X==1) THEN
            GLOB_Y=(Y-IDIM/2)*DELTAH
            GLOB_Z=(Z-IDIM/2)*DELTAH
            ZT(1) = EXP(-GLOB_Y**2-GLOB_Z**2)
         ELSE IF ((Y==1).OR.(Y==IDIM).OR.(Z==1).OR.(Z==IDIM)) THEN
            GLOB_X=3*(X-1)*DELTAH
            GLOB_Y=(Y-IDIM/2)*DELTAH
            GLOB_Z=(Z-IDIM/2)*DELTAH
            ZT(1) = EXP(-GLOB_Y**2-GLOB_Z**2)*EXP(-GLOB_X)
         ELSE
            ZT(1) = 0.D0
         ENDIF
         CALL PGEINS(B,ZT(1:1),DESC_A,IA)
         ZT(1)=0.D0
         CALL PGEINS(T,ZT(1:1),DESC_A,IA)
      END IF
   END DO
END DO

CALL BLACS_BARRIER(ICONTXT,'ALL')
T2 = TIMEF()

IF (MYPROW.EQ.0) THEN
   WRITE(0,*) '      pspins  time',TINS/1.D3
   WRITE(0,*) '    Insert time',(T2-T1)/1.D3
ENDIF

DEALLOCATE(ROW_MAT%AS,ROW_MAT%IA1,ROW_MAT%IA2)


CALL BLACS_BARRIER(ICONTXT,'ALL')
T1 = TIMEF()

CALL PSPASB(A,DESC_A,INFO=INFO,DUPFLAG=0,MTYPE='GEN  ')
```

```
  CALL BLACS_BARRIER(ICONTXT,'ALL')
  T2 = TIMEF()

  IF (MYPROW.EQ.0) THEN
     WRITE(0,*) '   Assembly  time',(T2-T1)/1.D3
  ENDIF

  CALL PGEASB(B,DESC_A)
  CALL PGEASB(T,DESC_A)
  RETURN
END SUBROUTINE CREATE_MATRIX
!
! Functions parameterizing the differential equation
!
FUNCTION A1(X,Y,Z)
  REAL(KIND(1.D0)) :: A1
  REAL(KIND(1.D0)) :: X,Y,Z
  A1=1.D0
END FUNCTION A1
FUNCTION A2(X,Y,Z)
  REAL(KIND(1.D0)) ::  A2
  REAL(KIND(1.D0)) :: X,Y,Z
  A2=2.D1*Y
END FUNCTION A2
FUNCTION A3(X,Y,Z)
  REAL(KIND(1.D0)) ::  A3
  REAL(KIND(1.D0)) :: X,Y,Z
  A3=1.D0
END FUNCTION A3
FUNCTION A4(X,Y,Z)
  REAL(KIND(1.D0)) ::  A4
  REAL(KIND(1.D0)) :: X,Y,Z
  A4=1.D0
END FUNCTION A4
FUNCTION B1(X,Y,Z)
  REAL(KIND(1.D0)) ::  B1
  REAL(KIND(1.D0)) :: X,Y,Z
  B1=1.D0
END FUNCTION B1
FUNCTION B2(X,Y,Z)
  REAL(KIND(1.D0)) ::  B2
  REAL(KIND(1.D0)) :: X,Y,Z
  B2=1.D0
END FUNCTION B2
FUNCTION B3(X,Y,Z)
  REAL(KIND(1.D0)) ::  B3
  REAL(KIND(1.D0)) :: X,Y,Z
  B3=1.D0
END FUNCTION B3
!
! Get iteration parameters from the command line
!
SUBROUTINE  GET_PARMS(ICONTXT,CMETHD,PREC,IDIM,ISTOPC,ITMAX,ITRACE)
  integer      :: icontxt
  Character*10 :: CMETHD, PREC
  Integer      :: IDIM, IRET, ISTOPC,ITMAX,ITRACE
  Character*40 :: CHARBUF
```

```
INTEGER      :: IARGC, NPROW, NPCOL, MYPROW, MYPCOL
EXTERNAL     IARGC
INTEGER      :: INTBUF(10), IP

CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

IF (MYPROW==0) THEN
   ! Read command line parameters
   IP=IARGC()
   IF (IARGC().GE.3) THEN
      CALL GETARG(1,CHARBUF)
      READ(CHARBUF,*) CMETHD
      CALL GETARG(2,CHARBUF)
      READ(CHARBUF,*) PREC

     ! Convert strings in array
      DO I = 1, LEN(CMETHD)
         INTBUF(I) = IACHAR(CMETHD(I:I))
      END DO
     ! Broadcast parameters to all processors
      CALL IGEBS2D(ICONTXT,'ALL',' ',10,1,INTBUF,10)

      DO I = 1, LEN(PREC)
         INTBUF(I) = IACHAR(PREC(I:I))
      END DO
     ! Broadcast parameters to all processors
      CALL IGEBS2D(ICONTXT,'ALL',' ',10,1,INTBUF,10)

      CALL GETARG(3,CHARBUF)
      READ(CHARBUF,*) IDIM
      IF (IARGC().GE.4) THEN
         CALL GETARG(4,CHARBUF)
         READ(CHARBUF,*) ISTOPC
      ELSE
         ISTOPC=1
      ENDIF
      IF (IARGC().GE.5) THEN
         CALL GETARG(5,CHARBUF)
         READ(CHARBUF,*) ITMAX
      ELSE
         ITMAX=500
      ENDIF
      IF (IARGC().GE.6) THEN
         CALL GETARG(6,CHARBUF)
         READ(CHARBUF,*) ITRACE
      ELSE
         ITRACE=0
      ENDIF
     ! Broadcast parameters to all processors
      CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,IDIM,1)
      CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ISTOPC,1)
      CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ITMAX,1)
      CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ITRACE,1)
      WRITE(6,*)'Solving matrix: ELL1'
      WRITE(6,*)'on  grid',IDIM,'x',IDIM,'x',IDIM
      WRITE(6,*)' with BLOCK data distribution, NP=',Np,&
           & ' Preconditioner=',PREC,&
           & ' Iterative methd=',CMETHD
```

```
          ELSE
           ! Wrong number of parameter, print an error message and exit
             CALL PR_USAGE(0)
             CALL BLACS_ABORT(ICONTXT,-1)
             STOP 1
          ENDIF
       ELSE
      ! Receive Parameters
          CALL IGEBR2D(ICONTXT,'ALL',' ',10,1,INTBUF,10,0,0)
          DO I = 1, 10
             CMETHD(I:I) = ACHAR(INTBUF(I))
          END DO
          CALL IGEBR2D(ICONTXT,'ALL',' ',10,1,INTBUF,10,0,0)
          DO I = 1, 10
             PREC(I:I) = ACHAR(INTBUF(I))
          END DO
          CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,IDIM,1,0,0)
          CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ISTOPC,1,0,0)
          CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ITMAX,1,0,0)
          CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ITRACE,1,0,0)
       END IF
       RETURN

     END SUBROUTINE GET_PARMS
     !
     !  Print an error message
     !
     SUBROUTINE PR_USAGE(IOUT)
       INTEGER :: IOUT
       WRITE(IOUT,*)'Incorrect parameter(s) found'
       WRITE(IOUT,*)' Usage:  pde90 methd prec dim &
            &[istop itmax itrace]'
       WRITE(IOUT,*)' Where:'
       WRITE(IOUT,*)'     methd:    CGSTAB TFQMR CGS'
       WRITE(IOUT,*)'     prec :    ILU DIAGSC NONE'
       WRITE(IOUT,*)'     dim       number of points along each axis'
       WRITE(IOUT,*)'               the size of the resulting linear '
       WRITE(IOUT,*)'               system is dim**3'
       WRITE(IOUT,*)'     istop     Stopping criterion  1, 2 or 3 [1]  '
       WRITE(IOUT,*)'     itmax     Maximum number of iterations [500] '
       WRITE(IOUT,*)'     itrace    0  (no tracing, default) or '
       WRITE(IOUT,*)'               >= 0 do tracing every ITRACE'
       WRITE(IOUT,*)'               iterations '
     END SUBROUTINE PR_USAGE

   END PROGRAM PDE90
```

Fortran 77 Sample Sparse Program

```
C
C This sample program shows how to build and solve a sparse linear
C system using the subroutines in the sparse section of Parallel
C ESSL. The matrix and RHS are generated
C in  parallel, so that there is no serial bottleneck.
C
C The program solves a linear system based on the partial differential equation
C
C   b1 dd(u)  b2 dd(u)    b3 dd(u)    a1 d(u)   a2 d(u)  a3 d(u)
C -   ------ -  ------ -  ------ -  -----  -  ------  -  ------ + a4 u
C      dxdx     dydy       dzdz        dx       dy         dz
C
C  = 0
C
C with  Dirichlet boundary conditions on the unit cube
C
C    0<=x,y,z<=1
C
C The equation is discretized with finite differences and uniform stepsize;
C the resulting  discrete  equation is
C
C ( u(x,y,z)(2b1+2b2+2b3+a1+a2+a3)+u(x-1,y,z)(-b1-a1)+u(x,y-1,z)(-b2-a2)+
C  + u(x,y,z-1)(-b3-a3)-u(x+1,y,z)b1-u(x,y+1,z)b2-u(x,y,z+1)b3)*(1/h**2)
C
C
C In this sample program the index space of the discretized
C computational domain is first numbered sequentially in a standard way,
C then the corresponding vector is distributed according to an HPF BLOCK
C distribution directive.
C
C Boundary conditions are set in a very simple way, by adding
C equations of the form
C
C   u(x,y,z) = rhs(x,y,z)
C
C
      Program PDE77
      USE F90SPARSE
      Implicit none


      EXTERNAL PART_BLOCK
C Input parameters
      Character*10 :: CMETHD, PREC
      Integer      :: IDIM
C Miscellaneous
      Integer, Parameter   :: IZERO=0, IONE=1
      Character, PARAMETER :: ORDER='R'
      REAL(KIND(1.D0)), POINTER :: B_COL(:), X_COL(:)
      INTEGER              :: NR, NNZ,IRCODE, NNZ1,  NRHS
      REAL(KIND(1.D0)), PARAMETER :: DZERO = 0.D0, ONE = 1.D0
      REAL(KIND(1.D0)) :: TIMEF, T1, T2, TPREC, TSOLVE, T3, T4
      REAL(KIND(1.D0)), POINTER :: DWORK(:)
      EXTERNAL          TIMEF
      LOGICAL, PARAMETER :: UPDATE=.TRUE., NOUPDATE=.FALSE.
C Sparse Matrices
      REAL(8), POINTER  :: AS(:),  PRCS(:)
      INTEGER, POINTER  :: DESC_A(:), IA1(:), IA2(:)
      INTEGER           :: INFOA(30)


C Dense Matrices
      REAL(KIND(1.D0)), POINTER :: B(:), X(:)
      INTEGER                   :: LB, LX, LDV, LDV1,IRET

      INTERFACE
         SUBROUTINE CREATE_MTRX_ELL1_BLOCK(PARTS,IDIM,
     +      AS,IA1,IA2,INFOA,B,T,DESC_A,ICONTXT)
         Implicit None
         external parts
         Integer                  :: IDIM
         Real(Kind(1.D0)),Pointer :: B(:), T(:), AS(:)
         integer                  :: infoa(30)
         INTEGER, POINTER         :: DESC_A(:), IA1(:),IA2(:)
         Integer                  :: ICONTXT
       end SUBROUTINE CREATE_MTRX_ELL1_BLOCK
      END INTERFACE

C Communications data structure
C BLACS parameters
      INTEGER            :: nprow, npcol, icontxt, iam, np, myprow,
     +   mypcol

C Solver parameters
      Integer            :: iter, itmax,ierr,itrace, methd,iprec,
     +   istopc, iparm(20)
      real(kind(1.d0))   :: err, eps, rparm(20)

C Other variables
      Integer            :: i,info
      INTEGER            :: INTERNAL, M,ii,nnzero

C Initialize BLACS
      CALL BLACS_PINFO(IAM, NP)
      CALL BLACS_GET(IZERO, IZERO, ICONTXT)

C Rectangular Grid,  Np x 1

      CALL BLACS_GRIDINIT(ICONTXT, ORDER, NP, IONE)
      CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

C
C  Get parameters
C
      CALL GET_PARMS(ICONTXT,CMETHD,PREC,IDIM,ISTOPC,ITMAX,ITRACE)

C
C  Allocate and fill in the coefficient matrix and the RHS
C
      CALL BLACS_BARRIER(ICONTXT,'All')
      T1 = TIMEF()
      CALL CREATE_MTRX_ELL1_BLOCK(PART_BLOCK,IDIM,
     +   AS,IA1,IA2,INFOA,B,X,DESC_A,ICONTXT)
      T2 = TIMEF() - T1

      CALL DGAMX2D(ICONTXT,'A',' ',IONE, IONE,T2,IONE,T1,T1,-1,-1,-1)
      IF (IAM.EQ.0) Write(6,*) 'Matrix creation Time : ',T2/1.D3

      LB   = SIZE(B)
      LX   = SIZE(X)
      LDV  = DESC_A(5)
      LDV1 = DESC_A(6)
      NNZ  = SIZE(AS)
      NNZ1 = SIZE(IA1)


      ALLOCATE(PRCS(2*NNZ+LDV+LDV1+31),STAT=IRCODE)
      IF (IRCODE /= 0) THEN
         WRITE(0,*) 'Allocation error'
         CALL BLACS_ABORT(ICONTXT,-1)
         STOP
      ENDIF

C
C  Prepare the preconditioning data structure
C
      SELECT CASE (PREC)
      CASE ('ILU')
         IPREC = 2
      CASE ('DIAGSC')
         IPREC = 1
      CASE ('NONE')
         IPREC = 0
      CASE DEFAULT
         WRITE(0,*) 'Unknown preconditioner'
         CALL BLACS_ABORT(ICONTXT,-1)
      END SELECT

      CALL BLACS_BARRIER(ICONTXT,'All')
      T1 = TIMEF()
      CALL PDSPGPR(IPREC,AS,IA1,IA2,INFOA,PRCS,SIZE(PRCS),DESC_A,IRET)
      TPREC = TIMEF()-T1

      CALL DGAMX2D(icontxt,'A',' ',IONE, IONE,TPREC,IONE,t1,t1,-1,-1,-1)

      IF (IAM.EQ.0) WRITE(6,*) 'Preconditioner Time : ',TPREC/1.D3
      if (iret.ne.0) then
         write(0,*) 'Error on preconditioner',iret
         call blacs_abort(icontxt,-1)
         stop
      endif

C
C Iteration parameters
C
      IF (CMETHD(1:6).EQ.'CGSTAB') Then
         METHD = 1
      ELSE IF (CMETHD(1:3).EQ.'CGS') Then
         METHD = 2
      ELSE IF (CMETHD(1:5).EQ.'TFQMR') THEN
         METHD = 3
      ELSE
         WRITE(0,*) 'Unknown method '
         CALL BLACS_ABORT(ICONTXT,-1)
      END IF
      EPS   = 1.D-9

      IPARM = 0
      RPARM = 0.D0
      IPARM(1) = METHD
      IPARM(2) = ISTOPC
      IPARM(3) = ITMAX
      IPARM(4) = ITRACE
      RPARM(1) = EPS

      NRHS = 1


      CALL BLACS_BARRIER(ICONTXT,'All')
      T1 = TIMEF()
      CALL PDSPGIS(AS,IA1,IA2,INFOA,NRHS,B,LB,X,LX,PRCS,
     +   DESC_A,IPARM,RPARM,INFO)
      CALL BLACS_BARRIER(ICONTXT,'All')
      TSOLVE = TIMEF() - T1
      ERR    = RPARM(2)
      ITER   = IPARM(5)

      IF (IAM.EQ.0) THEN
         WRITE(6,*) 'Time to Solve Matrix : ',TSOLVE/1.D3
         WRITE(6,*) 'Time per iteration : ',TSOLVE/(1.D3*ITER)
         WRITE(6,*) 'Number of iterations : ',ITER
         WRITE(6,*) 'Error on exit : ',ERR
         WRITE(6,*) 'INFO on exit:',INFO
      END IF

      CALL BLACS_GRIDEXIT(ICONTXT)
      CALL BLACS_EXIT(0)
      STOP


      END
C
C  Print an error message
C
      SUBROUTINE PR_USAGE(IOUT)
      INTEGER :: IOUT
      WRITE(IOUT,*)'Incorrect parameter(s) found'
      WRITE(IOUT,*)
     +   ' Usage: pde77 methd prec dim [istopc itmax itrace]'
      WRITE(IOUT,*)' Where:'
      WRITE(IOUT,*)'     methd:    CGSTAB TFQMR CGS'
      WRITE(IOUT,*)'     prec :    ILU DIAGSC NONE'
      WRITE(IOUT,*)'     dim       number of points along each axis'
      WRITE(IOUT,*)'               the size of the resulting linear '
      WRITE(IOUT,*)'               system is dim**3'
      WRITE(IOUT,*)'     istopc    Stopping criterion 1 2 or 3  [1]  '
      WRITE(IOUT,*)'     itmax     Maximum number of iterations [500]'
      WRITE(IOUT,*)'     itrace         0  (no tracing, default) or '
      WRITE(IOUT,*)'                    >= 0 do tracing every ITRACE'
      WRITE(IOUT,*)'                    iterations '
      RETURN
      END
C
C Functions parameterizing the differential equation
C
      FUNCTION A1(X,Y,Z)
      REAL(KIND(1.D0)) ::  A1
      REAL(KIND(1.D0)) :: X,Y,Z
      A1=1.D0
      END
      FUNCTION A2(X,Y,Z)
      REAL(KIND(1.D0)) ::  A2
      REAL(KIND(1.D0)) :: X,Y,Z

      A2=2.D1*Y
      END
      FUNCTION A3(X,Y,Z)
      REAL(KIND(1.D0)) ::  A3
      REAL(KIND(1.D0)) :: X,Y,Z
      A3=1.D0
      END
      FUNCTION A4(X,Y,Z)
      REAL(KIND(1.D0)) ::  A4
      REAL(KIND(1.D0)) :: X,Y,Z
      A4=1.D0
      END
      FUNCTION B1(X,Y,Z)
      REAL(KIND(1.D0)) ::  B1
      REAL(KIND(1.D0)) :: X,Y,Z
      B1=1.D0
      END
      FUNCTION B2(X,Y,Z)
      REAL(KIND(1.D0)) ::  B2
      REAL(KIND(1.D0)) :: X,Y,Z
      B2=1.D0
      END
      FUNCTION B3(X,Y,Z)
      REAL(KIND(1.D0)) ::  B3
      REAL(KIND(1.D0)) :: X,Y,Z
      B3=1.D0
      END

C
C  Subroutine to allocate and fill in the coefficient matrix and
C  the RHS.
C
      SUBROUTINE CREATE_MTRX_ELL1_BLOCK(PARTS,IDIM,
     +   AS,IA1,IA2,INFOA,B,T,DESC_A,ICONTXT)
C
C     the equation generated is:
C   b1 d d (u)  b2  d d (u)   b3  d d (u)   a1 d (u))  a2 d (u)))   a3d (u))  a4 u
C -   ------   -    ------  -     ------ -      ----- -  ------  -  ------ +
C      dx dx         dy dy         dz dz         dx        dy         dz
C
C =g(x,y,z)
C where g is the RHS extracted from exact solution:
C  f(x,y,z)=10.d0*X*Y*Z*(1-X)*(1-Y)*(1-Z)*EXP(X**4.5)
C boundary condition: Dirichlet
C    0< x,y,z<1
```

```
C discretized with finite differences; the discrete equation is
C  u(x,y,z)(2b1+2b2+2b3+a1+a2+a3)+u(x-1,y,z)(-b1-a1)+u(x,y-1,z)(-b2-a2)+
C  + u(x,y,z-1)(-b3-a3)-u(x+1,y,z)b1-u(x,y+1,z)b2-u(x,y,z+1)b3
C     !!this matrix is non symmetric
      USE F90SPARSE
      EXTERNAL PARTS

      Implicit None
      Integer                  :: IDIM
      Real(Kind(1.D0)),Pointer :: B(:), T(:), AS(:)
      integer                  :: infoa(20)
      INTEGER, POINTER         :: DESC_A(:), IA1(:),IA2(:)
      Integer                  :: ICONTXT
      Real(Kind(1.d0))         :: ZT(10),GLOB_X,GLOB_Y,GLOB_Z,
     +   ras(20)
      Integer                  :: M,N,NNZ,glob_row,nr,j
      integer                  :: ria1(20),ria2(20),rinfoa(30)
      Real(Kind(1.D0)),POINTER :: SOL(:)
      real(kind(1.d0)), external :: a1,a2,a3,b1,b2,b3
      Integer                  :: X,Y,Z,COUNTER,IA,I,NPROW,NPCOL,MYPROW
     +   ,MYPCOL,DOMAIN_INDEX
      Integer                  :: BOUND_COND_0YZ, BOUND_COND_1YZ,
     +   BOUND_COND_X0Z ,   BOUND_COND_X1Z ,   BOUND_COND_XY0 ,
     +   BOUND_COND_XY1,MP,ELEMENT,LDSCA,IRCODE, NNZ1
      REAL(KIND(1.D0))         :: DELTAH
      INTEGER                  :: GAP,INFO
      integer    :: prv(64), indx_owner, nv,inv
C deltah dimension of each grid cell
C deltat discretization time
      Real(Kind(1.d0)),Parameter   :: RHS=0.d0,ONE=1.d0,ZERO=0.d0
      Real(Kind(1.d0))   :: TIMEF, T1, T2,t3, TINS
      external            timef
C common area
      INTEGER DIM_BLOCK, NPROC


      CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

      NPROC = NPROW*NPCOL
      DELTAH=1.D0/(IDIM-1)



      M   = IDIM*IDIM*IDIM
      N   = M
      LDSCA = 3*N+31+3*NPROC
      DIM_BLOCK=(N+NPROC-1)/NPROC
      NNZ  = MAX(2,(N*7)/(NPROC))
      NNZ1 = MAX(2,(N*9)/(NPROC))+MAX(1,DIM_BLOCK)
      ALLOCATE(DESC_A(LDSCA),AS(NNZ),IA1(NNZ1),
     +   IA2(NNZ1),STAT=IRCODE)
      IF (IRCODE /= 0) THEN
         WRITE(0,*) 'Allocation error in CREATE'
         CALL BLACS_ABORT(ICONTXT,-1)
         STOP
      ENDIF
      INFOA(1) = NNZ
      INFOA(2) = NNZ1
      INFOA(3) = NNZ1
      DESC_A(11) = LDSCA
      CALL PADINIT(N,PARTS,DESC_A,ICONTXT)

      NR = DESC_A(5)
      ALLOCATE(B(NR),T(NR),STAT=IRCODE)
      IF (IRCODE /= 0) THEN
         WRITE(0,*) 'Allocation error in CREATE'
         CALL BLACS_ABORT(ICONTXT,-1)
         STOP
      ENDIF

      CALL PDSPINIT(AS,IA1,IA2,INFOA,DESC_A)

C
C We build an auxiliary matrix consisting of one row at a
C time in CSR mode
C
      RINFOA(4) = 1
      RINFOA(5) = 1
      RINFOA(6) = 1
      RINFOA(7) = N

      GAP = 1
      RIA2(1)=1
      TINS = 0.D0

      CALL BLACS_BARRIER(ICONTXT,'ALL')
      T1 = TIMEF()
C Loop over all rows which belongs to me; we have a BLOCK
C distribution !!
      DO GLOB_ROW = 1, N
         CALL PARTS(GLOB_ROW,N,NPROW,PRV,NV)
         DO INV = 1, NV
            INDX_OWNER = PRV(INV)
            IF (INDX_OWNER == MYPROW) THEN
               ELEMENT=1
C               GLOB_X, GLOB_Y, GLOB_X coordinates in current measure unit

C Compute Point Coordinates
               IF (MOD(GLOB_ROW,(IDIM*IDIM)).EQ.0) THEN
                  X = GLOB_ROW/(IDIM*IDIM)
               ELSE
                  X = GLOB_ROW/(IDIM*IDIM)+1
               ENDIF
               IF (MOD((GLOB_ROW-(X-1)*IDIM*IDIM),IDIM).EQ.0) THEN
                  Y = (GLOB_ROW-(X-1)*IDIM*IDIM)/IDIM
               ELSE
                  Y = (GLOB_ROW-(X-1)*IDIM*IDIM)/IDIM+1
               ENDIF
               Z = GLOB_ROW-(X-1)*IDIM*IDIM-(Y-1)*IDIM
               GLOB_X=X*DELTAH
               GLOB_Y=Y*DELTAH
               GLOB_Z=Z*DELTAH
               IF (X.EQ.1) THEN
                  RAS(ELEMENT)=ONE
                  RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                  ELEMENT=ELEMENT+1
```

```
                      ELSE IF (Y.EQ.1) THEN
                         RAS(ELEMENT)=ONE
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
                      ELSE IF (Z.EQ.1) THEN
                         RAS(ELEMENT)=ONE
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
                      ELSE IF (X.EQ.IDIM) THEN
                         RAS(ELEMENT)=ONE
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
                      ELSE IF (Y.EQ.IDIM) THEN
                         RAS(ELEMENT)=ONE
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
                      ELSE IF (Z.EQ.IDIM) THEN
                         RAS(ELEMENT)=ONE
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
                      ELSE
C           !             .....internal point......
C           !               (x-1,y,z)
                         RAS(ELEMENT)=-B1(GLOB_X,GLOB_Y,GLOB_Z)
     +                      -A1(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                         RIA1(ELEMENT)=(X-2)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
C           !               (x,y-1,z)
                         RAS(ELEMENT)=-B2(GLOB_X,GLOB_Y,GLOB_Z)
     +                      -A2(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-2)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
C           !               (x,y,z-1)
                         RAS(ELEMENT)=-B3(GLOB_X,GLOB_Y,GLOB_Z)
     +                      -A3(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z-1)
                         ELEMENT=ELEMENT+1
C           !               (x,y,z)
                         RAS(ELEMENT)=2*B1(GLOB_X,GLOB_Y,GLOB_Z)
     +                      +2*B2(GLOB_X,GLOB_Y,GLOB_Z)
     +                      +2*B3(GLOB_X,GLOB_Y,GLOB_Z)
     +                      +A1(GLOB_X,GLOB_Y,GLOB_Z)
     +                      +A2(GLOB_X,GLOB_Y,GLOB_Z)
     +                      +A3(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                         ELEMENT=ELEMENT+1
C           !               (x,y,z+1)
                         RAS(ELEMENT)=-B1(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                         RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z+1)
                         ELEMENT=ELEMENT+1
C           !               (x,y+1,z)
                         RAS(ELEMENT)=-B2(GLOB_X,GLOB_Y,GLOB_Z)
                         RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
```

```
                  RIA1(ELEMENT)=(X-1)*IDIM*IDIM+(Y)*IDIM+(Z)
                  ELEMENT=ELEMENT+1
C           !       (x+1,y,z)
                  RAS(ELEMENT)=-B3(GLOB_X,GLOB_Y,GLOB_Z)
                  RAS(ELEMENT) = RAS(ELEMENT)/(DELTAH*DELTAH)
                  RIA1(ELEMENT)=(X)*IDIM*IDIM+(Y-1)*IDIM+(Z)
                  ELEMENT=ELEMENT+1
               ENDIF
               RIA2(2)   = ELEMENT
               RINFOA(1) = 20
               RINFOA(2) = 20
               RINFOA(3) = 20
               RINFOA(4) = 1
               RINFOA(5) = 1
               RINFOA(6) = 1
C IA== GLOBAL ROW INDEX
               IA=(X-1)*IDIM*IDIM+(Y-1)*IDIM+(Z)
               T3 = TIMEF()
               CALL PDSPINS(AS,IA1,IA2,INFOA,DESC_A,
     +            IA,1,RAS,RIA1,RIA2,RINFOA)
               TINS = TINS + (TIMEF()-T3)
C   Build RHS
               IF (X==1) THEN
                  GLOB_Y=(Y-IDIM/2)*DELTAH
                  GLOB_Z=(Z-IDIM/2)*DELTAH
                  ZT(1) = EXP(-GLOB_Y**2-GLOB_Z**2)
               ELSE IF ((Y==1).OR.(Y==IDIM).OR.(Z==1).OR.(Z==IDIM)) THEN
                  GLOB_X=3*(X-1)*DELTAH
                  GLOB_Y=(Y-IDIM/2)*DELTAH
                  GLOB_Z=(Z-IDIM/2)*DELTAH
                  ZT(1) = EXP(-GLOB_Y**2-GLOB_Z**2)*EXP(-GLOB_X)
               ELSE
                  ZT(1) = 0.D0
               ENDIF
               CALL PDGEINS(1,B,NR,IA,1,1,1,ZT,1,DESC_A)
               ZT(1) = 0.D0
               CALL PDGEINS(1,T,NR,IA,1,1,1,ZT,1,DESC_A)
            ENDIF
         ENDDO
      ENDDO

      CALL BLACS_BARRIER(ICONTXT,'ALL')
      T2 = TIMEF()

      IF (MYPROW.EQ.0) THEN
         WRITE(0,*) '     pspins  time',TINS/1.D3
         WRITE(0,*) '   Insert time',(T2-T1)/1.D3
      ENDIF

      CALL BLACS_BARRIER(ICONTXT,'ALL')
      T1 = TIMEF()

      CALL PDSPASB(AS,IA1,IA2,INFOA,DESC_A,
     +   'GEN  ','DEF  ',0,INFO)

      CALL BLACS_BARRIER(ICONTXT,'ALL')
      T2 = TIMEF()
      IF (MYPROW.EQ.0) THEN
         WRITE(0,*) '   Assembly  time',(T2-T1)/1.D3
      ENDIF

      CALL PDGEASB(1,B,NR,DESC_A)
      CALL PDGEASB(1,T,NR,DESC_A)
      RETURN
      END
C
C  Get iteration parameters from the command line
C
      SUBROUTINE  GET_PARMS(ICONTXT,CMETHD,PREC,IDIM,
     +   ISTOPC,ITMAX,ITRACE)
      integer      :: icontxt
      Character*10 :: CMETHD, PREC
      Integer      :: IDIM, IRET, ISTOPC,ITMAX,ITRACE
      Character*40 :: CHARBUF
      INTEGER      :: IARGC, NPROW, NPCOL, MYPROW, MYPCOL
      EXTERNAL     IARGC
      INTEGER      :: INTBUF(10), IP

      CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

      IF (MYPROW==0) THEN
C Read command line parameters
         IP=IARGC()
         IF (IARGC().GE.3) THEN
            CALL GETARG(1,CHARBUF)
            READ(CHARBUF,*) CMETHD
            CALL GETARG(2,CHARBUF)
            READ(CHARBUF,*) PREC

C Convert strings in array
            DO I = 1, LEN(CMETHD)
               INTBUF(I) = IACHAR(CMETHD(I:I))
            END DO
C Broadcast parameters to all processors
            CALL IGEBS2D(ICONTXT,'ALL',' ',10,1,INTBUF,10)

            DO I = 1, LEN(PREC)
               INTBUF(I) = IACHAR(PREC(I:I))
            END DO
C Broadcast parameters to all processors
            CALL IGEBS2D(ICONTXT,'ALL',' ',10,1,INTBUF,10)

            CALL GETARG(3,CHARBUF)
            READ(CHARBUF,*) IDIM
            IF (IARGC().GE.4) THEN
               CALL GETARG(4,CHARBUF)
               READ(CHARBUF,*) ISTOPC
            ELSE
               ISTOPC=1
            ENDIF
            IF (IARGC().GE.5) THEN
               CALL GETARG(5,CHARBUF)
               READ(CHARBUF,*) ITMAX
            ELSE
               ITMAX=500
            ENDIF
            IF (IARGC().GE.6) THEN
               CALL GETARG(6,CHARBUF)
               READ(CHARBUF,*) ITRACE
            ELSE
               ITRACE=0
            ENDIF
C Broadcast parameters to all processors
            CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,IDIM,1)
            CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ISTOPC,1)
            CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ITMAX,1)
            CALL IGEBS2D(ICONTXT,'ALL',' ',1,1,ITRACE,1)
            WRITE(6,*)'Solving matrix: ELL1'
            WRITE(6,*)'on  grid',IDIM,'x',IDIM,'x',IDIM
            WRITE(6,*)' with BLOCK data distribution, NP=',Np,
     +         ' Preconditioner=',PREC,
     +         ' Iterative methd=',CMETHD
         ELSE
C Wrong number of parameter, print an error message and exit
            CALL PR_USAGE(0)
            CALL BLACS_ABORT(ICONTXT,-1)
            STOP 1
         ENDIF
      ELSE
C Receive Parameters
         CALL IGEBR2D(ICONTXT,'ALL',' ',10,1,INTBUF,10,0,0)
         DO I = 1, 10
            CMETHD(I:I) = ACHAR(INTBUF(I))
         END DO
         CALL IGEBR2D(ICONTXT,'ALL',' ',10,1,INTBUF,10,0,0)
         DO I = 1, 10
            PREC(I:I) = ACHAR(INTBUF(I))
         END DO
         CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,IDIM,1,0,0)
         CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ISTOPC,1,0,0)
         CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ITMAX,1,0,0)
         CALL IGEBR2D(ICONTXT,'ALL',' ',1,1,ITRACE,1,0,0)
      END IF
      RETURN

      END
```

Fortran 90 Sample Sparse Program (using the Harwell-Boeing exchange format)

```
@PROCESS FREE(F90) INIT(F90PTR)
!
! This sample program shows how to build and solve a sparse linear
! system using the subroutines in the sparse section of Parallel
! ESSL; the matrices are read from file using the Harwell-Boeing
! exchange format. Details on the format and sample matrices are
! available from
!
! http://math.nist.gov/MatrixMarket/
!
! The user can choose between different data distribution strategies.
! These are equivalents to the HPF BLOCK and  CYCLIC(N) distributions;
! they do not take into account the sparsity pattern of the input
! matrix.
!
PROGRAM HB_SAMPLE
  USE F90SPARSE
  USE MAT_DIST
  USE READ_MAT
  USE PARTRAND
  USE PARTBCYC
  IMPLICIT NONE

  ! Input parameters
  CHARACTER*40 :: CMETHD, PREC, MTRX_FILE
  CHARACTER*80 :: CHARBUF

  DOUBLE PRECISION DDOT
  EXTERNAL DDOT
  EXTERNAL PART_BLOCK

  INTEGER, PARAMETER    :: IZERO=0, IONE=1
  CHARACTER, PARAMETER  :: ORDER='R'
  REAL(KIND(1.D0)), POINTER,SAVE :: B_COL(:), X_COL(:), R_COL(:), &
       & B_COL_GLOB(:), X_COL_GLOB(:), R_COL_GLOB(:), B_GLOB(:,:)
  INTEGER              :: IARGC
  Real(Kind(1.d0)), Parameter :: Dzero = 0.d0, One = 1.d0
  Real(Kind(1.d0)) :: TIMEF, T1, T2, TPREC, R_AMAX, B_AMAX,bb(1,1)
  integer :: nrhs, nrow, nx1, nx2
  External IARGC, TIMEF
  integer bsze,overlap
  common/part/bsze,overlap

  ! Sparse Matrices
  TYPE(D_SPMAT) :: A, AUX_A
  TYPE(D_PRECN) :: APRC
  ! Dense Matrices
  REAL(KIND(1.D0)), POINTER ::  AUX_B(:,:) , AUX1(:), AUX2(:)

  ! Communications data structure
  TYPE(DESC_TYPE)    :: DESC_A

  ! BLACS parameters
  INTEGER            :: NPROW, NPCOL, ICTXT, IAM, NP, MYPROW, MYPCOL

  ! Solver paramters
  INTEGER            :: ITER, ITMAX, IERR, ITRACE, IRCODE, IPART,&
       & IPREC, METHD, ISTOPC
```

```
REAL(KIND(1.D0))   :: ERR, EPS
integer   iparm(20)
real(kind(1.d0)) rparm(20)

! Other variables
INTEGER            :: I,INFO,J
INTEGER            :: INTERNAL, M,II,NNZERO

! common area
INTEGER M_PROBLEM, NPROC

! Initialize BLACS
CALL BLACS_PINFO(IAM, NP)
CALL BLACS_GET(IZERO, IZERO, ICTXT)

! Rectangular Grid,  Np x 1

CALL BLACS_GRIDINIT(ICTXT, ORDER, NP, IONE)
CALL BLACS_GRIDINFO(ICTXT, NPROW, NPCOL, MYPROW, MYPCOL)

!
!  Get parameters
!
CALL GET_PARMS(ICTXT,MTRX_FILE,CMETHD,PREC,&
     & IPART,ISTOPC,ITMAX,ITRACE)

CALL BLACS_BARRIER(ICTXT,'A')
T1 = TIMEF()
! Read the input matrix to be processed and (possibly) the RHS
IF (IAM == 0) THEN
   CALL READMAT(MTRX_FILE, AUX_A, ICTXT,B=AUX_B)
   M_PROBLEM = AUX_A%M
   CALL IGEBS2D(ICTXT,'A',' ',1,1,M_PROBLEM,1)
   IF (SIZE(AUX_B,1).EQ.M_PROBLEM) THEN
      ! If any RHS were present, broadcast the first one
      NRHS = 1
      CALL IGEBS2D(ICTXT,'A',' ',1,1,NRHS,1)
      CALL DGEBS2D(ICTXT,'A',' ',M_PROBLEM,1,AUX_B(:,1),M_PROBLEM)
   ELSE
      NRHS = 0
      CALL IGEBS2D(ICTXT,'A',' ',1,1,NRHS,1)
   ENDIF
ELSE
   CALL IGEBR2D(ICTXT,'A',' ',1,1,M_PROBLEM,1,0,0)
   CALL IGEBR2D(ICTXT,'A',' ',1,1,NRHS,1,0,0)
   IF (NRHS.EQ.1) THEN
      ALLOCATE(AUX_B(M_PROBLEM,1), STAT=IRCODE)
      IF (IRCODE /= 0) THEN
         WRITE(0,*) 'Memory allocation failure in HB_SAMPLE'
         CALL BLACS_ABORT(ICTXT,-1)
         STOP
      ENDIF
      CALL DGEBR2D(ICTXT,'A',' ',M_PROBLEM,1,AUX_B,M_PROBLEM,0,0)
   ENDIF
END IF
IF (NRHS.EQ.1 ) THEN
   B_COL_GLOB =>AUX_B(:,1)
ELSE
```

```
   ALLOCATE(AUX_B(M_PROBLEM,1), STAT=IRCODE)
   B_COL_GLOB =>AUX_B(:,1)
   IF (IAM==0) THEN
      DO I=1, M_PROBLEM
         B_COL_GLOB(I) = REAL(I)*2.0/REAL(M_PROBLEM)
      ENDDO
   ENDIF
ENDIF
NPROC = NPROW

! Switch over different partition types
IF (IPART > 0 ) THEN
   WRITE(6,*) 'Partition type: CYCLIC(NB)'
   CALL SET_NB(IPART,0,0,ICTXT)
   CALL MATDIST(AUX_A, A, PART_BCYC, ICTXT, &
        & DESC_A,B_COL_GLOB,B_COL)
ELSE
   SELECT CASE (IPART)

   CASE (0)
      WRITE(6,*) 'Partition type: BLOCK'
      CALL MATDIST(AUX_A, A, PART_BLOCK, ICTXT, &
           & DESC_A,B_COL_GLOB,B_COL)
   CASE (-1)
      WRITE(6,*) 'Partition type: RANDOM'
      IF (IAM==0) THEN
         CALL BUILD_RNDPART(AUX_A,NP)
      ENDIF
      CALL DISTR_RNDPART(0,0,ICTXT)
      CALL MATDIST(AUX_A, A, PART_RAND, ICTXT, &
           & DESC_A,B_COL_GLOB,B_COL)
   CASE DEFAULT
      WRITE(6,*) 'Partition type: BLOCK'
      CALL MATDIST(AUX_A, A, PART_BLOCK, ICTXT, &
           & DESC_A,B_COL_GLOB,B_COL)
   END SELECT
ENDIF


CALL PGEALL(X_COL,DESC_A)
CALL PGEASB(X_COL,DESC_A)
T2 = TIMEF() - T1


CALL DGAMX2D(ICTXT, 'A', ' ', IONE, IONE, T2, IONE,&
     & T1, T1, -1, -1, -1)

IF (IAM.EQ.0) THEN
   WRITE(6,*) 'Time to Read and Partition Matrix : ',T2/1.D3
END IF

!
!  Prepare the preconditioning matrix. Note the availability
!  of optional parameters
!
IF (PREC(1:3) == 'ILU') THEN
   IPREC = 2
ELSE IF (PREC(1:6) == 'DIAGSC') THEN
```

```
   IPREC = 1
ELSE IF (PREC(1:4) == 'NONE') THEN
   IPREC = 0
ELSE
   WRITE(0,*) 'Unknown preconditioner'
   CALL BLACS_ABORT(ICTXT,-1)
END IF
CALL BLACS_BARRIER(ICTXT,'A')
T1 = TIMEF()
CALL PSPGPR(IPREC,A,APRC,DESC_A,INFO=INFO)
TPREC = TIMEF()-T1


CALL DGAMX2D(ICTXT,'A',' ',IONE, IONE,TPREC,IONE,T1,T1,-1,-1,-1)

IF (IAM.EQ.0) WRITE(6,*) 'Preconditioner Time : ',TPREC/1.D3
IF (INFO /= 0) THEN
   WRITE(0,*) 'Error in preconditioner :',INFO
   CALL BLACS_ABORT(ICTXT,-1)
   STOP
END IF

IPARM = 0
RPARM = 0.D0

EPS   = 1.D-8
RPARM(1) = EPS
IPARM(2) = ISTOPC
IPARM(3) = ITMAX
IPARM(4) = ITRACE
IF (CMETHD(1:6).EQ.'CGSTAB') Then
   IPARM(1)=1
ELSE IF (CMETHD(1:3).EQ.'CGS') THEN
   IPARM(1)=2
ELSE IF (CMETHD(1:5).EQ.'TFQMR') THEN
   IPARM(1)=3
ELSE
   WRITE(0,*) 'Unknown method '
   CALL BLACS_ABORT(ICTXT,-1)
END IF


CALL BLACS_BARRIER(ICTXT,'All')
T1 = TIMEF()
CALL PSPGIS(A,B_COL,X_COL,APRC,DESC_A,&
     & IPARM=IPARM,RPARM=RPARM,INFO=IERR)
CALL BLACS_BARRIER(ICTXT,'All')
T2 = TIMEF() - T1
CALL DGAMX2D(ICTXT,'A',' ',IONE, IONE,T2,IONE,T1,T1,-1,-1,-1)
ITER=IPARM(5)
ERR = RPARM(2)
IF (IAM.EQ.0) THEN
   WRITE(6,*) 'methd iprec istopc   : ',METHD, IPREC, ISTOPC
   WRITE(6,*) 'Number of iterations : ',ITER
   WRITE(6,*) 'Time to Solve Matrix : ',T2/1.D3
   WRITE(6,*) 'Time per iteration   : ',T2/(1.D3*ITER)
   WRITE(6,*) 'Error on exit        : ',ERR
END IF
```

```
  CALL PGEFREE(B_COL, DESC_A)
  CALL PGEFREE(X_COL, DESC_A)
  CALL PSPFREE(A, DESC_A)
  CALL PSPFREE(APRC, DESC_A)
  CALL PADFREE(DESC_A)
  CALL BLACS_GRIDEXIT(ICTXT)
  CALL BLACS_EXIT(0)

CONTAINS
  !
  ! Get iteration parameters from the command line
  !
  SUBROUTINE  GET_PARMS(ICONTXT,MTRX_FILE,CMETHD,PREC,IPART,&
       & ISTOPC,ITMAX,ITRACE)
    integer      :: icontxt
    Character*40 :: CMETHD, PREC, MTRX_FILE
    Integer      :: IRET, ISTOPC,ITMAX,ITRACE,IPART
    Character*40 :: CHARBUF
    INTEGER      :: IARGC, NPROW, NPCOL, MYPROW, MYPCOL
    EXTERNAL     IARGC
    INTEGER      :: INPARMS(20), IP

    CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)
    IF (MYPROW==0) THEN
       ! Read Input Parameters
       IF (IARGC().GE.3) THEN
          CALL GETARG(1,CHARBUF)
          READ(CHARBUF,*) MTRX_FILE
          CALL GETARG(2,CHARBUF)
          READ(CHARBUF,*) CMETHD
          CALL GETARG(3,CHARBUF)
          READ(CHARBUF,*) PREC
          IF (IARGC().GE.4) THEN
             CALL GETARG(4,CHARBUF)
             READ(CHARBUF,*) IPART
          ELSE
             IPART = 0
          ENDIF
          IF (IARGC().GE.5) THEN
             CALL GETARG(5,CHARBUF)
             READ(CHARBUF,*) ITMAX
          ELSE
             ITMAX  = 500
          ENDIF
          IF (IARGC().GE.6) THEN
             CALL GETARG(6,CHARBUF)
             READ(CHARBUF,*) ISTOPC
          ELSE
             ISTOPC = 1
          ENDIF
          IF (IARGC().GE.7) THEN
             CALL GETARG(7,CHARBUF)
             READ(CHARBUF,*) ITRACE
          ELSE
             ITRACE = 0
          ENDIF
```

```
      ! Convert strings to integers
        DO I = 1, 20
           INPARMS(I) = IACHAR(MTRX_FILE(I:I))
        END DO
      ! Broadcast parameters to all processors
        CALL IGEBS2D(ICTXT,'A',' ',20,1,INPARMS,20)

      ! Convert strings in array
        DO I = 1, 20
           INPARMS(I) = IACHAR(CMETHD(I:I))
        END DO
      ! Broadcast parameters to all processors
        CALL IGEBS2D(ICTXT,'A',' ',20,1,INPARMS,20)

        DO I = 1, 20
           INPARMS(I) = IACHAR(PREC(I:I))
        END DO
      ! Broadcast parameters to all processors
        CALL IGEBS2D(ICTXT,'A',' ',20,1,INPARMS,20)

      ! Broadcast parameters to all processors
        CALL IGEBS2D(ICTXT,'A',' ',1,1,IPART,1)
        CALL IGEBS2D(ICTXT,'A',' ',1,1,ITMAX,1)
        CALL IGEBS2D(ICTXT,'A',' ',1,1,ISTOPC,1)
        CALL IGEBS2D(ICTXT,'A',' ',1,1,ITRACE,1)

     ELSE
        CALL PR_USAGE(0)
        CALL BLACS_ABORT(ICTXT,-1)
        STOP 1
     END IF
  ELSE
   ! Receive Parameters
     CALL IGEBR2D(ICTXT,'A',' ',20,1,INPARMS,20,0,0)
     DO I = 1, 20
        MTRX_FILE(I:I) = ACHAR(INPARMS(I))
     END DO

     CALL IGEBR2D(ICTXT,'A',' ',20,1,INPARMS,20,0,0)
     DO I = 1, 20
        CMETHD(I:I) = ACHAR(INPARMS(I))
     END DO

     CALL IGEBR2D(ICTXT,'A',' ',20,1,INPARMS,20,0,0)
     DO I = 1, 20
        PREC(I:I) = ACHAR(INPARMS(I))
     END DO

     CALL IGEBR2D(ICTXT,'A',' ',1,1,IPART,1,0,0)
     CALL IGEBR2D(ICTXT,'A',' ',1,1,ITMAX,1,0,0)
     CALL IGEBR2D(ICTXT,'A',' ',1,1,ISTOPC,1,0,0)
     CALL IGEBR2D(ICTXT,'A',' ',1,1,ITRACE,1,0,0)

  END IF

END SUBROUTINE GET_PARMS
SUBROUTINE PR_USAGE(IOUT)
  INTEGER IOUT
```

```
    WRITE(IOUT, *) ' Number of parameters is incorrect!'
    WRITE(IOUT, *) ' Use: hb_sample mtrx_file methd prec [ptype &
         &itmax istopc itrace]'
    WRITE(IOUT, *) ' Where:'
    WRITE(IOUT, *) '     mtrx_file      is stored in HB format'
    WRITE(IOUT, *) '     methd          may be: CGSTAB CGS TFQMR'
    WRITE(IOUT, *) '     prec           may be: ILU DIAGSC NONE'
    WRITE(IOUT, *) '     ptype          Partition strategy default 0'
    WRITE(IOUT, *) '                   >0: CYCLIC(ptype)   '
    WRITE(IOUT, *) '                    0: BLOCK partition '
    WRITE(IOUT, *) '                   -1: Random partition  '
    WRITE(IOUT, *) '     itmax          Max iterations [500]        '
    WRITE(IOUT, *) '     istopc         Stopping criterion [1]      '
    WRITE(IOUT, *) '     itrace         0  (no tracing, default) or '
    WRITE(IOUT, *) '                    >= 0 do tracing every ITRACE'
    WRITE(IOUT, *) '                    iterations '
  END SUBROUTINE PR_USAGE
END PROGRAM HB_SAMPLE
```

Sample PARTS Subroutine

This section shows sample *parts* programs.

PART_BLOCK (Block Data Distribution)

```
C
C User defined function corresponding to an HPF  BLOCK partition
C
      SUBROUTINE PART_BLOCK(GLOBAL_INDX,N,NP,PV,NV)

      IMPLICIT NONE

      INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
      INTEGER, INTENT(OUT) :: NV
      INTEGER, INTENT(OUT) :: PV(*)
      INTEGER              :: DIM_BLOCK
      REAL(8), PARAMETER   :: PC=0.0D0
      REAL(8)              :: DDIFF
      INTEGER              :: IB1, IB2, IPV

      DIM_BLOCK = (N + NP - 1)/NP
      NV = 1
      PV(NV) = (GLOBAL_INDX - 1) / DIM_BLOCK

      IPV = PV(1)
      IB1 = IPV * DIM_BLOCK + 1
      IB2 = (IPV+1) * DIM_BLOCK

      DDIFF = DBLE(ABS(GLOBAL_INDX-IB1))/DBLE(DIM_BLOCK)
      IF (DDIFF < PC/2) THEN
C
C     Overlap at the beginning of a block, with the previous proc
C
         IF (IPV>0) THEN
            NV     = NV + 1
            PV(NV) = IPV - 1
         ENDIF
      ENDIF
      DDIFF = DBLE(ABS(GLOBAL_INDX-IB2))/DBLE(DIM_BLOCK)
      IF (DDIFF < PC/2) THEN
C
C     Overlap at the end of a block, with the next proc
C
         IF (IPV<(NP-1)) THEN
            NV     = NV + 1
            PV(NV) = IPV + 1
         ENDIF
      ENDIF

      RETURN
      END
```

PARTBCYC (Block-Cyclic Data Distribution)

```
@process free(f90)
MODULE PARTBCYC
  PUBLIC PART_BCYC, SET_NB
  PRIVATE
  INTEGER, SAVE :: BLOCK_SIZE

CONTAINS
  !
  ! User defined subroutine corresponding to an HPF  CYCLIC(NB)
  ! data distribution
  !
  SUBROUTINE PART_BCYC(GLOBAL_INDX,N,NP,PV,NV)

    IMPLICIT NONE

    INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
    INTEGER, INTENT(OUT) :: NV
    INTEGER, INTENT(OUT) :: PV(*)

    NV = 1
    PV(NV) = MOD((((GLOBAL_INDX+BLOCK_SIZE-1)/BLOCK_SIZE)-1),NP)
    RETURN
  END SUBROUTINE PART_BCYC

  SUBROUTINE SET_NB(NB, RROOT, CROOT, ICTXT)
    INTEGER    :: RROOT, CROOT, ICTXT
    INTEGER    :: N, MER, MEC, NPR, NPC

    CALL BLACS_GRIDINFO(ICTXT,NPR,NPC,MER,MEC)

    IF (.NOT.((RROOT>=0).AND.(RROOT<NPR).AND.&
      & (CROOT>=0).AND.(CROOT<NPC))) THEN
       WRITE(0,*) 'Fatal error in SET_NB: invalid ROOT  ',&
         & 'coordinates '
       CALL BLACS_ABORT(ICTXT,-1)
       RETURN
    ENDIF

    IF ((MER==RROOT).AND.(MEC==CROOT)) THEN
       IF (NB < 1) THEN
          WRITE(0,*) 'Fatal error in SET_NB: invalid NB'
          CALL BLACS_ABORT(ICTXT,-1)
          RETURN
       ENDIF
       CALL IGEBS2D(ICTXT,'A',' ',1,1,NB,1)
    ELSE
       CALL IGEBR2D(ICTXT,'A',' ',1,1,NB,1,RROOT,CROOT)
    ENDIF
    BLOCK_SIZE = NB

    RETURN
  END SUBROUTINE SET_NB
END MODULE PARTBCYC
```

PARTRAND (Random Data Distribution)

```
@process free(f90) init(f90ptr)
!
! Purpose:
!  Provide a set of subroutines to define a data distribution based on
!  a random number generator.
!  This partition does *not* generally give good performance; it may be
!  useful as a model to implement a graph partitioning based
!  distribution; to do this you need to alter the BUILD_RNDPART
!  subroutine to make it call your favorite graph partition subroutine
!  instead of the random number generator.
!
!  Subroutines:
!
!  BUILD_RNDPART(A,NPARTS): This subroutine will be called by the root
!    process to build define the data distribution mapping.
!      Input parameters:
!        TYPE(D_SPMAT) :: A   The input matrix. The coefficients are
!                             ignored; only the structure is used.
!        INTEGER       :: NPARTS  How many parts we are requiring to the
!                                 partition utility
!
!
!  DISTR_RNDPART(RROOT,CROOT,ICTXT): This subroutine will be called by
!      all processes to distribute the information computed by the root
!      process, to be used subsequently.
!
!
!  PART_RAND : The subroutine to be passed to PESSL sparse library;
!      uses information prepared by the previous two subroutines.
!
MODULE PARTRAND
  PUBLIC PART_RAND, BUILD_RNDPART, DISTR_RNDPART
  PRIVATE
  INTEGER, POINTER, SAVE :: RAND_VECT(:)

CONTAINS

  SUBROUTINE PART_RAND(GLOBAL_INDX,N,NP,PV,NV)

    INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
    INTEGER, INTENT(OUT) :: NV
    INTEGER, INTENT(OUT) :: PV(*)

    IF (.NOT.ASSOCIATED(RAND_VECT)) THEN
```

```
      WRITE(0,*) 'Fatal error in PART_RAND: vector RAND_VECT ',&
        & 'not initialized'
      RETURN
   ENDIF
   IF ((GLOBAL_INDX<1).OR.(GLOBAL_INDX > SIZE(RAND_VECT))) THEN
      WRITE(0,*) 'Fatal error in PART_RAND: index GLOBAL_INDX ',&
        & 'outside RAND_VECT bounds'
      RETURN
   ENDIF
   NV = 1
   PV(NV) = RAND_VECT(GLOBAL_INDX)
   RETURN
 END SUBROUTINE PART_RAND


 SUBROUTINE DISTR_RNDPART(RROOT, CROOT, ICTXT)
   INTEGER    :: RROOT, CROOT, ICTXT
   INTEGER    :: N, MER, MEC, NPR, NPC

   CALL BLACS_GRIDINFO(ICTXT,NPR,NPC,MER,MEC)

   IF (.NOT.((RROOT>=0).AND.(RROOT<NPR).AND.&
     & (CROOT>=0).AND.(CROOT<NPC))) THEN
      WRITE(0,*) 'Fatal error in DISTR_RNDPART: invalid ROOT  ',&
        & 'coordinates '
      CALL BLACS_ABORT(ICTXT,-1)
      RETURN
   ENDIF

   IF ((MER == RROOT) .AND.(MEC == CROOT)) THEN
      IF (.NOT.ASSOCIATED(RAND_VECT)) THEN
      WRITE(0,*) 'Fatal error in DISTR_RNDPART: vector RAND_VECT ',&
           & 'not initialized'
      CALL BLACS_ABORT(ICTXT,-1)
      RETURN
      ENDIF
      N = SIZE(RAND_VECT)
      CALL IGEBS2D(ICTXT,'All',' ',1,1,N,1)
      CALL IGEBS2D(ICTXT,'All',' ',N,1,RAND_VECT,N)
   ELSE
      CALL IGEBR2D(ICTXT,'All',' ',1,1,N,1,RROOT,CROOT)
      IF (ASSOCIATED(RAND_VECT)) THEN
      DEALLOCATE(RAND_VECT)
      ENDIF
      ALLOCATE(RAND_VECT(N),STAT=INFO)
      IF (INFO /= 0) THEN
      WRITE(0,*) 'Fatal error in DISTR_RNDPART: memory allocation ',&
           & ' failure.'
      RETURN
      ENDIF
      CALL IGEBR2D(ICTXT,'All',' ',N,1,RAND_VECT,N,RROOT,CROOT)
   ENDIF

   RETURN

 END SUBROUTINE DISTR_RNDPART
```

```
      SUBROUTINE BUILD_RNDPART(A,NPARTS)
        USE F90SPARSE
        TYPE(D_SPMAT) :: A
        INTEGER       :: NPARTS
        INTEGER       :: N, I, IB, II
        INTEGER, PARAMETER :: NB=512
        REAL(KIND(1.D0)), PARAMETER :: SEED=12345.D0
        REAL(KIND(1.D0)) :: XV(NB)

        N      = A%M

        IF (ASSOCIATED(RAND_VECT)) THEN
           DEALLOCATE(RAND_VECT)
        ENDIF

        ALLOCATE(RAND_VECT(N),STAT=INFO)

        IF (INFO /= 0) THEN
           WRITE(0,*) 'Fatal error in BUILD_RNDPART: memory allocation ',&
             & ' failure.'
           RETURN
        ENDIF

        IF (NPARTS.GT.1) THEN
           DO I=1, N, NB
              IB = MIN(N-I+1,NB)
              CALL DURAND(SEED,IB,XV)
              DO II=1, IB
                 RAND_VECT(I+II-1) = MIN(NPARTS-1,INT(XV(II)*NPARTS))
              ENDDO
           ENDDO
        ELSE
           DO I=1, N
           RAND_VECT(I) = 0
           ENDDO
        ENDIF

        RETURN

      END SUBROUTINE BUILD_RNDPART

    END MODULE PARTRAND
```

The READ_MAT Subroutine

```
@PROCESS FREE(F90) INIT(F90PTR)
!
!  READ_MAT subroutine reads a matrix and its right hand sides,
!  all stored in a BCS format file. The B field is optional,.
!
!  Character                            :: filename*20
!     On Entry: name of file to be processed.
!     On Exit : unchanged.
!
!  Type(D_SPMAT)                        :: A
!     On Entry: fresh variable.
!     On Exit : will contain the global sparse matrix as follows:
!        A%AS for coefficient values
!        A%IA1  for column indices
!        A%IA2  for row pointers
!        A%M    for number of global matrix rows
!        A%K    for number of global matrix columns
!
!  Integer                              :: ICTXT
!     On Entry: BLACS context.
!     On Exit : unchanged.
!
!  Real(Kind(1.D0)), Pointer, Optional  :: B(:,:)
!     On Entry: fresh variable.
!     On Exit:  will contain right hand side(s).
!
!  Integer, Optional                    :: inroot
!     On Entry: Index of root processor (default: 0)
!     On Exit : unchanged.
!
!  Real(Kind(1.D0)), Pointer, Optional  :: indwork(:)
!     On Entry/Exit: Double Precision Work Area.
!
!  Integer, Pointer, Optional           :: iniwork()
!     On Entry/Exit: Integer Work Area.
!
MODULE READ_MAT
  PUBLIC READMAT
CONTAINS
  SUBROUTINE READMAT (FILENAME, A, ICTXT, B, INROOT,&
       & INDWORK, INIWORK)

    USE F90SPARSE

    ! Parameters
    IMPLICIT NONE
    REAL(KIND(1.D0)), POINTER, OPTIONAL   :: B(:,:)
    INTEGER                               :: ICTXT
    TYPE(D_SPMAT)                         :: A
    CHARACTER                             :: FILENAME*(*)
    INTEGER, OPTIONAL                     :: INROOT
    REAL(KIND(1.0D0)), POINTER, OPTIONAL  :: INDWORK(:)
    INTEGER, POINTER, OPTIONAL            :: INIWORK(:)

  ! Local Variables
    INTEGER, PARAMETER          :: INFILE = 2
    CHARACTER                   :: MXTYPE*3, KEY*8, TITLE*72,&
         & INDFMT*16, PTRFMT*16, RHSFMT*20, VALFMT*20, RHSTYP
```

```
INTEGER                     :: INDCRD,  PTRCRD, TOTCRD,&
     & VALCRD, RHSCRD, NROW, NCOL, NNZERO, NELTVL, NRHS, NRHSIX
REAL(KIND(1.0D0)), POINTER  :: AS_LOC(:), DWORK(:)
INTEGER, POINTER            :: IA1_LOC(:), IA2_LOC(:), IWORK(:)
INTEGER                     :: D_ALLOC, I_ALLOC, IRCODE, I,&
     & J, LIWORK, LDWORK, ROOT, NPROW, NPCOL, MYPROW, MYPCOL


IF (PRESENT(INROOT)) THEN
   ROOT = INROOT
ELSE
   ROOT = 0
END IF

CALL BLACS_GRIDINFO(ICTXT, NPROW, NPCOL, MYPROW, MYPCOL)

IF (MYPROW == ROOT) THEN
   WRITE(*, *) 'Start read_matrix'

 ! Open Input File
   OPEN(INFILE,FILE=FILENAME, STATUS='OLD', ERR=901, ACTION="READ")
   READ(INFILE,FMT='(A72,A8,/,5I14,/,A3,11X,4I14,/,2A16,2A20)',&
        & END=902) TITLE, KEY, TOTCRD, PTRCRD,INDCRD, VALCRD,&
        & RHSCRD, MXTYPE, NROW, NCOL, NNZERO, NELTVL,&
        & PTRFMT, INDFMT, VALFMT, RHSFMT

   A%M    = NROW
   A%N    = NCOL
   A%FIDA = 'CSR'
   IF (RHSCRD>0) READ(INFILE, FMT='(A1,13X,2I14)',&
        & END=902)  RHSTYP, NRHS, NRHSIX

   IF (MXTYPE == 'RUA') THEN
      ALLOCATE(A%AS(NNZERO), A%IA1(NNZERO), A%IA2(NROW + 1),&
           & STAT = IRCODE)
      IF (IRCODE <> 0)   GOTO 993
      READ(INFILE,FMT=PTRFMT,END=902) (A%IA2(I), I=1,NROW+1)
      READ(INFILE,FMT=INDFMT,END=902) (A%IA1(I), I=1,NNZERO)
      READ(INFILE,FMT=VALFMT,END=902) (A%AS(I), I=1,NNZERO)

   ELSE IF (MXTYPE == 'RSA') THEN
    ! We are generally working with non-symmetric matrices, so
    ! we de-symmetrize what we are about to read
      ALLOCATE(A%AS(2*NNZERO),A%IA1(2*NNZERO),&
           & A%IA2(NROW+1),AS_LOC(2*NNZERO),&
           & IA1_LOC(2*NNZERO),IA2_LOC(NROW+1),STAT=IRCODE)
      IF (IRCODE <> 0)   GOTO 993
      READ(INFILE,FMT=PTRFMT,END=902) (IA2_LOC(I), I=1,NROW+1)
      READ(INFILE,FMT=INDFMT,END=902) (IA1_LOC(I), I=1,NNZERO)
      READ(INFILE,FMT=VALFMT,END=902) (AS_LOC(I), I=1,NNZERO)

      LDWORK = MAX(NROW + 1, 2 * NNZERO)
      IF (PRESENT(INDWORK)) THEN
         IF (SIZE(INDWORK) >= LDWORK) THEN
            DWORK => INDWORK
            D_ALLOC = 0
         ELSE
            ALLOCATE(DWORK(LDWORK), STAT = IRCODE)
```

```
                D_ALLOC = 1
             END IF
          ELSE
             ALLOCATE(DWORK(LDWORK), STAT = IRCODE)
             D_ALLOC = 1
          END IF
          IF (IRCODE <> 0)   GOTO 993

          LIWORK = NROW + 1
          IF (PRESENT(INIWORK)) THEN
             IF (SIZE(INIWORK) >= LIWORK) THEN
                IWORK => INIWORK
                I_ALLOC = 0
             ELSE
                ALLOCATE(IWORK(LIWORK), STAT = IRCODE)
                I_ALLOC = 1
             END IF
          ELSE
             ALLOCATE(IWORK(LIWORK), STAT = IRCODE)
             I_ALLOC = 1
          END IF
          IF (IRCODE <> 0)   GOTO 993

        ! After this call NNZERO contains the actual value for
        ! desymetrized matrix
          CALL DESYM(NROW, AS_LOC, IA1_LOC, IA2_LOC, A%AS, A%IA1,&
               & A%IA2, IWORK, DWORK, NNZERO, 1)

          DEALLOCATE(AS_LOC, IA1_LOC, IA2_LOC)
          IF (D_ALLOC == 1)   DEALLOCATE(DWORK)
          IF (I_ALLOC == 1)   DEALLOCATE(IWORK)
       ELSE
          WRITE(0,*) 'READ_MATRIX: matrix type not yet supported'
          CALL BLACS_ABORT(ICTXT, 1)
       END IF

     ! Read Right Hand Sides
       IF (PRESENT(B) .AND. (NRHS > 0)) THEN
          WRITE(0,*) 'Reading RHS'
          IF (RHSTYP == 'F') THEN
             ALLOCATE(B(NROW, NRHS), STAT = IRCODE)
             IF (IRCODE <> 0)   GOTO 993
             READ(INFILE,FMT=RHSFMT,END=902) ((B(I,J), I=1,NROW),J=1,NRHS)
          ELSE !(RHSTYP <> 'F')
             WRITE(0,*) 'READ_MATRIX: unsupported RHS type'
          END IF
       END IF

       CLOSE(INFILE)
       WRITE(*,*) 'End READ_MATRIX'
    END IF

    RETURN

  ! Open failed
901 WRITE(0,*) 'READ_MATRIX: Could not open file ',&
         & INFILE,' for input'
    CALL BLACS_ABORT(ICTXT, 1)
  ! Unexpected End of File
902 WRITE(0,*) 'READ_MATRIX: Unexpected end of file ',INFILE,&
         & ' during input'
    CALL BLACS_ABORT(ICTXT, 1)

  ! Allocation Failed
993 WRITE(0,*) 'READ_MATRIX: Memory allocation failure'
    CALL BLACS_ABORT(ICTXT, 1)

  END SUBROUTINE READMAT
END MODULE READ_MAT
```

The MAT_DIST Subroutine

```
@process free(f90) init(f90ptr)
MODULE MAT_DIST
  PUBLIC MATDIST
CONTAINS
  SUBROUTINE MATDIST (A_GLOB, A, PARTS, ICONTXT, DESC_A,&
       & B_GLOB, B, INROOT)
  !
  ! An utility subroutine to distribute a matrix among processors
  ! according to a user defined data distribution, using PESSL
  ! sparse matrix subroutines.
  !
  !  Type(D_SPMAT)                            :: A_GLOB
  !     On Entry: this contains the global sparse matrix as follows:
  !        A%FIDA =='CSR'
  !        A%AS for coefficient values
  !        A%IA1  for column indices
  !        A%IA2  for row pointers
  !        A%M    for number of global matrix rows
  !        A%K    for number of global matrix columns
  !     On Exit : undefined, with unassociated pointers.
  !
  !  Type(D_SPMAT)                            :: A
  !     On Entry: fresh variable.
  !     On Exit : this will contain the local sparse matrix.
  !
  !       INTERFACE PARTS
  !         !   .....user passed subroutine.....
  !         SUBROUTINE PARTS(GLOBAL_INDX,N,NP,PV,NV)
  !           IMPLICIT NONE
  !           INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
  !           INTEGER, INTENT(OUT) :: NV
  !           INTEGER, INTENT(OUT) :: PV(*)
  !
  !       END SUBROUTINE PARTS
  !       END INTERFACE
  !     On Entry:  subroutine providing user defined data distribution.
  !        For each GLOBAL_INDX the subroutine should return
  !        the list  PV of all processes owning the row with
  !        that index; the list will contain NV entries.
  !        Usually NV=1; if NV >1 then we have an overlap in the data
  !        distribution.
  !
  !  Integer                                  :: ICONTXT
  !     On Entry: BLACS context.
  !     On Exit : unchanged.
  !
  !  Type (DESC_TYPE)                  :: DESC_A
  !     On Entry: fresh variable.
  !     On Exit : the updated array descriptor
  !
  !  Real(Kind(1.D0)), Pointer, Optional      :: B_GLOB(:)
  !     On Entry: this contains right hand side.
  !     On Exit :
  !
  !  Real(Kind(1.D0)), Pointer, Optional      :: B(:)
  !     On Entry: fresh variable.
  !     On Exit : this will contain the local right hand side.
  !
  !  Integer, Optional    :: inroot
  !     On Entry: specifies processor holding A_GLOB. Default: 0
  !     On Exit : unchanged.
  !
  !

    Use F90SPARSE

    Implicit None

  ! Parameters
    Type(D_SPMAT)              :: A_GLOB
    Real(Kind(1.D0)), Pointer  :: B_GLOB(:)
    Integer                    :: ICONTXT
    Type(D_SPMAT)              :: A
    Real(Kind(1.D0)), Pointer  :: B(:)
    Type (DESC_TYPE)           :: DESC_A
    INTEGER, OPTIONAL          :: INROOT
    INTERFACE PARTS
     !   .....user passed subroutine.....
       SUBROUTINE PARTS(GLOBAL_INDX,N,NP,PV,NV)
         IMPLICIT NONE
         INTEGER, INTENT(IN)  :: GLOBAL_INDX, N, NP
         INTEGER, INTENT(OUT) :: NV
         INTEGER, INTENT(OUT) :: PV(*)

       END SUBROUTINE PARTS
    END INTERFACE


  ! Local variables
    Integer                     :: NPROW, NPCOL, MYPROW, MYPCOL
    Integer                     :: IRCODE, LENGTH_ROW, I_COUNT, J_COUNT,&
         & K_COUNT, BLOCKDIM, ROOT, LIWORK, NROW, NCOL, NNZERO, NRHS,&
         & I,J,K, LL, INFO
    Integer, Pointer            :: IWORK(:)
    CHARACTER                   :: AFMT*5, atyp*5
    Type(D_SPMAT)               :: BLCK



  ! Executable statements
```

```
      IF (PRESENT(INROOT)) THEN
         ROOT = INROOT
      ELSE
         ROOT = 0
      END IF


      CALL BLACS_GRIDINFO(ICONTXT, NPROW, NPCOL, MYPROW, MYPCOL)

      IF (MYPROW == ROOT) THEN
       ! Extract information from A_GLOB
         IF (A_GLOB%FIDA /= 'CSR') THEN
            WRITE(0,*) 'Unsupported input matrix format'
            CALL BLACS_ABORT(ICONTXT,-1)
         ENDIF
         NROW = A_GLOB%M
         NCOL = A_GLOB%N
         IF (NROW /= NCOL) THEN
            WRITE(0,*) 'A rectangular matrix ? ',NROW,NCOL
            CALL BLACS_ABORT(ICONTXT,-1)
         ENDIF
         NNZERO = Size(A_GLOB%AS)
         NRHS   = 1
       ! Broadcast informations to other processors
         CALL IGEBS2D(ICONTXT, 'A', ' ', 1, 1, NROW, 1)
         CALL IGEBS2D(ICONTXT, 'A', ' ', 1, 1, NCOL, 1)
         CALL IGEBS2D(ICONTXT, 'A', ' ', 1, 1, NNZERO, 1)
         CALL IGEBS2D(ICONTXT, 'A', ' ', 1, 1, NRHS, 1)
      ELSE !(MYPROW <> root)
       ! Receive informations
         CALL IGEBR2D(ICONTXT, 'A', ' ', 1, 1, NROW, 1, ROOT, 0)
         CALL IGEBR2D(ICONTXT, 'A', ' ', 1, 1, NCOL, 1, ROOT, 0)
         CALL IGEBR2D(ICONTXT, 'A', ' ', 1, 1, NNZERO, 1, ROOT, 0)
         CALL IGEBR2D(ICONTXT, 'A', ' ', 1, 1, NRHS, 1, ROOT, 0)
      END IF

    ! Allocate integer work area
      LIWORK = MAX(NPROW, NROW + NCOL)
      ALLOCATE(IWORK(LIWORK), STAT = IRCODE)
      IF (IRCODE <> 0) THEN
         WRITE(0,*) 'MATDIST Allocation failed'
         RETURN
      ENDIF


      IF (MYPROW == ROOT) THEN
         WRITE (*, FMT = *) 'Start matdist'
      ENDIF

      CALL PADALL(NROW,PARTS,DESC_A,ICONTXT)
      CALL PSPALL(A,DESC_A,NNZ=NNZERO/NPROW)
      CALL PGEALL(B,DESC_A)

    ! Prepare the local
      ALLOCATE(BLCK%AS(NCOL),BLCK%IA1(NCOL),BLCK%IA2(2),STAT=IRCODE)
      IF (IRCODE /= 0) THEN
         WRITE(0,*) 'Error on allocating BLCK'
         CALL BLACS_ABORT(ICONTXT,-1)
```

```
     STOP
  ENDIF

  BLCK%M    = 1
  BLCK%N    = NCOL
  BLCK%FIDA = 'CSR'
  Do I_COUNT = 1, NROW
     CALL PARTS(I_COUNT,NROW,NPROW,IWORK, LENGTH_ROW)
   ! Here processors are counted 1..NPROW
     DO J_COUNT = 1, LENGTH_ROW
        K_COUNT = IWORK(J_COUNT)
        IF (MYPROW == ROOT) THEN
           BLCK%IA2(1) = 1
           BLCK%IA2(2) = 1
           DO J = A_GLOB%IA2(I_COUNT), A_GLOB%IA2(I_COUNT+1)-1
              BLCK%AS(BLCK%IA2(2)) = A_GLOB%AS(J)
              BLCK%IA1(BLCK%IA2(2)) = A_GLOB%IA1(J)
              BLCK%IA2(2) =BLCK%IA2(2) + 1
           ENDDO
           LL = BLCK%IA2(2) - 1
           IF (K_COUNT == MYPROW) THEN
              BLCK%INFOA(1) = LL
              BLCK%INFOA(2) = LL
              BLCK%INFOA(3) = 2
              BLCK%INFOA(4) = 1
              BLCK%INFOA(5) = 1
              BLCK%INFOA(6) = 1
              CALL PSPINS(A,I_COUNT,1,BLCK,DESC_A)
              CALL PGEINS(B,B_GLOB(I_COUNT:I_COUNT),DESC_A,I_COUNT)
           ELSE
              CALL IGESD2D(ICONTXT,1,1,LL,1,K_COUNT,0)
              CALL IGESD2D(ICONTXT,LL,1,BLCK%IA1,LL,K_COUNT,0)
              CALL DGESD2D(ICONTXT,LL,1,BLCK%AS,LL,K_COUNT,0)
              CALL DGESD2D(ICONTXT,1,1,B_GLOB(I_COUNT),1,K_COUNT,0)
              CALL IGERV2D(ICONTXT,1,1,LL,1,K_COUNT,0)
           ENDIF
        ELSE IF (MYPROW /= ROOT) THEN
           IF (K_COUNT == MYPROW) THEN
              CALL IGERV2D(ICONTXT,1,1,LL,1,ROOT,0)
              BLCK%IA2(1) = 1
              BLCK%IA2(2) = LL+1
              CALL IGERV2D(ICONTXT,LL,1,BLCK%IA1,LL,ROOT,0)
              CALL DGERV2D(ICONTXT,LL,1,BLCK%AS,LL,ROOT,0)
              CALL DGERV2D(ICONTXT,1,1,B_GLOB(I_COUNT),1,ROOT,0)
              CALL IGESD2D(ICONTXT,1,1,LL,1,ROOT,0)
              CALL PSPINS(A,I_COUNT,1,BLCK,DESC_A)
              CALL PGEINS(B,B_GLOB(I_COUNT:I_COUNT),DESC_A,I_COUNT)
           ENDIF
        ENDIF
     END DO
  END DO


! Default storage format for sparse matrix; we do not
! expect duplicated entries.
  AFMT = 'DEF'
  ATYP = 'GEN'
  CALL PSPASB(A,DESC_A,INFO=INFO,MTYPE=ATYP,STOR=AFMT,DUPFLAG=2)
```

```
    CALL PGEASB(B,DESC_A)

    DEALLOCATE(BLCK%AS,BLCK%IA1,BLCK%IA2,IWORK)

    IF (MYPROW == root) Write (*, FMT = *) 'End matdist'

    RETURN

  END SUBROUTINE MATDIST
END MODULE MAT_DIST
```

The DESYM Subroutine

```
      SUBROUTINE DESYM(NROW,A,JA,IA,AS,JAS,IAS,IAW,WORK,NNZERO,
     +   VALUE)
      IMPLICIT NONE
C      .. Scalar Arguments ..
      INTEGER NROW,NNZERO,VALUE,INDEX
C     .. Array Arguments ..
      DOUBLE PRECISION A(*),AS(*),WORK(*)
      INTEGER IA(*),IAS(*),JAS(*),JA(*),IAW(*)
C     .. Local Scalars ..
      INTEGER I,IAW1,IAW2,IAWT,J,JPT,K,KPT,LDIM,COUNT,JS,BUFI
C      REAL*8  BUF
C     ..

      DO I=1,NROW
         IAW(I)=0
      END DO
C    ....Compute element belonging to each row in output matrix.....
      DO I=1,NROW
         DO J=IA(I),IA(I+1)-1
            IAW(I)=IAW(I)+1
            IF (JA(J).NE.I) IAW(JA(J))=IAW(JA(J))+1
         END DO
      END DO

      IAS(1)=1
      DO I=1,NROW
         IAS(I+1)=IAS(I)+IAW(I)
         IAW(I)=0
      END DO


C
C     .....Computing values array AS and column array indices JAS....
C
      DO I=1,NROW
         DO J=IA(I),IA(I+1)-1
            IF (VALUE.NE.0) THEN
               AS(IAS(I)+IAW(I))=A(J)
            ENDIF
            JAS(IAS(I)+IAW(I))=JA(J)
            IAW(I)=IAW(I)+1
            IF (I.NE.JA(J)) THEN
               IF (VALUE.NE.0) THEN
                  AS(IAS(JA(J))+IAW(JA(J)))=A(J)
```

```
                 ENDIF
                 NNZERO=NNZERO+1
                 JAS(IAS(JA(J))+IAW(JA(J)))=I
                 IAW(JA(J))=IAW(JA(J))+1
              END IF
           END DO
        END DO

C       ......Sorting output arrays by column index......
C       .....the IAS index not must be modified.....
C
        DO I=1,NROW
           CALL ISORTX(JAS(IAS(I)),1,IAS(I+1)-IAS(I),IAW)
           INDEX=IAS(I)-1
           IF (VALUE.NE.0) THEN
              DO J=1,IAS(I+1)-IAS(I)
                 WORK(J)=AS(IAW(J)+INDEX)
              END DO
              DO J=1,IAS(I+1)-IAS(I)
                 AS(J+INDEX)=WORK(J)
              END DO
           ENDIF
C           ....column indices are already sorted by ISORTX...
        ENDDO
        RETURN

        END
```

Sample Makefiles and Run Script

You can use the message passing makefile and run script with the message passing sample thermal diffusion and sparse linear algebraic equations programs.

You can use the HPF makefile and run script with the HPF sample thermal diffusion programs.

Makefile (Message Passing)

```
#  Makefile to build the diffusion program, sparse solver and utility routines
#

#  add rule for making mod files
.SUFFIXES:  .mod

#
# Compilers and such
#

CC=mpcc
FORT=mpxlf
LINK=mpxlf

LDFLAGS =  -lblacs -lessl -lpessl $(LIB)
FCOPT = -O3 -C -qsource -qxref -qattr $(INCLUDE)

# default for include and lib directories
```

```
INCLUDE=
LIB=

## DISTRIBUTED DATA samples

# OBJS list objects module used in the diffusion program
#
OBJS = main.o scalemod.o param.o diffusion.o fourier.o

BASEOBJS = broadcast.o create.o delete.o init.o scatter_gather.o \
           pdata.o northsouth.o eastwest.o index.o

UTILOBJS = $(BASEOBJS) cdata.o putilities.o

UTILLIB = libputils.a

LIBOBJS = $(UTILLIB)(broadcast.o) $(UTILLIB)(create.o) \
          $(UTILLIB)(delete.o) $(UTILLIB)(init.o)      \
          $(UTILLIB)(scatter_gather.o) $(UTILLIB)(pdata.o)      \
          $(UTILLIB)(northsouth.o) $(UTILLIB)(eastwest.o)      \
          $(UTILLIB)(index.o) $(UTILLIB)(cdata.o)      \
          $(UTILLIB)(putilities.o)


distribute: diffusion pdgexmp image simple

#
#  Rule for building diffusion program
#  power libraries
diffusion: $(OBJS)
 $(LINK) -o diffusion $(OBJS) -lpessl -lblacs -lessl

#  power2 libraries
#diffusion: $(OBJS)
# $(LINK) -o diffusion $(OBJS) -lpesslp2 -lblacsp2 -lesslp2


pdgexmp:  pdgexmp.o  $(UTILLIB)
 $(LINK) -o pdgexmp pdgexmp.o  -L . -lputils $(LDFLAGS)

image:  image.o  $(UTILLIB)
 $(LINK) -o image image.o -L . -lputils $(LDFLAGS)

simple:  simple.o  $(UTILLIB)
 $(LINK) -o simple simple.o  -L . -lputils $(LDFLAGS)

#rule to create the library

$(UTILLIB): $(LIBOBJS)
 ar rv $(UTILLIB) $%

# rules to create library objects

init.o: init.f pdata.o
 xlf -c $(FFLAGS) init.f

exchange.o: init.f pdata.o
 xlf -c $(FFLAGS) exchange.f
```

```
cdata.o: cdata.f pdata.o
 xlf -c $(FFLAGS) cdata.f

create.o: create.f pdata.o
 xlf -c $(FFLAGS) create.f

broadcast.o: broadcast.f pdata.o
 xlf -c $(FFLAGS) broadcast.f

delete.o: delete.f pdata.o
 xlf -c $(FFLAGS) delete.f

scatter_gather.o: scatter_gather.f pdata.o
 xlf -c $(FFLAGS) scatter_gather.f

putilities.o: putilities.f $(BASEOBJS) cdata.o
 xlf -c $(FFLAGS) putilities.f

eastwest.o: eastwest.f pdata.o
 xlf -c $(FFLAGS) eastwest.f

northsouth.o: northsouth.f pdata.o
 xlf -c $(FFLAGS) northsouth.f

$(UTILLIB)(broadcast.o): broadcast.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(create.o): create.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(delete.o): delete.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(init.o): init.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(scatter_gather.o): scatter_gather.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(pdata.o): pdata.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(northsouth.o): northsouth.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(eastwest.o): eastwest.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(index.o): index.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(cdata.o): cdata.o
 ar rv $(UTILLIB) $%

$(UTILLIB)(putilities.o): putilities.o
 ar rv $(UTILLIB) $%
```

```
#
#  List of object module dependencies for distributed data sample.
main.o: main.f scalemod.o param.o  diffusion.o fourier.o
diffusion.o: diffusion.f scalemod.o param.o
fourier.o: fourier.f diffusion.o scalemod.o param.o
scalemod.o: scalemod.f param.o
param.o: param.f
pdgexmp.o: pdgexmp.f $(UTILLIB)
simple.o: simple.f $(UTILLIB)
image.o: image.f $(UTILLIB)


##  SPARSE MATRIX samples

# HBOBJS and PARTOBJS list objects module used in the SPARSE programs.
#
HBOBJS=read_mat.o mat_dist.o desym.o
PARTOBJS= part_block.o  partbcyc.o partrand.o

sparse: hb_sample pde90 pde77

pde90: pde90.o part_block.o
 $(LINK) $(LDFLAGS)  pde90.o part_block.o -o pde90



pde77: pde77.o  part_block.o
 $(LINK) $(LDFLAGS)  pde77.o  part_block.o -o pde77

hb_sample: $(HBOBJS) hb_sample.o $(PARTOBJS)
 $(LINK) $(LDFLAGS)  hb_sample.o -o hb_sample \
 $(HBOBJS) $(PARTOBJS)

#
#  List of object module dependencies for sparse matrix sample.
$(HBOBJS) hb_sample.o: read_mat.mod mat_dist.mod  part_bcyc.mod partrand.mod
part_bcyc.mod: partbcyc.o

#
#  Rule to clean executable and program
cleanall:
 rm -f *.lst *.o *.mod diffusion core image pdgexmp simple hb_sample pde90 pde77 libputils.a
#
clean:
 /bin/rm -f *.o *.mod *.lst

#
# definitions for compiles
.f.mod:
 $(FORT) $(INCLUDE) $(FCOPT) -c $<
.c.o:
 $(CC) $(INCLUDE) $(CCOPT) -c  $<
.f.o:
 $(FORT) $(INCLUDE) $(FCOPT)  -c  $<
```

Makefile (HPF)

```
#
#  Makefile to build the diffusion program.
#
# OBJS list objects module used in the program
#
OBJS = main.o param.o diffusion.o fourier.o

# For Power2 compilation
#FFLAGS = -qreport=hpflist -C -O -qarch=pwrx
FFLAGS = -qreport=hpflist -C -O
# For Power2 libraries
#LDFLAGS = -lblacsp2 -lpesslp2 -lpesslhpfp2 -lesslp2
LDFLAGS = -lblacs -lpessl -lpesslhpf -lessl

#
#  Default rule for compiling Fortran modules.
.f.o:
     xlhpf -c $(FFLAGS) $<

#
#  Rule for building diffusion program
#  power libraries
diffusion:     $(OBJS)
     xlhpf -o diffusion $(OBJS) $(LDFLAGS)


#
#  Rule to clean executable and program
clean:
     rm -f *.o *.mod diffusion core
#
#  List of object module dependencies.
main.o: main.f param.o  diffusion.o fourier.o
diffusion.o: diffusion.f param.o
fourier.o: fourier.f diffusion.o param.o
param.o: param.f
```

Run Script

```
#!/bin/ksh
#
# USING THE FORTRAN EXAMPLE
# Copy the files from /usr/lpp/pessl.rte.common/example/fortran to a directory that
# is part of a shared file system (e.g. NFS mounted).  You can not run
# the program from a private file system.
#
# You must have the same userid on the home node and each remote node.
#
# You must have remote execution authority on all the nodes.
#
# Invoke 'make'.
#
# In run.script, edit EXAMP_PATH below to point to the working directory
# which contains the files.
#
```

```
# Invoke 'run.script'.
#

#
# Script file to execute a sample program.
# The first argument is the name of the sample to run
# The remaining arguments are any needed to by the sample
#

#
#  Set the number of processor to be 8.
export MP_PROCS=8
#
#  Set the program to run in user space.
export MP_EUILIB=us
#
#  Use the high speed switch.
export MP_EUIDEVICE=css0
#
#  Use the poe resource manager.
export MP_RESD=yes
#
#  Use the resource pool 0, this may need to be changed
#  depending on installation defaults used.
export MP_RMPOOL=0
#
#  Do not use a hostfile list.
export MP_HOSTFILE=NULL
#
#  Use low information output level.
export MP_INFOLEVEL=1
#
#
export MP_PGMMODEL=spmd
#
#  Standard output is not node ordered.
export MP_STDOUTMODE=unordered
#
#  Retry node allocation every 60 seconds.
export MP_RETRY=60
#
#  Retry node allocation five times.
export MP_RETRYCOUNT=5
#
#
export MP_CSS_INTERRUPT=yes
#
export MP_PULSE=0
#
#  Label standard io by processor number.
export MP_LABELIO=yes
poe $*
```

Glossary

This glossary defines terms and abbreviations used in this publication. If you do not find the term you are looking for, refer to the index portion of this book. This glossary includes terms and definitions from:

- *IBM Dictionary of Computing,* New York: McGraw Hill (1-800-2MC-GRAW), 1994.
- *American National Standard Dictionary for Information Systems,* ANSI X3.172-1990, copyright 1990 by the American National Standards Institute (ANSI). Copies may be purchased from the American National Standards Institute, 11 West 42nd Street, New York, New York 10036. Definitions are identified by the symbol (A) after the definition.
- *Information Technology Vocabulary,* developed by Subcommittee 1, Joint Technical Committee 1, of the International Organization for Standardization and the International Electrotechnical Commission (ISO/IEC JTC1/SC1). Definitions from published sections of these vocabularies are identified by the symbol (I) after the definition. Definitions taken from draft international standards, committee drafts, and working papers being developed by ISO/IEC JTC1/SC1 are identified by the symbol (T) after the definition, indicating that final agreement has not yet been reached among the participating National Bodies of SC1.

IBM is grateful to the American National Standards Institute (ANSI) for permission to reprint its definitions from the American National Standard *Vocabulary for Information Processing* (Copyright 1970 by American National Standards Institute, Incorporated), which was prepared by Subcommittee X3K5 on Terminology and Gloassary of the American National Standards Committee X3. ANSI definitions are preceded by an asterisk (*).

A

address. A character or group of characters that identifies a register, a device, a particular part of storage, or some other data source or destination.

AIX. Abbreviation for Advanced Interactive Executive, IBM's licensed version of the UNIX operating system. AIX is particularly suited to support technical computing applications, including high function graphics and floating point computations.

Amd. Berkely Software Distribution automount daemon.

APAR. Authorized Program Analysis Report. A report of a problem caused by a suspected defect in a current unaltered release of a program.

application. The use to which a data processing system is put; for example, a computational chemistry application, a signal processing application.

application data. The data that is produced using an application program.

argument. A parameter passed between a calling program and a SUBROUTINE subprogram, a FUNCTION subprogram, or a statement function.

array. An ordered set of data items identified by a single name.

array descriptor. Contains the information required to establish the mapping between a global data structure and its corresponding process and memory location.

array element. A data item in an array, identified by the array name followed by a subscript indicating its position in the array.

array name. The name of an ordered set of data items that make up an array.

assignment statement. A statement that assigns a value to a variable or array element. It is made up of a variable or array element, followed by an equal sign (=), followed by an expression. The variable, array element, or expression can be character, logical, or arithmetic. When the assignment statement is processed, the expression to the right of the equal sign replaces the value of the variable or array element to the left.

B

bandwidth. The total available bit rate of a digital channel.

Basic Linear Algebra Communication Subprograms (BLACS). A standard set of public domain subroutines that perform message passing (communications) between processes.

Basic Linear Algebra Subprograms (BLAS). A standard set of public domain mathematical subroutines that perform linear algebra operations.

BLACS. Basic Linear Algebra Communication Subprograms.

BLAS. Basic Linear Algebra Subprograms.

C

cache. A high-speed buffer.

character constant. A string of one or more alphanumeric characters enclosed in apostrophes. The delimiting apostrophes are not part of the value of the constant.

character expression. An expression in the form of a single character constant, variable, array element, substring, function reference, or another expression enclosed in parentheses. A character expression is always of type character.

character type. The data type for representing strings of alphanumeric characters; in storage, one byte is used for each character.

client. * (1) A function that requests services from a server, and makes them available to the user. * (2) A term used in an environment to identify a machine that uses the resources of the network.

cluster. A group of processors interconnected through a high speed network that can be used for high performance computing.

column-major order. A sequencing method used for storing multidimensional arrays according to the subscripts of the array elements. In this method the leftmost subscript position varies most rapidly and completes a full cycle before the next subscript position to the right is incremented.

CMI. Centralized Management Interface provides a series of SMIT menus and dialogues used for defining and querying the SP system configuration.

complex conjugate even data. Complex data that has its real part even and its imaginary part odd.

complex constant. An ordered pair of real or integer constants separated by a comma and enclosed in parentheses. The first real constant of the pair is the real part of the complex number; the second is the imaginary part.

complex type. The data type for representing an approximation of the value of a complex number. A data item of this type consists of an ordered pair of real data items separated by a comma and enclosed in parentheses. The first item represents the real part of the complex number; the second represents the imaginary part.

constant. An unvarying quantity. The four classes of constants specify numbers (arithmetic), truth values (logical), character data (character), and hexadecimal data.

D

daemon. A process, not associated with a particular user, that performs system-wide functions such as administration and control of networks, execution of time-dependent activities, line printer spooling, etc.

default. An alternative value, attribute, or option that is assumed when none has been specified.

dataless workstation. A workstation that has local disks which may be used for **swap**, **tmp**, and **usr** file systems.

data distribution. The method in which global data structures are divided among processes. Three types of data distribution are: cyclic, block-cyclic, and block distribution.

data type. The structural characteristics, features and properties of data that may be directly specified by a programming language; for example, integers, real numbers in Fortran; arrays in APL; linked lists in LISP; character string in SNOBOL.

decimation. The formation of a sequence containing every n-th element of another sequence.

dimension of an array. One of the subscript expression positions in a subscript for an array. In Fortran, an array may have from one to seven dimensions. Graphically, the first dimension is represented by the rows, the second by the columns, and the third by the planes. Contrast with rank. See also extent of a dimension.

direct access storage. A storage device in which the access time is in effect independent of the location of the data. (A)

diskless workstation. A computer workstation with its own processor, keyboard, graphics system and monitor but no local disk system. The system relies on disk resources which are found in the network either on a dedicated server or shared over the entire network resources.

divide-by-zero exception. The condition recognized by a processor that results from running a program that attempts to divide by zero.

DNS. Domain Name Server is a hierarchical name service which maps high level machine names to IP addresses.

double precision. Synonym for long-precision.

DWM. Diskless Workstation Manager is operating-system software that initializes and maintains resources for diskless clients and diskless servers.

E

Ethernet. Ethernet is the standard hardware for TCP/IP LANs in the UNIX marketplace. It is a 10 megabit per second baseband type network that uses the contention based CSMA/CD (collision detect) media access method.

expression. A notation that represents a value: a primary appearing alone, or combinations of primaries and operators. An expression can be arithmetic, character, logical, or relational.

extent of a dimension. The number of different integer values that may be represented by subscript expressions for a particular dimension in a subscript for an array.

external function. A function defined outside the program unit that refers to it. It may be referred to in a procedure subprogram or in the main program, but it must not refer to itself, either directly or indirectly. Contrast with statement function.

EXTRINSIC (HPF). Indicates that the procedure is HPF-conforming. Such a procedure is referred to as an HPF procedure. This is the default; any procedure that does not specify the EXTRINSIC attribute and is compiled with the `-qhpf` option is considered to be an HPF procedure.

EXTRINSIC (HPF_LOCAL). Indicates a procedure in HPF that is targeted to a single processor, with many copies executing on different processors. Such a procedure is referred to as a local procedure. The programming style in which many copies of the same program run on multiple processors is often referred to as SPMD (single program, multiple data.)

EXTRINSIC (HPF_SERIAL). Indicates a procedure in HPF that is targeted to a single processor, with only one instance of the procedure executing on only one processor. Such a procedure is referred to as a serial procedure. (Serial procedures are useful for code written in other languages or current XL Fortran programs that you do not wish to recode.)

F

file. A set of related records treated as a unit, for example, in stock control, a file could consist of a set of invoices.

file server. A centrally located computer that acts as a storehouse of data and applications for numerous users of a local area network.

foreign host. Any host on the network other than the local host.

FTP. File transfer protocol.

function. In Fortran, a procedure that is invoked by referring to it in an expression and that supplies a value to the expression. The value supplied is the value of the function. See also external function, intrinsic function, and statement function. Contrast with subroutine.

function reference. A Fortran source program reference to an intrinsic function, to an external function, or to a statement function.

G

general matrix. A matrix with no assumed special properties such as symmetry. Synonym for matrix.

global. (1) Pertaining to that which is defined in one subdivision of a computer program and used in at least one other subdivision of the computer program. (2) Pertaining to information available to more than one program or subroutine. (3) Contrast with local.

H

home directory. The directory associated with an individual user.

host. A computer connected to a network, and providing an access method to that network. A host provides end-user services.

I

integer constant. A string of decimal digits containing no decimal point and expressing a whole number.

integer expression. An arithmetic expression whose values are of integer type.

integer type. An arithmetic data type capable of expressing the value of an integer. It can have a positive, negative, or 0 value. It must not include a decimal point.

Internet. The collection of worldwide networks and gateways which function as a single, cooperative virtual network.

internet address. A unique 32-bit address assigned to hosts connected to a TCP/IP network.

intrinsic function. A function, supplied by Fortran, that performs mathematical or character operations.

IP. Internet protocol.

K

kernel. The core portion of the UNIX operating system which controls the resources of the CPU and allocates them to the users. The kernel is memory-resident, is said to run in "kernel mode" and is protected from user tampering by the hardware.

L

LAN. Acronym for Local Area Network, a data network located on the user's premises in which serial transmission is used for direct data communication among data stations.

latency. The time interval between the instant at which an instruction control unit initiates a call for data transmission and the instant at which the actual transfer of data begins. Latency is related to the hardware characteristics of the system and to the different layers of software that are involved in initiating the task of packing and transmitting the data.

leading dimension. For a two-dimensional array, an increment used to find the starting point for the matrix elements in each successive column of the array.

local. Pertaining to that which is defined and used only in one subdivision of a computer program. Contrast with global.

local host. The computer to which a user's terminal is directly connected.

logical constant. A constant that can have one of two values: true or false. The form of these values in Fortran is: .TRUE. and .FALSE. respectively.

logical expression. A logical primary alone or a combination of logical primaries and logical operators. A logical expression can have one of two values: true or false.

logical type. The data type for data items that can have the value true or false and upon which logical operations such as .NOT. and .OR. can be performed. See also "data type."

long-precision. Real type of data of length 8. Contrast with single precision and short-precision.

M

main program. In Fortran, a program unit, required for running, that can call other program units but cannot be called by them.

mainframe. A large computer to which other computers can be connected, so that they can share facilities that the large computer provides; for example, it could be a System/370 or System/390 computing system to which personal computers are attached, so that they can upload and download programs and data.

mask. To use a pattern of characters to control the retention or elimination of portions of another pattern of characters. (I)

matrix. A rectangular array of elements, arranged in rows and columns, that may be manipulated according to the rules of matrix algebra. (A) (I)

menu. A display of a list of available functions for selection by the user.

message passing. The method of communication among processor nodes operating in parallel with distributed memory.

MPI. A Message Passing Interface standard.

MPMD (Multiple Program - Multiple Data). A parallel programming model in which different, but related, programs are run on different sets of data.

N

name. In Fortran, a string of up to six alphanumeric characters, the first of which must be alphabetic. Used to identify a constant, a variable, an array, a function, a subroutine, or a common block.

network. An interconnected group of nodes, lines, and terminals. A network provides the ability to transmit data to and receive data from other systems and users.

NFS. Network file system. NFS allows different systems (UNIX or non-UNIX), different architectures, or vendors connected to the same network, to access remote files in a LAN environment as though they were local files.

node. In a network, the point where one or more functional units interconnect transmission lines. A computer location defined in a network.

nodeid. The specific symbolic name assigned to a node during network definition.

O

overflow exception. A condition caused by the result of an arithmetic operation having a magnitude that exceeds the largest possible number.

P

parallel processing. A multiprocessor architecture which allows processes to be allocated to tightly coupled multiple processors in a cooperative processing environment, allowing concurrent execution of tasks.

parameter. * (1) A variable that is given a constant value for a specified application and that may denote the application. (2) An item in a menu for which the operator specifies a value or for which the system provides a value when the menu is interpreted. (3) A name in a procedure that is used to refer to an argument that is passed to the procedure. (4) A particular piece of information that a system or application program needs to process a request.

pipe. A UNIX utility allowing the output of one command to be the input of another. Represented by the | symbol. It is also referred to as filtering output.

port. (1) An endpoint for communication between devices, generally referring to physical connection. (2) A 16-bit number identifying a particular TCP or UDP resource within a given TCP/IP node.

primary. An irreducible unit of data; a single constant, variable, array element, function reference, or expression enclosed in parentheses.

process. * (1) A unique, finite course of events defined by it purpose or by its effect, achieved under defined conditions. * (2) Any operation or combination of operations on data. * (3) A function being performed or waiting to be performed. * (4) A program in operation. For example, a daemon is a system process that is always running on the system.

process grid. A way to view a parallel machine as a logical one- or two-dimensional rectangular grid of processes.

program exception. The condition recognized by a processor that results from running a program that improperly specifies or uses instructions, operands, or control information.

protocol. A set of semantic and syntactic rules that defines the behavior of functional units in achieving communication.

PTF. Program Temporary Fix. A temporary solution or by-pass of a problem diagnosed by IBM as resulting from a defect in a current unaltered release of the program. A report of a problem caused by a suspected defect in a current unaltered release of a program.

R

rank. In Fortran, the number of dimensions of an array. It is zero for scalar.

real constant. A string of decimal digits that expresses a real number. A real constant must contain either a decimal point or a decimal exponent and may contain both. For example, the real constant 0.36819E+2 has the value +36.819.

real type. An arithmetic data type, capable of approximating the value of a real number. It can have a positive, negative, or 0 value.

remote host. *See foreign host.*

row-major order. A sequencing method used for storing multidimensional arrays according to the subscripts of the array elements. In this method the rightmost subscript position varies most rapidly and completes a full cycle before the next subscript position to the left is incremented.

RISC. Reduced Instruction Set Computing (RISC), the technology for today's high performance personal computers and workstations, was invented in 1975.

S

scalar. (1) A quantity characterized by a single number. (A) (I) (2) Contrast with vector.

scope. (1) The portion of a computer program within which the definition of a variable remains unchanged.

(2) In Appendix A on page 995, for the broadcast topologies and global operations, scope can equal `'all'`, `'row'`, or `'column'`.

server. (1) A function that provides services for users. A machine may run client and server processes at the same time. (2) A machine that provides resources to the network. It provides a network service, such as disk storage and file transfer, or a program that uses such a service.

ScaLAPACK (Scalable Linear Algebra Package). A scalable linear algebra library for distributed memory concurrent computers. The library was jointly developed by the University of Tennessee, Knoxville, Oak Ridge National Laboratory, and the University of California, Berkeley.

shape of an array. The extents of all the dimensions of an array listed in order. For example, the shape of a three-dimensional array that has four rows, five columns, and three planes is (4,5,3) or 4 by 5 by 3.

shell. The shell is the primary user interface for the UNIX operating system. It serves as command language interpreter, programming language, and allows foreground and background processing. There are three different implementations of the shell concept: Bourne, C and Korn.

short-precision. Real type data of length 4. Contrast with double precision and long-precision.

single precision. Synonym for short-precision.

size of an array. The number of elements in an array. This is the product of the extents of its dimensions.

SMIT. The System Management Interface Toolkit is a set of menu driven utilities for AIX that provides functions such as transaction login, shell script creation, automatic updates of object data base, etc.

SMP. Symmetric Multi-Processing.

SPMD (Single Program - Multiple Data). A parallel programming model in which different processors execute the same program on different sets of data.

statement. The basic unit of a program, that specifies an action to be performed, or the nature and characteristics of the data to be processed, or information about the program itself. Statements fall into two broad classes: executable and nonexecutable.

statement function. A procedure specified by a single statement that is similar in form to an arithmetic, logical, or character assignment statement. The statement must appear after the specification statements and before the first executable statement. In the remainder of the program it can be referenced as a function. A statement function may be referred to only in the program unit in which it is defined. Contrast with external function.

statement label. A number of from one through five decimal digits that is used to identify a statement. Statement labels can be used to transfer control, to define the range of a DO, or to refer to a FORMAT statement.

statement number. See “statement label.”

stride. The increment used to step through array storage to select the vector or matrix elements from the array.

subprogram. A program unit that is invoked by another program unit in the same program. In Fortran, a subprogram has a FUNCTION, SUBROUTINE, or BLOCK DATA statement as its first statement.

subscript. (1) A symbol that is associated with the name of a set to identify a particular subset or element. (A) (2) A subscript expression or set of subscript expressions, enclosed in parentheses and used with an array name to identify a particular array element.

subscript expression. An integer expression in a subscript whose value and position in the subscript determine the index number for the corresponding dimension in the referenced array.

System Administrator. The user who is responsible for setting up, modifying, and maintaining the computing system.

T

tar. Tape ARchive, is a standard UNIX data archive utility for storing data on tape media.

TCP. Acronym for Transmission Control Protocol, a stream communication protocol that includes error recovery and flow control.

TCP/IP. Acronym for Transmission Control Protocol/Internet Protocol, a suite of protocols designed to allow communication between networks regardless of the technologies implemented in each network.

Telnet. Terminal Emulation Protocol, a TCP/IP application protocol that allows interactive access to foreign hosts.

thread. A thread is the element that is scheduled, and to which resources such as execution time, locks, and queues may be assigned. There may be one or more threads in a process, and each thread is executed by the operating system concurrently.

thread-safe. A subroutine which may be called from multiple threads of the same process simultaneously.

thread-tolerant. A library is thread-tolerant if it can be called from a single thread of a multithreaded application. However, multiple simultaneous calls to the thread-tolerant library from different threads of a single process causes unpredictable results.

transaction. An exchange between the user and the system. Each activity the system performs for the user is considered a transaction.

transfer. To send data from one place and to receive the data at another place. Synonymous with move.

transmission. * The sending of data from one place for reception elsewhere.

type declaration. The explicit specification of the type of a constant, variable, array, or function by use of an explicit type specification statement.

U

UDP. User Datagram Protocol.

underflow exception. A condition caused by the result of an arithmetic operation having a magnitude less than the smallest possible nonzero number.

URL. Uniform Resource Locator.

user. Anyone who requires the services of a computing system.

V

variable. (1) A quantity that can assume any of a given set of values. (A) (2) A data item, identified by a name, that is not a named constant, array, or array element, and that can assume different values at different times during program processing.

vector. A one-dimensional ordered collection of numbers.

W

working directory. A collection of files to be manipulated by an FTP operation.

workstation. A workstation is a single-user, high-performance microcomputer (or even a minicomputer) which has been specialized in some way, usually for graphics output. Such a machine has a screen and a keyboard, but is also capable of extensive processing of your input before it is passed to the host. Likewise, the host's responses may be extensively processed before being passed along to your screen. A workstation may be intelligent enough to do much or all the processing itself.

X

X Window System. A product developed at MIT that gives users windows into applications and processes not located only or specifically on their own console or computer system.

Bibliography

This bibliography lists the publications that you may need to use with ESSL and describes how to obtain them.

References

Text books and articles covering the mathematical aspects of ESSL are listed in this section, as well as several software libraries available from other companies. They are listed alphabetically as follows:

- Publications are listed by the author's name. IBM publications that include an order number, other than an *IBM Technical Report* can be ordered through the Subscription Library Services System (SLSS). The non-IBM publications listed here should be obtained through publishers, bookstores, or professional computing organizations.
- Software libraries are listed by their product name. Each reference includes the names, addresses, and phone numbers of the companies from which they can be obtained.

Each citation in the text of this book is shown as a number enclosed in square brackets. It indicates the number of the item listed in the bibliography. For example, reference [1] cites the first item listed below.

1. Agarwal, R. C.; Cooley, J. W. September 1987. “Vectorized Mixed Radix Discrete Fourier Transform Algorithms.” *IEEE Proceedings,* Vol. 75-9:1283-1292.
2. Agarwal, R. C.; Gustavson, F.; Joshi, M.; Zubair, M. February 1995. “A Scalable Parallel Block Algorithm for Band Cholesky Factorization.” SIAM Conference on Parallel Processing.
3. Agarwal, R. C.; Gustavson, F.; Zubair, M. May 1994. “An Efficient Parallel Algorithm for the Three-Dimensional FFT NAS Parallel Benchmark.” *Proceedings of IEEE SHPCC '94,* 129-133.
4. Anderson, E.; Bai, Z.; Bischof, C.; Demmel, J.; Dongarra, J.; DuCroz, J.; Greenbaum, S.; Hammarling, A.; McKenney, S.; Ostrouchov, S.; Sorensen, D. 1995. “LAPACK User's Guide, Second Edition.” SIAM, Philadelphia, Pa.
5. Anderson, E.; Bai, Z.; Bischof, C.; Demmel, J.; Dongarra, J.; DuCroz, J.; Greenbaum, S.; Hammarling, A.; McKenney, S.; Sorensen, D. May 1990. “LAPACK: A Portable Linear Algebra Library for High-Performance Computers.” University of Tennessee, Technical Report CS-90-105.
6. Anderson, E.; Benzoni, A.; Dongarra, J.; Moulton, S.; Ostrouchov, S.; Tourancheau, B.; van de Geijn, R. 1991. “Basic Linear Algebra Communication Subprograms.” *Sixth Distributed Memory Computing Conference Proceedings* IEEE Computer Society Press.
7. Arioli, M.; Duff, I. S.; Ruiz, M. 1992. “Stopping Criteria for Iterative Solvers,” *SIAM Journal of Matrix Analysis Application*, Vol. 13, pp. 138-144.
8. Bailey, D.; Barton, J.; Lasinski, T.; Simon, H.. 1991. “NAS Parallel Benchmarks.” Report RNR-91-002, Revision 2. NASA Ames Research Center, Moffett Field, CA.
9. Barrett, R.; Berry, M.; Chan, T. F.; Demmel, J.; Donato, J.; Dongarra, J.; Eijkhout, V.; Pozo, R; Romine, C.; van der Vorst, H. 1994. “Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods,” *SIAM*. Philadelphia, PA.
10. Blackford, L. S.; Choi, J.; Cleary, A.; D'Azevedo, E.; Demmel, J.; Dhillon, I.; Dongarra, J.; Hammarling, S.; Henry, G.; Petitet, A.; Stanley, K.; Walker, D.; Whaley, R. C. 1997. “ScaLAPACK Users' Guide,” *SIAM*. Philadelphia, PA.
11. Brainerd, W. S.; Goldberg, C. H.; Adams, J. C. 1990. *Programmer's Guide to Fortran 90* Intertext Publications and McGraw-Hill Book Company, New York, N.Y.
12. Cerioni, F.; Colajanni, M.; Filippone, S.; Maiolatesi, S. 1996. “A Proposal for Parallel Sparse BLAS,” in Proceedings of PARA '96. Edited by J. Wasniewski, J. Dongarra, K. Madsen, and D. Olesen. *Springer-Verlag Lecture Notes in Computer Science*, No. 1184, pp. 166-175.
13. Choi, J.; Demmel, J.; Dhillon, I.; Dongarra, J. J.; Ostrouchov, L. S.; Petitet, A.; Walker, D.; Whaley, R. C.; Stanley, K. March 1995. “Installation Guide for ScaLAPACK,” *LAPACK Working Note 93* University of Tennessee, Technical Report CS-95-280.
14. Choi, J.; Dongarra, J. J.; Walker, D. W. 1994. “PB-BLAS: A Set of Parallel Block Basic Linear Algebra Subprograms,” *Technical Report ORNL/TM-12468* Oak Ridge National Laboratory, Mathematical Sciences Section, Oak Ridge, Tennessee.
15. Choi, J.; Dongarra, J. J.; Walker, D. W. 1994. “PB-BLAS: Reference Manual,” *Technical Report ORNL/TM-12469* Oak Ridge National Laboratory, Mathematical Sciences Section, Oak Ridge, Tennessee.
16. Choi, J.; Dongarra, J. J.; Ostrouchov, S.; Petitet, A. P.; Walker, D. W.; Whaley, R. C. September 1994. “The Design and Implementation of the

ScaLAPACK LU, QR, and Cholesky Factorization Routines," *LAPACK Working Note 80* University of Tennessee, Technical Report CS-94-246.

17. Choi, J.; Dongarra, J. J.; Ostrouchov, S.; Petitet, A. P.; Walker, D. W.; Whaley, R. C. May 1995. "A Proposal for a Set of Parallel Basic Linear Algebra Subprograms," *LAPACK Working Note 100* Soogsil University, University of Tennessee, and Oak Ridge National Laboratory.
18. Choi, J.; Dongarra, J. J.; Walker, D. W. 1994. "SCALAPACK Reference Manual I: Parallel Factorization Routines (LU, QR, and Cholesky)," *Technical Report ORNL/TM-12471* Oak Ridge National Laboratory, Mathematical Sciences Section, Oak Ridge, Tennessee.
19. Choi, J.; Dongarra, J. J.; Walker, D. W. 1994. "SCALAPACK II: Parallel Reduction Routines (HRD, TRD, and BRD)," *Technical Report ORNL/TM-12472* Oak Ridge National Laboratory, Mathematical Sciences Section, Oak Ridge, Tennessee.
20. Choi, J.; Dongarra, J. J.; Walker, D. W. 1994. "SCALAPACK Reference Manual II: Parallel Reduction Routines (HRD, TRD, and BRD)," *Technical Report ORNL/TM-12473* Oak Ridge National Laboratory, Mathematical Sciences Section, Oak Ridge, Tennessee.
21. Choi, J.; Dongarra, J. J.; Walker, D. February 1995. "The Design of a Parallel Dense Linear Algebra Software Library: Reduction to Hessenberg, Tridiagonal, and Bidiagonal Form," *LAPACK Working Note 92* University of Tennessee, Technical Report CS-95-275.
22. Choi, J.; Dongarra, J. J.; Pozo, R.; Walker, D. 1992. "ScaLAPACK: A Scalable Linear Algebra Library for Distributed Memory Concurrent Computers." *Proceedings of the Fourth Symposium on the Frontiers of Massively Parallel Computation (FRONTIERS '92)* IEEE Computer Society Press.
23. Choi, J.; Dongarra, J. J.; Pozo, R.; Walker, D. November 1992. "ScaLAPACK: A Scalable Linear Algebra Library for Distributed Memory Concurrent Computers," University of Tennessee, Technical Report CS-92-181.
24. Demmel, J. W.; Dhillon, I.; Ren, H. March 1994. "On the correctness of Parallel Bisection in Floating Point," *LAPACK Working Note 70* University of Tennessee, Technical Report CS-94-228.
25. Demmel, J. W.; Kahan, W. February 1988. "Computing Small Singular Values of Bidiagonal Matrices with Guaranteed High Relative Accuracy," *LAPACK Working Note 3* Argonne National Laboratory, MCS-TM-110.
26. Demmel, J. W.; Stanley, K. September 1994. "The Performance of Finding Eigenvalues and Eigenvectors of Dense Symmetric Matrices on Distributed Floating Point," *LAPACK Working Note 86* University of Tennessee, Technical Report CS-94-254.
27. Dongarra, J. J.; DuCroz, J.; Hammarling, S.; Duff, I. March 1990. "A Set of Level 3 Basic Linear Algebra Subprograms." *ACM Transactions on Mathematical Software*, 16(1):1–17.
28. Dongarra, J. J.; DuCroz, J.; Hammarling, S.; Hanson, R. J. March 1988. "An Extended Set of FORTRAN Basic Linear Algebra Subprograms." *ACM Transactions on Mathematical Software*, 14(1):1–17.
29. Dongarra, J. J.; DuCroz, J.; Hammarling, S.; Hanson, R. J. March 1988. "Algorithm 656. An Extended Set of Basic Linear Algebra Subprograms: Model Implementation and Test Programs." *ACM Transactions on Mathematical Software*, 14(1):18–32.
30. Dongarra, J. J.; DuCroz, J.; Hammarling, S. 1996. "A Proposal for Fortran 90 BLAS," *LAPACK Working Note* University of Tennessee, Oak Ridge National Laboratory, and Numerical Algorithms Group Ltd.
31. Dongarra, J. J.; DuCroz, J.; Hammarling, S.; Wasniewski, J.; Zemla, A. August 1995. "A Proposal for a Fortran 90 Interface for LAPACK," *LAPACK Working Note 101* University of Tennessee, Numerical Algorithms Group Ltd., Technical University of Denmark, and Polish Academy of Sciences.
32. Dongarra, J. J.; van de Geijn, R. A. 1991. "Two Dimensional Basic Linear Algebra Communication Subprogram," *LAPACK Working Note 37* University of Tennessee, Technical Report CS-91-138.
33. Dongarra, J. J.; van de Geijn, R. A.; Whaley, R. C. December 1993 and June 1994. *A User's Guide to the BLACS*, Oak Ridge National Laboratory, Oak Ridge, Tennessee.
34. Dongarra, J. J.; Walker, D. 1993. "The Design of Linear Algebra Libraries for High Performance Computers," *LAPACK Working Note 58* University of Tennessee, Technical Report CS-93-188.
35. Duff, I. S.; Marrone, M.; Radicati, G.; and Vittoli, C. March 1997. "Basic Linear Algebra Subprograms for Sparse Matrices: a User Level Interface." Technical Report RAL-TR-95-049, Department for Computation and Information, Atlas Centre, Rutherford Appleton Laboratory, Oxon OX11 0QX. (To appear in *ACM Transactions on Mathematical Software*.)
36. Filippone, S.; Sales, M. L. 1994. "Experiences in Numerical Software on IBM Distributed Memory Architectures." *Parallel Scientific Computing*

207–218. Edited by J. Dongarra and J. Wasnievski. Springer-Verlag, New York, Heidelberg, Berlin.

37. Golub, G. H.; Van Loan, C. F. 1990. *Matrix Computations*, John Hopkins University Press, Baltimore, Maryland.

38. Gropp, W.; Lusk, E.; Skjellum, A. 1994. *Using MPI: Portable Parallel Programming with the Message Passing Interface.* The MIT Press, Cambridge, MA; London, England. (This book is also orderable through SLSS by specifying publication number SR28-5757.)

39. Gupta, A.; Gustavson, F.; Joshi, M.; Toledo, S. June, 20, 1996. “The Design, Implementation, and Evaluation of a Banded Linear Solver for Distributed-Memory Parallel Computers.” *IBM Research Report* RC 20481.

40. Gupta, A.; Gustavson, F.; Joshi, M.; Toledo, S. 1996. “The Design, Implementation, and Evaluation of a Banded Linear Solver for Distributed-Memory Parallel Computers.” *Applied Parallel Computing, Industrial Problems and Optimization.* Edited by Dongarra, J.; Madsden, K.; Washniewski, J. Parallel Computing, Third International Workshop, PARA'96 Lyngby, Denmark, August 1996 Proceedings Lecture Notes in Computer Science, Springer-Verlag, 1996.

41. *High Performance Fortran Language Specification* High Performance Fortran Forum; November 10, 1994; Version 1.1.

42. Holian, B. L.; Percus, O. E.; Warnock, T. T.; Whitlock, P. A. August 1994. “Pseudorandom Number Generator for Massively Parallel Molecular-Dynamics Simulations.” *Physical Review E,* 50(2).

43. Kelley, C. T. 1995. “Iterative Methods for Linear and Nonlinear Equations” *SIAM.* Philadelphia, PA.

44. Koelbel, C.; Loveman, D.; Schreiber, R.; Steele Jr., G.; Zosel, M. 1994. *The High Performance Fortran Handbook.* The MIT Press, Cambridge, MA; London, England.

45. Metcalf, M.; Reid, J. 1994. *Fortran 90 Explained.* Oxford University Press, U.S.A.

46. *Message Passing Interface Forum, MPI: A Message Passing Interface Standard, Version 1.1.* June 6, 1995. University of Tennessee, Knoxville, Tennessee. This document can be obtained at the URL, http://www.mcs.anl.gov/Projects/mpi/index.html

47. Percus, O. E.; Kalos, M. H. 1989. “Random Number Generators for MIMD Parallel Processors.” *Journal of Parallel and Distributed Computing,* 6:477-497.

48. Percus, O. E.; Percus, J. K. July 1988. “Long Range Correlations in Linear Congruential Generators.” *Journal of Computational Physics,* 77(1).

49. Percus, O. E.; Percus, J. K. 1992. “An Expanded Set of Correlation Tests for Linear Congruential Random Number Generators.” *Combinatorics, Probability and Computing,* 1:161-168.

50. Percus, O. E.; Percus, J. K. 1992. “Intrinsic Relations in the Structure of Linear Congruential Generators Modulo 2^{β}.” *Statistics and Probability Letters,* 15:381-383.

51. Sun, Xian-He; Zhang, Hong; Ni, Lionel M. March 1992. “Efficient Tridiagonal Solvers on Multicomputers.” *IEEE Transactions on Computers,* Vol. 41, No. 3.

52. Whaley, R. Clint. May 1994. “Basic Linear Algebra Communication Subprograms: Analysis and Implementation Across Multiple Parallel Architectures,” *LAPACK Working Note 73* University of Tennessee, Technical Report CS-94-234.

Parallel ESSL Publications

This section lists the publications for each major task that you perform when using the ESSL products.

You can order the full set of hardcopy publications for Parallel ESSL product through the Subscription Library Services System (SLSS) by specifying:

- The subject code 82
- The program number 5765-C41

You can also order individual hardcopy publications by specifying the individual order numbers. For example, to order a copy of the *Parallel ESSL Version 2 Release 1.1 Guide and Reference*, specify the order number SA22-7273. Contact your IBM Marketing Representative or Systems Engineer to order manuals through SLSS.

Evaluation and Planning

ESSL Products General Information, GC23-0529—provides detailed information helpful in evaluating and planning for all the ESSL products: Parallel ESSL, ESSL for AIX, and ESSL/370.

Installation

Parallel ESSL Installation Memo, GI10-0607—describes how to install Parallel ESSL on AIX. It is a packing list for the Parallel ESSL product when it is shipped. (One copy is delivered with each Parallel ESSL product.)

Application Programming

Parallel ESSL Version 2 Guide and Reference, SA22-7273—contains guidance information for designing, coding, and running programs using Parallel ESSL. It contains complete reference information for coding calls to the subroutines. This manual is available in HTML and PostScript format on the Parallel ESSL product medium.

ESSL Version 3 Guide and Reference, SA22-7272—contains guidance information for designing, coding, and running programs using ESSL for AIX. It contains complete reference information for coding calls to the subroutines. It is also available in HTML and PostScript format on the ESSL for AIX product medium.

ESSL Version 2.2 Guide and Reference, SC23-0526—contains guidance information for designing, coding, and running programs using ESSL/370. It contains complete reference information for coding calls to the subroutines.

Related Publications

The related publications listed below may be useful to you when using Parallel ESSL.

AIX for the IBM RS/6000

IBM AIX Calls and Subroutines References for the IBM RS/6000, (all volumes) SC23-2198

AIX Version 4 Release 2 for the IBM RS/6000

IBM AIX Version 4.1 and 4.2 Commands and Reference, (all volumes) SBOF-1851

IBM Version 4.1 and 4.2 General Programming Concepts: Writing and Debugging Programs, SC23-2533

IBM AIX Version 4.1 and 4.2 System Management Guide: Operating System and Devices, SC23-2525

AIX Version 4 Release 3 for the IBM RS/6000

For the latest updates, visit the RS/6000 web site at: **http://www.rs6000.ibm.com/resource/aix_resource/Pubs**

AIX Version 4.3 Commands and Reference, (all volumes) SBOF-1877

AIX Version 4.3 General Programming Concepts:Writing and Debugging Programs, SC23-4128

AIX Version 4.3 System Guide: Operating System and Devices, SC23-4126

XL Fortran

On the XL Fortran Version 5 Release 1.1 CD ROM and the XL High Performance Fortran Version 1 Release 3.1 tape, you may reference the following document available in both PostScript and HTML formats, which describes the changes for those releases: *Changes to XL Fortran for AIX and XL High Performance Fortran for AIX*

IBM XL Fortran for AIX User's Guide Version 5.1, SC09-2606

IBM XL Fortran for AIX Language Reference Version 5.1, SC09-2607

IBM Optimization and Tuning Guide for the XL Fortran, and XL C, SC09-1545

XL HPF

IBM XL High Performance Fortran for AIX Version 1 Release 3 Language Reference and User's Guide, SC09-2631

Workstation Processors

IBM RS/6000 POWERstation and POWERserver Hardware Technical Reference Information—General Architectures, SA23-2643

Parallel Environment Version 2

IBM Parallel Environment for AIX: Hitchhiker's Guide, GC23-3895

IBM Parallel Environment for AIX: Installation Guide, GC28-1981

IBM Parallel Environment for AIX: Operation and Use, Volume 1, SC28-1979

IBM Parallel Environment for AIX: Operation and Use, Volume 2, SC28-1980

IBM Parallel Environment for AIX: MPI Programming and Subroutine Reference, GC23-3894

IBM Parallel Environment for AIX: MPL Programming and Subroutine Reference, GC23-3893

IBM Parallel Environment for AIX: Messages, GC28-1982

Parallel System Support Programs Version 2

IBM Parallel System Support Programs for AIX: Administration Guide, GC23-3897

IBM Parallel System Support Programs for AIX: Installation and Migration Guide, GC23-3898

IBM Parallel System Support Programs for AIX: Diagnosis and Message Reference, GC23-3899

IBM Parallel System Support Programs for AIX: Command and Technical Reference, GC23-3900

Parallel System Support Programs Version 3

IBM Parallel System Support Programs for AIX: Administration Guide, SA22-7348

IBM Parallel System Support Programs for AIX: Installation and Migration Guide, GA22-7347

IBM Parallel System Support Programs for AIX: Diagnosis Guide, GA22-7350

IBM Parallel System Support Programs for AIX: Message Reference, GA22-7350

IBM Parallel System Support Programs for AIX: Command and Technical Reference, SA22-7351

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